Analysis of two-body decays of charm mesons using the quark-diagram scheme: Addendum on hairpin diagrams

Ling-Lie Chau
Department of Physics, University of California, Davis, California 95616

Hai-Yang Cheng
Institute of Physics, Academia Sinica, Nankang, Taipei, 11529, Taiwan

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We further develop our previous work [Phys. Rev. D **36**, 137 (1987)]. We incorporate in this addendum quark-diagram amplitudes of the hairpin graphs.

The quark-diagram scheme has provided a useful model-independent way to analyze two-body charm-meson decays $D \rightarrow PP$ (P denotes pseudoscalar mesons) and $D \rightarrow PV$ (V denotes vector mesons), and to make predictions.[1,2] We incorporate in this addendum quark-diagram amplitudes of charmed-meson decay into two vector mesons and also the hairpin graphs.

The hairpin contributions are those diagrams in which a quark-antiquark pair is created from vacuum to form a color- and flavor-singlet final-state meson; they appear in charm decays involving a SU(3) singlet in the final products, e.g., $D^0 \rightarrow \bar{K}^0\phi$, $\bar{K}^0\omega$, and $\bar{K}^0\eta'$. There are four different types of hairpin diagrams, (c_h), (d_h), (e_h), and (f_h), corresponding to the quark diagrams W exchange (c), W annihilation (d), horizontal W loop (e), and vertical W loop (f), with a hairpin quark-antiquark line. In Tables I and II hairpin amplitudes are included for two-body decays of D^+, D^0, D_s^+. Hairpin contributions to $D \rightarrow VP$ have been discussed in Ref. 3 but with coefficients different from ours. For example, if the hairpin amplitude in $D^+ \rightarrow K^+\phi$ is denoted by d_h, then the hairpin contribution to $D^+ \rightarrow K^{*+}\eta_0$ will be $3d_h'$ since all $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ quark-antiquark pairs created from vacuum can form η_0 while ϕ is made of the $s\bar{s}$ pair only. Reference 3 obtains the hairpin amplitudes $\frac{1}{2}d_h$ and d_h', respectively, in $D^+ \rightarrow K^+\phi$ and $K^{*+}\eta_0$. We believe that the relative magnitude of the hairpin diagrams is not properly described in Ref. 3. We also incorporate SU(3)-breaking and final-state-interaction effects in the quark-diagram

TABLE I. Charm-meson decays into a vector boson and a pseudoscalar meson. The experimental data are from the following. (I) Mark III Collaboration, R. Baltrusaitis *et al.*, Phys. Rev. Lett. **55**, 150 (1985); **56**, 2136 (1986); **56**, 2140 (1986); J. Adler *et al.*, Phys. Lett. B **196**, 107 (1987); Phys. Rev. Lett. **60**, 89 (1988). (II) CLEO Collaboration, A. Chen *et al.*, Phys. Rev. Lett. **51**, 634 (1983); C. Bebek *et al.*, *ibid.* **56**, 1893 (1986). (III) ARGUS Collaboration, H. Albrecht *et al.*, Z. Phys. C **33**, 359 (1987); Phys. Lett. B **195**, 102 (1987). (IV) HRS Collaboration, M. Derrick *et al.*, Phys. Rev. Lett. **54**, 2568 (1985). (V) TASSO Collaboration, M. Althoff *et al.*, Phys. Lett. **136B**, 130 (1984); W. Braunschweig *et al.*, Z. Phys. C **35**, 317 (1987). (VI) E691 Collaboration, J. C. Anjos *et al.*, Phys. Rev. Lett. **60**, 897 (1988); **62**, 125 (1989); in *Lepton and Photon Interactions*, proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, West Germany, 1987, edited by R. Rückl and W. Bartel [Nucl. Phys. B, Proc. Suppl. **3** (1987)]; J. P. Cumalat *et al.*, Phys. Lett. B **210**, 253 (1988). (VII) LEBC-EHS Collaboration, M. Aguilar-Benitez *et al.*, Z. Phys. C **36**, 559 (1987).

	Experimental branching ratio B_r (%)	Quark-mixing factor ($V_{us}V_{cs}^* \simeq -V_{ud}V_{cd}^* \simeq s_1c_1$ used)	Amplitudes with SU(3) symmetry	Amplitudes with SU(3) breaking and final-state interactions
		(a) D^+ decays		
$\bar{K}^{*0}\pi^+$	$5.9\pm1.9\pm2.5^{(\mathrm{I})}$	$(c_1)^2\times$	$(a'+b')$	$\rightarrow(a'+b')e^{i\delta_{3/2}^{\bar{K}^*\pi}}$
$\rho^+\bar{K}^0$	$6.9\pm0.8\pm2.3^{(\mathrm{I})}$	$(c_1)^2\times$	$(a+b)$	$\rightarrow(a+b)e^{i\delta_{3/2}^{\rho\bar{K}}}$
$\phi\pi^+$	$0.77\pm0.22\pm0.11^{(\mathrm{I})}$ $0.68\pm0.07\pm0.12^{(\mathrm{VI})}$	$(s_1c_1)\times$	$(b'+\tilde{d}_h)$	$\rightarrow(b'+\tilde{d}_h+\delta e_h)e^{i\delta^{\phi\pi}}$
$\omega\pi^+$		$-(1/\sqrt{2})(s_1c_1)\times$	$(a'+b'+d+d'+2d_h')$	$\rightarrow[a'+b'+d+d'+(\delta e+\delta e')+2d_h'+2\delta e_h]e^{i\delta^{\omega\pi}}$
$\bar{K}^{*0}K^+$	$0.44\pm0.20\pm0.11^{(\mathrm{I})}$ $0.56\pm0.08\pm0.10^{(\mathrm{VI})}$	$(s_1c_1)\times$	$(a'-\tilde{d})$	$\rightarrow(a'-\tilde{d}+\delta e)e^{i\delta_1^{\bar{K}^*K}}$
$K^{*+}\bar{K}^0$		$(s_1c_1)\times$	$(a-\tilde{d}')$	$\rightarrow(a-\tilde{d}'+\delta e')e^{i\delta_1^{\bar{K}^*K}}$
$\rho^+\pi^0$		$(1/\sqrt{2})(s_1c_1)\times$	$(a+b-d+d')$	$\rightarrow[(a+b)(e^{i\delta_0^{\rho\pi}}\pm e^{i\delta_2^{\rho\pi}})/2+(a'+b')(e^{i\delta_0^{\rho\pi}}\mp e^{i\delta_2^{\rho\pi}})/2$
$\rho^0\pi^+$	$0.07\pm0.05\pm0.01^{(\mathrm{VI})}$	$(1/\sqrt{2})(s_1c_1)\times$	$(a'+b'+d-d')$	$\mp(d-d')e^{i\delta_1^{\rho\pi}}\pm2(\delta e-\delta e')e^{i\delta_1^{\rho\pi}}]$
$\rho^+\eta_8$		$-(1/\sqrt{6})(s_1c_1)\times$	$(a+3b+d+d')$	$\rightarrow[a+3b+d+d'+(\delta e+\delta e')$ $+2d_h'-2\tilde{d}_h'+2\delta e_h'-2\delta\tilde{e}_h']e^{i\delta_1^{\rho\eta_8}}$
$\rho^+\eta_0$		$-(1/\sqrt{3})(s_1c_1)\times$	$(a+d+d'+3d_h')$	$\rightarrow[a+d+d'+(\delta e+\delta e')+2d_h'+\tilde{d}_h'+3\delta e_h']e^{i\delta_1^{\rho\eta_0}}$

TABLE I. (*Continued*).

	Experimental branching ratio B_r (%)	Quark-mixing factor ($V_{us}V_{cs}^{*} \simeq -V_{ud}V_{cd}^{*} \simeq s_1 c_1$ used)	Amplitudes with SU(3) symmetry	Amplitudes with SU(3) breaking and final-state interactions
		(b) D^0 decays		
$\phi\bar{K}^0$	$0.99\pm0.24\pm0.14^{(\mathrm{III})}$ $0.86^{+0.50+0.31}_{-0.41-0.17}{}^{(\mathrm{I})}$ $0.91\pm0.33\pm0.13^{(\mathrm{II})}$	$(c_1)^2\times$	$(c'+c_h)$	$\rightarrow(\tilde{c}'+\tilde{c}_h)e^{i\delta^{\phi\bar{K}}}$
$\omega\bar{K}^0$	$3.2\pm1.3\pm0.8^{(\mathrm{I})}$	$(1/\sqrt{2})(c_1)^2\times$	$(b+c+2c_h)$	$\rightarrow(b+c+2c_h)e^{i\delta^{\omega\bar{K}}}$
$K^{*-}\pi^+$	$5.2\pm0.3\pm1.5^{(\mathrm{I})}$	$(c_1)^2\times$	$(a'+c')$	$\rightarrow[(a'+c')-\frac{1}{3}(a'+b')(1-e^{i\Delta'_{K^*\pi}})]e^{i\delta_{1/2}^{\bar{K}^*\pi}}$
$\bar{K}^{*0}\pi^0$	$2.6\pm0.3\pm0.7^{(\mathrm{I})}$	$(1/\sqrt{2})(c_1)^2\times$	$(b'-c')$	$\rightarrow[(b'-c')-\frac{2}{3}(a'+b')(1-e^{i\Delta_{\bar{K}^*\pi}})]e^{i\delta_{1/2}^{\bar{K}^*\pi}}$
$\bar{K}^{*0}\eta_8$		$(1/\sqrt{6})(c_1)^2\times$	$(b'+c'-2c)$	$\rightarrow(b'+c'-2\tilde{c}+2c_h'-2\tilde{c}_h')e^{i\delta^{\bar{K}^*\eta_8}}$
$\bar{K}^{*0}\eta_0$		$(1/\sqrt{3})(c_1)^2\times$	$(b'+c'+c+3c_h')$	$\rightarrow(b'+c'+\tilde{c}+2c_h+\tilde{c}_h')e^{i\delta^{\bar{K}^*\eta_0}}$
ρ^+K^-	$10.8\pm0.4\pm1.7^{(\mathrm{I})}$	$(c_1)^2\times$	$(a+c)$	$\rightarrow[(a+c)-\frac{1}{3}(a+b)(1-e^{i\Delta_{\rho\bar{K}}})]e^{i\delta_{1/2}^{\rho\bar{K}}}$
$\rho^0\bar{K}^0$	$0.75\pm0.09\pm0.47^{(\mathrm{I})}$	$(1/\sqrt{2})(c_1)^2\times$	$(b-c)$	$\rightarrow[(b-c)-\frac{2}{3}(a+b)(1-e^{i\Delta_{\rho\bar{K}}})]e^{i\delta_{1/2}^{\rho\bar{K}}}$
$\phi\pi^0$		$(1/\sqrt{2})(s_1c_1)\times$	$(b'-c_h)$	$\rightarrow(b'-\tilde{c}_h+\delta e_h)e^{i\delta^{\phi\pi}}$
$\phi\eta_8$		$(1/\sqrt{6})(s_1c_1)\times$	$(b'-2\tilde{c}'-2\tilde{c})$	$\rightarrow[b'-2\tilde{c}'-2\tilde{c}-2(\delta f+\delta f')$
				$+2c_h-2\tilde{c}_h+2\delta e_h-2\delta\tilde{e}_h]e^{i\delta^{\phi\eta_8}}$
$\phi\eta_0$		$(1/\sqrt{3})(s_1c_1)\times$	$(b'+\tilde{c}'+\tilde{c}+2c_h')$	$\rightarrow[b'+\tilde{c}'+\tilde{c}+(\delta f+\delta f')$
				$+2\tilde{c}_h'+\delta c_h+2\delta e_h+2\delta f_h+2\delta f_h']e^{i\delta^{\phi\eta_0}}$
$\omega\pi^0$		$\frac{1}{2}(s_1c_1)\times$	$(b-b'+c+c'-2c_h)$	$\rightarrow(b-b'+c+c'+\delta e+\delta e'-2c_h+2\delta e_h)e^{i\delta^{\omega\pi}}$
$\omega\eta_8$		$(1/\sqrt{12})(s_1c_1)\times$	$(-3b-b'-c-c')$	$\rightarrow[-3b-b'-c-c'+\delta e+\delta e'+2\delta f+2\delta f'$
				$-2c_h+2\tilde{c}_h+2\delta\tilde{e}_h-2\delta\tilde{e}_h]e^{i\delta^{\omega\eta_8}}$
$\omega\eta_0$		$(1/\sqrt{6})(s_1c_1)\times$	$(b+c+c'+3c_h)$	$\rightarrow[b+c+c'+(\delta e+\delta e'+2\delta f+2\delta f')+2c_h+\tilde{c}_h$
				$+3\delta e_h+3\delta e_h'+6\delta f_h+6\delta f_h']e^{i\delta^{\omega\eta_0}}$
$\bar{K}^{*0}K^0$		$-(s_1c_1)\times$	$(c-c')$	$\rightarrow\{(c-c')-\delta f-\delta f'$
				$+[(a'+c')+(\tilde{c}-\tilde{c}')-\delta e](1-e^{i\Delta_{\bar{K}^*K}})/2\}e^{i\delta_0^{\bar{K}^*K}}$
$K^{*0}\bar{K}^0$	$<0.55^{(\mathrm{I})}$	Same as $\bar{K}^{*0}K^0$ but with primed and unprimed amplitudes interchanged		
$K^{*-}K^+$	$0.8\pm0.5^{(\mathrm{I})}$	$(s_1c_1)\times$	$(a'+c')$	$\rightarrow\{(a'+c')+\delta e+\delta f+\delta f'$
				$-[(a'+c')+(\tilde{c}-c')-\delta e](1-e^{i\Delta_{\bar{K}^*K}})/2\}e^{i\delta_0^{\bar{K}^*K}}$
$K^{*+}K^-$		Same as $K^{*-}K^+$ but with primed and unprimed amplitudes interchanged		
$\rho^-\pi^+$		$-(s_1c_1)\times$	$(a'+c')$	$\rightarrow[-a'(2e^{i\delta_0^{\rho\eta}}\pm3e^{i\delta_1^{\rho\pi}}+e^{i\delta_2^{\rho\eta}})/3$
$\rho^+\pi^-$		$-(s_1c_1)\times$	$(a+c)$	$-a(2e^{i\delta_0^{\rho\eta}}\mp3e^{i\delta_1^{\rho\pi}}+e^{i\delta_2^{\rho\eta}})/3$
				$-(b+b')(e^{i\delta_0^{\rho\pi}}-e^{i\delta_2^{\rho\pi}})/3$
				$-(c'-\delta e)(e^{i\delta_0^{\rho\pi}}\pm3e^{i\delta_1^{\rho\pi}}+2e^{i\delta_2^{\rho\pi}})/3$
				$-(c-\delta e')(e^{i\delta_0^{\rho\pi}}\mp3e^{i\delta_1^{\rho\pi}}+2e^{i\delta_2^{\rho\pi}})/3$
				$-(\delta f+\delta f')e^{i\delta_0^{\rho\pi}}]$
$\rho^0\pi^0$		$\frac{1}{2}(s_1c_1)\times$	$(b+b'-c-c')$	$\rightarrow[(a+a')(e^{i\delta_0^{\rho\pi}}-e^{i\delta_2^{\rho\pi}})/3$
				$+(b+b')(e^{i\delta_0^{\rho\pi}}+2e^{i\delta_2^{\rho\pi}})/6$
				$+(c+c')(e^{i\delta_0^{\rho\pi}}-4e^{i\delta_2^{\rho\pi}})/6$
				$-(\delta e+\delta e')(e^{i\delta_0^{\rho\pi}}-4e^{i\delta_2^{\rho\pi}})/6$
				$+(\delta f+\delta f')e^{i\delta_0^{\rho\pi}}]$
$\rho^0\eta_8$		$(1/\sqrt{12})(s_1c_1)\times$	$(-3b+b'+c+c')$	$\rightarrow(-3b+b'+c+c'+\delta e+\delta e'$
				$+2c_h'-2\tilde{c}_h')e^{i\delta^{\rho\eta_8}}$
$\rho^0\eta_0$		$(1/\sqrt{6})(s_1c_1)\times$	$(b'+c+c'+3c_h')$	$\rightarrow(b'+c+c'+\delta e+\delta e'+2c_h'$
				$+\tilde{c}_h'+3\delta e_h')e^{i\delta^{\rho\eta_8}}$

TABLE I. (*Continued*).

	Experimental branching ratio B_r (%)	Quark-mixing factor ($V_{us}V_{cs}^{*} \simeq -V_{ud}V_{cd}^{*} \simeq s_1c_1$ used)	Amplitudes with SU(3) symmetry	Amplitudes with SU(3) breaking and final-state interactions
		(c) D_s^+ decays		
$\phi\pi^+$	$3.3\pm1.1^{(IV)}$; $4.4\pm1.1^{(II)}$ $3.3\pm1.6\pm0.4^{(V)}$ $3.2\pm0.7\pm0.5^{(III)}$	$(c_1)^2\times$	$(a'+d_h)$	$\rightarrow(a'+\tilde{d}_h)e^{i\delta^{\phi\pi}}$
$\omega\pi^+$		$(1/\sqrt{2})(c_1)^2\times$	$(d+d'+2d_h)$	$\rightarrow(d+d'+2d_h)e^{i\delta_1^{\omega\pi}}$
$K^{*+}\bar{K}^0$		$(c_1)^2\times$	$(b+d')$	$\rightarrow(b+\tilde{d}')e^{i\delta_1^{K^*K}}$
$\bar{K}^{*0}K^+$	$3.6\pm2.1^{(I)}$; $5.0\pm1.3^{(III)}$ $(0.87\pm0.13)B(D_s\rightarrow\pi\phi^+)^{(VI)}$	$(c_1)^2\times$	$(b'+d)$	$\rightarrow(b'+\tilde{d})e^{i\delta_1^{K^*K}}$
$\rho^+\pi^0$		$(1/\sqrt{2})(c_1)^2\times$	$(d-d')$	$\rightarrow(d-d')e^{i\delta_1^{\rho\pi}}$
$\rho^0\pi^+$	$\leq 0.22B(D_s\rightarrow\pi\phi^+)^{(III)}$ $\leq 0.08B(D_s\rightarrow\pi\phi^+)^{(VI)}$	$(1/\sqrt{2})(c_1)^2\times$	$(d'-d)$	$\rightarrow(d'-d)e^{i\delta_1^{\rho\pi}}$
$\rho^+\eta_8$		$(1/\sqrt{6})(c_1)^2\times$	$(-2a+d+d')$	$\rightarrow(-2a+d+d'+2d_h'-2\tilde{d}_h')e^{i\delta^{\rho\eta_8}}$
$\rho^+\eta_0$		$(1/\sqrt{3})(c_1)^2\times$	$(a+d+d'+3d_h')$	$\rightarrow(a+d+d'+2d_h'+\tilde{d}_h')e^{i\delta^{\rho\eta_0}}$
ϕK^+		$(s_1c_1)\times$	$(a'+b'+\tilde{d}+d_h)$	$\rightarrow(a'+b'+\tilde{d}+\delta e+\tilde{d}_h+\delta e_h)e^{i\delta^{\phi K}}$
ωK^+		$-(1/\sqrt{2})(s_1c_1)\times$	$(b'-d'-2d_h)$	$\rightarrow(b'-d'-\delta e'-2d_h-2\delta e_h)e^{i\delta^{\omega K}}$
$K^{*0}\pi^+$		$-(s_1c_1)\times$	$(a'-d)$	$\rightarrow[(a'-d-\delta e)-(2a'+b-d-\delta e)$ $\times\frac{1}{3}(1-e^{-i\Delta_{K^*\pi}})]e^{\delta_{3/2}^{K^*\pi}}$
$K^{*+}\pi^0$		$-(1/\sqrt{2})(s_1c_1)\times$	$(b-d)$	$\rightarrow[(b-d+\delta e)-(2a'+b-d-\delta e)$ $\times\frac{2}{3}(1-e^{-i\Delta_{K^*\pi}})]e^{\delta_{3/2}^{K^*\pi}}$
ρ^+K^0		$-(s_1c_1)\times$	$(a-d')$	$\rightarrow[(a-d')-(2a+b'-d'-\delta e')\frac{1}{3}(1-e^{-i\Delta_{\rho K}})]e^{i\delta_{3/2}^{\rho K}}$
ρ^0K^+		$(1/\sqrt{2})(s_1c_1)\times$	$(b'+d')$	$\rightarrow[(b'+d')-(2a+b'-d'-\delta e')\frac{2}{3}(1-e^{-i\Delta_{\rho K}})]e^{i\delta_{3/2}^{\rho K}}$
$K^{*+}\eta_8$		$(1/\sqrt{6})(s_1c_1)\times$	$(-2a+3b+d-2\tilde{d}')$	$\rightarrow(-2a-3b+d+\tilde{d}'+\delta e-2\delta e'$ $+2d_h'-2\tilde{d}_h'+2\delta e_h'-2\delta\tilde{e}_h')e^{i\delta^{K^*\eta_8}}$
$K^{*+}\eta_0$		$(1/\sqrt{3})(s_1c_1)\times$	$(a+d+\tilde{d}'+3d_h')$	$\rightarrow[a+d+\tilde{d}'+(\delta e-\delta e')+2d_h'+\tilde{d}_h'+3\delta e_h']e^{i\delta^{K^*\eta_8}}$

scheme so that $D^+\rightarrow K^{*+}\eta_8$, for instance, receives hairpin contributions proportional to $(2d_h'-2\tilde{d}_h')$; and the hairpin diagram contribution to $D^+\rightarrow K^+\phi$ changes from d_h to $\tilde{d}_h$, and to $D^+\rightarrow K^{*+}\eta_0$ changes from $3d_h'$ to $2d_h'+\tilde{d}_h'$. These effects were not considered in Ref. 3.

The hairpin diagram in the contributions for the vector bosons $D\rightarrow VP$ decays are expected to be suppressed owing to the Okubu-Zweig-Iizuka rule. However, this rule does not apply to $D\rightarrow PP$. In the $1/N_c$ (N_c being the number of colors) approach,[4] the hairpin diagram in $D\rightarrow PP, VP$ is suppressed by a factor of N_c^2 and N_c^3, respectively, relative to the leading factorization amplitudes. Since $N_c=3$ in the real world, *a priori* the hairpin amplitude in $D\rightarrow PP$ decays is not necessarily suppressed.[5] Therefore, it is important to perform a model-independent analysis to see if there is any experimental evidence for the existence of hairpin diagrams. Here we provide a framework for future detailed data analysis.

In summary, we have further developed the model-independent quark-diagram scheme to include the hairpin diagrams. They can be useful in analyzing future data of $D\rightarrow VV$, and in finding out the possible contributions of hairpin diagrams.

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[4] A. J. Buras, J.-M. Gérard, and R. Rückl, Nucl. Phys. **B268**, 16 (1986).

[5] For a recent thorough analysis, see L. L. Chau, and H. Y. Cheng, Phys. Lett. (to be published).

TABLE II. Charm-meson decays to two pseudoscalars.

	Experimental branching ratio B_r (%)	Quark-mixing factor ($V_{us}V_{cs}^* \simeq -V_{ud}V_{cd}^*$ $\simeq s_1c_1$ used)	Amplitudes with SU(3) symmetry	Amplitudes with SU(3) breaking and final-state interactions
		(a) D^+ decays		
$\bar{K}^0\pi^+$	$3.2\pm0.5\pm0.2^{(\mathrm{I})}$	$(c_1)^2\times$	$(a+b)$	$\rightarrow(a+b)e^{i\delta_{3/2}^{\bar{K}\pi}}$
$\bar{K}^0K^+$	$1.01\pm0.32\pm0.18^{(\mathrm{I})}$	$(s_1c_1)\times$	$(a-d)$	$\rightarrow(a-\tilde{d}+\delta e)e^{i\delta_1^{KK}}$
$\pi^0\pi^+$	$\leq0.48^{(\mathrm{I})}$	$(1/\sqrt{2})(s_1c_1)\times$	$(a+b)$	$\rightarrow(a+b)e^{i\delta_2^{\pi\pi}}$
$\eta_8\pi^+$		$-(1/\sqrt{6})(s_1c_1)\times$	$[(a+b)+2(b+d)]$	$\rightarrow[a+b+2(b+d)+2\delta e$ $+2d_h-2\tilde{d}_h+2\delta e_h-2\delta\tilde{e}_h]e^{i\delta^{\eta\pi}}$
$\eta_0\pi^+$		$-(1/\sqrt{3})(s_1c_1)\times$	$(a+2d+3d_h)$	$\rightarrow[(a+2d)+2\delta e+2d_h+\tilde{d}_h+3\delta e_h]e^{i\delta^{\eta\pi}}$
		(b) D^0 decays		
$K^-\pi^+$	$4.2\pm0.4\pm0.4^{(\mathrm{I})}$ $4.0^{+2.1}_{-1.0}\pm0.2^{(\mathrm{VII})}$ $4.5\pm0.8\pm0.5^{(\mathrm{IV})}$	$(c_1)^2\times$	$(a+c)$	$\rightarrow[(a+c)-(a+b)\frac{1}{3}(1-e^{i\Delta_{K\pi}})]e^{i\delta_{1/2}^{K\pi}}$
$\bar{K}^0\pi^0$	$1.9\pm0.4\pm0.2^{(\mathrm{I})}$	$(1/\sqrt{2})(c_1)^2\times$	$(b-c)$	$\rightarrow[(b-c)-(a+b)\frac{2}{3}(1-e^{i\Delta_{K\pi}})]e^{i\delta_{1/2}^{K\pi}}$
$\bar{K}^0\eta_8$		$(1/\sqrt{6})(c_1)^2\times$	$(b-c)$	$\rightarrow(b+c-2\tilde{c}+2c_h-2\tilde{c}_h)e^{i\delta^{K\eta_8}}$
$\bar{K}^0\eta_0$		$(1/\sqrt{3})(c_1)^2\times$	$(b+2c+3c_h)$	$\rightarrow(b+c+\tilde{c}+2c_h+\tilde{c}_h)e^{i\delta^{K\eta_0}}$
$\bar{K}^0\eta$	$1.5\pm0.7\pm0.2^{(\mathrm{I})}$	$A(D^0\rightarrow\bar{K}^0\eta_8)\cos\theta+A(D^0\rightarrow\bar{K}^0\eta_0)\sin\theta,\ \theta\approx20°$		
$K^0\bar{K}^0$	$0.20\pm0.16^{(\mathrm{VI})}$	$(s_1c_1)\times$	(0)	$\rightarrow[(c-\tilde{c}-2\delta f)$ $+(a+\tilde{c}-\delta e)\frac{1}{2}(1-e^{i\Delta_{K\bar{K}}})]e^{i\delta_0^{K\bar{K}}}$
K^-K^+	$0.51\pm0.09\pm0.06^{(\mathrm{I})}$ $0.33\pm0.01^{(\mathrm{III})}$	$(s_1c_1)\times$	$(a+c)$	$\rightarrow[(a+c)+(\delta e+2\delta f)$ $-(a+\tilde{c}-\delta e)\frac{1}{2}(1-e^{i\Delta_{K\bar{K}}})]e^{i\delta_0^{K\bar{K}}}$
$\pi^+\pi^-$	$0.14\pm0.04\pm0.03^{(\mathrm{I})}$ $0.5^{+1.2}_{-0.2}\pm0.04^{(\mathrm{VII})}$	$-(s_1c_1)\times$	$(a+c)$	$\rightarrow[(a+c)+(\delta e+2\delta f)$ $-(a+b)\frac{1}{3}(1-e^{i\Delta_{\pi\pi}})]e^{i\delta_0^{\pi\pi}}$
$\pi^0\pi^0$	$<0.3^{(\mathrm{I})}$	$(\sqrt{2})\frac{1}{2}(s_1c_1)\times$	$(b-c)$	$\rightarrow[(b-c)+(\delta e+2\delta f)$ $-(a+b)\frac{2}{3}(1-e^{i\Delta_{\pi\pi}})]e^{i\delta_0^{\pi\pi}}$
$\eta_8\eta_8$		$-(\sqrt{2})\frac{1}{2}(s_1c_1)\times$	$(b-c)$	$\rightarrow[(b-c)+2(-\delta c/6-\delta e/6-\delta f)]e^{i\delta^{\eta_8\eta_8}}$
$\pi^0\eta_8$		$-(1/\sqrt{3})(s_1c_1)\times$	$(b-c)$	$\rightarrow(b-c-\delta e-2c_h+2\tilde{c}_h$ $-2\delta e_h+2\delta\tilde{e}_h)e^{i\delta^{\pi\eta_0}}$
$\pi^0\eta_0$		$(1/\sqrt{6})(s_1c_1)\times$	$(b+2c-3c_h)$	$\rightarrow(b+2c+2\delta e-2c_h-\tilde{c}_h+3\delta e_h)e^{i\delta^{\pi\eta_0}}$
$\eta_8\eta_0$		$(1/\sqrt{2})(s_1c_1)\times$	$(b+2c)$	$\rightarrow(b+2c+2\delta e+2c_h-2\tilde{c}_h$ $+2\delta e_h-2\delta\tilde{e}_h)e^{i\delta^{\eta_0\eta_8}}$
$\eta_0\eta_0$		$(\sqrt{2})(1/\sqrt{3})(s_1c_1)\times$	$(3c_h)$	$\rightarrow(\delta c+\delta e+3\delta f+2c_h+\tilde{c}_h$ $+3\delta e_h+3\delta f_h)e^{i\delta^{\eta_0\eta_0}}$
		(c) D_s^+ decays		
$\bar{K}^0K^+$	$3.7\pm2.0^{(\mathrm{I})}$	$(c_1)^2\times$	$(b+d)$	$\rightarrow(b+\tilde{d})e^{i\delta_1^{K\bar{K}}}$
$\pi^0\pi^+$		$(c_1)^2\times$	(0)	$\rightarrow(0)$
$\eta_8\pi^+$		$-(\sqrt{2}/\sqrt{3})(c_1)^2\times$	$(a-d)$	$\rightarrow(a-d-2d_h+2\tilde{d}_h)e^{i\delta^{\pi\eta_8}}$
$\eta_0\pi^+$		$(1/\sqrt{3})(c_1)^2\times$	$(a+2d+3d_h)$	$\rightarrow(a+2d+2d_h+\tilde{d}_h)e^{i\delta^{\pi\eta_0}}$
$\eta\pi^+$	$(2.5\pm0.8\pm0.8)$ $\times B(D_s^+\rightarrow\phi\pi^+)^{(\mathrm{I})}$	$A(D_s^+\rightarrow\eta_8\pi^+)\cos\theta+A(D_s^+\rightarrow\eta_0\pi^+)\sin\theta,\ \theta\approx20°$		
$\eta'\pi^+$		$-A(D_s^+\rightarrow\eta_8\pi^+)\sin\theta+A(D_s^+\rightarrow\eta_0\pi^+)\cos\theta$		
$K^0\pi^+$		$-(s_1c_1)\times$	$(a-d)$	$\rightarrow[(a-d-\delta e)$ $-(2a+b-d-\delta e)\frac{1}{3}(1-e^{-i\Delta_{K\pi}})]e^{i\delta_{3/2}^{K\pi}}$
$K^+\pi^0$		$(1/\sqrt{2})(s_1c_1)\times$	$(b+d)$	$\rightarrow[(b+d+\delta e)$ $-(2a+b-d-\delta e)\frac{2}{3}(1-e^{-i\Delta_{K\pi}})]e^{i\delta_{3/2}^{K\pi}}$
$K^+\eta_8$		$-(1/\sqrt{6})(s_1c_1)\times$	$[2(a+b)+(b+d)]$	$\rightarrow[2(a+b)+b-d+2\tilde{d}+\delta e]e^{i\delta^{K\eta_8}}$
$K^+\eta_0$		$(1/\sqrt{3})(s_1c_1)\times$	$(a+2d+3d_h)$	$\rightarrow(a+d+\tilde{d}+2\delta e+2d_h+\tilde{d}_h+3\delta e_h)e^{i\delta^{K\eta_0}}$