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New metastable states in supercritical QED

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It is shown that new metastable charge-neutral states exist in the supercritical phase of QED around a large-Z nucleus. They are the vibration modes of the induced electron cloud and therefore do not exist in the normal phase. Under the adiabatic approximation it is argued that the states mimic the stable particle states and may be responsible for the peak structure in e^+e^- spectra found in heavy-ion-collision experiments.

In previous papers[1,2] we developed a framework for nonperturbative treatment of QED in strong external fields. The theory is cast into the form of an effective two-dimensional fermion theory by appealing to the importance of the lowest partial waves of the electrons and the photons. Then, by using the bosonization technique the theory is further converted into a two-dimensional boson theory, rendering its semiclassical analysis feasible. By constructing such a framework we aim at understanding the physics around a highly charged source created, for example, in heavy-ion-collision experiments. In particular, we have suggested that the nonperturbative aspect of QED might be responsible for the peak structure in the electron and the positron spectra found in these experiments.[3,4]

In this paper we wish to report on the existence of new metastable states in the supercritical phase of QED, based on our understanding of its ground state obtained in Ref. 1. Essentially they are the vibration modes of the electron cloud induced around a highly charged nucleus in the supercritical ground state. We show that in certain circumstances the width of the states is of the order of 1–10 keV, which suggests an interesting possibility that these states might be the cause of the sharp e^+e^- peaks in heavy-ion experiments. We also discuss the conditions under which these states can imitate the experimental data. We should mention that a similar treatment of the states has been given in Ref. 5 relying on the bosonized framework of Ref. 1. Our treatment and conclusions, however, differ from theirs on several crucial points.

The bosonized Hamiltonian obtained in Ref. 1 takes the form

$$H=\int dr\left\{\sum_m \tfrac{1}{2}(\Pi_m^2+P_m^2+\Phi_m'^2+Q_m'^2)-\sum_{m,\delta}\frac{1}{2\pi r^2}N_\mu\cos\left[\sqrt{\pi}\left(\Phi_m+Q_m-\delta\int_r^\infty ds[\Pi_m(s)-P_m(s)]\right)\right]\right.$$
$$\left.-\sum_m\frac{c\mu m_0}{\pi}N_\mu[\cos(2\sqrt{\pi}\Phi_m)+\cos(2\sqrt{\pi}Q_m)]+\frac{e^2}{8\pi r^2}\left[\Theta(r,t)-\frac{1}{\sqrt{\pi}}\sum_m(\Phi_m+Q_m)\right]^2\right\}, \qquad (1)$$

when expressed in terms of the "physical" Bose variables. Here Π_m and P_m denote the canonical conjugates of the Bose fields Φ_m and Q_m, respectively. The index m $(=\pm\frac{1}{2})$ represents the z component of the angular momentum and δ is the chirality signature and takes values ± 1. $\Theta(r,t)$ in Eq. (1) indicates the external charge contained within radius r.

The Hamiltonian (1) is normal ordered at the bare value μ of the mass of the Bose fields. It is a small but arbitrary mass which is necessary to define the bosonization in the interaction representation. The key issue for our problem is to renormal order the Hamiltonian with respect to the physical masses. Unfortunately, however, the dynamics of the full system (1) is not under control because of the severe nonlinearity of the canonical momenta. Therefore, we appeal to the following physical argument to determine the renormal-ordering point. At radii large compared with the electron Compton wave length, $r\gg m_e^{-1}$, the centrifugal barrier and the Coulomb interaction terms are negligible. In this case the theory reduces to two decoupled sine-Gordon theories. Then we choose the renormal-ordering point in such a way that with the corresponding renormalized coefficient of the mass (cosine) term the theory reproduces the observed electron mass.[6]

On the other hand, at sufficiently smaller radii compared to m_e^{-1} the mass term is unimportant. We restrict ourselves in this paper to the discussion of small fluctuations around the supercritical ground-state configuration with $\Phi_m^{\rm cl}=Q_m^{\rm cl}$ and vanishing canonical momenta. This is a natural parity-invariant ground state and is known to provide a reasonable description of the supercritical vacuum.[1] Then the fluctuations ϕ_m+q_m and ϕ_m-q_m (the small letters represent the quantum fluctuation) around this ground-state decouple from each other. We further restrict our discussion to the ϕ_m+q_m mode since it is free from the nonlinearity of the canonical momenta. We use the simple notation $\psi_m=(1/\sqrt{2})(\Phi_m+Q_m)$. After freezing the $\phi_m - q_m$ mode we have an effective Hamiltonian written in terms of the ψ_m degrees of freedom. We propose to renormal order this Hamiltonian with respect to the tree-level boson mass κ/r where $\kappa^2 = 2 + e^2/2\pi^2$ for $\psi_+ = (1/\sqrt{2})(\psi_{1/2} + \psi_{-1/2})$ and $\kappa^2 = 2$ for $\psi_- = (1/\sqrt{2}) \times (\psi_{1/2} - \psi_{-1/2})$, where the tree masses are obtained by expanding the $r^{-2}\times$cosine term. Needless to say this means only the first approximation of an exact procedure but we believe that it is a reasonable thing to do.

The renormal-ordering formula for ψ_m fields can readily be worked out in the manner of Ref. 7 as

$$N_\mu \exp[i\beta\psi_m(r,t)] = \exp\{(\beta^2/4\pi)[\psi(\nu+1)+\gamma]\} \times N_{\kappa/r}\exp[i\beta\psi_m(r,t)] , \quad (2)$$

where $\nu = (\kappa^2 + \frac{1}{4})^{1/2} - \frac{1}{2}$, γ is the Euler constant, and $\psi(z)$ indicates the digamma function. The renormalization factor in (2) plays a crucial role in our discussion. The readers may feel somewhat curious about the renormal ordering with respect to radius-dependent masses. But it is known in the discussion of the magnetic-monopole–fermion system that this procedure is necessary to obtain the physically sensible (μ-independent) result in the bosonized framework.[8] We are planning to explore it in more detail in a separate paper.[9] In particular we will demonstrate that in this system the renormalization factor in (2) is indispensable to reproduce the correct answer derived by other methods such as the path integral.

We now argue that by collecting the centrifugal barrier and the mass terms renormal ordered in respective regions of their importance, we have an effective Hamiltonian which is usable for the analysis including the region $r \sim m_e^{-1}$. Since the renormal ordering is affected only by the short-distance behavior of the theory, the inclusion of the soft mass term does not modify the renormal-ordering coefficient in (2). For details see Ref. 9.

Using (2) in the ψ Hamiltonian and adding the mass term

$$(\pi/4)m_e^2[\cos(2\sqrt{\pi}\Phi_m) + \cos(2\sqrt{\pi}Q_m)]$$

we obtain an effective Hamiltonian for the discussion of small fluctuations around the symmetric ($\Phi_m^{\rm cl} = Q_m^{\rm cl}$) ground state. We expand around the classical configuration, $\Phi_m = \Phi_m^{\rm cl} + \phi_m$, etc., and keep only the quadratic terms of small fluctuations. If we use the stationary ansatz $\psi_\pm(r,t) = \exp(-i\omega_\pm t)\psi_\pm(r)$, then $\psi_\pm(r)$ obey the Schrödinger-type equations with the potential

$$V(r) = [2C(\kappa)r^{-2} + \pi^2 m_e^2]\cos(2\sqrt{\pi}\Phi^{\rm cl}) + 4\alpha/(\pi r^2) \quad (\text{for } \psi_+) , \quad (3)$$

where $C(\kappa)$ denotes the renormalization factor in (2) and the Coulomb term on the right-hand side of (3) appears only in the ψ_+ equation.

In Fig. 1 the shape of the potential in the Schrödinger-type equation is depicted for a spherically symmetric uniformly charged external source (to which we confine our considerations in this paper) with radius $R = 35$ fm and $Z = 170$. The pocket structure arises where the background field develops the soliton configuration, and therefore it is characteristic of the supercritical ground state.

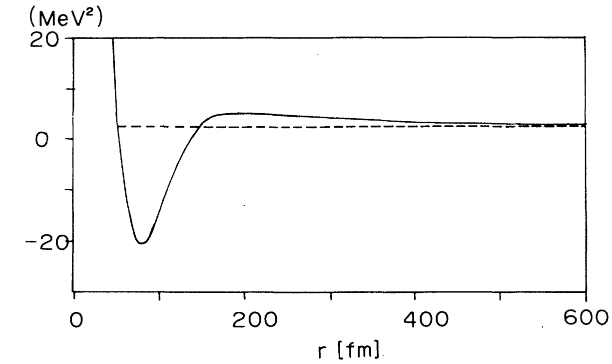

FIG. 1. The potential (3) which is felt by the fluctuation mode is depicted with the parameters $Z = 170$ and $R = 35$ fm. The dashed line indicates ω^2, the energy squared of the metastable states.

We employ the WKB approximation to evaluate the energies of the states trapped in this potential well. The result for ψ_+ is presented in Fig. 2. We realize that the state exists in a wide region of the parameters of the external source. The Z dependence of the energy level is sizable for small R but is very weak for R greater than ~ 30 fm.

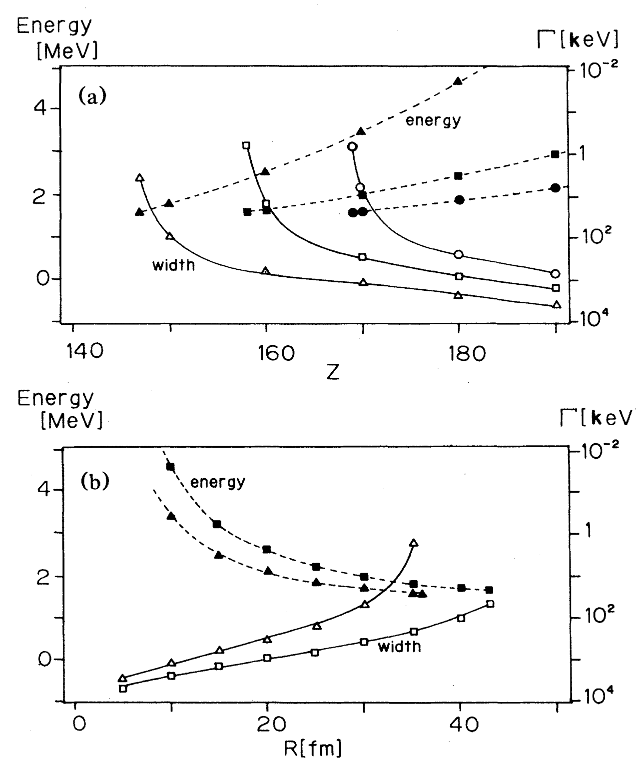

FIG. 2. The energy levels and the widths of the metastable states are plotted as (a) functions of Z for $R = 10$ fm (△), 22 fm (□), and 34 fm (○); (b) functions of R for $Z = 170$ (△), and 180 (□). The solid (open) symbols indicate the energies (widths) of the states. The left (right) ordinate is for the energy (width) plotted in units of MeV (keV) in linear (inverse-logarithmic) scale. The solid and the dashed curves are to guide the eye.

The splitting between ψ_+ and ψ_- states is tiny, of the order of 10 keV, in our calculation but this quantity is very sensitive to the behavior of the wave function near the origin and, therefore, the WKB estimate is not very reliable. Also the results of the energy level in the case of small R or large Z may suffer from the error of $\sim 20\%$ level. We, however, restrict ourselves to the WKB approximation since we are primarily interested in the qualitative systematics of the state over a wide range of the source parameters. We should emphasize that the existence of these localized states is solely due to the renormalization factor in (2) whose numerical value is $C(2)=e^{1/2}=1.65$. [We have ignored the $O(\alpha)$ term.]

The natural mechanism of the decay of these states is that they first tunnel through the barrier depicted in Fig. 1 and then decay into the soliton-antisoliton pair via the sine-Gordon interaction. We exploit the WKB approximation to evaluate the lifetime of the states. The standard formula for the decay width reads $\Gamma=c(r_b-r_a)^{-1}T$, where T is the usual tunneling probability and r_a and r_b are the roots of the equation $V(r)=\omega^2$ as shown in Fig. 1. We have approximated the velocity of the particle trapped in the potential well as nearly equal to c.

The computed results of the decay width are summarized in Fig. 2. We observe that some of them are very narrow, less than the order of ~ 10 keV, which suggests an interesting possibility of identifying them as the origin of the peak structure in heavy-ion collisions.

We should mention that there exist peculiar cases in which the states are absolutely stable within our treatment. To see this we note that from (3) the potential height at spatial infinity is $(\pi m_e)^2$ which is greater than $(2m_e)^2$. Then there is a region $2m_e < \omega < \pi m_e$ for absolutely stable states which is, nevertheless, above the e^+e^- threshold. While this may look curious there is a good reason for this to occur in our present framework. It is due to the fact that the small fluctuation feels the tree-level elementary boson mass πm_e, not the true boson mass $2m_e$.[10] Therefore, there is nothing wrong with these states as far as our framework of working with the small fluctuation around the solitonic ground state is concerned.

Let us make a brief comment on the quantum numbers of these states. By examining the transformation properties under the parity and charge conjugation worked out in Ref. 1, it is easy to show that the ψ_+ (ψ_-) mode has $PC=+-$ $(++)$. Since the ψ_+ mode is nothing but the charge density it must be rotationally scalar. This can be explicitly verified by constructing the rotation generators in terms of the Bose variables. Similarly, one can show that the ψ_- mode is rotationally vector.

We now examine the question of how these metastable states could be produced in the environment of heavy-ion collisions. In order to facilitate their long lifetime we would need a molecular-type intermediate nuclear state to form with lifetime longer than $\sim 10^{-19}$ sec.[11] Since there exists experimental evidence for such states in not-so-heavy-ion collisions[12] and this is the situation conventionally assumed in the theory of spontaneous positron creation,[12] it is a reasonable thing to assume.

In such circumstance the adiabatic approximation may apply since the collision time scale is so large compared with the characteristic time scale m_e^{-1} of particle creation.[13] The phase transition from the normal to the supercritical ground states may occur, which is signaled by the spontaneous positron creation associated with the induction of the electron cloud around the molecular-type giant nucleus. In the environment of the collision of two heavy and highly charged nuclei it is conceivable that the metastable states as small vibration modes of the electron cloud are excited with sizable probability. They then decay by tunneling through the potential barrier as we have described before with long enough lifetimes which can mimic the sharp e^+e^- peaks. Because they exist as metastable excitations only in the supercritical ground state the heavy-ion-collision experiments are the only possible place to create them. The experimental feature that they are emitted at rest in the center-of-mass frame[9] is also very natural from our viewpoint.

How about the observed (approximate) Z independence[3,14] of the peak energies? We describe two different mechanisms to realize it. Let us suppose that the molecular nucleus has discrete levels whose sizes are comparable to a few tens of fm. From Fig. 2(a) one can see that the states with long lifetime are at the largest size of the nuclear level, beyond which the state ceases to exist. Therefore, they form a sharp peak over the background coming from the broad states with other nuclear levels as well as the dynamic positrons. Importantly, the energy of the states near the terminal point is nearly independent of Z to very good accuracy, and is about 1.6 MeV [Fig. 2(a)]. The other possibility relies on the dynamical assumption that the adiabatic approximation holds through the whole collision process even when two nuclei start to separate. Then the energy of the metastable state decreases until the terminal point in Fig. 2(b). Again the experimentally measured energy is very insensitive to the total charge of the external source.

We should mention that our preceding argument is not quite complete for the explanation of the momentum balance between e^- and e^+. The strong Coulomb attraction or repulsion can split their energies after the state decays into e^+ and e^-. Two different mechanisms for avoiding this problem are built into our model. First, the states with long lifetime always tunnel through the barrier to large radii, of the order of ~ 2000 fm, so that they feel only weak Coulomb fields. Second, during the destruction period of the supercritical vacuum when the molecular-type nucleus decays the background value of the Bose fields may be disturbed and they may well be off their stationary value. Then one can show that the elementary boson has a lower mass than that in the normal vacuum, thereby remaining stable against decay into e^+ and e^-.

We note that our picture implies the occurrence of spontaneous positron production as a necessary ingredient to create the supercritical ground state. This is important because it has immediate experimental consequences. Namely, this picture can be checked by observing possible correlations between sharp e^+e^- peaks and the excess of the positron yield due to spontaneous positron creation.[15]

Finally, we make several remarks on some important points which nevertheless have not been discussed in depth in this note.

(1) The renormalization factor in (2) does affect the critical value of Z at which the normal and the supercritical ground states degenerate. With larger centrifugal barrier we have a larger value $Z_{cr} \sim 170$ for $R = 1.2 \times (2.5Z)^{1/3}$ fm.[9]

(2) To answer the question, how many metastable states exist, we have to investigate the $\phi_m - q_m$ fluctuation. This sector contains "ρ" and "π" mesons in quark-model language. If they exist as narrow states then there may be room for the light state found in the $\gamma\gamma$ channel.[16]

(3) In view of the nuclear democracy realized in the sine-Gordon theory[6] the fermionic description of the metastable states may not be transparent. In contrast, they fit into the very simple bosonic description as shown here.

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[10]We emphasize that this fact itself does not mean any inconsistency in our present framework. For instance in the usual (1+1)-dimensional soliton field theories the zero mode, which is necessary to recover translational invariance, consistently arises only with this tree-level boson mass.

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[12]W. Greiner, B. Müller, and J. Rafelski, *Quantum Electrodynamics of Strong Fields* (Springer, New York, 1985).

[13]Here we investigate the different (quite opposite) hierarchy of the time scales from the one discussed in Ref. 2.

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[15]We should note, however, that the positrons from the spontaneous mechanism do not necessarily form a sharp peak over the dynamic positron background. It arises only if the energy difference between the two ground states is precisely fixed to a unique value in every collision which may or may not be true in real physical situations. It depends, for example, on the level structure of the molecular-type nuclear complex.

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