Neutron diffusion and nucleosynthesis in the Universe with isothermal fluctuations produced by quark-hadron phase transition

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Nucleosynthesis in the Universe with isothermal baryon-density fluctuations is investigated. It is shown that neutron diffusion during nucleosynthesis plays an important role. Systematic calculations in the wide ranges of parameters suggest that $\Omega_B=1$ is marginally consistent with the light-element abundances (D, ^{3}He, ^{4}He, ^{7}Li) if the Hubble constant is as small as $H_0=45$ km Mpc^{-1}sec^{-1} or the mean lifetime of neutrons is shorter than 860 sec. Moreover it is found that the abundances of CNO elements in the low-density and neutron-rich region are very small.

I. INTRODUCTION

When the Universe cools down to $T\sim 100$ MeV, the quark-hadron phase transition from an unconfined quark phase to a confined hadronic phase is expected to occur. Some recent investigations raise the possibility that isothermal baryon-density fluctuations are generated during the quark-hadron phase transitions[1–6] if the phase transition is a first-order one, as suggested by lattice QCD simulations. When nuggets of stable strange matter are formed, highly concentrated baryon clouds are left as dark matter and they may enable $\Omega_B=1$. This nugget scenario was criticized by several authors[2,7] and it is open to debate as to whether or not stable quark nuggets are really formed. Even if the phase transition, however, proceeds keeping chemical equilibrium between two phases, isothermal density fluctuations with rather high-density contrast would be formed to affect primordial nucleosynthesis significantly.[2,5–10]

The effects of these isothermal density fluctuations on primordial nucleosynthesis are summarized by the following two points: (1) The abundances of elements sensitively depend on the local baryon-photon ratio and the final average abundances of the elements are much different from those in the uniform universe; (2) when the temperature falls down to ~ 1 MeV, the weak interactions among nucleons cannot catch up with cosmic expansion and decouple. Then the neutron diffusion length becomes significantly longer than the proton diffusion length and neutron-proton segregation occurs.[8] The neutron-to-proton ratio is directly reflected in the abundances of elements.

Earlier works[8–11] which studied nucleosynthesis in this model showed that the light-element abundances are greatly modified in particular by the effect of (2): The ^{4}He overproduction in the high-density region is suppressed by the escape of neutrons and also the synthesized abundance of D is enhanced in the low-density region by neutrons which come from the high-density region. The most important of these results is that, in this scenario, an $\Omega_B=1$ universe can be consistent with the observations of the abundances of light elements, except ^{7}Li, provided that the baryon-density contrast between high- and low-density regions is sufficiently large. If the ^{7}Li problem is removed, the dark matter can be baryonic and no more speculative candidates, such as photinos, gravitinos, etc., are necessary. However they assumed that the neutron diffusion stops before the onset of nucleosynthesis and it was criticized that the neutron diffusion during nucleosynthesis is important.[12–17]

Since the neutron diffusion length keeps growing during nucleosynthesis,[8] neutron diffusion cannot be neglected unless the fluctuation scale is so large that diffusion effects are essentially negligible. It is crucial to discover whether neutron diffusion lasts after the onset of nucleosynthesis since nucleosynthesis in the high-density region proceeds faster than in the low-density region and neutrons may diffuse back to the high-density region from the low-density region. We pointed out that neutrons are expected to diffuse back from the low-density zone to the high-density zone rapidly as soon as nucleosynthesis in the high-density zone begins and the amounts of synthesized elements turn out to be very small in the low-density zone because of the lack of neutrons if the scale of density fluctuations is small enough.[17] For larger scales, the time scale of neutron diffusion is so long that a large amount of ^{4}He is expected to be produced in the low-density zone but neutrons diffusing back into the high-density zone may have no small effects on the abundances of elements in the high-density zone such as the reduction of the ^{7}Be abundance.[12–16] Moreover, Applegate, Hogan, and Scherrer (1988) suggested that heavy elements are formed via a primordial r process which may take place in the neutron-rich regions.

To examine the effects of the nucleon diffusion after the onset of nucleosynthesis, we calculate nucleosynthesis in the inhomogenous universe incorporating the neutron and proton diffusions in the wide ranges of parameters using the extended reaction network including up to ^{32}S. In Sec. II we show the scheme of our calculations, especially, that of the treatment of nucleon diffusion. In Sec. III the primordial abundances of light elements, which are compared with the calculated abundances, are briefly summarized. The calculated results are shown and the dependences of the abundances of elements on the parameters are analyzed in Sec. IV. Section V is devoted to conclusions.

II. FLUX-LIMITED DIFFUSION OF NUCLEON

Towards the end of the QCD-phase transition, nearly isothermal density fluctuations of baryons are formed. Though the density fluctuations have characteristic scale and density contrast which are characterized by the physics of the phase transition, actual scales and density contrasts spread around the characterisitc value and they have various density profiles and topologies. Hence general treatments of density fluctuations are far beyond the present ability of computer simulations. In order to see the dependences of the abundances of elements on the scale and the density contrast of fluctuations, and in order to find behavior of the abundances in the wide ranges of four-dimensional parameter space (d, R, f_v, and Ω_B), we adopt a two-zone model and assume that the Universe consists of two regions: a high-density region whose volume fraction is f_v and a low-density region. The average baryon density is then

$$\rho_B = f_v \rho_h + (1-f_v)\rho_l \ , \tag{1}$$

where ρ_h and ρ_l are the baryon densities in the high- and low-density regions, respectively. We assume that the high-density zone is spherically symmetric and surrounded by the low-density zone. The topology of density fluctuation is, however, no matter in the two-zone approximation.

The density contrast evolves with the cosmic expansion and dumps drastically at the onset of neutron diffusion after the weak interactions among nucleons are effectively frozen out. We take the initial density contrast prior to the effective onset of the neutron diffusion as a parameter:

$$R \equiv \frac{\rho_h^0}{\rho_l^0} \ . \tag{2}$$

In order to incorporate nucleon diffusions, diffusion terms are added to the equations which govern the changes of the nucleon abundances:

$$\frac{dY_i^h}{dt} = -\frac{Y_i^h - \frac{\rho_l}{\rho_h} Y_i^l}{f_v \tau_i} \ , \tag{3}$$

$$\frac{dY_i^l}{dt} = -\frac{Y_i^l - \frac{\rho_h}{\rho_l} Y_i^h}{(1-f_v)\tau_i} \ , \tag{4}$$

where the abundances are given by Y, which is defined with the mass factions X by $Y_i = X_i / A_i$, and τ_i is the time scale of diffusion. The diffusion time scale τ_i is given by

$$\tau_i = \frac{d^2}{6D_i}\left[1+\frac{\lambda_i}{d}\right] \ , \tag{5}$$

where d is the physical size of high-density zone, D_i is the diffusion coefficient, and λ_i is the mean free path of neutrons or protons. The second flux-limiter term in the parentheses is added to avoid the overestimation of diffusion. When the mean free path of nucleons becomes longer than the length scale of density fluctuations, the diffusion approximation breaks down and the diffusion time scale might give a shorter time scale than that of free streaming unless the flux-limiter term is involved. This difficulty is not removed by shortening of time steps of the calculations or by dividing the space into fine meshes. Though there are several schemes in the flux-limited diffusions,[18] the simplest one employed here is sufficient for the case in question. Furthermore, we define the diffusion time scale by the physical values in the high-density zone. We may also define it taking an appropriate average of physical values of two zones. It is, anyway, an unavoidable approximation of the same order in two-zone models whichever scheme is employed.

For the diffusion coefficient of neutrons, we use that given by Applegate, Hogan, and Scherrer.[8] The diffusion coefficient of neutron (D_n) consists of the neutron diffusion coefficient for electron scattering (D_{ne}) and that for neutron-proton scattering (D_{np}):

$$D_n^{-1} = D_{ne}^{-1} + D_{np}^{-1} \ . \tag{6}$$

These two diffusion coefficients are given by

$$D_{ne} = 3.95\times 10^9 \frac{e^{1/x}}{xf(x)} \frac{\text{cm}^2}{\text{sec}} \ , \tag{7}$$

$$D_{np} = \frac{6.53\times 10^2}{1-X_n} \frac{T_e^{1/2}}{\eta \sigma_{np} T_v^3} \frac{\text{cm}^2}{\text{sec}} \ , \tag{8}$$

where $x = T/m_e c^2$, $f(x) = 1+3x+3x^2$, X_n is the neutron fraction in the high-density region, T_e and T_v are the electron and neutrino temperatures in MeV, σ_{np} is the neutron-proton cross section in fm^2, and η is the baryon-to-photon ration in the high-density region.

We found that the proton diffusion significantly affected the abundances of light elements, especially that of ^{7}Li when the diffusion length of protons became as long as the fluctuation scale at low temperatures.[19] The proton-diffusion coefficient given by Applegate, Hogan, and Scherrer[8] cannot be applied in such a low-temperature range for several reasons. At first, the diffusion coefficient given by them becomes too large at low temperatures because the distribution function of electrons does not give a correct value. Second, the contribution from proton-proton self-collisions must be taken into account since their contribution becomes the same order as that by electron-proton collisions when electron-positron pairs disappear. The drag force of photons on electrons, moreover, reduces the electron mobility. This prevents protons from diffusing easily when the Debye length becomes longer than a mean separation of protons.[8]

The proton-diffusion coefficient (D_p) is interpolated as a sum of three components: the coefficient for electron-proton scattering (D_{pe}), that for proton-proton self-scattering (D_{pp}), and the effective diffusion coefficient due to the photon drag on electrons ($D_{e\gamma}$),

$$D_p^{-1}=D_{pe}^{-1}+D_{pp}^{-1}+D_{e\gamma}^{-1} . \tag{9}$$

The coefficient for electron-proton scattering is improved to give a correct value at the low-temperature limit taking the electron distribution function to be a Boltzmann distribution:

$$D_{pe}=\begin{cases}D_{pe1} & (x>1) ,\\ D_{pe2} & (x<1) ,\end{cases} \tag{10}$$

$$D_{pe1}=\frac{2.56\times10^4}{\Lambda_e}\frac{xe^{1/x}}{g(x)}\frac{\text{cm}^2}{\text{sec}} , \tag{11}$$

$$D_{pe2}=\frac{3T^2}{8\pi\alpha^2 n_e\Lambda_e}\left[\frac{T}{m_e}\right]^{1/2}=\frac{1.07\times10^4}{x^{1/2}\eta\Lambda_e}\frac{\text{cm}^2}{\text{sec}} , \tag{12}$$

where $x=T/m_e$, $g(x)=1+2x+2x^2$, and Λ_e is the Coulomb logarithm cutoff for electrons. It is given by[20]

$$\Lambda_e=\begin{cases}\Lambda_{e1} & (e^2/\hbar\bar{v}_{\text{rel}}<1) ,\\ \Lambda_{e2} & (e^2/\hbar\bar{v}_{\text{rel}}>1) ,\end{cases} \tag{13}$$

$$\Lambda_{e1}=\ln(\lambda_D m_e\bar{v}_{\text{rel}}) , \tag{14}$$

$$\Lambda_{e2}=\ln\left[\frac{\lambda_D m_e\bar{v}_{\text{rel}}^2}{\alpha}\right] , \tag{15}$$

where $\bar{v}_{\text{rel}}$ is a mean relative velocity of particles and λ_D is the Debye shielding length defined by

$$\lambda_D=\left[\frac{4\pi\alpha n_e}{T}\right]^{-1/2} . \tag{16}$$

The coefficient for proton-proton scattering is given by

$$D_{pp}=\frac{3T^2}{8\pi\alpha^2 n_p\Lambda_p}\left[\frac{T}{\mu}\right]^{1/2}=\frac{3.53\times10^2}{x^{1/2}\eta\Lambda_p}\frac{\text{cm}^2}{\text{sec}} , \tag{17}$$

where $\mu=M_p/2$ is the reduced mass of a proton-proton pair and Λ_p is the Coulomb logarithm cutoff for protons, which is given by

$$\Lambda_p=\begin{cases}\Lambda_{p1} & (e^2/\hbar\bar{v}_{\text{rel}}<1) ,\\ \Lambda_{p2} & (e^2/\hbar\bar{v}_{\text{rel}}>1) ,\end{cases} \tag{18}$$

$$\Lambda_{p1}=\ln(\lambda_D\mu\bar{v}_{\text{rel}}) , \tag{19}$$

$$\Lambda_{e2}=\ln\left[\frac{\lambda_D\mu\bar{v}_{\text{rel}}^2}{\alpha}\right] . \tag{20}$$

The effective diffusion coefficient due to the photon drag on electrons ($D_{e\gamma}$) is given by

$$D_{e\gamma}^{-1}=\frac{\sigma_T}{3\pi^2T^2}\int\frac{E^4}{e^{E/T}-1}dE$$
$$=3.24\times10^{-4}x^3\ \frac{\text{cm}^2}{\text{sec}} . \tag{21}$$

When the temperature is high enough ($T>m_e$) and electron-position pairs can be created easily, it might not be a good approximation to take the elctron mobility due to a photon drag as the proton mobility. However, D_{pe} dominates D_p at such a high temperature and a contribution from $D_{e\gamma}$ is negligible.

The background universe is assumed to be described by the Einstein equation for the uniform and isotropic Friedmann-Robertson-Walker model:

$$\left[\frac{1}{a}\frac{da}{dt}\right]^2=\tfrac{8}{3}\pi G\rho , \tag{22}$$

where ρ is the total energy density of the Universe and it is dominated by radiation and neutrinos. The neutron lifetime and the number of neutrino species are taken as $\tau_n=898$ sec (Ref. 21) and $N_\nu=3$, respectively. Thus intrinsic parameters of the calculation are the average baryon-density parameter Ω_B, the volume fraction of the high-density region f_v, the initial ratio of the baryon density of the two regions R, and the physical size of the high-density zone d at $T=1$ MeV. The calculated ranges are $\Omega_B=0.1-1$ ($H_0=50$ km Mpc^{-1}sec^{-1}, $T_0=2.7$ K), $f_v=0$–0.5, $R=1-10\,000$, and d ($T=1$ MeV)$=10^3$–10^8 cm. Possible changes of the abundances due to the uncertainties in τ_n and H_0 and their influences on the conclusions will be addressed in the last section.

Our nucleosynthesis network is large enough to include up to ^{32}S (Ref. 22). Thermonuclear reaction rates are taken from Refs. 23–25 and some of them are updated[26] according to the recent experimental results.

III. PRIMORDIAL ABUNDANCE OF LIGHT ELEMENTS

In this section , we summarize the bounds on the primordial abundances of the light elements which are the key elements when we examine the compatibility of models.

For the primordial abundance of ^{4}He, Steigman, Gallagher, and Schramm[27] have found $Y_p=0.235 \pm0.012(3\sigma)$ by extrapolating the correlation between the ^{4}He abundance and the carbon abundance of H II regions. This value is consistent with other recent estimates.[28,29] On the basis of these estimates, we adopt

$$0.22<{}^4\text{He}<0.26 . \tag{23}$$

For the primordial abundances of D and ^{3}He the following vaues are taken:[30]

$$\text{D/H}>1\times10^{-5} , \tag{24}$$

$$(\text{D}+{}^3\text{He})/\text{H}<1\times10^{-4} . \tag{25}$$

Since Spite and Spite[31] claimed that the extreme Population II stars give the best guess on the primordial abundance of ^{7}Li, it has been recognized generally that the ^{7}Li abundance also provides the crucial test of the consistency of primordial nucleosynthesis. Two recent independent studies[32,33] obtained the same result, ^{7}Li/H$=1.2 \pm 0.9\times 10^{-10}$ (3σ), which is consistent with ^{7}Li/H$\approx 1.1\times 10^{-10}$ obtained by Spite and Spite (Ref. 31). Though these values may give the best estimate of the primordial ^{7}Li abundance, we rather adopt the following loose bound, which is inferred from observations of Population I stars, lest unnecessary ambiguities should be invoked:

$$^7\mathrm{Li/H} < 1\times 10^{-9}\ . \qquad (26)$$

IV. RESULTS OF CALCULATIONS

Figure 1 shows the dependence of the abundances of D, ^{3}He, ^{4}He, and ^{7}Li on the fluctuation scale $d(T=1$ MeV). In the case that the fluctuation scale at $T=1$ MeV is smaller than 10^3 cm, nucleon densities in both regions become equal before the onset of nucleosynthesis since neutrons diffused out of the high-density region into the low-density region are converted to protons and more neutrons diffuse out of the high-density region till the nucleon densities in the two zones become equal. Thus the abundances of elements are equal to those in the uniform $\Omega_B=1$ universe for this range of d.

Furthermore, we find that the behavior of the abundances in the case $d(T=1$ MeV$)\sim 5\times 10^3$ cm is similar to that in our previous calculation,[17] in which the neutron diffusion rate is taken large enough to maintain the neutron densities in both regions equal. For fluctuations with a scale of this order, the neutron diffusion is so rapid that neutrons once diffused out of the high-density region diffuse back as soon as the nucleosynthesis in the high-density region begins. Hence little amount of elements are synthesized in the low-density region, and the average abundances are essentially decided by those in the high-density region. The D abundance, therefore, shows a dip around $d(T=1$ MeV$)=5\times 10^3$ cm and the abundances of ^{4}He and ^{7}Li increase in the range $d(T=1$ MeV$)=10^3$–10^4 cm though deviations from the abundances in the uniform case are small.

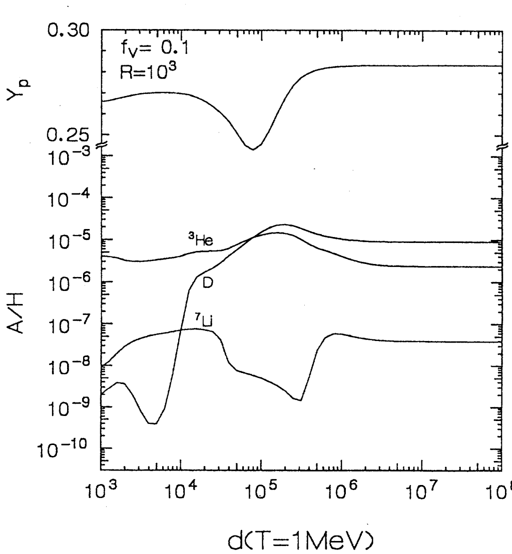

FIG. 1. The abundances of D, ^{3}He, and ^{7}Li plotted against the scale of density fluctuations $d(T=1$ MeV) for $\Omega_B=1$, $f_v=0.1$, and $R=1000$.

The most interesting point is the behavior of the abundances in the range 10^4 cm$\lesssim d(T=1$ MeV$)\lesssim 10^6$ cm. For density fluctuations in this range, the diffusion time scale of neutrons is long enough and the nucleosynthesis in the low-density region starts before neutrons diffuse back to the high-density region. Hence the low-density region becomes neutron rich and neutron-rich nucleosynthesis occurs there. Since the excessive neutrons decay into protons, the effective neutron fraction decreases and the ^{4}He abundance turns out to be small in consequence. Thus we find the ^{4}He abundance is consistent with the primordial abundance (23) at d $(T=1$ MeV$)\sim 10^5$ cm. The decrease of the ^{7}Li abundance in the range 10^4–10^6 cm comes from the destruction of ^{7}Be in the high-density region by neutron diffusion [via $^7\mathrm{Be}(n,p)^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$]. The scale, where the ^{7}Li abundance takes a minimum value, is somewhat longer than that where the ^{4}He abundance has a dip. This difference of scale is crucial in finding the parameter region in which the light element abundances are consistent with primordial abundances. On the other hand, the increase of the D abundance in the range $d(T=1$ MeV$)\gtrsim 10^4$ cm is essentially due to the segregation of the high- and low-density regions.

We find that the proton diffusion rate is very small and almost negligible. In our previous work,[19] we overestimated the proton diffusion since we used the proton diffusion coefficient given by Applegate, Hogan, and Scherrer[8] beyond its applicable range. Comparing the results in this work with those of previous work and the abundances when the proton diffusion is switched off, we find that the effect of the proton diffusion is almost negligible.

In Fig. 2 we show the abundances of elements for $d=10^{5.5}$ cm and $\Omega_B=1$. The D abundance increases with increasing R and satisfy the lower bound on the primordial abundance (24) in the ranges $R\gtrsim 100$ and $0.07<f_v<0.36$. The increase of D abundance with R is due to the decrease of baryon density in the low-density region with increasing R. For such a large scale of fluctuations, the high-density region and the low-density region are essentially segregated and the D abundances depend on the baryon density of the low-density region although D is produced further in the low-density region by neutrons diffused out of the high-density region.

It is interesting that the ^{4}He abundance decreases with increasing R in the range $f_v\gtrsim 0.2$ and takes the minimum value at $f_v=0.3$–0.4 [Fig. 2(c)]. This behavior of the ^{4}He abundance is seen in the narrow range around $d=10^5$ cm where the ^{4}He abundance has a dip as shown in Fig. 1. In the case of Fig. 2(c), the ranges $0.2\lesssim f_v\lesssim 0.5$ and $R\gtrsim 100$ are consistent with the primordial ^{4}He abundance (23).

The ^{7}Li abundance also decreases with increasing R in the range $R\gtrsim 100$ because of the escape of a proton from the high-density region and the destruction of ^{7}Be in the high-density region by neutrons diffusing back from the low-density region [Fig. 2(d)]. We find the narrow region $R \gtrsim 5000$ and $f_v \sim 0.1$, where the ^{7}Li abundance is marginally consistent with the primordial abundance (26). The ^{4}He abundance, however, is too large to be consistent with the primordial abundance in this region, as shown in Fig. 2(b).

After searching for regions where all the light-element abundances are consistent with the primordial abundance

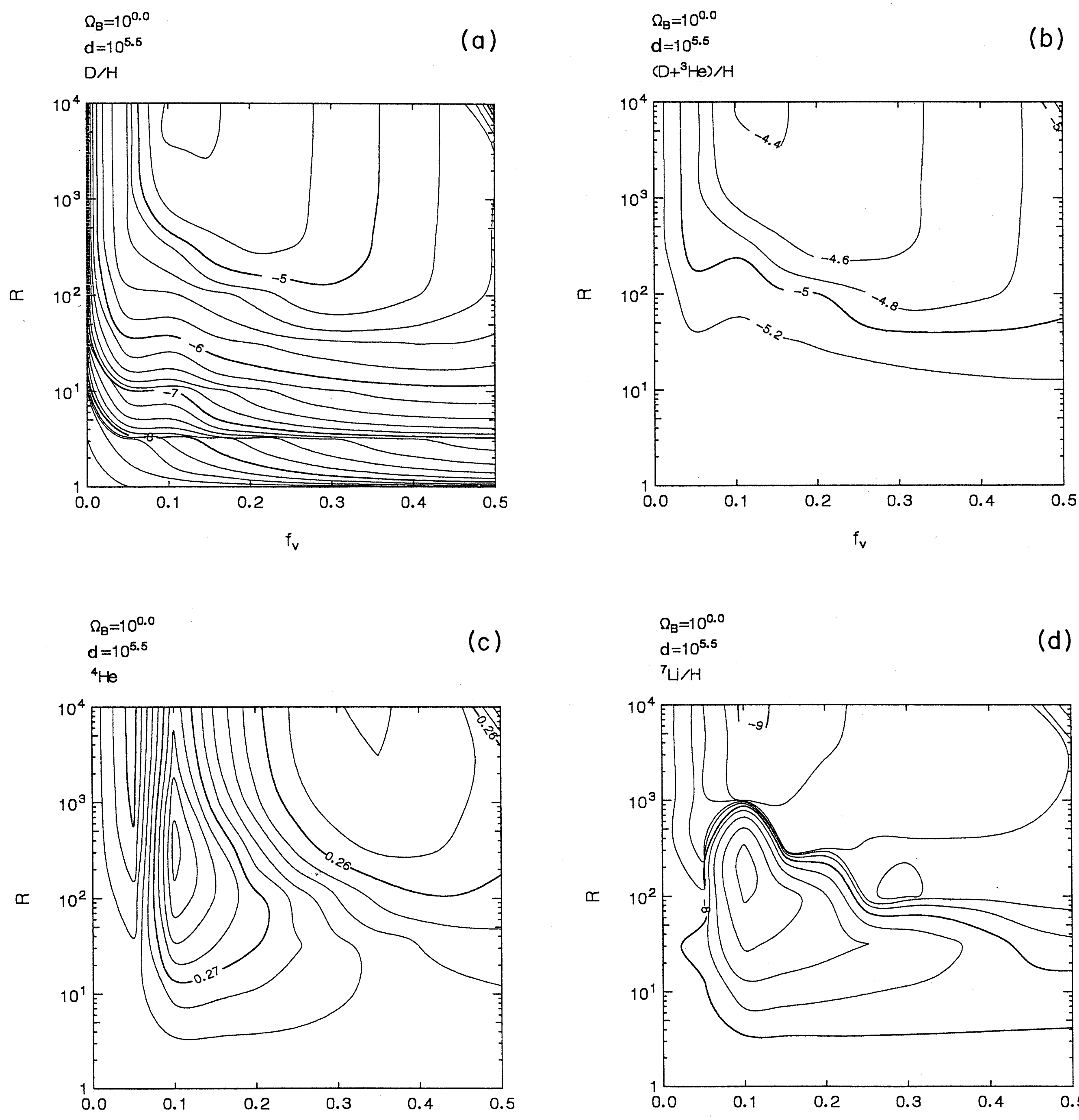

FIG. 2. The average abundances of D, ^{3}He, ^{4}He, and ^{7}Li on the R-f_v plane for $\Omega_B = 1$ and $d(T=1\ \text{MeV}) = 10^{5.5}$ cm. (a) $\log_{10}$ (D/H). (b) $\log_{10}$ (D+^{3}He)/H. (c) The mass fraction of ^{4}He(Y). (d) $\log_{10}$ (^{7}Li/H).

varying the scale of fluctuations, we find that $\Omega_B = 1$ ($H_0 = 50$ km $\text{Mpc}^{-1}\text{sec}^{-1}$ and $T_0 = 2.7$ K cannot be consistent with the primordial abundances of all the light elements simultaneously.

How large can we take the density parameter of baryonic matter then? One example, in which we find a region consistent with the abundances of light elements, is displayed in Fig. 3 [$\Omega_B = 10^{-0.4}$ and $d(T=1$

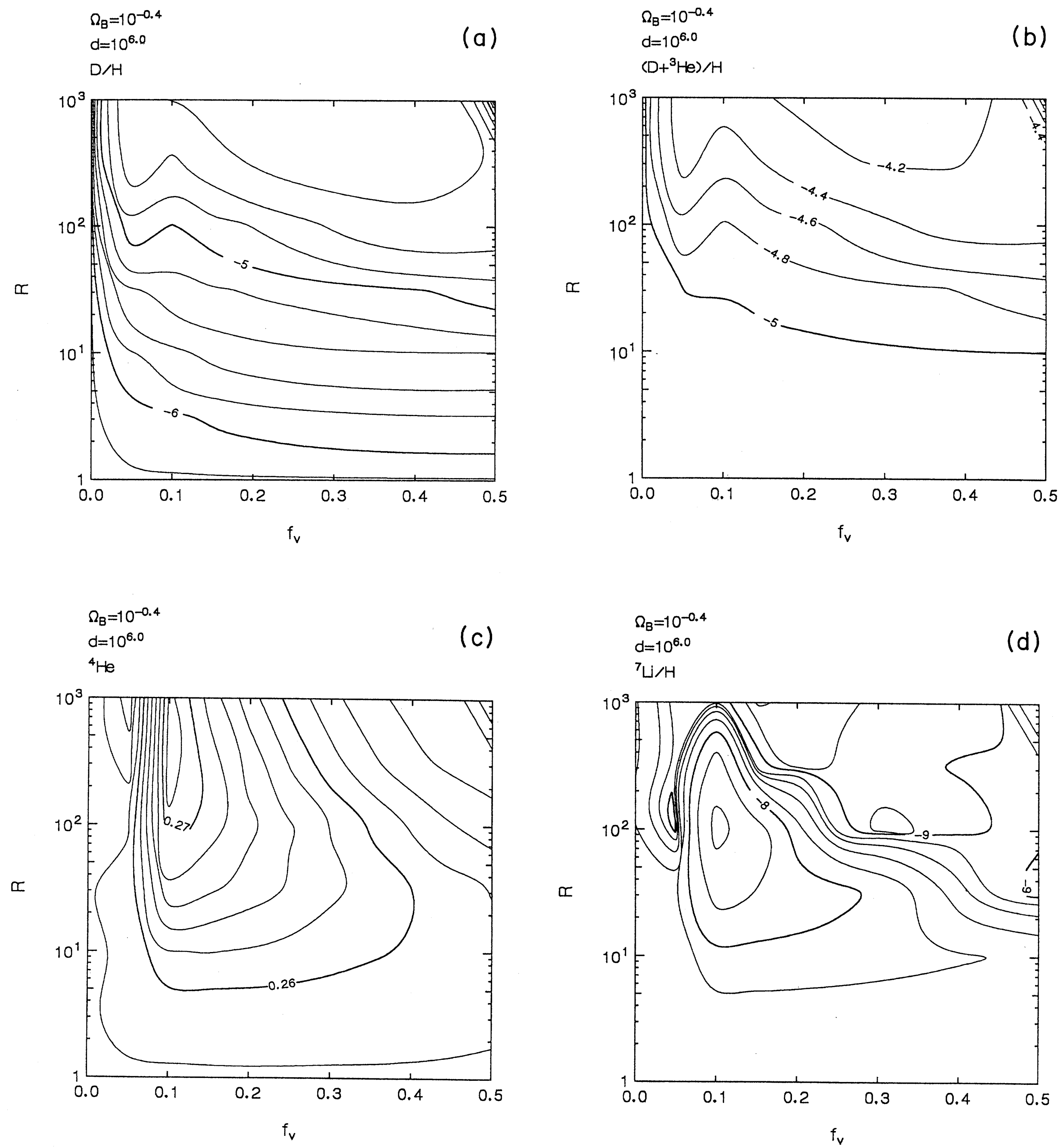

FIG. 3. The average abundances of D, ^{3}He, ^{4}He, and ^{7}Li on the R-f_v plane for $\Omega_B = 10^{-0.4}$ and d ($T = 1$ MeV)$= 10^6$ cm.

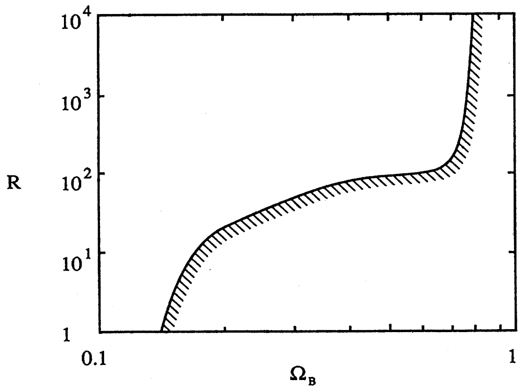

FIG. 4. Constraints on R and $\Omega_B = 1$ ($H_0 = 50$ km Mpc^{-1}sec^{-1} and $T_0 = 2.7$ K). Hatched region is excluded by the constraints $Y < 0.26$, D/H $> 10^{-5}$, and ^{7}Li/H $< 10^{-9}$.

MeV) $= 10^6$ cm]. For a small value of Ω_B of this order, the ^{4}He abundance is consistent with the primordial abundance (23) even in a uniform universe. The D abundance is, however, too small and the ^{7}Li abundance is too large to be consistent with the primordial abundances. If density fluctuations with appropriate scale and density contrast survive until the era of nucleosynthesis, this contradiction is removed as shown in Fig. 3. We find the region $R > 100$ and $0.2 \lesssim f_v \lesssim 0.4$ is consistent with the primordial abundances of D, ^{3}He, ^{4}He, and ^{7}Li. Thus we find that the density parameter of baryon can be as large as 0.4.

Sweeping out in the parameter space of R, f_v, d ($T = 1$ MeV), and Ω_B to find regions consistent with the light element abundances, we derive constraints on R and Ω_B shown in Fig. 4. For large values of R, the abundances of elements become insensitive to R as long as $Rf_v \gtrsim 1$. A sharp rise in the excluded region at $\Omega_B = 0.8$ is due to this reason. Thus we find the maximum value of Ω_B is 0.8 provided that $H_0 = 50$ km Mpc^{-1}sec^{-1}, $T_0 = 2.7$ K, and $\tau_n = 898$ sec.

V. DISCUSSION AND CONCLUSIONS

As is shown in Fig. 4, $\Omega_B = 1$ is not consistent with the abundances of light elements. However, the ^{4}He abundance depends on the lifetime of free decay of neutrons as $\Delta Y \sim 0.0032$ ($\Delta\tau_n / 16$ sec) (Refs. 26 and 34). Therefore, $\Omega_B = 1$ can be consistent with all the light-element abundances if the lifetime of neutron-free decay is somewhat smaller than 898 sec employed in this study. We find that there really appears a region consistent with both the ^{7}Li abundance and the ^{4}He abundance around $R = 10\,000$, $f_v = 0.1$, and $d(T = 1$ MeV) $= 10^{5.5}$ cm if the lifetime of neutron-free decay is as short as 860 sec which is well within the standard deviation from the mean value reported recently by Last *et al.*,[35] $\tau_n = 876 \pm 21$ sec.

Furthermore, $\Omega_B = 0.8$ ($H_0 = 50$ km Mpc^{-1}sec^{-1}) corresponds to $\Omega_B = 1$ ($H_0 = 45$ km Mpc^{-1}sec^{-1}). We cannot necessarily exclude the present value of Hubble's constant as small as $H_0 = 40$ km sec^{-1} Mpc^{-1} (Ref. 36).

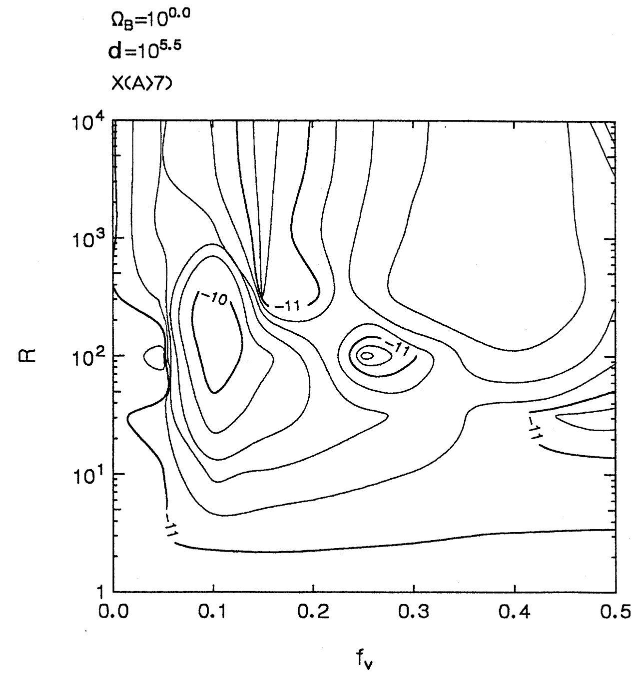

FIG. 5. The mass fraction of heavy elements ($A > 7$) on the R-f_v plane for $\Omega_B = 1$ and $d(T = 1$ MeV) $= 10^{5.5}$ cm.

Hence we cannot necessarily exclude $\Omega_B = 1$. The consistent region is, however, limited to narrow ranges of the parameters and we need tuning of these parameters. In addition, our calculations are based on a two-zone model and actual density fluctuations have various topological shapes and scales. Therefore, our results may be said to be the most optimistic case and the situation will become worse in the context of whether or not $\Omega_B = 1$ is allowed when density fluctuations are treated more realistically and generally.

After this paper was submitted, we received a paper written by Kurki-Suonio and Matzner.[37] They performed multidimensional calculations and concluded that all the abundances of D, ^{3}He, and ^{4}He disagree with observations for $\Omega_B = 1$. Their conclusion is, however, based on the calculations for a few sets of parameters though calculated abundances themselves are essentially consistent with our results. In fact, the dependence of the ^{4}He abundance on the scale of density fluctuations show a good agreement though their ^{4}He abundance is systematically greater than ours by about $\Delta Y \approx 0.01$. This discrepancy seems to be due to the problem in their treatment of nucleosynthesis since the ^{4}He abundance in uniform cases is also greater than ours and the results given by other authors (see, for instance, Yang *et al.*[38]). Good agreement in the dependence on the fluctuation scale, however, might suggest that the two-zone model gives satisfactory results. The ^{7}Li abundance, on the other hand, shows disagreement in the dip around $d(T = 1$ MeV) $= 10^5$–10^6 cm (which corresponds to the scale in their notation, $r_i \sim 10$–100 light hours) and the D abundance calculated by us is systematically greater than that given by them. Our results show a deeper dip in the ^{7}Li abundance in this range than theirs. The difference in the treatment of the proton diffusion seems to cause these disagreements. They describe that the proton diffusion causes no appreciable change in other abundances except for D. The proton diffusion, however, affects also the ^{7}Li abundance if the diffusion coefficient given by Applegate, Hogan, and Scherrer[8] is used as shown in the previous section. The proton diffusion is prevented by the photon drag on electrons and it is a better approximation to neglect the proton diffusion.

Mathews, Fuller, Alcock, and Kajino[16] also carried out multidimensional calculations. Since their initial density profile is a specific one and only a few results are shown, we cannot compare their results with ours quantitatively. However, the dependence of the abundances on the fluctuation scales is similar to our results.

Finally, we would like to comment on the primordial r process proposed by Applegate, Hogan, and Scherrer.[14] We show a typical example of the behavior of the heavy element abundance on R-f_v plane for $\Omega_B=1$ and $d(T=1\ \mathrm{MeV})=10^{5.5}$ cm in Fig. 5. The mass fraction of heavy elements in this case is at most 10^{-10} and it is dominated by ^{11}B produced in the high-density region. The abundances of CNO elements are, for instance, $X(^{12}\mathrm{C})=1.3\times10^{-11}$, $X(^{14}\mathrm{N})=5.3\times10^{-12}$, $X(^{16}\mathrm{O})=1.1\times10^{-13}$ in the case that $f_v=0.1$ and $R=1000$. The total mass fraction of heavy elements does not exceed 10^{-9} whatever set of the parameters is taken as long as $\Omega_B\leq1$. It must, furthermore, be stressed that the abundance of heavy elements in the neutron-rich and low-density region is very small; $X(^{12}\mathrm{C})$ is 100 times smaller than the abundances in the high-density region and the abundances of elements heavier than ^{12}C are negligibly small. Hence the abundances of seed elements for the r process are very small and final abundance of the r-process elements will not be as large as expected.

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