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Effective electromagnetic form factor of the neutrino

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The conceptual difficulties affecting the definition of the neutrino electromagnetic form factor are briefly reviewed in the context of the standard model. It is then shown that the radiative corrections $\Delta^{(\nu;l)}(q^2)$ and $\Delta^{(\nu;h)}(q^2)$ relevant to ν-lepton and ν-hadron scattering in the renormalization scheme in which $\sin^2\theta_W \equiv 1-m_W^2/m_Z^2$, can be separated into two finite and gauge-invariant parts. One part $\Delta^{(\nu)}(q^2)$ is target independent and the associated function $f(q^2)\equiv -q^2\Delta^{(\nu)}(q^2)/(2m_W^2)$ can be interpreted as an effective electromagnetic form factor of the neutrino in the framework of the low-energy theory derived from the standard model at invariant mass scales $\ll m_W^2$. The observability of this quantity and its dependence on the top-quark mass and the neutrino flavor are discussed. Updated estimates of the hadronic contributions to $\Delta^{(\nu)}(q^2)$ are given. A strategy to search for ν structure beyond the standard model is outlined.

I. INTRODUCTION

As neutrinos are some of the most intriguing particles of nature, it is not surprising that physicists have long been interested in their properties. In particular, attempts to evaluate their electromagnetic form factor $f(q^2)$ go back to the pioneering work of Bernstein and Lee[1] who placed particular emphasis in the case $g_W=1$ (g_W is the gyromagnetic factor of the W). As their analysis was carried out in the framework of a nonrenormalizable theory, they employed a nonperturbative approach by considering the sum of all Feynman diagrams of arbitrary order in e^2 but only second order in the weak coupling g. Applications to ν-electron scattering were discussed in Refs. 2 and 3. In the standard model (SM), e^2 and g^2 are, of course, intimately related, $g_W=2$, and moreover the theory is renormalizable; so, it is natural to search for a perturbative answer. As $f(q^2)$ must vanish as $q^2\to 0$, it has long been recognized however that, in the context of the SM, virtual photons interacting with neutrinos give contributions to scattering processes which are generally of the same order of magnitude as radiative corrections to Z^0 mediated amplitudes, $Z^0-\gamma$ mixing graphs, and box diagrams involving exchanges of pairs of W's or Z^0's. Because of this fact and associated problems of non-Abelian gauge invariance discussed in Sec. II and Appendix A, the separate consideration of $f(q^2)$ lost much of its appeal for many years. Recently, however, there has been a revival of interest in the concept. Thus, an experimental paper[4] has set limits on the neutrino mean-square charge radius

$$\langle r^2\rangle \equiv 6\left.\frac{\partial f(q^2)}{\partial q^2}\right|_{q^2=0} \tag{1}$$

by comparing $\sigma(\nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e)$ with other weak-interaction processes not involving neutrinos. On the other hand, the theoretical situation in the framework of the SM, as portrayed in the literature, is somewhat confusing. Thus, for example, a paper of long standing[5] concluded that $\langle r^2\rangle$ is divergent and not a physical quantity in the present theory. On the other hand, more recently, some theorists claimed to have either estimated[6] or accurately calculated[7] $\langle r^2\rangle$ in the SM. A different point of view was taken by Lucio, Rosado, and Zepeda[8] who emphasized the problems of non-Abelian gauge invariance involved in the definition of $\langle r^2\rangle$ and proposed a generalization, the "electroweak radius" $\langle r^2\rangle_{\rm EW}$ proportional to $1-\kappa^{(\nu;l)}(0)$, where $\kappa^{(\nu;l)}$ is a radiative correction factor in the ν-lepton scattering amplitude introduced in Refs. 9 and 10. Because $\kappa^{(\nu;l)}(q^2)$ is an observable quantity, $\langle r^2\rangle_{\rm EW}$ satisfies the crucial requirement of being finite and gauge invariant. In contrast, Ref. 7 considered only the proper $\bar{\nu}\nu\gamma$ vertex diagrams. As these contributions are, by themselves, ξ dependent (ξ is the parameter that specifies the gauge within the class of renormalizable gauges), the corresponding results cannot be identified with physical, observable quantities in the SM. It has also been suggested that relatively large nonzero values of $\langle r^2\rangle$ may arise from "nongauge" $WW\gamma$ interactions associated with compositeness.[11] Unfortunately, the theoretical status of the corresponding one-loop calculations seems very uncertain. For example, the electromagnetic gauge invariance and the scheme independence of the regularization procedure is not clear and, at least by elementary power counting, the degree of divergence increases very rapidly with the number of loops so that the neglect of higher-order terms is very questionable.

In this paper we restrict ourselves mainly to the SM. We first discuss once more the difficulties that arise in defining $f(q^2)$ as a physical quantity by focusing on the non-Abelian gauge dependence of a class of contributions to neutrino scattering processes. Restricting then the discussion to the domain in which the kinematical variables are small relative to m_W^2, we point out that the radiative corrections $\Delta^{(\nu;l)}(q^2)$ and $\Delta^{(\nu;h)}(q^2)$, relevant to ν-lepton and ν-hadron scattering[9,10] in the renormalization scheme[12] in which $\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2$, can be separated into two finite and gauge-invariant parts. One part, $\Delta^{(\nu)}(q^2)$, is independent of the nature of the target. As we will see, $\Delta^{(\nu)}(q^2)$ cannot be thought of as arising from a $\bar{\nu}\nu\gamma$ interaction in the context of the SM but it does admit that interpretation in the framework of the effective low-energy theory which emerges at invariant-mass scales $\ll m_W^2$ when the intermediate-vector-boson degrees of freedom are integrated out. This leads to the concept of the effective electromagnetic form factor $f(q^2) = -q^2\Delta^{(\nu)}(q^2)/(2m_W^2)$ of the neutrino, a finite, gauge-invariant function that can be associated with all "low energy" neutrino scattering processes. Comparison of $f(q^2)$ and the associated mean-square radius with the results reported by other authors, as well as its dependence on the ν flavor and the top-quark mass m_t, are briefly discussed.

A second, more practical, aim of this paper is to provide an updated estimate of the uncertainties in the hadronic contributions to $f(q^2)$ or, equivalently, $\kappa(q^2)$ as these quantities are important for the precise determination of $\sin^2\theta_W$ from ν-lepton scattering processes. Finally, a strategy to search for ν structure beyond the SM is briefly discussed.

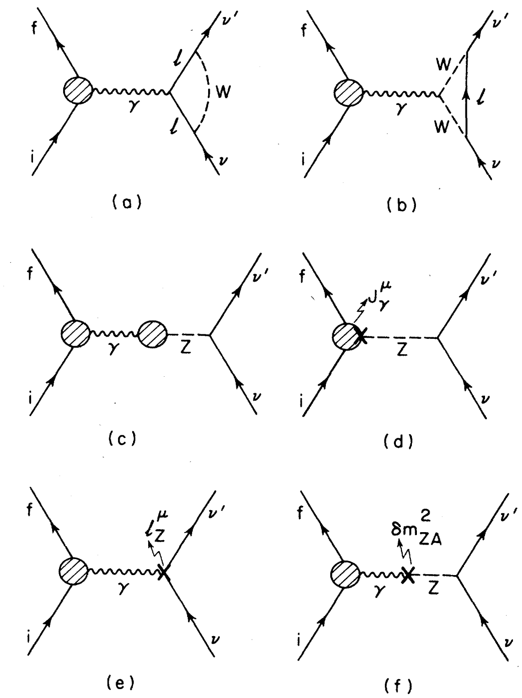

FIG. 1. Electromagnetic proper vertex, γZ mixing, and related counterterm diagrams in ν scattering.

II. GAUGE PROPERTIES OF RADIATIVE CORRECTIONS TO NEUTRAL-CURRENT ν SCATTERING PROCESSES

A general and convenient discussion of radiative corrections to neutral-current ν scattering processes in the domain in which the kinematical variables are much smaller than m_W^2 has been given in Ref. 9. The analysis in that work was carried out in the simple renormalization framework of Ref. 12, in which the basic renormalized parameters of the theory are taken to be e (the conventionally defined charge of the positron) and the physical masses m_W, m_Z, while the weak-interaction angle is defined by $\sin^2\theta_W = 1 - m_W^2/m_Z^2$. For definiteness the particles from which the ν's are scattered, as well as those emerging in the final state (with the exception of photons) will be referred to as the target system.

At first sight it might seem that in order to evaluate the electromagnetic form factor of the neutrino one should consider the sum of all Feynman diagrams in which the ν-scattering process is mediated by a photon. Examples, depicted in Figs. 1(a)–1(c) include $\bar{\nu}\nu\gamma$ proper vertex diagrams and the γZ self-energies. One must also include the counterterm diagrams of Figs. 1(d)–1(f) as described in detail in Ref. 9. However, a closer examination reveals that this procedure is not sufficient to obtain a sensible answer. For instance, in order to obtain a convergent result in the renormalizable gauges, it is necessary to include the part proportional to $\langle f|J_\gamma^\mu|i\rangle$ in the vertex correction to the Z^0 exchange amplitude [Figs. 2(a) and 2(b)] where J_γ^μ stands for the electromagnetic current associated with quarks and leptons and $|i\rangle$ and $|f\rangle$ represent the initial and final states of the target system. With these additions, the answer is convergent in the renormalizable gauges but, alas, not gauge invariant. Indeed, as shown in detail in Appendix A, what happens is that the W-W box diagram [Fig. 2(c)] (which is convergent in the renormalizable gauges) contains ξ-dependent parts necessary to cancel identical terms arising from Fig. 1(b). In order to analyze the W-W box diagram in the ξ gauge, it is convenient to write the corresponding amplitude in compact form:

$$\Delta M_{W,W} = \left[\frac{-ig}{\sqrt{2}}\right]^4 \int \frac{d^4k}{(2\pi)^4} D_{\mu\alpha}(k) D_{\nu\beta}(k-q) \int d^4x\, e^{ik\cdot x}\langle f|T[J_W^{\mu}(x) J_W^{\nu\dagger}(0)]|i\rangle \times \int d^4x'\, e^{-ik\cdot x'}\langle \nu|T[l_W^{\alpha\dagger}(x') l_W^{\beta}(0)]|\nu\rangle \,, \tag{2}$$

where

$$D_{\mu\alpha}(k) = -i[g_{\mu\alpha} - k_\mu k_\alpha(1-1/\xi)(k^2 - m_W^2/\xi)^{-1}](k^2 - m_W^2)^{-1}$$

is the W propagator, l_W^β the leptonic current coupled to W, and J_W^μ the corresponding current in the target system. As indicated in Appendix A, the ξ-dependent parts can be greatly simplified, for arbitrary q^2, by using the appropriate Ward identities. However, as we are interested in this section in the domain $|q^2| \ll m_W^2$ (as well as $m^2 \ll m_W^2$, where m stands for a generic lepton or quark mass either in the neutrino or target lines) it is sufficient to set $q = m = 0$ in Eq. (2) and retain the leading terms as $k \to \infty$ in the two-current correlation functions. This leads to

$$\int d^4x\, e^{ik\cdot x}\langle f|T[J_W^\mu(x)J_W^{\nu\dagger}(0)]|i\rangle = -i\frac{k_\lambda}{k^2}(g^{\mu\lambda}g^{\nu\rho}-g^{\nu\mu}g^{\lambda\rho}+g^{\lambda\nu}g^{\mu\rho})\langle f|J_\rho^{(3)}|i\rangle - \frac{\epsilon^{\mu\lambda\nu\rho}k_\lambda}{k^2}\langle f|J_\rho^{(0)}|i\rangle + \cdots \;, \quad (3)$$

where $J_\rho^{(3)}$ and $J_\rho^{(0)}$ are the third current of $SU(2)_L$ and the $V-A$ current of U(1), respectively. Specifically, if the target is a hadron, $J_\rho^{(3)} \equiv \bar\psi C_3 \gamma_\rho a_- \psi$ and $J_\rho^{(0)} \equiv \bar\psi 1 \gamma_\rho a_- \psi$ where ψ is a column vector $\psi \equiv (uctdsb)^T$, $a_- \equiv (1-\gamma_5)/2$, 1 is the 6×6 unit matrix and, calling I the 3×3 unit matrix,

$$C_3 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} .$$

In deriving Eq. (3) we have neglected small corrections induced perturbatively by the strong interactions.[13] If the target is a lepton, $J_\rho^{(3)}$ and $J_\rho^{(0)}$ are replaced by the corresponding leptonic currents. The result for

$$\int d^4x' e^{-ik\cdot x'}\langle \nu'|T[l_W^{\alpha\dagger}(x')l_W^\beta(0)]|\nu\rangle$$

is obtained from the right-hand side (RHS) of Eq. (3) by identifying $J_\rho^{(3)}$ and $J_\rho^{(0)}$ with the leptonic currents and interchanging $\mu \leftrightarrow \nu$.

Inserting Eq. (3) and the corresponding expression for

$$\int d^4x' e^{-ik\cdot x'}\langle \nu'|T[l_W^{\alpha\dagger}(x')l_W^\beta(0)]|\nu\rangle$$

into Eq. (2), the ξ-dependent parts of the propagators, proportional to $k_\mu k_\alpha$ and $k_\nu k_\beta$, cancel when contracted with the $\epsilon^{\mu\lambda\nu\rho}$ terms. Thus, the terms involving $\epsilon^{\mu\lambda\nu\rho}$ tensors are separately gauge invariant. On the other hand, a simple calculation shows that the contribution to $\Delta M_{W,W}$ arising from the first term in Eq. (3) is given by

$$\Delta M_{W,W}^{(3)} = \left[\frac{g}{\sqrt{2}}\right]^4 \langle f|J_\rho^{(3)}|i\rangle L^\rho \int \frac{d^4k}{(2\pi)^4(k^2-m_W^2)^2}\left[\frac{5}{2}\frac{1}{k^2} - \frac{2\left[1-\frac{1}{\xi}\right]}{\left[k^2-\frac{m_W^2}{\xi}\right]} + \frac{\left[1-\frac{1}{\xi}\right]^2\frac{k^2}{4}}{\left[k^2-\frac{m_W^2}{\xi}\right]^2}\right] , \quad (4)$$

where $L^\rho \equiv \bar u_\nu' \gamma^\rho a_- u_\nu$ and the superscript 3 on the LHS reminds us that Eq. (4) is the part of the WW box diagram proportional to the $J_\rho^{(3)}$ current. Evaluation of the integrals leads to

$$\Delta M_{W,W}^{(3)} = \frac{-ig^4}{64\pi^2 m_W^2}\langle f|J_\rho^{(3)}|i\rangle L^\rho[\tfrac{5}{2}+g(\xi)] , \quad (5)$$

where $g(\xi) = -2 + \frac{3}{2}\ln\xi/(\xi-1) + \frac{1}{4}(1+1/\xi)$. As expected $g(1)=0$ ('t Hooft–Feynman gauge) and $g(0)=\infty$ (unitary gauge).

A similar analysis shows that the Z-Z box diagrams are separately gauge invariant, a fact that can be readily understood from the associated Ward identities. In fact, as the equal time commutator of J_Z^0 and J_Z^μ vanishes (J_Z^μ is the fermionic current coupled to Z_μ), contractions with the k four-vectors from the propagators also lead to a null result when terms proportional to current divergences are neglected.

In summary, examination of the box diagrams shows that, to leading order in $1/m_W^2$, their gauge-dependent part is contained in Eq. (5). Recalling that

$$J_\rho^{(3)} = 2(J_Z + \sin^2\theta_W J_\gamma)_\rho ,$$

detailed examination (see Appendix A) shows that the term proportional to

$$g(\xi)\langle f|J_\gamma^\rho|i\rangle$$

in Eq. (5) cancels against an identical contribution from Fig. 1(b). We conclude that the minimal set of contributions proportional to $\langle f|J_\gamma^\rho|i\rangle$ which is finite and gauge invariant includes Figs. 1(a)–1(f), and the corresponding terms arising from Figs. 2(a) and 2(b) and Eq. (5) with $J_\rho^{(3)} \to 2\sin^2\theta_W(J_\gamma)_\rho$. Combination of these results leads to

$$\delta M = -\frac{ie^2}{2m_W^2}\langle f|J_\gamma^\rho|i\rangle L_\rho \Delta^{(\nu)}(q^2) , \quad (6)$$

where

$$\Delta^{(\nu)}(q^2)=\frac{\alpha}{2\pi s^2}\left\{R_l(q^2)-\tfrac{1}{9}(26c^2+\tfrac{35}{4}-\tfrac{19}{2}s^2)-\frac{1}{2s^2}(\tfrac{5}{6}-3c^2)\ln c^2+\tfrac{5}{4}+\frac{c^2}{2s^2}[I_1(c^2)-I_2(c^2)]\right.$$
$$\left.+\frac{1}{2s^2}\left[H(\xi)-c^2H\left[\frac{\xi}{c^2}\right]\right]\right\}+\frac{c}{s}\frac{A_{\gamma Z}^{(h;l)}(q^2)}{q^2}+\frac{c^2}{s^2}\mathrm{Re}\left[\frac{A_{ZZ}^{(h;l)}(m_Z^2)}{m_Z^2}-\frac{A_{WW}^{(h;l)}(m_W^2)}{m_W^2}\right]. \qquad (7)$$

In Eq. (7),

$$s^2\equiv\sin^2\theta_W,\quad c^2\equiv\cos^2\theta_W,\quad \xi\equiv m_H^2/m_Z^2\,,$$
$$R_l(q^2)=\tfrac{1}{3}+2\int_0^1 dx\, x(1-x)\ln\{m_W^2/[m_l^2-q^2x(1-x)]\}$$

(m_l is the mass of the charged lepton associated with ν_l), $H(\xi)$, $I_1(c^2)$, and $I_2(c^2)$ are functions discussed in detail in Refs. 9 and 12 (numerically, for $s^2=0.23$, $I_1=1.768$, $I_2=0.147$) and $A_{\gamma Z}^{(h;l)}$, $A_{ZZ}^{(h;l)}$, $A_{WW}^{(h;l)}$ represent the sum of the hadronic and leptonic contributions to the γZ, ZZ, and WW self-energies, respectively. Their detailed expression for general values of m_t can be obtained from Ref. 9. We note that $R_l(q^2)$ involves m_l; as a consequence, $\Delta^{(\nu)}(q^2)$ depends on the neutrino flavor.

An important property of $\Delta^{(\nu)}(q^2)$ is its independence of the quantum numbers or other properties of the target. It differs from the functions $\Delta^{(\nu;h)}(q^2)$ and $\Delta^{(\nu;l)}(q^2)$, appropriate to ν-hadron and ν-lepton scattering,[9] in that the latter include contributions to the Z-Z and W-W box diagrams arising from the $\epsilon^{\mu\lambda\nu\rho}$ terms in the two-current correlation functions. In the case of hadronic targets, the $\epsilon^{\mu\lambda\nu\rho}$ contributions can be expressed in terms of J_γ^ρ, J_Z^ρ and two additional induced currents, not present at the tree level.[9] Furthermore, the coefficients of $\langle f|J_\gamma^\rho|i\rangle$ and $\langle f|J_Z^\rho|i\rangle$ in these contributions are different from hadronic and leptonic targets. Taking this into account we may write

$$\Delta^{(\nu;t)}(q^2)=\Delta^{(\nu)}(q^2)+\Delta^{(t)}\,, \qquad (8)$$

where $t=l,h$ specifies whether we are considering a leptonic or hadronic target. We also note that the small effects induced perturbatively by the strong interactions in the case of hadronic targets[13] are separately gauge invariant and, if retained, can be incorporated into $\Delta^{(t)}$. Neglecting such small contributions, comparison with Ref. 9 leads to

$$\Delta^{(h)}=\frac{\alpha}{2\pi s^2}\left[\frac{a_\gamma}{c^2}-\frac{5}{4}\right], \qquad (9a)$$

$$\Delta^{(l)}=\frac{\alpha}{2\pi s^2}\left[\frac{c_\gamma}{c^2}-\frac{5}{4}\right], \qquad (9b)$$

where a_γ and c_γ are numerical constants given in Eqs. (19e) and (20f) of that work. Numerically, for $s^2=0.23$, $a_\gamma=1.15$, and $c_\gamma=1.56$. Thus, employing $\hat{\alpha}(m_W)\simeq\frac{1}{128}$, $\Delta^{(h)}=1.3\times10^{-3}$, $\Delta^{(l)}=4.2\times10^{-3}$. Using Jegerlehner's analysis of hadronic vacuum-polarization contributions[14] (see Appendix B), we find in the case of the muon neutrino ν_μ, for $m_t=45$ GeV and $m_H=100$ GeV: $\Delta^{(\nu)}(0)=(3.3\pm2)\times10^{-3}$. Recalling Eq. (8) and the numerical value of $\Delta^{(l)}$ we see that $\Delta^{(\nu_\mu;l)}(0)=(7.5\pm2)\times10^{-3}$ and correspondingly $\kappa^{(\nu_\mu;l)}(0)\equiv1-\Delta^{(\nu_\mu;l)}(0)=0.9925\pm0.0020$ (Ref. 15).

For other neutrino flavors one must add, at $q^2=0$, $(\alpha/3\pi s^2)\ln(m_\mu/m_l)$ which equals 1.79×10^{-2} for the electron neutrino ν_e and -0.95×10^{-2} for the τ neutrino ν_τ. Thus $\Delta^{(\nu)}(0)=(2.12\pm0.2)\times10^{-2}$ for ν_e and $\Delta^{(\nu)}(0)=-(6.2\pm2)\times10^{-3}$ for ν_τ. For values of $|q^2|\gg4m_\tau^2$ the flavor dependence becomes negligible and $\Delta^{(\nu)}(q^2)$ approaches a single universal function for all neutrino species. The m_t dependence of $\kappa(0)$, and therefore $\Delta^{(\nu)}(0)$, can be obtained from Ref. 9. It is illustrated numerically in Ref. 16 and Table I of this paper.

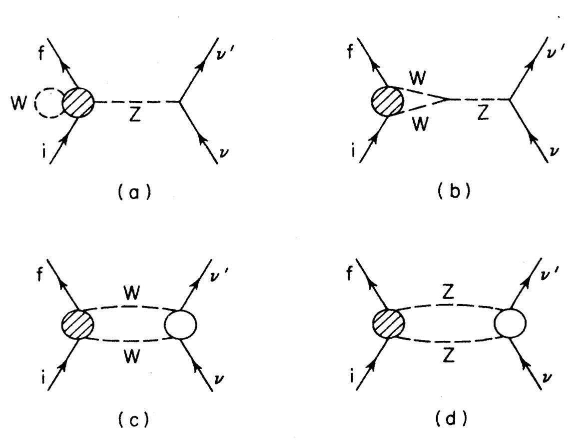

FIG. 2. Relevant vertex corrections to Z^0-mediated amplitudes and box diagrams in ν scattering.

III. EFFECTIVE ELECTROMAGNETIC FORM FACTOR OF THE NEUTRINO

As explained in Refs. 9 and 12, by a suitable redefinition of the Z_μ and A_μ fields the counterterm dia-

TABLE I. $\Delta^{(\nu)}(0)$ for $\nu=\nu_\mu$ and $m_H=100$ GeV, as a function of m_t. For $\nu=\nu_e$ and $\nu=\nu_\tau$, $+1.79$ and -0.95 must be added to all entries, respectively. The estimated error in the second column is ±0.20.

m_t (GeV)	$10^2\Delta^{(\nu)}(0)$
45	0.33
60	0.39
90	−1.04
120	−2.16
150	−3.29
180	−4.53

gram of Fig. 1(d) can be effectively transformed into additional contributions to Figs. 1(e) and 1(f). The latter can be obviously thought of as arising from $\bar{\nu}\nu\gamma$ photon interactions. The same, however, is not true for the contributions of Figs. 2(a)–2(c). Thus, we reach the conclusion that $\Delta^{(\nu)}(q^2)$ cannot be identified diagrammatically with an electromagnetic form factor of the neutrino in the context of the complete theory describing the SM.

Consider, however, the effective low-energy theory derived from the SM by integrating out the intermediate boson degrees of freedom. The remaining underlying fields are then quarks, gluons, leptons, and the photon. The theory is described by $\mathcal{L}_{\rm QCD}$ plus $\mathcal{L}_{\rm QED}$ plus nonrenormalizable four-Fermi interactions, inversely proportional to m_W^2, describing the charged- and neutral-current weak interactions. Because Eq. (6) is proportional to $\langle f|J_\gamma^\rho|i\rangle$ with a target-independent cofactor $\Delta^{(\nu)}(q^2)$, it is apparent that in this effective low-energy theory δM may be interpreted as arising from a nonrenormalizable $\bar{\nu}\nu\gamma$ interaction of the form

$$\mathcal{L}_{\bar{\nu}\nu\gamma}=-e(\bar{\psi}_\nu\gamma_\mu a_-\psi_\nu)\Delta^{(\nu)}(-\partial^2)\partial_\nu F^{\nu\mu}/(2m_W^2)\ , \qquad (10)$$

where $F^{\nu\mu}\equiv\partial^\nu A^\mu-\partial^\mu A^\nu$ is the field-strength tensor, e is the positron charge, and $\Delta^{(\nu)}(-\partial^2)$ the operator obtained by substituting $q^2\to-\partial^2$ in $\Delta^{(\nu)}(q^2)$. On the other hand, the target-dependent parts Δ^t cannot be described by a term analogous to Eq. (10); thus, they must be treated as explicit renormalizations of $\sin^2\theta_W$ in the effective four-Fermi interactions of the low-energy theory.

Integrating Eq. (10) by parts in the action and writing $\mathcal{L}_{\bar{\nu}\nu\gamma}=-ej_\gamma^\mu A_\mu$, where j_γ^μ is the effective electromagnetic current of the neutrino, we have the identification

$$j_\gamma^\mu=\Delta^{(\nu)}(-\partial^2)(\partial^2 g^{\mu\lambda}-\partial^\mu\partial^\lambda)(\bar{\psi}_\nu\gamma_\lambda a_-\psi_\nu)/(2m_W^2)\ . \qquad (11a)$$

Defining the effective electromagnetic form factor of the neutrino by the relation

$$\langle\nu'|j_\gamma^\mu|\nu\rangle=f(q^2)\bar{u}_{\nu'}\gamma^\mu\left[\frac{1-\gamma_5}{2}\right]u_\nu \qquad (11b)$$

we obtain

$$f(q^2)=-\frac{q^2}{2m_W^2}\Delta^{(\nu)}(q^2)\ . \qquad (11c)$$

The corresponding effective mean-square charge radius is

$$\langle r^2\rangle=6\frac{\partial f(q^2)}{\partial q^2}\bigg|_{q^2=0}=-\frac{3}{m_W^2}\Delta^{(\nu)}(0)\ . \qquad (11d)$$

Inserting $m_W=81$ GeV, $\langle r^2\rangle=-17.8\Delta^{(\nu)}(0)(10^{-16}\ {\rm cm})^2$. Employing the $\Delta^{(\nu)}(0)$ values of Sec. II, which correspond to $m_t=45$ GeV and $m_H=100$ GeV, we obtain

$$\langle r^2\rangle\simeq\begin{cases}-(37.7\pm3.6)\times(10^{-17}\ {\rm cm})^2 & \text{for } \nu_e\ ,\\ -(5.9\pm3.6)\times(10^{-17}\ {\rm cm})^2 & \text{for } \nu_\mu\ ,\\ +(11.0\pm3.6)\times(10^{-17}\ {\rm cm})^2 & \text{for } \nu_\tau\ .\end{cases} \qquad (12)$$

In Eq. (11b) we have used a different normalization than Refs. 1–3. As a consequence, our $f(q^2)$ and $\langle r^2\rangle$ are defined to be a factor of 2 larger than in those papers. Because the probability of disintegration into virtual particles via electroweak radiative corrections is very small, these results should not be interpreted as describing a physical size for the neutrino (cf. Ref. 2). Note also that the neutrino charge density $\rho(r)$ is not positive definite; as a consequence, $\langle r^2\rangle$ can be positive or negative. We also point out that the replacement $\Delta^{(\nu)}(q^2)\to\Delta^{(\nu)}(0)$ is valid when $-q^2\ll4m_\mu^2$ for $l=\mu$, τ and $-q^2\ll4m_e^2$ for $l=e$. For larger values of $-q^2$, the contributions from Figs. 1(a) and 1(c) vary significantly with q^2 and one should use the complete expressions.

Our results are of the same order of magnitude as those presented by other authors in the past, but the physics behind them and the detailed answers are significantly different. In particular we have included the large hadronic contributions to Figs. 1(c)–1(f). Although separately finite and gauge invariant, there is no theoretical reason for excluding them in the definition of $f(q^2)$ or $\langle r^2\rangle$. In fact, for $m_t=45$ GeV we find large cancellations between these hadronic contributions and the characteristic $\ln(m_W/m_l)$ terms arising from Figs. 1(a) and 1(b). As a consequence of these and other differences, our $\langle r^2\rangle$ are generally smaller, in absolute value, than the results of calculations that retain only the $\ln(m_W/m_l)$ contributions. This is particularly true for ν_μ where our $|\langle r^2\rangle|$ value is smaller by a factor of $\simeq7$. For ν_e, the $\ln(m_W/m_e)$ contribution is more dominant and our $\langle r^2\rangle$ is only smaller by a factor $\simeq2$.

It also should be stressed that our $\langle r^2\rangle$ values depend sensitively on m_t, particularly for ν_μ and ν_τ. For example, for $m_t=90$ GeV and $m_H=100$ GeV, we find

$$\langle r^2\rangle_{\nu_e}\simeq-(13.4\pm3.6)\times(10^{-17}\ {\rm cm})^2\ ,$$

$$\langle r^2\rangle_{\nu_\mu}\simeq+(18.5\pm3.6)\times(10^{-17}\ {\rm cm})^2\ ,$$

$$\langle r^2\rangle_{\nu_\tau}\simeq+(35.4\pm3.6)\times(10^{-17}\ {\rm cm})^2\ ,$$

values significantly different from Eq. (12). Again, this sensitive dependence can be traced to the fact that for some values of m_l and m_t there are important cancellations of rather large terms; relatively moderate changes in the hadronic contributions due to a shift in m_t can then induce substantial modifications in $\langle r^2\rangle$. The m_t dependence can be derived by combining Eq. (11d) and Table I.

There appears also to be considerable confusion in the literature regarding the sign of $\langle r^2\rangle$. We point out that in our calculation the $\ln(m_W/m_l)$ terms from Figs. 1(a) and 1(b) have the same sign as in the classical paper of Ref. 1. This is as it should be because these terms are finite and gauge-invariant contributions from Fig. 1(a) and do not depend on the underlying theory describing the electromagnetic properties of W.

How can $f(q^2)$ be measured? By performing solely experiments on ν-lepton and ν-hadron scattering, the experimental physicist can determine the effective parameter $\sin^2\theta^{\rm eff}(q^2)\equiv\kappa^{(\nu;t)}(q^2)\sin^2\theta_W(t=l,h)$. In order to find $\kappa^{(\nu;t)}(q^2)$ one needs to extract $\sin^2\theta_W\equiv1-m_W^2/m_Z^2$ from other observables such as m_W, m_Z, or μ decay, which do not involve ν scattering. One then obtains $\Delta^{(\nu;t)}(q^2) \equiv 1-\kappa^{(\nu;t)}(q^2)$ and, using (8), (9), and (11c), determines $f(q^2)$. Clearly, measurements of $\sin^2\theta_W$ with relative errors small in comparison with $\Delta^{(\nu)}(0)$ are needed, both in ν scattering and the other observables. Table I indicates that for $l=\mu$ one requires relative errors in $\sin^2\theta_W$ of less than 1% if $m_t \lesssim 90$ GeV and less than 4.5% if $m_t \simeq 180$ GeV.

The effect of $f(q^2)$ or $\langle r^2\rangle$ in the ν-scattering cross sections can be readily obtained by noting that it amounts to a renormalization of $\sin^2\theta_W$: one simply replaces in the tree-level cross sections

$$\sin^2\theta_W \rightarrow \sin^2\theta_W[1-\Delta^{(\nu)}(q^2)] = \sin^2\theta_W\left[1+\frac{2m_W^2 f(q^2)}{q^2}\right]$$

in the general q^2 case or $\sin^2\theta_W \rightarrow \sin^2\theta_W(1+\frac{1}{3}m_W^2\langle r^2\rangle)$ in the small-q^2 case. Thus, for example, neglecting terms proportional to $m_e^2/(p_1\cdot p_2)$ and assuming $-q^2 \ll 4m_\mu^2$, the total cross section for $\overset{(-)}{\nu}_\mu e$ scattering is of the form

$$\sigma(\nu_\mu e)=\sigma^0(\nu_\mu e)+\frac{2G_\mu^2}{3\pi}(p_1\cdot p_2)s_W^2 m_W^2\langle r^2\rangle \times\left[-1+\frac{8s_W^2}{3}+\tfrac{4}{9}s_W^2 m_W^2\langle r^2\rangle\right], \quad (13a)$$

$$\sigma(\bar{\nu}_\mu e)=\sigma^0(\bar{\nu}_\mu e)+\frac{2G_\mu^2(p_1\cdot p_2)}{3\pi}s_W^2 m_W^2\langle r^2\rangle \times\left[\frac{-1+8s_W^2}{3}+\tfrac{4}{9}s_W^2 m_W^2\langle r^2\rangle\right], \quad (13b)$$

where σ^0 includes all the contributions (lowest order and radiative corrections) not contained in $\langle r^2\rangle$ and p_1 and p_2 are the four-momenta of the initial ν_μ and e. Complete expressions for $\sigma(\nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e)$ can be found in Ref. 10.

Let us finally consider the possibility that, for some unknown reason associated with physics beyond the SM, the ν has an additional electromagnetic interaction described phenomenologically by a mean-square charge radius $\langle r^2\rangle_{\rm nph}$ (the subscript reminds us that we are considering here structure effects associated with new physics). In this case, the effective phenomenological parameter $\sin^2\theta^{\rm eff}(q^2)$ determined in $\nu_\mu e$ scattering no longer equals the SM value $\kappa^{(\nu_\mu;e)}(q^2)\sin^2\theta_W$ but rather

$$\sin^2\theta^{\rm eff}(q^2)=\kappa^{(\nu_\mu;e)}(q^2)\sin^2\theta_W(1+\tfrac{1}{3}m_W^2\langle r^2\rangle_{\rm nph}) . \quad (14a)$$

Thus

$$\langle r^2\rangle_{\rm nph}=\frac{3}{m_W^2}\left[\frac{\sin^2\theta^{\rm eff}(q^2)}{\kappa^{(\nu_\mu;e)}(q^2)\sin^2\theta_W}-1\right] . \quad (14b)$$

Therefore, by measuring $\sin^2\theta^{\rm eff}(q^2)$ in $\nu_\mu e$ scattering, determining $\sin^2\theta_W$ from m_W, m_Z, or μ decay [see previous discussion concerning the measurability of $f(q^2)$] and employing the SM calculation of $\kappa^{(\nu_\mu;e)}(q^2)$, the experimental physicist can attempt to determine $\langle r^2\rangle_{\rm nph}$ via Eq. (14b) and, in this way, search for ν structure associated with new physics.

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APPENDIX A: ξ-DEPENDENT CONTRIBUTIONS

In this appendix we give, in integral form, the ξ-dependent part of the contributions proportional to $\langle f|J_\gamma^\mu|i\rangle$ arising from Figs. 1 and 2. The q^2 values are arbitrary but terms proportional to lepton or quark masses have been neglected (the cancellation of the latter requires the inclusion of additional diagrams in which unphysical Higgs scalars are coupled to the external fermions). Extensive use has been made of the Ward identities associated with the algebra of the electroweak currents. The ξ-dependent parts have been defined so that they cancel individually for $\xi=1$ ('t Hooft–Feynman gauge). Defining

$$N^{\lambda\delta}\equiv(\xi^{-1}-1)\frac{g^2e^2}{2}L^\lambda\langle f|J_\gamma^\delta|i\rangle ,$$

$$A_{\lambda\delta}\equiv\int\frac{d^nk}{(2\pi)^n}\frac{g_{\lambda\delta}}{(k^2-m_W^2)(k^2-m_W^2/\xi)} ,$$

$$B_{\lambda\delta}\equiv\int\frac{d^nk}{(2\pi)^n}\frac{k_\lambda k_\delta-g_{\lambda\delta}m_W^2}{(k^2-m_W^2)(k^2-m_W^2/\xi)[(k+q)^2-m_W^2]} ,$$

$$C_{\lambda\delta}\equiv\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_W^2)(k^2-m_W^2/\xi)[(k+q)^2-m_W^2]}\left[2g_{\lambda\delta}+\frac{(\xi^{-1}-1)k_\lambda k_\delta}{(k+q)^2-m_W^2/\xi}\right] ,$$

we obtain

$$(1a)=-N^{\lambda\delta}\frac{1}{q^2}A_{\lambda\delta}\ ,$$
$$(1b)=-N^{\lambda\delta}\frac{1}{q^2}(-2A_{\lambda\delta}+2B_{\lambda\delta}+q^2C_{\lambda\delta})\ ,$$
$$(1c)=N^{\lambda\delta}\frac{1}{q^2-m_Z^2}\frac{1}{q^2}[q^2(-A_{\lambda\delta}+2B_{\lambda\delta}+q^2C_{\lambda\delta})+(q^2-m_Z^2)(-A_{\lambda\delta}+2B_{\lambda\delta})]\ ,$$
$$(2a)=-N^{\lambda\delta}\frac{1}{q^2-m_Z^2}A_{\lambda\delta}\ ,$$
$$(2b)=-N^{\lambda\delta}\frac{1}{q^2-m_Z^2}(-2A_{\lambda\delta}+2B_{\lambda\delta}+q^2C_{\lambda\delta})\ ,$$
$$(2c)=N^{\lambda\delta}C_{\lambda\delta}\ .$$

The left-hand sides in the above equation indicate the appropriate diagrams in Figs. 1 and 2. In the renormalization scheme of Ref. 12, the counterterms appearing in Figs. 1(d)–1(f) are gauge invariant so that these diagrams are ξ independent. As expected the sum of (1a) through (2c) vanishes. In particular we note that the finite contribution (2c) from the W-W box diagram is needed to cancel an identical term from the $\bar{\nu}\nu\gamma$ proper vertex (1b). In the limit $q\rightarrow 0$, (2c) coincides with the last two terms of Eq. (4) when the substitution $J_\rho^{(3)}\rightarrow 2\sin^2\theta_W(J_\gamma)_\rho$ is made.

APPENDIX B: HADRONIC VACUUM-POLARIZATION UNCERTAINTIES

There are computational uncertainties in the hadronic loop corrections to γ, W, and Z propagators as well as γ-Z mixing. They stem from low-frequency contributions where QCD perturbation theory is not applicable and present ignorance regarding the value of the top-quark mass m_t. The latter is presumably temporary, since we anticipate m_t will be determined in the not too distant future. Once m_t is known, the top-quark contribution to loop effects will be reliably known (provided m_t is not so large as to invalidate perturbation theory). The dependence of $\Delta^{(\nu)}(0)$ on m_t is illustrated in Table I. Low-frequency hadronic loop effects are another matter. There, we are fortunate that they can be related to measured cross sections, $\sigma(e^+e^-\rightarrow\text{hadrons})$, via dispersion relations. One finds, in fact, that existing data provides quite a precise determination of the low-frequency part with relatively small uncertainties. In this appendix we update the hadronic corrections to $\Delta^{(\nu)}(0)$ and $\kappa^{(\nu_\mu;l)}(0)$ (Ref. 17) by employing the results of a detailed phenomenological analysis of $e^+e^-\rightarrow$hadrons data by Jegerlehner.[14]

The hadronic radiative corrections in Eq. (7) at $q^2=0$ can be parametrized by

$$\Delta^{(\nu)}(0)_{\text{hadronic}}=\frac{4\pi\alpha}{s^2}[\Delta\pi^{3\gamma}_{(5)}(m_Z^2)-s^2\Delta\pi^{\gamma\gamma}_{(5)}(m_Z^2)]+\frac{\alpha}{2\pi s^2}\left[\left[\frac{2}{3}-\frac{16s^2}{9}\right]\left[\ln\frac{m_t}{m_Z}+\frac{5}{6}\right]\right]-\left[\frac{3c^2\ln c^2}{2s^2}\right]\frac{\hat{\alpha}(m_W)}{2\pi s^2}+G(m_t^2/m_Z^2)\ , \quad \text{(B1)}$$

where $G(m_t^2/m_Z^2)$ is a complicated function of the top-quark mass that has been normalized such that $G(0)=0$ and $\hat{\alpha}(m_W)\simeq\frac{1}{128}$ is the running fine-structure constant. The first term represents contributions from the five known quark flavors u, d, s, c, and b to γ-Z mixing. We have followed the notation of Jegerlehner where

$$(q_\mu q_\nu-q^2g_{\mu\nu})\pi^{i\gamma}(q^2)\equiv i\int d^4x\, e^{iq\cdot x}\langle 0|T|[J^i_\mu(x)J^\gamma_\nu(0)]|0\rangle\ ,$$
$$\Delta\pi^{i\gamma}(m_Z^2)\equiv\text{Re}\pi^{i\gamma}(m_Z^2)-\pi^{i\gamma}(0)\ , \quad \text{(B2)}$$

with J^γ_ν the hadronic component of the electromagnetic current and J^3_μ the $SU(2)_L$ hadronic component of the weak neutral current. [In Ref. 17 a similar notation was used: $\pi^{\gamma Z}(q^2)\equiv\pi^{3\gamma}(q^2)-s^2\pi^{\gamma\gamma}(q^2)$ was denoted as $\hat{\Pi}^{\gamma Z}(q^2)$.]

$\Delta\pi^{\gamma\gamma}_{(5)}(m_Z^2)$ can be determined by employing the dispersion relation

$$\Delta\pi^{\gamma\gamma}_{(5)}(m_Z^2)=\frac{m_Z^2}{12\pi^2}\left[\int_{4m_\pi^2}^{E_1^2}+\int_{E_1^2}^{\infty}\right]\frac{ds\,R(s)}{s(s-m_Z^2)}\ ,$$
$$R(s)\equiv\frac{\sigma(e^+e^-\rightarrow\text{hadrons})}{\sigma(e^+e^-\rightarrow\mu^+\mu^-)}\ , \quad \text{(B3)}$$

with the understanding that weak-neutral-current contributions to $R(s)$ at large s have been subtracted. Employing existing measurements of $R(s)$ up to $E_1=40$ GeV and computing $R(s)$ perturbatively at higher energies, Jegerlehner found, for $m_Z=91.84$ GeV (corresponding to $\sin^2\theta_W=0.23$ and $m_t=45$ GeV),

$$\Delta\pi^{\gamma\gamma}_{(5)}(m_Z^2)=-0.3108\pm0.0028\pm0.0075\ , \quad \text{(B4)}$$

where the errors are statistical and systematic, respectively. We note that $\Delta\pi^{\gamma\gamma}_{(5)}(m_Z^2)$ can be used to obtain the hadronic corrections to the W and Z mass shift parameter $\Delta r_{u.d.s.c.b}=-4\pi\alpha\Delta\pi^{\gamma\gamma}_{(5)}(m_Z^2)\simeq 2.85\%$.

In the case of $\Delta\pi^{3\gamma}(m_Z^2)$, the direct experimental analog of $R(s)$ does not exist. However, assuming $SU(3)_{\text{flavor}}$ symmetry for u, d, and s and an Okubo-Zweig-Iizuka (OZI) suppression rule for heavy flavors c and b, a strategy already suggested in Sec. III C of Ref. 9, Jegerlehner has shown that

$$\Delta\pi^{3\gamma}_{(5)}(m_Z^2)=\tfrac{1}{2}\Delta\pi^{\gamma\gamma}(m_Z^2)_{u,d,s}+\tfrac{3}{8}\Delta\pi^{\gamma\gamma}(m_Z^2)_c+\tfrac{3}{4}\Delta\pi^{\gamma\gamma}(m_Z^2)_b\ , \quad \text{(B5)}$$

where the subscripts denote the different flavor contributions. Dividing $R(s)$ into its distinct flavor contributions then gives, via (B5),

$$\Delta\pi^{3\gamma}_{(5)}(m_Z^2) = -0.1481 \pm 0.0013 \pm 0.0035 \ . \qquad \text{(B6)}$$

The errors in (B6) and (B4) are of course highly correlated, since they were obtained from the same data. Therefore, one finds (for $s^2 = 0.23$)

$$\Delta\pi^{3\gamma}_{(5)}(m_Z^2) - s^2\Delta\pi^{\gamma\gamma}_{(5)}(m_Z^2) = -0.0766 \pm 0.0019 \qquad \text{(B7)}$$

for the combination relevant in (B1). Combining that result with $G(m_t^2/m_Z^2) = 3.83 \times 10^{-3}$ for $m_t = 45$ GeV, $s^2 = 0.23$, and the other terms in (B1) then leads to

$$\Delta^{(\nu)}(0)_{\text{hadronic}} = -0.0195 \pm 0.0008 \qquad (m_t = 45 \text{ GeV}) \ . \qquad \text{(B8)}$$

That value represents an approximate shift of -2.7×10^{-3} when compared with our previous analysis.[15,17]

Of course, the error in (B8) is only experimental. In order to estimate the theoretical error induced in the analysis of the u,d,s, contributions by the violation of $SU(3)_{\text{flavor}}$ symmetry we may write in Eq. (B2) for $i = 3$

$$(J^3_\mu)_{u,d,s} = \tfrac{1}{2}(J^\gamma_\mu)_{u,d,s} - \tfrac{1}{12}(J^0_\mu)_{u,d,s} + \cdots \ , \qquad \text{(B9)}$$

where the subscripts indicate that we are only considering here the contribution of the u,d,s flavors, $J^0_\mu \equiv \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s$ is a SU(3) singlet, and the ellipsis stands for axial-vector terms which give vanishing contributions to (B2). The first term in (B9) corresponds to the first term in (B5). The effect of the symmetry breaking J^0_μ may be estimated in perturbation theory with effective constituent masses $m_u = m_d \neq m_s$, leading to an additional contribution of $\ln(m_s/m_u)/72\pi^2$ to $\Delta\pi^{3\gamma}(m_Z^2)$. For $m_s/m_u \lesssim 4$ this amounts to $\lesssim 2.0 \times 10^{-3}$ and correspondingly to an error of $\lesssim 8 \times 10^{-4}$ in (B8). In addition, there are uncertainties associated with QCD perturbation theory for the high-frequency contributions and higher-order effects that have not been calculated. Given those uncertainties, we deem it prudent at this time to increase the total uncertainty in $\Delta^{(\nu)}(0)$ (for fixed m_t and $\sin^2\theta_W$) to ± 0.0020. Combining the results of this appendix with Eq. (7) we find in the case of ν_μ, for $m_t = 45$ GeV, $m_H = 100$ GeV, and $s^2 = 0.23$:

$$\Delta^{(\nu)}(0) = 0.0033 \pm 0.0020 \qquad (\nu_\mu;\ m_t = 45 \text{ GeV};\ m_H = 100 \text{ GeV}) \qquad \text{(B10)}$$

and, correspondingly, $\kappa^{(\nu_\mu;l)}(0) = 1 - \Delta^{(\nu)}(0) - \Delta^{(l)} = 0.9925 \pm 0.0020$. Even with our increase in the errors, the uncertainty in $\kappa^{(\nu_\mu;l)}(0)$ is relatively small and well below the precision of ongoing or contemplated experiments.

We note that the above analysis can be carried over to radiative corrections to atomic parity violation. The value of $\kappa_{\text{PV}}(0)$ previously employed[15,17] is shifted up by $\simeq 0.002$ and the uncertainty is estimated to be ± 0.0020. (Experiments with cesium are aiming for a few percent uncertainty in the determination of $\sin^2\theta_W$.)

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