Extended inflation with induced gravity

Frank S. Accetta
Center for Theoretical Physics, Department of Physics, Yale University, New Haven, Connecticut 06520

Jeffrey J. Trester
Department of Physics, Yale University, New Haven, Connecticut 06520

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We consider a recently proposed extended model of inflation which improves upon the original old inflation scenario by achieving a graceful exit from the false-vacuum phase. In this paper extended inflation is generalized to include a potential $V(\phi)$ for the Brans-Dicke-type field ϕ. We find that whereas a graceful exit can still be had, the inclusion of a potential places constraints on the percolation time scale for exiting the inflationary phase. Additional constraints on $V(\phi)$ and the false-vacuum energy density ρ_F from density and gravitational-wave perturbations are discussed. For initially small values of ϕ the false vacuum undergoes power-law inflation, while for initially large values of ϕ the expansion is exponential. Within true-vacuum regions slow-rolling inflation can occur. As a result, this model generically leads to multiple episodes of inflation. We discuss the significance these multiple episodes of inflation may have on the formation of large-scale structure and the production of voids.

I. INTRODUCTION

Inflation has proven to be so natural an outcome of the application of particle physics to cosmology that inflationary scenarios have been raised to the level of a paradigm for any complete model of the early Universe.[1] In model building, one generically employs some variation on "new" inflation,[2] wherein a scalar field is displaced from the minimum of its potential and slowly evolves to that minimum. During this slow-rolling evolution, the (nearly) constant potential energy density of the field dominates the expansion rate of the Universe and the scale factor grows exponentially. The Universe executes a "graceful exit" from inflation when the scalar field begins to oscillate about its minimum and finally decays into radiation, reheating the Universe. It is straightforward in these scenarios to show how an initially smooth region of size H_0^{-1} can be made large enough to encompass the entire observable Universe.

In contrast, Guth's original inflation scenario[3] assumed that the Universe undergoes a strongly first-order phase transition with the false metastable vacuum dominated by the constant energy-density difference between the false and true vacua. As with new inflation, this constant energy density implies that the scale factor for the false vacuum will exponentially grow. The subsequent nucleation and collision of true-vacuum bubbles was suggested as a means by which a region could be formed to contain the observed Universe. However, as is well known, the rate of nucleation of true-vacuum bubbles cannot keep up with the exponential growth of the false vacuum so that for old inflation there is no graceful transition to a radiation-dominated universe.[4]

Recently, the old inflation scenario has been reconsidered by La and Steinhardt[5] in the context of a Brans-Dicke theory of gravity.[6] They note that if the Universe were to be in a metastable false-vacuum state with energy density ρ_F, provided by additional physics, and were to undergo a strongly first-order phase transition, the scale factor in the false vacuum would not grow exponentially but rather as $a(t) \propto t^{\omega+1/2}$ (for t sufficiently large) where ω is the Brans-Dicke parameter. Since $\omega > 500$ for Brans-Dicke theory not to conflict with experiment,[7] the scale factor undergoes strong power-law growth. Since $p(t)=\exp[-\xi(t^4-t_b^4)]$ is the probability that if bubble nucleation starts at t_b, at time t a point will still be in the false vacuum, a comparison of the power-law growth in the volume of the false vacuum to $p(t)$ shows that for $\xi t^4 > 1$ where ξ is a parameter depending on the ratio of the bubble nucleation rate to the expansion rate of the Universe, $p(t)$ is decreasing much more rapidly than the volume is increasing. The true vacuum percolates the Universe.

The important new ingredient that "extended inflation" adds to the original inflation scenario is an inflationary epoch "soft" enough to allow percolation. For the Brans-Dicke theory, extended inflation arises because there is a massless scalar field [with dimensions of (mass)^2] in the theory which regulates the value of the effective gravitational constant, $G_{\text{eff}} \approx \Phi^{-1}$, and which evolves during inflation. Since $H \approx (G_{\text{eff}} \rho_F)^{1/2}$ and $\Phi \approx \rho_F t^2/\omega^2$, H decreases inversely with time, $H \approx \omega t^{-1}$. Thus the scale factor grows as a large power of $t\,[a(t) \sim t^{\omega}]$. Without a potential to provide a fixed vacuum expectation value for ϕ, percolation is guaranteed to occur since the power-law growth of the false vacuum continues indefinitely while the ratio of the nucleation rate to the expansion rate increases with time.

Power-law inflation has also been demonstrated for slow-rolling transitions in induced-gravity theories.[8] Induced gravity bears a strong resemblance to Brans-Dicke theory though the motivations behind the two are rather different. Induced gravity is based on the observation in gauge theories that dimensionful coupling constants which arise in a low-energy effective theory can be expressed in terms of vacuum expectation values of scalar fields. It has been suggested that similarly, gravity arises as a symmetry-breaking phenomenon induced in an effective action derived from some defining action.[9] The key difference between induced gravity and Brans-Dicke theory is the existence of a potential $V(\phi)$ for the scalar field ϕ. Neglecting the potential, the two theories can be mapped into one another by a simple field redefinition—since $\mathcal{L}_{BD}=\Phi R+\omega(\partial\Phi)^2/\Phi$, setting $\Phi=\epsilon\phi^2/2$ and $\omega=(4\epsilon)^{-1}$ takes $\mathcal{L}_{BD}\to\mathcal{L}_{IG}=\epsilon\phi^2 R/2+(\partial\phi)^2/2$. The difference arising from the inclusion of a potential is important. Because of $V(\phi)$, at low energies G is strongly anchored at its presently measured value—for low energies the theory is identical to general relativity with the gravitational constant $G=(8\pi\epsilon\langle\phi\rangle^2)^{-1}$. Only at very high energies does the theory deviate from general relativity.[10] It has been shown in a Ginzburg-Landau model of induced gravity with $V(\phi)=\frac{1}{8}\lambda_\phi(\phi^2-v^2)^2$, that the symmetry-breaking transition can be inflationary. In particular, a slow-rolling transition for $\phi<v$, where v is the vacuum expectation value of ϕ, leads to strong power-law growth, $a(t)\propto t^{\epsilon^{-1}/4}$ where $\epsilon\ll 1$, while for $\phi>v$ ("chaotic" inflation[11]), the growth is exponential.

In this paper we comment on the consequences to the extended inflation scenario of including a symmetry-breaking potential (for global scale invariance) for the ϕ field. It is our hope that by including a potential we will uncover effects which have a general applicability to extended inflation and which do not in general depend upon the particular choice of potential (see Sec. II). The specific choice of induced gravity for our analysis allows us to draw on familiar results concerning inflation in a particular scalar-tensor theory of gravity.[12]

In addition to the field ϕ, we introduce a second scalar field σ whose potential $V(\phi)$ (see Fig. 1), which we leave unspecified for now, has a global minimum at σ_0 and a local minimum corresponding to a false-vacuum state at $\sigma=0$ with energy density $\rho_F=V(\sigma=0)=\frac{1}{8}\lambda_\sigma\sigma_0^4$. As we shall see, our results are similar to that for slow-rolling inflation with induced gravity. For $\phi<(\lambda_\sigma\sigma_0^4/\lambda_\phi v^2)^{1/2}=\phi'$, using $V(\phi)$ given above, the growth in the scale factor is governed by the false-vacuum energy of the σ field and the Universe undergoes power-law inflation while for $\phi>\phi'$, $V(\phi)$ dominates and the scale factor grows exponentially. Indeed, in the absence of percolation, the late-time evolution of ϕ implies that $a(t)$ grows exponentially regardless of the initial value of ϕ (there is a limiting stationary value for ϕ). This places constraints on the time scale for percolating a true vacuum. If these constraints are not satisfied, a second episode of inflation must (and in this model can) occur.

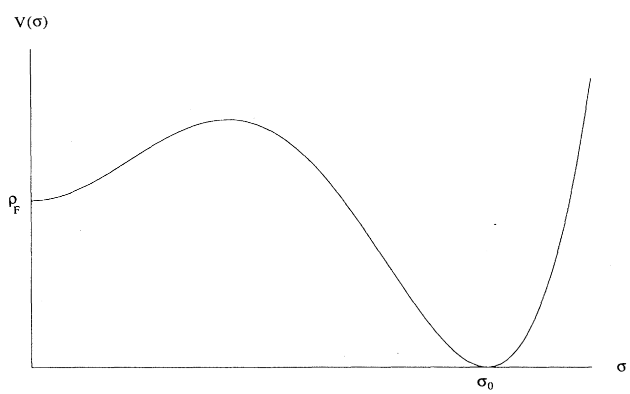

FIG. 1. The potential $V(\sigma)$ exhibiting a false-vacuum state at $\sigma=0$ with vacuum energy $\rho_F=V(\sigma=0)=\frac{1}{8}\lambda_\sigma\sigma_0^4$ and a global minimum at $\sigma=\sigma_0$.

The paper is arranged as follows. In Sec. II we present a model for induced gravity plus a scalar field σ whose potential $V(\sigma)$ exhibits a false-vacuum state. We derive equations of motion for the scale factor $a(t)$ and ϕ and present approximate analytical solutions to these equations in the inflationary false-vacuum state. In Sec. III we discuss the possible scenarios for the inflationary false-vacuum state and derive constraints on percolation. Section IV discusses the evolution of the true-vacuum state and in Sec. V we derive constraints on the model due to the production of density and gravitational-wave perturbations. We conclude in Sec. VI with some final remarks on possible implications of the model, particularly for the formation of large-scale structure and voids.

II. A MODEL FOR INDUCED-GRAVITY EXTENDED INFLATION

The scenario for extended inflation considered in Ref. 5 involves a scalar-tensor theory of gravity with a potential $V(\phi)$ for the scalar field which is identically zero. The lack of a nontrivial potential is the reason for the well-known constraints on Brans-Dicke models from present-day (low-energy) observations. If $V(\phi)$ is not identically zero, these constraints may not be relevant to low-energy physics. At the classical level, a globally scale-invariant theory described by the Lagrangian $\mathcal{L}_{IG}=\epsilon\phi^2R/2+(\partial\phi)^2/2$ allows a potential of the form $V(\phi)\sim\phi^4$. The renormalized one-loop correction to this potential[13] introduces a mass scale into the theory thereby breaking the global scale invariance by allowing ϕ to have a nonzero vacuum expectation value. The behavior of the theory with this Coleman-Weinberg potential in curved space $V=\frac{1}{4}\lambda_\phi\phi^4\{\ln[\max(H^2,\phi^2)/v^2]-\frac{1}{2}\}+\frac{1}{8}\lambda_\phi v^4$ is similar to that with the phenomenological Ginzburg-Landau potential mentioned previously both for the slow-rolling as well as for the present extended inflation scenario. Models with other potentials can be envisioned. A monotonic potential which asymptotically approaches zero from above would clearly not affect the scenario of Ref. 5 so long as ρ_F is sufficiently large compared to $V(\phi)$. This is due to the fact that the evolution of ϕ will always be dominated by ρ_F in the false σ vacuum. Of course the steepness of such a potential would be limited in the true vacuum by the magnitude of $\dot{G}/G$. In general, though a potential with minima (of which there could be several, not all degenerate) at nonzero values of ϕ and for which $V(\phi)>\rho_F$ over some range of ϕ would affect the evolution of the false vacuum.

A particularly simple model which captures some of these ideas is based on the action[14]

$$S=\int d^4x\sqrt{-g}\,[-\tfrac{1}{2}\epsilon\phi^2R-\tfrac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi-\tfrac{1}{8}\lambda_\phi(\phi^2-v^2)^2-\tfrac{1}{2}g_{\mu\nu}\partial^\mu\sigma\partial^\nu\sigma-V(\sigma)]\ , \quad (1)$$

where ϵ,λ_ϕ are dimensionless coupling constants and $v=\epsilon^{-1/2}$ is the vacuum expectation value of ϕ which we consider to be a fundamental scalar field. In this model, the effective value of the gravitational constant, $G_{\rm eff}=(8\pi\epsilon\phi^2)^{-1}$ can vary at early times. Depending on the initial value of ϕ, $G_{\rm eff}$ can be either greater or less than the value measured today, $G_N=(8\pi)^{-1}$.

Restricting to a Robertson-Walker metric with scale factor $a(t)$ which assures that ϕ is spatially homogeneous, the Ricci scalar $R=-6[\ddot{a}(t)/a(t)+\dot{a}(t)^2/a(t)^2+k/a(t)^2]$ with k the curvature signature, overdots denote derivatives with respect to time, and $\sqrt{-g}=a(t)^3r^2\sin\theta/(1-kr^2)^{1/2}$. Substituting into the action and varying, we obtain equations of motion for $a(t)$, ϕ, and σ:

$$H^2\left[1+\frac{2\dot{\phi}/\phi}{H}\right]=\frac{1}{3\epsilon\phi^2}[\tfrac{1}{2}\dot{\phi}^2+V(\phi)+\tfrac{1}{2}\dot{\sigma}^2+V(\sigma)]-k/a(t)^2\ , \quad (2a)$$

$$\ddot{\phi}+3H\dot{\phi}+\frac{\dot{\phi}^2}{\phi}+\frac{1}{1+6\epsilon}\left[\frac{\dot{\sigma}^2}{\phi}+V'(\phi)-\frac{4}{\phi}[V(\phi)+V(\sigma)]\right]=0\ , \quad (2b)$$

$$\ddot{\sigma}+3H\dot{\sigma}+V'(\sigma)=0\ , \quad (2c)$$

where $H=\dot{a}(t)/a(t)$ is the expansion rate, primes denote derivatives with respect to the ϕ and σ fields in Eqs. (2b) and (2c), respectively, and $V(\phi)=\frac{1}{8}\lambda_\phi(\phi^2-v^2)^2$. $V(\phi)$ is shown in Fig. 2. Comparing Eqs. (2b) and (2c), the equation of motion for ϕ differs from the usual one by the terms $\dot{\phi}^2/\phi$ and $\{\dot{\sigma}^2/\phi-(4/\phi)[V(\phi)+V(\sigma)]\}/(1+6\epsilon)$ and the factor $1/(1+6\epsilon)$. Since $(3\epsilon\phi^2)^{-1}=(8\pi G_{\rm eff}/3)$ the equation for H differs from the standard one by the additional term $(2\dot{\phi}/\phi)H$.

In the false-vacuum state, $\sigma=0$, $\dot{\sigma}=0$, $V(\sigma=0)=\frac{1}{8}\lambda_\sigma\sigma_0^4$, and we assume $\lambda_\sigma\sigma_0^4>\lambda_\phi v^4$ so that as $\phi\to0$, $V(\sigma=0)$ dominates the energy density. We will make the following assumptions that the evolution of ϕ is "friction dominated" (similar to those imposed in slow-rolling inflation) and establish their validity shortly:

$$\tfrac{1}{2}\dot{\phi}^2\ll V(\phi)+V(\sigma)\ , \quad (3a)$$

$$|\dot{\phi}^2/\phi|\ll 3H\dot{\phi}\ , \quad (3b)$$

$$\ddot{\phi}\ll 3H\dot{\phi}\ . \quad (3c)$$

In the regime where Eqs. (3a)–(3c) are valid, Eqs. (2a) and (2b) reduce to

$$H^2=\frac{1}{3\epsilon\phi^2}[V(\phi)+V(\sigma)]-k/a(t)^2\ , \quad (4a)$$

and

$$3H\dot{\phi}=\frac{1}{1+6\epsilon}\left[\frac{4}{\phi}[V(\phi)+V(\sigma)]-V'(\phi)\right]\ . \quad (4b)$$

In terms of Eq. (4b), the potential ϕ evolves on is

$$U(\phi)=\tfrac{1}{4}\lambda_\phi v^2\phi^2-\tfrac{1}{2}\lambda_\sigma\sigma_0^4\ln\phi\ . \quad (5)$$

$U(\phi)$ is shown in Fig. 3. For small values of ϕ, the evolution of the field is controlled by the $V(\sigma=0)/\phi$ term (the logarithm term) while for large values of ϕ the quadratic term dominates. This potential has a minimum at

$$\phi'=\left[\frac{\lambda_\sigma\sigma_0^4}{\lambda_\phi v^2}\right]^{1/2}=\left[\epsilon^{-1}\frac{V(\sigma=0)}{V(\phi=0)}\right]^{1/2}.$$

Comparing ϕ' to v we see that $\phi'>v$ so long as we impose

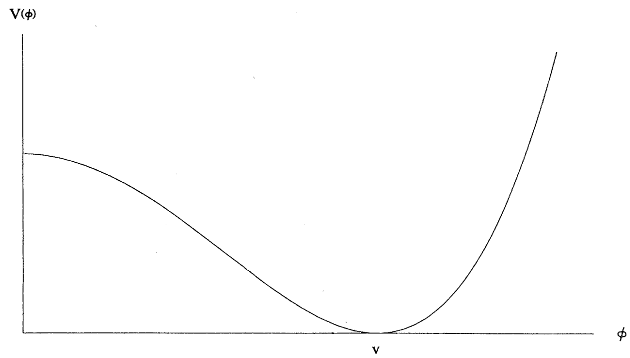

FIG. 2. The potential $V(\phi)=(\lambda_\phi/8)(\phi^2-v^2)^2$ with symmetry-breaking minimum at $\phi=v$.

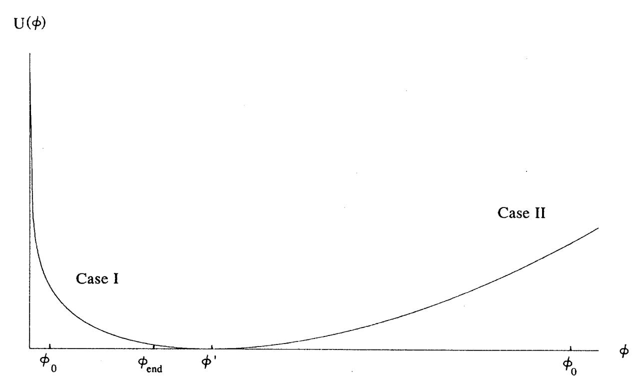

FIG. 3. The potential $U(\phi)=\frac{1}{4}\lambda_\phi v^2\phi^2-\frac{1}{2}\lambda_\sigma\sigma_0^4\ln(\phi)$ with minimum $\phi'=(\lambda_\sigma\sigma_0^4/\lambda_\phi v^2)^{1/2}$ and $V(\phi')=0$. The arbitrary initial value of ϕ,ϕ_0, is displayed both for $\phi<\phi'$ and for $\phi>\phi'$. These regions correspond to cases I and II, respectively. $\phi_{\rm end}$ is the value of ϕ at the end of percolation and may have any value from ϕ_0 to ϕ' when $\phi<\phi'$.

$\lambda_\sigma\sigma_0^4 > \lambda_\phi v^4$. Note that when $\phi=\phi'$, there is a constant energy density

$$\rho(\phi') \equiv V(\phi') + V(\sigma) = \tfrac{1}{8}\lambda_\phi[(\phi'^2 - v^2)^2 + v^2\phi'^2] \ . \tag{6}$$

As a result, if ϕ were to come to rest at ϕ', the false-vacuum state would grow exponentially.

Solutions to Eqs. (4a) and (4b) are straightforward to obtain [here and in the rest of the paper we will assume $V(\sigma=0) > V(\phi=0)$ so that $\lambda_\sigma\sigma_0^4 > \lambda_\phi v^4$ and $k=0$]. If ϕ_0 is the initial value of ϕ at $t=0$ and $\phi_0 < \phi'$ then

$$\phi = \phi_0 + (\tfrac{2}{3}\lambda_\sigma\epsilon)^{1/2}\sigma_0^2 t \ , \tag{7a}$$

while, for $\phi_0 > \phi'$,

$$\phi = \phi_0 - (\tfrac{2}{3}\lambda_\phi\epsilon)^{1/2} v^2 t \ . \tag{7b}$$

Since

$$H^2 = \frac{1}{3\epsilon\phi^2}\left[\frac{\lambda_\phi}{8}(v^2-\phi^2)^2 + \frac{\lambda_\sigma}{8}\sigma_0^4\right] , \tag{8}$$

for $\phi < v < \phi'$, as ϕ evolves from $\phi=\phi_0$ to ϕ the scale factor grows like

$$\begin{aligned} a/a_0 \simeq\ & (\phi/\phi_0)^{\epsilon^{-1}[1+(\lambda_\phi v^4)/2(\lambda_\sigma\sigma_0^4)]/4} \\ & \times\exp\left[\frac{\epsilon^{-1}}{32}\frac{\lambda_\phi}{\lambda_\sigma\sigma_0^4}(\phi_0^4-\phi^4) + \frac{\epsilon^{-1}}{8}\frac{\lambda_\phi v^2}{\lambda_\sigma\sigma_0^4}(\phi_0^2-\phi^2)\right] \\ \approx\ & (\phi/\phi_0)^{\epsilon^{-1}/4}\exp\left[\frac{\epsilon^{-1}}{8\phi'^2}(\phi_0^2-\phi^2)\right] . \end{aligned} \tag{9a}$$

The combination of power-law and exponential behaviors of $a(t)$ is a direct consequence of the variable dependence of H on ϕ. For $\phi \approx \phi_0 \ll (v,\phi')$, while $\phi_0 > (\frac{2}{3}\lambda_\sigma\epsilon)^{1/2}\sigma_0^2 t$, the scale factor behaves as $\ln[a(t)/a_0] \sim [V(\sigma=0)/3\epsilon\phi_0^2]^{1/2} t$, so in this region one recovers the exponential solution discussed by La and Steinhardt. Starting from some initial value $\phi_0 \ll v$ for the ϕ field, the total increase in the size of the scale factor as ϕ evolves from ϕ_0 to ϕ' is

$$\ln(a'/a_0) = \tfrac{1}{4}\epsilon^{-1}\ln(\phi'/\phi_0) + \tfrac{1}{8}\epsilon^{-1}(\phi_0^2 - \phi'^2)/\phi'^2 \ , \tag{9b}$$

where $a' \equiv a(\phi=\phi')$. When $\phi > \phi'$, $H \approx (\lambda_\phi/24\epsilon)^{1/2}\phi$ so that

$$\begin{aligned} a(t)/a_0 &= \frac{1}{8}\left[\frac{\lambda_\sigma\sigma_0^4}{\lambda_\phi v^4}\right]^{1/2}\frac{\epsilon^{-1}}{\phi' v}(\phi_0^2-\phi^2) \\ &= \exp[\tfrac{1}{8}(\phi_0^2-\phi^2)] \ . \end{aligned} \tag{10}$$

Returning to the assumptions in Eqs. (3a)–(3c), it is straightforward to show that our analysis is consistent for $|\phi-\phi'| \geq \epsilon^{1/2}\phi'$.

When $\phi \approx \phi' \gg v$ and $|\phi-\phi'| \geq \epsilon^{1/2}\phi'$, $H \sim (\lambda_\sigma\sigma_0^4/24)^{1/2}\phi/\phi'$ (using $v=\epsilon^{-1/2}$). In this regime, if $\phi \to \phi'$ from the left of ϕ',

$$a'/a \approx \exp\left[\frac{\lambda_\sigma\sigma_0^4}{12\phi'}(t'^2 - t^2)\right] .$$

If ϕ approaches ϕ' from the right,

$$a'/a \approx \exp(\tfrac{1}{12}|t'^2 - t^2|) \ .$$

In the next section we will consider in more detail the evolution of ϕ in the false σ vacuum.

III. PHYSICS OF THE FALSE σ VACUUM: INFLATION AND PERCOLATION

Our discussion in this section will divide neatly into "prenucleation" and "postnucleation" physics corresponding to false and true σ-vacuum physics. We begin first by enumerating the "prenucleation" scenarios.

Case I. $\phi < \phi'$, $|\phi-\phi'| \geq \epsilon^{1/2}\phi'$. Here the expansion rate is dominated by the false-vacuum energy of σ. First, consider $\phi_0 \sim \phi$. As we have pointed out, here the expansion is exponential. Once $\phi_0 < (\frac{2}{3}\lambda_\sigma\epsilon)^{1/2}\sigma_0^2 t$, the scale factor grows as a power law. In particular, in the region $\phi_0 \ll \phi \ll \phi'$, the scale factor grows as

$$a/a_0 \approx [(\tfrac{2}{3}\epsilon\lambda_\sigma\sigma_0^4)^{1/2}/\phi_0]^{\epsilon^{-1}/4} t^{\epsilon^{-1}/4} \ , \tag{11}$$

so that $a(t)$ grows as a strong power of t. The total time required for ϕ to evolve from ϕ_0 to ϕ' is

$$t' = (\tfrac{2}{3}\lambda_\phi v^2\epsilon)^{-1/2} \ . \tag{12}$$

In this time the scale factor grows by

$$\ln[a(t')/a_0] \approx \tfrac{1}{4}\epsilon^{-1}[\ln(\phi'/\phi_0) - \tfrac{1}{2}] \ . \tag{13}$$

Case II. $\phi > \phi'$, $|\phi-\phi'| \geq \epsilon^{1/2}\phi'$. Here the expansion rate is dominated by the potential $V(\phi)$. During the time that $\phi \approx \phi_0 \gg \phi'$, Eq. (10) implies that the scale factor grows exponentially:

$$a(t)/a_0 \approx \exp\left[\left[\frac{\lambda_\phi}{24}\right]^{1/2}\epsilon^{-1/2}\phi_0 t\right] . \tag{14}$$

The total time for evolving from $\phi=\phi_0$ to $\phi=\phi'$ is

$$t' = (\tfrac{2}{3}\lambda_\phi\epsilon)^{-1/2}\frac{\phi'}{v^2}\left[\frac{\phi_0}{\phi'} - 1\right] . \tag{15}$$

Clearly, t' can be made arbitrarily large by suitable choice of ϕ_0.

If percolation does not occur before t' in case I, or for $t > t'$ in case II, we must consider the additional evolution for ϕ in the false σ vacuum.

Case III. $\phi \approx \phi'$, $|\phi-\phi'| \approx \epsilon^{1/2}\phi'$. Once $|\phi-\phi'| \approx \epsilon^{1/2}\phi'$, ϕ will begin to oscillate about ϕ' with frequency $m_\phi^f = (\frac{3}{4}\lambda_\phi)^{1/2} v$ where m_ϕ^f is the mass of the ϕ particle in the false vacuum at $\phi=\phi'$. In a complete theory, one would expect couplings of ϕ to other fields. The rate for ϕ to decay to another state X (with mass $\ll m_\phi^f$) is $\Gamma_{\phi\to XX} \approx g^2 m_\phi$, where g is the relevant coupling.[15] The decay lifetime is then $\tau_\phi \approx (g^2 m_\phi^f)^{-1} = [(3\lambda_\phi/4)^{1/2} g^2 v]^{-1}$. In general, if τ_ϕ is the lifetime of the decay channel with the largest branching ratio, then ϕ will oscillate about ϕ'

from $t=t'$ to $t=\tau_\phi$.

The behavior of $a(t)$ will be determined in this regime by the coherent oscillations of ϕ (see Ref. 16). Matter domination [where $a(t)\propto t^{2/3}$] occurs if anharmonic terms in the potential, having magnitude δ, satisfy $\delta \le \phi_{\rm max}^{-1}$ where $\phi_{\rm max}^{-1}$ is the amplitude of the oscillations. Strictly speaking, soon after oscillations have begun, such terms cannot be ignored in our discussion if $\phi' > v$. In this case the energy density will in general fall off more rapidly than for a matter-dominated universe, i.e., $a(t)\propto t^n$, $n \ge \frac{2}{3}$. Indeed right after oscillations have begun, if $\phi \rightarrow \phi'$ from values less than $\phi', n \sim \epsilon^{-1}/4$, while for late times, due to damping via expansion, $n \sim \frac{2}{3}$. Similar behavior for $\phi \rightarrow \phi'$ from $\phi > \phi'$ can be expected. As a result, we will assume that during this epoch the expansion rate of the false vacuum is "oscillation dominated" by the ϕ field condensate and grows as $a(t)\propto t^n$. For the sake of discussion n will be assumed to take on values from $n \sim \epsilon^{-1}/4$ to $n \sim \frac{2}{3}$. During oscillation domination, because of damping, n is time dependent and $\dot n$ will be monotonically decreasing [$a(t)$ will increase less strongly with time]. However, "matter domination," $n \sim \frac{2}{3}$, will occur only if percolation of a true vacuum takes a sufficiently long time. Finally, note that since strong damping of ϕ oscillations would require $g^2 > (24\epsilon)^{-1/2}\phi'/v \gg 1$, ϕ field oscillations should be weakly damped.

Case IV. $\phi=\phi'$. If ϕ comes to rest at ϕ' before a graceful exit is achieved, then from Eq. (6), $\rho(\phi')=\frac{1}{8}\lambda_\phi[(\phi'^2-v^2)^2+v^2\phi'^2]$, and the false vacuum will begin to exponentially expand:

$$a(t)/a'=\exp\left[\left[\frac{\lambda_\phi}{24\epsilon}\right]^{1/2}[\phi'^2(1-v^2/\phi'^2)^2+v^2]^{1/2}t\right] . \tag{16}$$

If $\phi' \gg v$ this reduces to

$$a(t)/a'=\exp\left[\left[\frac{\lambda_\phi}{24\epsilon}\right]^{1/2}\phi' t\right] =\exp\{[\tfrac{1}{3}V(\sigma=0)]^{1/2}t\} . \tag{17}$$

Since $\phi' > v$, the ϕ field will start out on $V(\phi)$ with $\phi_0 > v$ upon nucleation of a true σ vacuum bubble.

With this outline of "prenucleation" physics in hand, we can go on to discuss its implication for the percolation of a true vacuum. The rate at which true-vacuum bubbles are nucleated is governed by $\Gamma(t)=Ae^{-S}$, the tunneling rate per unit time per unit volume, where S is the Euclidean action and A is $O((\text{mass})^4)\sim\sigma_0^4$. The probability that a point p is in the false vacuum at time t given that bubble nucleation started at t_b with rate $\Gamma(t)$ is

$$p(t)=\exp\left[-\int_{t_b}^{t}dt'\Gamma(t')a^3(t')\frac{4\pi}{3}\left[\int_{t'}^{t}\frac{dt''}{a(t'')}\right]^3\right] . \tag{18}$$

Percolation occurs if nucleated bubbles of the true vacuum merge to form an infinite connected region. For our purposes, this will be true if $p(t)<1/e$. For case I, when $\phi<\phi'$, Eq. (18) reduces to

$$p(t)=\exp\left[-\frac{64\pi}{3}\epsilon^3\Gamma(t^4-t_b^4)\right] , \tag{19}$$

where we have taken $\Gamma(t)=\Gamma$ to be a constant during inflation and have dropped terms $O(1/\epsilon)$. Percolation occurs when $(64\pi/3)\epsilon^3\Gamma(t^4-t_b^4)>1$. This leads to a percolation time scale of

$$t_{\rm end}=(\tfrac{64}{3}\pi\epsilon^3\Gamma)^{-1/4} \tag{20}$$

and we are assuming that inflation is essentially over by this time. A measure of the nucleation rate to the expansion rate is given by Γ/H^4. By Eq. (8), H is not a constant as in the standard inflationary scenario, but rather depends on ϕ. When $\phi<v<\phi'$, $H\approx(\lambda_\sigma\sigma_0^4/24\epsilon\phi^2)^{1/2}$. Since ϕ increases linearly in time Γ/H^4 grows as $\phi(t)^4\propto t^4$. While for $v \ll \phi$, Γ/H^4 decreases as $\phi(t)^4\propto t^4$. It is for this reason that there is a constraint on percolation. In what follows, we will approximate H by $\chi=(\lambda_\sigma\sigma_0^4/24\epsilon\phi_{\rm end}^2)^{1/2}$ choosing $\phi=\phi_{\rm end}$, the value of ϕ at the end of extended inflation, so that a convenient measure of the nucleation rate to the expansion rate is Γ/χ^4. In terms of this, $t_{\rm end}=[(\pi\lambda_\sigma^2\sigma_0^8\epsilon/27\phi_{\rm end}^4)\Gamma/\chi^4]^{-1/4}$. Setting $\phi_{\rm end}\equiv\phi(t=t_{\rm end})$, ϕ at $t_{\rm end}$ is

$$\phi_{\rm end}=\phi_0+(\tfrac{2}{3}\lambda_\sigma\epsilon)^{1/2}\sigma_0^2 t_{\rm end} =\phi_0\left[1-\left[\frac{12\epsilon}{\pi}\right]^{1/4}\left[\frac{\Gamma}{\chi^4}\right]^{-1/4}\right]^{-1} \tag{21a}$$

for $t_{\rm end}<t'$ and

$$\phi_{\rm end}\approx\phi' \tag{21b}$$

for $t_{\rm end}\approx t'$. If $t_{\rm end}<t'$ then nucleation at a sufficiently rapid rate occurs while $\phi<\phi'$, $|\phi-\phi'|\le\epsilon^{1/2}\phi'$ (case I). Therefore we require

$$\frac{12}{\pi}\epsilon\left[\frac{\phi_{\rm end}^4}{\phi'^4}\right]<\frac{\Gamma}{\chi^4} \tag{22}$$

in order for a graceful exit from inflation to occur before ϕ reaches ϕ'. For case I, the requirement that $a(t)>e^{60}$ by $t_{\rm end}$ implies that

$$240<\epsilon^{-1}[\ln(\phi_{\rm end}/\phi_0)-\tfrac{1}{2}] . \tag{23}$$

In particular, if $t_{\rm end}\approx t'$, $240<\epsilon^{-1}[\ln(\phi'/\phi_0)-\frac{1}{2}]$. Clearly, the necessary number of e-folds can easily be met without strongly constraining any of the parameters involved.

A few remarks are in order concerning Eq. (22). One should keep in mind that as written, χ is a function of $\phi_{\rm end}$ inasmuch as varying $\phi_{\rm end}$ changes $\chi\propto\phi_{\rm end}^{-1}$. Thus, Eqs. (22) and (25) below are (not surprisingly) constraints on ϕ'. In the limit where $\phi'\rightarrow\infty$, $V(\phi)/V(\sigma=0)\rightarrow 0$ and Eq. (22) simply requires Γ/χ^4 to be nonzero. This is just another reflection of the observation that in this limit percolation will occur at some finite time regardless of the magnitude of Γ/χ^4. Since $\phi'^2=\epsilon^{-1}V(\sigma=0)/V(\phi=0)$, ϕ' can be made sufficiently large by dialing down ϵ and/or $V(\phi)$.

For cases I and III, the total time required to evolve from ϕ_0 and come to rest at ϕ' is simply

$$t_{\rm tot}=t'+\tau_\phi =(\lambda^{1/2}v)^{-1}[(\tfrac{2}{3}\epsilon)^{-1/2}+(\tfrac{3}{4}g^4)^{-1/2}] \ . \qquad (24)$$

For $g^4>\epsilon$, $t_{\rm tot}\approx(\lambda_\phi^{1/2}v)^{-1}(\frac{2}{3}\epsilon)^{-1/2}$, so that the dominant contribution to the total time is due to the evolution of ϕ from ϕ_0 to ϕ'. For $g^4<\epsilon$, $t_{\rm tot}\approx[(\frac{3}{4}\lambda_\phi)^{1/2}v]^{-1}g^{-2}$, and the dominant contribution to the total time is due to the oscillation of ϕ about ϕ'. If dominated by evolution from ϕ_0 to ϕ', percolation occurs when Eq. (22) is satisfied,

$$\frac{12}{\pi}\epsilon\left[\frac{\phi_{\rm end}^4}{\phi'^4}\right]<\frac{\Gamma}{\chi^4} ,$$

and $\phi_{\rm end}$ is given by Eq. (21). Oscillation domination implies that [assuming $a(t)\propto t^n$ and that $\Gamma(t)$ is constant in this regime]

$$\frac{4\pi}{3}(n-1)^{-3}\left[\frac{1}{4}+\frac{3}{2(n+1)}-\frac{1}{3n+1}-\frac{3}{n+3}\right]\Gamma\tau_\phi^4\geq 1$$

and percolation occurs if

$$\frac{243}{\pi}(n-1)^3\left[\frac{1}{4}+\frac{3}{2(n+1)}-\frac{1}{3n+1}-\frac{3}{n+3}\right]^{-1}\times g^8\epsilon^2\left[\frac{\phi_{\rm end}^4}{\phi'^4}\right]<\frac{\Gamma}{\chi^4} . \qquad (25)$$

In this case $\phi_{\rm end}\approx\phi'$. When $n=\epsilon^{-1}/4$, $(243/16\pi)g^8\epsilon^{-1}<\Gamma/\chi^4$, while for $n=\frac{2}{3}$, corresponding to matter domination, $(5\,940/\pi)g^8\epsilon^2<\Gamma/\chi^4$. Since the oscillations of the ϕ field are damped during expansion, $\dot{n}$ will monotonically decrease with time and for late times the $n\sim\frac{2}{3}$ should be accurate (though by this time the true vacuum may have percolated).

For cases II and III, the total time to evolve from ϕ_0 and come to rest at ϕ' is

$$t_{\rm tot}=t'+\tau_\phi =(\lambda^{1/2}v)^{-1}\left[(\tfrac{2}{3}\epsilon)^{-1/2}\frac{\phi'}{v}\left[\frac{\phi_0}{\phi'}-1\right]+(\tfrac{3}{4}g^4)^{-1/2}\right] . \qquad (26)$$

Since for case II the expansion is dominated by the energy density of ϕ and not the false vacuum of σ, the question of percolation does not become important until oscillations around ϕ' set in—i.e., when the σ false vacuum becomes important. As a result, percolation occurs if Eq. (25) is satisfied:

$$\frac{243}{\pi}(n-1)^3\left[\frac{1}{4}+\frac{3}{2(n+1)}-\frac{1}{3n+1}-\frac{3}{n+3}\right]^{-1}\times g^8\epsilon^2\left[\frac{\phi_{\rm end}^4}{\phi'^4}\right]<\frac{\Gamma}{\chi^4} .$$

Here we are using the fact that for times well after the exponential growth phase, an oscillation-dominated universe has $a(t)\propto t^n$. Exponential rather than power-law growth due to an initially large value of ϕ is acceptable if the Universe enters an oscillation-dominated phase of expansion for a sufficiently long time (long enough to percolate) while in the σ false vacuum. In fact, if $n>1$ in this phase for a long enough time (so that $a\approx e^{60}$), successful inflation can be accomplished completely during oscillation domination.

If the above constraints are satisfied, a sufficiently large connected region of true vacuum can be formed by the uniform coalescence of bubbles to encompass the presently observed Universe. The latent heat of the false vacuum, which upon nucleation of a single bubble was transformed to the surface energy density and motion of the bubble walls is thermalized. If this occurs rapidly compared to the expansion rate, the percolated region is reheated to a temperature $T_{\rm RH}\sim\rho_F^{1/4}=(\lambda_\sigma/8)^{1/4}\sigma_0$ for $\phi_{\rm end}<\phi'$ or $T_{\rm RH}\sim\rho(\phi')^{1/4}=(\lambda_\phi/8)^{1/4}[(\phi'^2-v^2)^2+v^2\phi'^2]^{1/4}$ for $\phi_{\rm end}\sim\phi'$. Limits on $T_{\rm RH}$ are discussed in Sec. V.

While curvature terms with $k\neq 0$ will be strongly suppressed after inflation, the fact that ϕ has nonzero kinetic energy means that it is possible for $\Omega\equiv\rho/\rho_C\neq 1$. In the regime where $\phi<\phi'$ and $|\phi-\phi'|\geq\epsilon^{1/2}\phi'$—case I—$\dot{\phi}$ is a constant so if percolation occurs for $\phi<\phi'$, from Eq. (2a) with $k=0$,

$$\Omega=(1+20\epsilon)/(1+37\epsilon/3)$$

up to terms of order ϵ^2. Substituting $\epsilon=(4\omega)^{-1}$ implied by the field redefinition $\Phi\rightarrow\epsilon\phi^2/2$, we find $\Omega=(1+5/\omega)/(1+37/12\omega)$. This is to be compared to the Brans-Dicke scenario[5] where $\Omega=1+4/3\omega$ at the end of inflation. In the Brans-Dicke scenario, the deviation from unity of Ω is expected to persist to the present epoch due to the constant evolution of the Brans-Dicke scalar. For a matter-dominated universe, the value today is $\Omega=1+5/6\omega$ while in the induced gravity scenario the fact that today $\phi=v$ means that $\Omega=1$.

Finally, we note that if none of the percolation constraints discussed above are satisfied, then ϕ will ultimately come to rest at $\phi=\phi'$ and case IV tells us that the false σ vacuum will begin to exponentially expand. In order to recover from this, sufficient inflation must occur within a single bubble.

IV. PHYSICS OF THE TRUE σ VACUUM

In this section, we discuss briefly some aspects of "postnucleation" physics. We consider a simplified case in order to gain some insight into the behavior of the ϕ field inside a true σ vacuum bubble. In particular, we wish to point out that there can be additional inflation due to a slow-rollover transition for the ϕ field within a given bubble. This would be especially relevant if the false vacuum fails to percolate, thereby requiring a second round of inflation. For the present discussion we will not attempt to find an explicit general-relativistic solution of the coupled ϕ and σ equations. To simplify our discussion, we will assume that in the interior of a bubble of the true vacuum, σ relaxes rapidly to $\sigma=\sigma_0$

compared to the evolution time scale for ϕ on $V(\phi)$, that ϕ is sufficiently slowly varying spatially well inside the bubble that such derivatives can be neglected, and for the moment we put aside the question of the efficiency of thermalization of bubble-wall energy. Under these assumptions, we reproduce the physics from the standard induced gravity inflation model[8] within a given bubble. However bubble nucleation adds some new wrinkles. Because bubble nucleation occurs at a constant rate Γ, we expect different initial values of ϕ in bubbles nucleated at different times.[17] However, since bubbles are nucleated at different times they will not only have different values of ϕ within them, they will themselves have a distribution of sizes with respect to H (which itself depends on ϕ). If the bubbles are all of the same size, all nucleated at $t \approx t_{\rm end}$ with $\phi \approx \phi_{\rm end}$ within the bubbles, then the Universe will be homogeneous after percolation. If a sizable fraction of bubbles are nucleated early and/or with a nontrivial distribution in sizes, maintainence of homogeneity may be problematical. Still, what might at first appear to be a possible difficulty for the scenario may in fact yield some unexpected benefits concerning large-scale structure. We will return to this question in Sec. VI. For the present discussion we will make no special assumption concerning the distribution of bubble sizes or the initial value of ϕ within them.

In any given bubble, when $|\phi_b - v| \geq \epsilon^{1/2} v$,

$$\phi_b(t) = \phi_n \pm (\tfrac{2}{3}\epsilon\lambda_\phi)^{1/2} v^2 (t \mp t_n) , \tag{27}$$

where ϕ_n is the initial value of ϕ_b at nucleation time t_n and $t \geq t_n$. This leads to the following behavior for the scale factor when $\phi_n < v$:

$$a(t)/a_n \approx [\phi_n + (\tfrac{2}{3}\lambda_\phi)^{1/2} v/\phi_n]^{\epsilon^{-1/4}} (t - t_n)^{\epsilon^{-1/4}} , \tag{28}$$

where $a_n \equiv a(t = t_n)$. The total growth in the scale factor from $\phi = \phi_n$ when $t = t_n$ to $\phi = v$ when $t = t_e$ (the time at which inflation ends) is

$$\ln[a(t_e)/a_n] \approx \tfrac{1}{4}\epsilon^{-1}[\ln(v/\phi_n) - \tfrac{1}{2}] , \tag{29}$$

which holds if $\phi_n < v, |\phi_b - v| \geq \epsilon^{1/2} v$. The total time needed to evolve from ϕ_n to v is

$$t = (\tfrac{2}{3}\epsilon\lambda_\phi)^{-1/2} v^{-2} (v - \phi_n) . \tag{30}$$

This is essentially the same as Eq. (12), the time to evolve from ϕ_0 to ϕ'. In that case $\dot\phi$ is greater. When $\phi_n > v$,

$$a(t)/a_n \approx \exp[(\lambda_\phi/24)^{1/2} \epsilon^{-1} (\phi_n/v) t] , \tag{31}$$

so that

$$\ln[a(t_e)/a_n] = \tfrac{1}{8}\epsilon^{-1}[(\phi_n^2/v^2 - 1) - 2\ln(\phi_n/v)] . \tag{32}$$

Clearly, by inspection of Eqs. (29) and (32), if ϵ and ϕ_n are sufficiently small, a single bubble of true σ vacuum can inflate via a slow-rolling transition of ϕ, controlled in this case by $V(\phi)$, to encompass the observed Universe. This will be the case if $\epsilon \leq \frac{1}{240}[\ln(v/\phi_n) - \frac{1}{2}]$, $\frac{1}{240}[\ln(v/\phi_n) + \frac{1}{2}(\phi_n^2/v^2 - 1)]$ for $\phi_n < v$, $\phi_n > v$, respectively. The fact that $\phi' > v$ implies that the Universe can recover from case IV, where ϕ comes to rest at ϕ', by inflating a single bubble if $\epsilon \leq \frac{1}{240}[\ln(v/\phi') + \frac{1}{2}(\phi'^2/v^2 - 1)]$.

V. CONSTRAINTS FROM DENSITY AND GRAVITATIONAL-WAVE PERTURBATIONS

In the extended inflation scenario we expect density and gravitational-wave perturbations to be generated by both the evolution of ϕ and collisions of bubble walls. For the present discussion we will focus on perturbations due to the evolution of ϕ in the false vacuum. These will be the relevant perturbations to consider if the distribution of bubbles is strongly peaked around small bubbles (see the end of Sec. VI and Ref. 23). Quantum fluctuations in ϕ produce density fluctuations with amplitude[18]

$$(\delta\rho/\rho)_H = (H^2/\dot\phi)|_{\phi_N} \tag{33}$$

when they cross inside the horizon during the post-inflation epoch,[19] while gravitational-wave perturbations produced during inflation have amplitude[20]

$$h_{\rm GW} = H/m_{\rm Pl}|_{\phi_N} . \tag{34}$$

Density and gravitational-wave perturbations on the present horizon scale ($N \approx 60$) were generated during the prenucleation phase (assuming no inflation with $N \geq 60$ occurred in the postnucleation phase) about 60 e-folds prior to the end of the inflation. Density perturbations have amplitude

$$(\delta\rho/\rho)_H \approx \tfrac{1}{24}\epsilon^{-1}\phi_N^{-2}\dot\phi_N^{-1}[\lambda_\phi\phi_N^4(v^2/\phi_N^2 - 1)^2 + \lambda_\sigma\sigma_v^4] . \tag{35}$$

For this to be consistent with both the microwave background and galaxy formation, $(\delta\rho/\rho)_H \approx \delta \times 10^{-4}$ with $\delta \leq 1$. The amplitude for gravitational-wave perturbations is

$$h_{\rm GW} \approx 0.04\epsilon^{-1}\lambda_\phi^{1/2}\left[\frac{V(\sigma=0)}{V(\phi=0)}\right]^{1/2} \times \left[\frac{\phi_N^2}{\phi'^2}\left[\frac{v^2}{\phi_N^2} - 1\right]^2 + \frac{v^2}{\phi_N^2}\right]^{1/2} . \tag{36}$$

For gravitational-wave perturbations to be consistent with the quadrupole anisotropy of the microwave background, $h_{\rm GW} \leq 3 \times 10^{-5}$ on the present horizon scale. The value of $\phi = \phi_N$, N e-folds before the end of inflation can be obtained by formally solving Eq. (9b) using $\phi_{\rm end}$ instead of ϕ':

$$N = \tfrac{1}{4}\epsilon^{-1}\ln(\phi_{\rm end}/\phi_N) + \tfrac{1}{8}\epsilon^{-1}(\phi_N^2 - \phi_{\rm end}^2)/\phi_{\rm end}^2 ,$$

which can be solved in the limiting cases where $\epsilon \gg (4N)^{-1}$ and $\epsilon \ll (4N)^{-1}$:

I. $\epsilon \gg (4N)^{-1} \sim \frac{1}{240}$. Scales of interest were produced when in the following situations.

(a) $\phi < v < \phi'$ and $|\ln(\phi_N/\phi_{\rm end})| \approx -4N\epsilon - \frac{1}{2}$, $\phi_N = (\frac{2}{3}\lambda_\sigma\epsilon)^{1/2}\sigma_0^2$. In this regime, density perturbations have the amplitude

$$(\delta\rho/\rho)_H \approx 0.05\epsilon^{-3/2}\lambda_\phi^{1/2}\left[\frac{V(\sigma=0)}{V(\phi=0)}\right]^{1/2} \times\left[\frac{\phi_N^2}{\phi'^2}\left[\frac{v^2}{\phi_N^2}-1\right]^2+\frac{v^2}{\phi_N^2}\right], \quad (37)$$

so that

$$V(\sigma=0)/\phi_{\rm end}^4 \approx 5\times10^{-7}\epsilon^3\delta^2\exp[-2(480\epsilon+1)] .$$

The constraint from gravitational-wave perturbations implies that

$$V(\sigma=0)/\phi_{\rm end}^2 \lesssim 7\times10^{-8}\epsilon^{3/2}\exp(-480\epsilon-1) .$$

(b) $\phi>\phi'$ and $\phi_N/\phi_{\rm end}\approx\phi_N/\phi'\approx(8N\epsilon)^{1/2}$, $\dot\phi=(\frac{2}{3}\lambda_\phi\epsilon)^{1/2}v^2$. In this regime, density perturbations have the amplitude

$$(\delta\rho/\rho)_H \approx 0.05\epsilon^{-3/2}\lambda_\phi^{1/2}\left[\frac{V(\sigma=0)}{V(\phi=0)}\right] \times\left[\frac{\phi_N^2}{\phi'^2}\left[\frac{v^2}{\phi_N^2}-1\right]^2+\frac{v^2}{\phi_N^2}\right], \quad (38)$$

which implies that

$$\lambda_\phi \approx 2\times10^{-11}\delta^2\epsilon\left[\frac{V(\phi=0)}{V(\sigma=0)}\right]^2 ,$$

while the false σ vacuum energy density is

$$\rho_F \lesssim 10^{-13}\epsilon^{-1}m_{\rm Pl}^4$$

from the gravitational-wave limits.

II. $\epsilon \ll (4N)^{-1}\sim\frac{1}{240}$. Scales of interest were produced when $v<\phi\sim\phi'$ so that $\phi_{\rm end}\approx\phi'$ and $|\ln(\phi_N/\phi')| \approx 2(N\epsilon)^{1/2}$. When $\phi\to\phi'$ from values less than ϕ', we find, from Eq. (37),

$$\rho_F \approx 8\times10^{-10}\delta^2\epsilon m_{\rm Pl}^4 ,$$

while for $\phi\to\phi'$ from values greater than ϕ', Eq. (38) implies

$$\lambda_\phi \approx 4\times10^{-6}\epsilon^3\delta^2\exp[-8(N\epsilon)^{1/2}]\left[\frac{V(\phi=0)}{V(\sigma=0)}\right]^2 .$$

In either case, constraining $h_{\rm GW}$ limits the false σ vacuum energy density:

$$\rho_F \lesssim \times10^{-10}m_{\rm Pl}^4 ,$$

consistent with the density perturbation constraint for $\phi\to\phi'$, $\phi<\phi'$.

To summarize these results, density and gravitational-wave perturbations produced by the evolution of ϕ from values $\phi>\phi'$, where $V(\phi)$ dominates, place constraints on the self-coupling λ_ϕ entering into the potential $V(\phi)$, requiring that λ_ϕ be small. These constraints are similar to those found in the standard induced gravity scenario. In the limit where λ_ϕ vanishes, we are still left with constraints on ρ_F. On the other hand, evolution for $\phi<\phi'$ mainly constrains $V(\sigma=0)$ and $V(\phi=0)$.

For $\epsilon \gg (4N)^{-1}$, if $\phi<v<\phi'$, $\epsilon\approx\frac{1}{30}$ implies that $V(\sigma=0)/\phi_{\rm end}^2 \lesssim 2\times10^{-17}$ from the limit on $h_{\rm GW}$. Using this, if $\phi_{\rm end}\sim\phi'$ then $V(\phi=0)\lesssim(4\times10^{14}\ {\rm GeV})^4$. We obtain similar results from the density perturbation constraint in this regime. On the other hand, when $\phi>\phi'$, a constraint on the false σ vacuum energy density is available: $\rho_F\lesssim(2\times10^{16}\ {\rm GeV})^4$. However, if we liberally choose $V(\phi=0)/V(\sigma=0)\lesssim 0.1$, $\lambda_\phi\approx 7\delta^2\times10^{-15}$. When $\epsilon\ll(4N)^{-1}$, if ϕ evolves from values greater than ϕ', $\lambda_\phi\lesssim 5\delta^2\times10^{-18}$ assuming $\epsilon\approx10^{-3}$ and $V(\phi=0)/V(\sigma=0)\lesssim 0.1$ while $\rho_F\lesssim(4\times10^{16}\ {\rm GeV})^4$. Our results are consistent with those on ρ_F, $\rho_F\lesssim(10^{17}\ {\rm GeV})^4$, from $h_{\rm GW}$ for the Brans-Dicke scenario.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have considered the effects on the extended inflation scenario of including a symmetry-breaking potential for the Brans-Dicke field. Potentials for Brans-Dicke-type fields arise for a number of reasons. In superstring theories (see Ref. 10) the dilaton field ϕ (which coherently couples to matter) must have some expectation value due to singularities in the equations. Quantum mechanically ϕ is a pseudo-Goldstone boson which cannot have a mass due to supersymmetry. After supersymmetry breaking, a nontrivial potential can arise. Furthermore, these theories predict values of the Brans-Dicke parameter ω which vanish, or are at most of order unity. To avoid conflict with experiment nontrivial potentials once again are expected to exist, generated nonperturbatively.

The existence of a potential for ϕ places constraints on the percolation time scale for the Universe to transit from the false-vacuum inflationary phase to the true-vacuum Robertson-Walker phase. For evolution in the region $\phi<\phi'$, the Universe undergoes power-law inflation $[a(t)\propto t^{\epsilon^{-1}/4}]$ and percolation occurs if $(12/\pi)\epsilon(\phi_{\rm end}^4/\phi'^4)<\Gamma/\chi^4$ while in the oscillation-dominated regime, $\phi_{\rm end}\approx\phi'$, $a(t)\propto t^n$ and for $n=\epsilon^{-1}/4$, percolation occurs if $(243/16\pi)g^8\epsilon^{-1}<\Gamma/\chi^4$. When $n=\frac{2}{3}$, corresponding to matter domination, percolation occurs if $(5{,}940/\pi)g^8\epsilon^2<\Gamma/\chi^4$. Starting with values of ϕ greater than ϕ', the Universe will undergo exponential growth and bubble nucleation and percolation will not occur until the Universe is oscillation dominated with $\phi\approx\phi'$. Since the percolation constraints for fixed Γ/χ^4 depend upon g and $\phi'^4=[\epsilon^{-1}V(\sigma=0)/V(\phi=0)]^2$, the extended inflation scenario can be implemented in the presence of a potential for the ϕ field if g, ϵ, and $V(\phi=0)/V(\sigma=0)$ are sufficiently small. For example, if $\phi\to2$ gravitons, then $g^2\approx(3/32\pi)\lambda_\phi$ and dialing down g will (all else fixed) increase ϕ'. Additional constraints on $V(\phi)$ and ρ_F arise by considering density and gravitational-wave perturbations produced by the evolution of ϕ. This will be the dominant source of such perturbations if only very small bubbles (compared to the horizon size) are produced and thermalize.

In the absence of percolation, $\phi=\phi'$ and the false σ vacuum will enter an epoch of exponential inflation from which it will not be able to recover as a whole. This is not fatal so long as $\epsilon\lesssim\frac{1}{240}[\ln(v/\phi')+\frac{1}{2}(\phi'^2/v^2-1)]$. In this case a single bubble can exponentially grow to encompass the entire observable Universe.

In the event that percolation of true σ vacuum is successful, to a first approximation, it should occur rapidly with the average value of $\phi_n \sim \phi_{\rm end}$ at the end of the false-vacuum inflationary epoch. If reheating of the true-vacuum phase by thermalization of wall energy is efficient, $T_{RH} \sim \rho_F^{1/4} \sim 10^{17}$ GeV and the ϕ symmetry can be restored [since $V(\sigma=0) > V(\phi=0)$], effectively resetting ϕ and leading to a round of induced gravity inflation in the percolated region. Inflation in the true vacuum can easily be great enough to make the previous round of inflation irrelevant [we are assuming that $V(\phi=0)$ is not too many orders of magnitude less than $V(\sigma=0)$]. Indeed, it is quite possible to have either a mild ($N<60$) or strong ($N>60$) second episode of inflation. On the other hand, if reheating is such that $T_{\rm RH} < V(\phi_n)^{1/4} \approx V(\phi_{\rm end})^{1/4}$, the percolated region will undergo an additional bout of power-law inflation if $\phi < v$ or exponential inflation if $\phi > v$ when $|\phi - v| \geq \epsilon^{1/2} v$. Depending on $\phi_{\rm end}$ this second round of inflation can likewise be either mild or strong. In both cases where the Universe is initially reheated after the first episode of inflation, after the second episode of inflation, the Universe is reheated to a temperature $T_{\rm RH} \approx [(\tau_\phi^t)^{-1} m_{\rm Pl}]^{1/2} \approx g(\lambda_\phi/\epsilon)^{1/2} 3\times 10^{18}$ GeV where τ_ϕ^t is the lifetime of ϕ in the true vacuum. This temperature must be high enough so that both nucleosynthesis and baryogenesis can take place. For example, the maximum reheat temperature from limits on $h_{\rm GW}$ when $\epsilon \gg (4N)^{-1}$, with evolution in the region $\phi < \phi'$ and $\phi_{\rm end} \sim \phi'$, $g=1$ is $[\rho_\phi(\phi_{\rm end})]^{1/4} \approx 5\times 10^{14}$ GeV. This is sufficient for baryogenesis; however it is not generic and we would expect in general a lower $T_{\rm RH}$ (see Refs. 8 and 15).

Given that our model undergoes multiple episodes of inflation, it is intriguing to consider its possible consequences for the formation of large-scale structure. In Sec. V we discussed the generation of density perturbations in the false vacuum. Density perturbations will also be generated within the percolated region as ϕ evolves there. These multiple episodes of inflation will in general lead to different spectra of density perturbations which can have power on different scales. Multiple episodes of inflation have been invoked in the past[21] to solve the problem that the spectrum of adiabatic density perturbations from inflation has too little power on large scales once it is normalized to observational data on small scales. In the "double inflation" scenario, an initial round of inflation can determine the spectrum on large scales while a second round of inflation would be responsible for a small-scale structure. The scale which separates large and small scales is $\lambda_{\rm seed} \approx (3\times 10^{-4} - 3)$Mpc, $M_{\rm seed} \approx 10^2 - 10^{13} M_\odot$. In this approach one requires an amplitude for large-scale fluctuations of order $10^{-5}-10^{-4}$, corresponding to $\delta \sim 0.1-1$ in Sec. V. While on small scales, the amplitude of perturbations is then $\sim 0.01-0.1$ which can result in the production of primordial black holes, early structure formation, and a host of other exotic phenomena. The second episode of inflation must last 40–50 e-folds which is possible if $\epsilon \approx (1/4N)[\ln(v/\phi_{\rm end}) - \frac{1}{2}]$, $(1/4N)[\ln(v/\phi_{\rm end}) + \frac{1}{2}(\phi_{\rm end}^2/v^2 - 1)]$ for $\phi_{\rm end} < v$, $\phi_{\rm end} > v$, respectively, and $N=40-50$. One particularly interesting variant on this model involves the production of the recently observed bubblelike structures in the distribution of galaxies.[22] In our scenario, there is an initial episode of power-law (case I $\phi < \phi'$) or exponential (case II $\phi > \phi'$) inflation which smoothes out the Universe over large scales ($\gg \lambda_{\rm seed}$). In case I percolation can occur prior to $\phi = \phi'$ while for case II percolation occurs only after oscillation domination. For either scenario, bubble nucleation occurs over a finite time with $\phi_n \neq v$ and slow-rolling transitions will take place within these bubbles (in particular ϕ_n will depend upon when a particular bubble is nucleated). On scales $\sim \lambda_{\rm seed}$, there would be density perturbations of order unity which may lead to bubblelike structures in the present Universe of order $(10-30)h^{-1}$ Mpc. The appearance of a voidlike structure would depend upon the bubble distribution function. If it is too sharply peaked around many small bubbles (compared to H) the Universe would appear essentially homogenous and any significant density perturbations will be due to the evolution of ϕ. If it allows too many large bubbles, unacceptable inhomogeneities would result. Since H is ϕ dependent and therefore changes in time as ϕ evolves, the distribution of bubbles with respect to H from the beginning to the end of the false σ vacuum inflationary epoch may be such as to yield a distribution of a sufficient number of large bubbles significant enough to generate the observed voids yet not so large or so many as to produce unacceptable inhomogeneities.

Since completing this work we have learned that La and Steinhardt[23] have investigated bubble percolation in extended inflation in quantitative detail and have shown that the bubble distribution function can admit a safe (so as not to distort the microwave background) but nonnegligible number of large bubbles. The bubble distribution function is not flat (though the exponent of the distribution function still needs to be accurately determined). They suggest extended inflation may generate a bubble distribution which can account for the large-scale void structure and/or provide new seeds for galaxy formation.

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[14]We will work in units where $\hbar=c=k_B=1$ and all energies are measured in units of $m_{\rm Pl}/(8\pi)^{1/2}$ where the Planck mass $m_{\rm Pl}=G^{-1/2}=1.22\times10^{19}$ GeV. Our conventions are those of S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), for the metric signature $(-+++)$ and definition of the Ricci tensor.

[15]Decays of ϕ into two gravitons is the dominant decay mode with $\tau_\phi^{-1}=\epsilon m_\phi^3/m_{\rm Pl}^2$ [G. Segrè and S. Barr, Penn Report No. UPR-0387T (unpublished)]. This may make reheating in a slow-rollover scenario via ϕ decays (as mentioned in Secs. IV and VI) problematical if the branching ratios to other particles (which thermalize) are small. Of course this does not affect reheating via thermalization of wall energy.

[16]For a discussion of coherent field oscillations, see M. S. Turner, Phys. Rev. D **28**, 1243 (1983).

[17]Inside a true σ vacuum bubble $\rho\neq0$ if $\phi\neq v$ and pressure balance implies that strictly $v_{\rm wall}<c$ especially for bubbles nucleated $\sim t_b$. This will affect our estimate of $t_{\rm end}$. However, by assumption, $\rho_\sigma\gg\rho_\phi$ so that any corrections should be small.

[18]As in Ref. 8 we have assumed that the usual formulas for density and gravitational-wave perturbations hold, with the replacement of G by $G_{\rm eff}=(8\pi\epsilon\phi^2)^{-1}$. A more detailed calculation for variable G has been done by F. Lucchin, S. Matarresse, and M. D. Pollock, Phys. Lett. **167B**, 163 (1986).

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