Linear sigma model with parity doubling

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Recent lattice-gauge-theory simulations at finite temperatures have suggested that chiral-symmetry restoration at finite temperatures entails parity doubling of the baryon spectrum. We show that a natural extension of the Gell-Mann–Lévy model incorporates this effect. Predictions of this candidate effective model for the hadronic component of high-density and high-temperature nuclear matter are discussed. The model suggests a parametrization of the dependence of the baryon-doublet masses on the quark mass. This parametrization is compared with the recent lattice results.

I. INTRODUCTION

Two recent simulations of QCD at finite temperatures have studied the screening spectrum of plasma hadronic modes and found spectral evidence for chiral-symmetry restoration at finite temperatures. Among other chiral multiplets a finite-mass parity-doubled baryon multiplet was found.[1,2] Although a direct observation of the corresponding real-time plasma modes was not possible in those simulations, the multiplet patterns of the corresponding real-time modes must nonetheless be mirrored in the screening spectrum. Since with the fermion scheme used in these simulations the full continuous chiral symmetry is recovered only in the continuum limit, one must be cautious about translating these first lattice results directly to the continuum. Nonetheless, these results have reopened the old problem of whether it is possible to construct a sensible effective meson-nucleon theory with parity doubling.

Chiral parity doubling does not appear naturally in currently popular composite models of the hadrons. Nonrelativistic quark models fail, since the large constituent-quark mass term breaks chiral symmetry badly. We are not aware of a formulation of the Skyrme model that permits baryons to survive chiral-symmetry restoration. The chiral bag model in its usual formulation links confinement and chiral-symmetry breaking.[3,4] Again, we are not aware of a high-temperature or -density version of the bag model that incorporates chiral-symmetry restoration. Therefore, we revert to an old-fashioned meson field-theoretic approach.[5]

Of course, it is desirable that the model be as nearly consistent as possible with QCD. An obstacle to parity doubling arising from QCD is suggested by the 't Hooft anomaly condition.[6] At zero temperature and chemical potential this condition requires that exact chiral symmetry be realized with massless fermions. However, at finite temperature or chemical potential the constraint is inconclusive.[7]

Therefore, we proceed to formulate a model intended to produce parity doubling only at a finite temperature or chemical potential. Although typical textbook models of parity doubling[8] lead to a vanishing g_A for the nucleon, we find that a suitable but simple extension of the Gell-Mann–Lévy linear sigma model produces parity doubling without apparent strong disagreement with low-temperature phenomenology. As we show below, the model predicts under certain conditions that g_A and the nucleon mass decrease abruptly close to the chiral phase transition. Thus it offers a new mechanism for the quenching of g_A in dense nuclear matter.

II. PARITY-DOUBLING MODEL

A. Single-doublet model

For simplicity we consider SU(2)×SU(2) chiral symmetry. Represent the nucleon N and its would-be parity partner N' by a doubled Dirac spinor

$$\Psi = \begin{pmatrix} \psi_2 \\ \psi_1 \end{pmatrix} .$$

We also define $\overline{\Psi} = (\overline{\psi}_1 \overline{\psi}_2)$. The spinors ψ_1 and ψ_2 are four-component Dirac matrices and two-component flavor spinors. Under an infinitesimal SU(2)× SU(2) transformation let the spinor transform as

$$\delta\Psi = \tfrac{1}{2} i \boldsymbol{\alpha}\cdot\boldsymbol{\tau}\Psi + \tfrac{1}{2} i \boldsymbol{\beta}\cdot\boldsymbol{\tau}\rho_3\gamma_5\Psi ,$$

where $\boldsymbol{\tau}$ are the usual Pauli matrices, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ parametrize the infinitesimal transformation and $(1,\rho_1,\rho_2,\rho_3)$ are Pauli spinors acting on the upper-lower spinor basis. Thus, the upper and lower components of the spinor transform in the opposite way under SU(2) chiral transformations. The invariant sigma model Lagrangian is then

$$\mathcal{L} = \overline{\Psi} i\gamma\cdot\partial\Psi - g_1\overline{\Psi}(\sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\pi}\rho_3\gamma_5)\Psi + g_2\overline{\Psi}(\rho_3\sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\pi}\gamma_5)\Psi - im_0\overline{\Psi}\rho_2\gamma_5\Psi + \mathcal{L}_M(\sigma,\boldsymbol{\pi}) ,$$

where $\mathcal{L}_M$ breaks the symmetry as usual at low temperature and density. The term proportional to m_0 mixing the upper and lower components is new. Since it is chirally invariant without the help of the meson field, it gives rise to nucleon masses when the chiral symmetry is restored. Although such a natural extension of the sigma model has probably been rediscovered at other times in the long history of the model, we were able to find only one previous reference.[9]

By not including quark and gluon fields we are assuming that the high-temperature and/or high-density phase of QCD consists of hadronic modes.[10] Extensions including other baryon doublets are discussed below.

The tree-level mass eigenstates of the theory are easily found in terms of σ_0, the vacuum or thermal expectation value of σ. They are given by

$$\Psi_+ = N\begin{bmatrix} \psi_+ \\ e^{-\delta}\gamma_5\psi_+ \end{bmatrix}, \quad \Psi_- = N\begin{bmatrix} -e^{-\delta}\gamma_5\psi_- \\ \psi_- \end{bmatrix},$$

where the Dirac plane-wave spinors satisfy $\gamma\cdot p\psi_\pm(p) = m_\pm\psi_\pm(p)$, for masses

$$m_\pm = \mp g_2\sigma_0 + [(g_1\sigma_0)^2 + m_0^2]^{1/2} \tag{2.1}$$

and we have defined $\sinh\delta = g_1\sigma_0/m_0$. It is convenient to normalize the plane-wave spinors to $\Psi^\dagger(p)\Psi(p) = 1$ and the component spinors to $\psi_i^\dagger(p)\psi_i(p) = 1$ so that $N^{-2} = 1 + e^{-2\delta}$. The Lagrangian is of course also invariant under the usual discrete symmetries $\mathcal{C}\Psi_\alpha\mathcal{C}^{-1} = \rho_3 C_{\alpha\beta}\overline{\Psi}_\beta$, $\mathcal{T}\Psi(\mathbf{x},t)\mathcal{T}^{-1} = T\Psi(\mathbf{x},t)$, and $\mathcal{P}\Psi(\mathbf{x},t)\mathcal{P}^{-1} = \rho_3 P\Psi(-\mathbf{x},t)$, where C, T, and P are the usual Dirac matrices for charge conjugation, time reversal, and parity. Therefore the intrinsic parity and (with our conventions) the charge conjugation of the mass eigenstate Ψ_- is opposite that of Ψ_+.

B. Characteristic spectrum and couplings

In the broken-symmetry phase the tree-level spectrum consists of a nucleon N of mass m_+ and an opposite parity N' of mass $m_- > m_+$ $(g_2 > 0)$. If chiral symmetry is restored, then the spectrum consists of a chiral multiplet of opposite-parity states of mass m_0.

The axial-vector charges in the basis of mass eigenstates are

$$g_A = \begin{bmatrix} \tanh\delta & -1/\cosh\delta \\ -1/\cosh\delta & -\tanh\delta \end{bmatrix},$$

where the first row and column refer to the nucleon and the second to the N'. Thus the axial-vector charge of the nucleon vanishes only when the chiral symmetry is restored. The model also predicts at the tree level that for the nucleon $g_A < 1$ at any temperature or density. To recover the experimental value of approximately 1.22 it is, of course, well known that one should add more states to the model—in particular, the $\Delta(1232)$ isobar. These adjustments in the axial-vector charge are summarized in the Adler-Weisberger relation, which, with all resonances included, is known to give satisfactory agreement.

The model also predicts that the axial-vector charge of the would-be parity partner of the nucleon is negative. Just as for the nucleon, introducing isospin-$\frac{3}{2}$ states similarly increases the magnitude of this negative axial-vector charge.

The pion-nucleon coupling constants satisfy Goldberger-Treiman relations, of course. At the tree level they are $g_{\pi NN} = g_{ANN}m_+/\sigma_0$, $g_{\pi N'N'} = g_{AN'N'}m_-/\sigma_0$, and $g_{\pi NN'} = g_{ANN'}(m_+ - m_-)/2\sigma_0$.

C. Parameter values

What is the parity partner of the nucleon? Perhaps the most natural assignment is the $N^*(1535)$. However, it could also be an undiscovered broad $\pi N\, S_{11}$ resonance, in analogy with the elusive σ meson.[11] Its contribution to the phase shift might be masked by the presence of the narrower $N^*(1535)$ and $N^*(1650)$.

If it is the $N^*(1535)$, then the four parameters of the model can be fixed in terms of the pion decay constant f_π, the decay width $\Gamma_{N\pi}$ for $N^*(1535) \to N + \pi$, and the masses of the N and N^*. We have

$$\Gamma_{N\pi} = 3q(E_N + m)g_{\pi NN'}^2/4\pi m',$$

where E_N, q, m, and m' are the energy, momentum, mass of the proton, and mass of the N^*. The experimental width is rather small for an S-wave decay: namely, $\Gamma_{N\pi} \approx 70$ MeV. Thus the transition coupling is small: $g_{\pi NN'} \approx 0.70$. Using $m_+ = 939$ MeV, $m_- = 1540$ MeV, and $\sigma_0 = 93$ MeV, at the tree level we get the parameter values

$$\sinh\delta = 5.5, \quad g_1 = 13.0, \quad g_2 = 3.2, \quad m_0 = 270 \text{ MeV}. \tag{2.2}$$

The mass mixing and Goldberger-Treiman relations also give a useful formula for m_0, the nucleon mass upon symmetry restoration:

$$m_0 = \sigma_0 g_{\pi NN'}(m_- + m_+)/(m_- - m_+).$$

For the given N and N^* masses, the ratio in parentheses is a factor of about 4, so the small decay rate requires a rather small symmetric mass.

D. Consequences for high-density nuclear matter

Are these results of any relevance to neutron-star phenomonology? The answer depends on assumptions made about the mechanism and critical density of chiral-symmetry restoration. Prior to the phase transition, some density-dependent shifts in both the nucleon axial-vector charge and mass are expected. These shifts arise both from axial-vector charge and mass renormalization due to the high-density environment, as well as from a possible decrease in σ_0 (a partial restoration of chiral symmetry) presaging the phase transition. It is noteworthy that this partial restoration affects, among other things, the neutrino cooling rate of neutron stars through both the decrease of g_A and of m_+. For example, neutrino emissivity in the Urca process is proportional to the combination $(3g_A^2 + 1)m_+^{*2}$ in the nonrelativistic limit, where m_+^* is an effective mass of the nucleon.[12] The quenching of g_A in nuclei[13] might be partly attributed to this restoring effect of chiral symmetry.

E. Extended models

Will qualitative and numerical predictions of the model survive when more doublets are added? Consider adding more $I=\frac{1}{2}$ doublets. The dimensions of the matrix g_A are increased. Current algebra requires simply that the tree-level axial-vector charge matrix be unitary. Thus if mixing is strong enough, it could change the sign of g_A. Furthermore, if these higher-mass states mix strongly with the nucleon when $\sigma_0 \neq 0$ and weakly when $\sigma_0=0$, then the predicted value of m_0 could be higher. However, if these states are added following the same doublet pattern, strong mixing also leads to large meson decay widths. Typical meson decay widths for the known excited baryons are all of the order 100 MeV or less.[14] For S-wave decays mixing must be only a few percent and for P-wave decays, not more than 20–30%. Thus if we are to associate the added doublets with known baryon resonances, strong mixing appears to be ruled out.

Thus, it appears that if the parity partner of the nucleon is the $N^*(1535)$, then a negative axial-vector charge for the parity partner, a sharp decrease of g_A at the chiral phase transition, and a rather low value for the symmetric phase mass are necessary consequences of our parity-doubling mechanism. The predicted sign of g_A for the odd-parity partner is probably only of interest for comparison with predictions of other models, for example, the bag model,[15] since the odd-parity N^* states decay strongly, making it virtually impossible to determine the weak-decay amplitudes experimentally.

F. Comparison with numerical simulations

The symmetric nucleon screening mass found in the lattice simulations is several times the temperature of the phase transition.[1,2] Since this temperature is expected to be at least 100–200 MeV, the predicted tree-level value (2.2) for the nucleon mass seems rather small in comparison. Collisional processes, which are expected to produce shifts only of the order of the temperature in these masses, cannot account for the discrepancy. However, for the same size lattices, other simulations overestimate the zero-temperature nucleon mass to ρ mass ratio by about 30% (Ref. 16). It is generally believed that the overestimate is a coarse-graining effect. Indeed, the baryon correlation lengths are less than the lattice spacing in present simulations. However, we would need a substantially larger decrease in the high-temperature screening mass with improved lattice simulations to achieve agreement.

III. EXPLICIT CHIRAL-SYMMETRY BREAKING

If we assume a conventional form for the meson self-interaction, we may then parametrize the dependence of the nucleon and meson masses on a symmetry-breaking parameter. Thus in the spontaneously broken phase we may take $\mathcal{L}_M=\frac{1}{2}|\partial_\mu\Phi|^2-V(\Phi)$, where

$$V(\Phi)=m^2|\Phi|^2/2+\lambda|\Phi|^4/4!-\epsilon\sigma\ ,$$

$|\Phi|^2=\omega^2+\boldsymbol{\pi}^2$ and ϵ is a symmetry-breaking parameter. For present purposes, we assume that m^2 is temperature and density independent. Then in the spontaneously broken phase ($m^2<0$), to order ϵ,

$$m_\sigma^2=2|m^2|+3m_\pi^2,\quad m_\pm=m_\pm(0)+d_\pm m_\pi^2\ , \tag{3.1}$$

where $d_\pm=(\mp g_2+g_1\tanh\delta)\sigma_0(0)/2|m^2|$.

Then in the symmetric phase ($m^2>0$) to lowest nontrivial order in ϵ,

$$\begin{aligned} &m_\sigma^2-m^2=3(m_\pi^2-m^2)=\lambda\epsilon^2/2m^4\ ,\\ &(m_--m_+)/2=g_2\epsilon/m^2\ ,\\ &[(m_++m_-)/2]^2=m_0^2+(g_1\epsilon/m^2)^2\ . \end{aligned} \tag{3.2}$$

Assuming the differences between screening masses and Lagrangian masses are small, we may proceed to fit the numerical results[1] to (3.1) and (3.2). Such a fit is amusing, since it tests the approximate validity of chiral perturbation theory in this form and gives a determination of some of the parameters of the doublet model. Using the values quoted in Ref. 1 we obtain at $\beta=5.10$ (spontaneously broken phase) fits

$$m_+=2.13(4)+0.42(7)m_\pi^2\ ,$$

$$m_-=2.37(8)+0.71(17)m_\pi^2$$

in lattice scale units with $\chi^2=1.2$ and 3.2 for one degree of freedom, respectively. The quality of the fit and the values at $m_\pi^2=0$ are essentially the same as in Ref. 1. Using the expression (3.1), we find $g_2/g_1=0.26(17)$ and $\sinh\delta=5.2$. The agreement with (2.2) is fortuitous, of course.

Fitting the expression (3.2) at $\beta=5.25$ in the chirally restored phase, assuming ϵ is proportional to the bare-quark mass m_q, gives

$$(m_--m_+)/2=0.81(30)m_q\ ,$$

$$[(m_++m_-)/2]^2=[1.79(3)]^2+2.1(2)\times10^2m_q^2\ ,$$

with $\chi^2/N_{\rm DF}=1.4/2$ and 4.4/1, respectively. In the latter case the fit is considerably improved over the crude linear fit of Ref. 1 with the same number of parameters, and the value $m_0=1.79(3)$ is higher than the value 1.56(4) quoted there. The slope parameters then give $g_2/g_1=0.056(21)$. It would be interesting to use this new parametrization with more refined lattice measurements.

IV. CONCLUSIONS

We have shown that it is possible to incorporate parity doubling in a linear sigma model without apparent serious contradiction with low-temperature observation. This model may help in understanding the effects of chiral-symmetry restoration at high temperature and density, both in heavy-ion collisions and neutron stars. We have considered two possible assignments for the odd-parity partner to the nucleon: an as yet undiscovered broad S-wave πN resonance and the

$N^*(1535)$. In the latter case mixing of the doublets must be small, the axial-vector charge of this N^* is probably negative, the axial-vector charge of the nucleon falls abruptly with chiral-symmetry restoration, and the masses in the chirally restored phase are probably rather low (couple hundred MeV). Chiral perturbation theory based upon this mechanism offers a parametrization of the dependence of the parity-doublet masses upon quark mass that may be compared with results of numerical simulations. We find satisfactory agreement with numerical results of Ref. 1. Further improvements include elucidating temperature- and density-dependent renormalization effects and extending the model to SU(3)$\times$SU(3). Further progress in treating the phenomenology of the chiral phase transition requires a better understanding of the underlying mechanism of this transition.

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