Two-γ decay widths of glueballs

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Using a nonrelativistic gluon bound-state model, we compute $\Gamma(G\rightarrow\gamma\gamma)$, where G is a pseudoscalar, tensor, or scalar digluon, starting from the amplitudes of the process $\gamma\gamma\rightarrow g^*g^*$ at threshold, the g^*'s being massive constituent gluons. Those amplitudes are obtained, at lowest order in perturbative QCD, by deriving them from a QED calculation performed many years ago by Constantini, de Tollis, and Pistoni. The unknown parameter (the digluon wave function, or its first or second derivative, at the origin) is determined by using measured values of $\Gamma(J/\psi\rightarrow G\gamma)$. Our predictions are compared, for various glueball candidates, with present experimental limits.

I. INTRODUCTION

The existence of bound states consisting only of gluons (two or more), called gluonia or glueballs, is one of the crucial tests of quantum chromodynamics. Various theoretical models have been used to predict the spectra of such systems: the MIT bag model, potential models, lattice gauge theory, QCD sum rules, and the flux-tube model;[1] most of them agree in predicting a number of gluonium states (mostly two-gluon bound states) in the range of 1–2 GeV.

Experimentally, those states have been intensively searched for over the last years, mostly through reactions that are supposed to involve a gluon-rich environment: radiative J/ψ decay and diffractive hadron-hadron scattering (involving double-Pomeron exchange).[2,3] Two serious candidates, the ι or $\eta(1440)$ and the θ or $f_2(1720)$, respectively, a $J^{PC}=0^{-+}$ and a 2^{++} state, appear to have emerged from various experiments; others, such as the ξ or $X(2220)$, are subject to doubt. Production of glueballs in photon-photon collisions has been looked for, but so far only upper limits have been obtained.[4]

In this work, using a nonrelativistic gluon bound-state model, we are trying to give numerical predictions for $\gamma\gamma\rightarrow$gluonium, more precisely for the 2γ decay width of various glueball candidates. An experimental check of those predictions would be all the more interesting as a rather complex type of QCD diagram [i.e., the quark box shown in Fig. 1(b) below] is involved in the reaction mechanism used in our calculations.

In Sec. II we shall explain the model we use. Section III shows how that model is applied to the reactions $\gamma\gamma\leftrightarrow\eta(1440)$, $f_2(1720)$, and $X(2220)$. In Sec. IV our results are normalized, for the various glueball candidates considered, by eliminating the only free parameter of the model, i.e., the radial wave function (or its first or second derivative) at the origin, with the help of experimental data on radiative J/ψ decay. In Sec. V the numerical values thus obtained are discussed and compared with present experimental limits.

II. A MODEL FOR GLUONIUM PRODUCTION IN $\gamma\gamma$ COLLISIONS

Limiting ourselves to gluonium states G made up of two gluons, we shall use a nonrelativistic gluon bound-state model, completely analogous to the nonrelativistic quark bound-state model, as described for instance in Refs. 5 and 6, to compute the production or decay of ordinary mesons. In the corresponding gluonium model, the Bethe-Salpeter wave function of the two-gluon system is reduced to its nonrelativistic form. If one goes to the rest frame of that system, the wave function is assumed to be sharply damped when the components of the relative four-momentum between the gluons become large on the scale of the glueball mass M; i.e., the mean values of those components are negligible compared to M. In addition to total spin (J) and parity (P) of the two-gluon state, one defines its orbital angular momentum (L) [connected with P through the relation $P=(-1)^L$], its intrinsic spin (S), as well as the component (Λ) of J on the production or decay axis of G in a given reaction.

A simple relation [analogous to formula (2.12) of Ref. 6] then connects the helicity amplitudes of any production or decay process $ab\leftrightarrow G$ with those of the corresponding scattering process $ab\leftrightarrow g^*g^*$ (where we use the symbol g^* for gluons with a mass of about $M/2$):

$$\mathcal{M}^{\lambda_a\lambda_b}_{ab\leftrightarrow G} =(-i)^L\left[\frac{2}{M}\right]^{L+1/2}\frac{(2L+1)!!}{L!}\left[\left[\frac{d}{dr}\right]^L R_L(r)\right]_{r=0} \times\lim_{\beta\rightarrow 0}\frac{1}{(\beta)^L}\int\frac{d(\cos\theta)}{2}\sum_{\lambda\lambda'}\chi^{LSJ\Lambda}_{\lambda\lambda'}(\theta)\mathcal{M}^{\lambda_a\lambda_b,\lambda\lambda'}_{ab\leftrightarrow g^*g^*}(\theta)\ , \quad (1)$$

where $\lambda_a,\lambda_b,\lambda,\lambda'$ are, respectively, the helicities of particles a,b and of the gluons (by angular momentum conservation, $\lambda_a-\lambda_b=\Lambda$); $R_L(r)$ is the radial part of the wave function of the two-gluon system in configuration space [normalized so that $\int R^2(r)r^2dr=1$]; θ is the scattering angle in the c.m. frame and β is the gluon velocity in that frame; finally the function $\chi^{LSJ\Lambda}_{\lambda\lambda'}(\theta)$ corresponds to the projection of the λ,λ' helicity state of the two spin-1 gluons onto the L,S,J,Λ state of the G, and is given by

$$\chi^{LSJ\Lambda}_{\lambda\lambda'}(\theta)=\left[\frac{2L+1}{4\pi}\right]^{1/2}\langle 11\lambda-\lambda'|11S\bar{\Lambda}\rangle \times\langle LS0\bar{\Lambda}|LSJ\bar{\Lambda}\rangle d^{J}_{\Lambda\bar{\Lambda}}(\theta)\ , \tag{2}$$

where the first factor on the right-hand side is a normalization factor of the angular part of the wave function; the second and third factors are Clebsch-Gordan coefficients in the most current notation (with $\bar{\Lambda}=\lambda-\lambda'$), while the last factor is a Wigner rotation matrix element.[7]

In particular for $L=0$ one gets, from (1),

$$\mathcal{M}^{\lambda_a\lambda_b}_{ab\leftrightarrow G}=\left[\frac{2}{M}\right]^{1/2}R_0(0)\lim_{\beta\to 0}\int\frac{d(\cos\theta)}{2} \times\sum_{\lambda\lambda'}\chi^{0JJ\Lambda}_{\lambda\lambda'}(\theta)\mathcal{M}^{\lambda_a\lambda_b,\lambda\lambda'}_{ab\leftrightarrow g^*g^*}(\theta)\ , \tag{3}$$

where the projection function is reduced to

$$\chi^{0JJ\Lambda}_{\lambda\lambda'}=\frac{1}{\sqrt{4\pi}}\langle 11\lambda-\lambda'|11J\bar{\Lambda}\rangle d^{J}_{\Lambda\bar{\Lambda}}(\theta)\ . \tag{4}$$

For $L=1$ formula (1) becomes

$$\mathcal{M}^{\lambda_a\lambda_b}_{ab\leftrightarrow G}=-3i\left[\frac{2}{M}\right]^{3/2}R'_1(0) \times\lim_{\beta\to 0}\frac{1}{\beta}\int\frac{d(\cos\theta)}{2} \times\sum_{\lambda\lambda'}\chi^{1SJ\Lambda}_{\lambda\lambda'}(\theta)\mathcal{M}^{\lambda_a\lambda_b,\lambda\lambda'}_{ab\leftrightarrow g^*g^*}(\theta) \tag{5}$$

and, for $L=2$,

$$\mathcal{M}^{\lambda_a\lambda_b}_{ab\leftrightarrow G}=-\frac{15}{2}\left[\frac{2}{M}\right]^{5/2}R''_2(0) \times\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2} \times\sum_{\lambda\lambda'}\chi^{2SJ\Lambda}_{\lambda\lambda'}(\theta)\mathcal{M}^{\lambda_a\lambda_b,\lambda\lambda'}_{ab\leftrightarrow g^*g^*}(\theta)\ . \tag{6}$$

Specializing to gluonium production in photon-photon collisions (or two-photon decay of gluonia [Fig. 1(a)]), we are now led to determine the $\beta\to 0$ limit of the helicity amplitudes of the process $\gamma\gamma\leftrightarrow g^*g^*$, as represented in lowest-order perturbative QCD by the diagrams of Fig. 1(b). These amplitudes can actually be taken from a calculation performed many years ago by Constantini, de Tollis, and Pistoni, of the analogous reaction in QED—with massive photons instead of massive gluons and intermediate quarks instead of electrons—where they used a double dispersion relation.[8] Their remarkable work (a generalization of the computation by de Tollis[9] of the reaction $\gamma\gamma\to\gamma\gamma$ with all four photons on their mass shell) contains all the expressions needed, using formulas (29), (30), (9), and Appendix III of that paper.

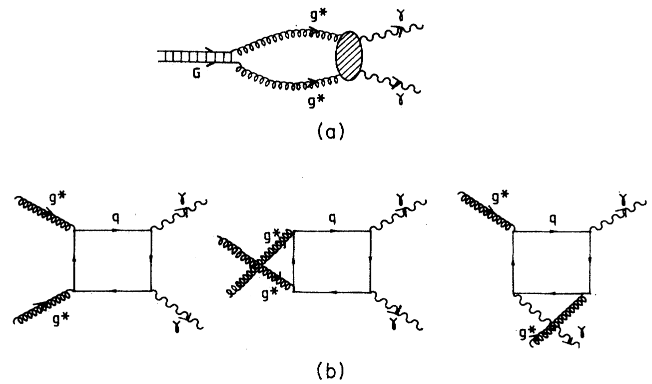

FIG. 1. (a) General representation of the process $G\to\gamma\gamma$ in the bound-state model. (b) Lowest-order Feynman graphs for $\gamma\gamma\to g^*g^*$; three additional graphs (giving the same contribution) are derived therefrom by inverting the arrow on the quark lines in the box.

However, it is by no means trivial to go to the $\beta\to 0$ limit, since many of the terms listed in Appendix III of Ref. 8 become divergent in that limit. One must expand each of these terms, which contain transcendental functions, into a power series in β, up to a certain order (depending on the L values one needs to consider), thereby canceling all divergences of that kind; one checks, incidentally, that mass singularities are canceled as well. We have performed that expansion up to order 2; the corresponding expressions of the 36 helicity amplitudes are given in Appendix A.

III. APPLICATION TO $\gamma\gamma\leftrightarrow\eta(1440)$, $f_2(1720)$, AND $X(2220)$

A. $\gamma\gamma\leftrightarrow\eta(1440)$

The $\eta(1440)$ being assumed to be a $J^P=0^-$ state, its additional quantum numbers are restricted to the values: $L=1$, $S=1$, $\Lambda=0$. With those values, one derives, from formula (5),

$$\mathcal{M}^{\lambda_\gamma\lambda'_\gamma}_{\gamma\gamma\leftrightarrow\eta(1440)}=\frac{3i}{\sqrt{\pi}}\frac{1}{M^{3/2}}R'_1(0)\lim_{\beta\to 0}\frac{1}{\beta} \times\int\frac{d(\cos\theta)}{2}(\mathcal{M}^{\lambda_\gamma\lambda'_\gamma,++}-\mathcal{M}^{\lambda_\gamma\lambda'_\gamma,--})\ , \tag{7}$$

where the photon helicities $\lambda_\gamma,\lambda'_\gamma$ are themselves restricted to the values $+,+$ and $-,-$; notice that we are using a simplified notation, writing $\mathcal{M}^{\lambda_\gamma\lambda'_\gamma,\lambda\lambda'}$ for $\mathcal{M}^{\lambda_\gamma\lambda'_\gamma,\lambda\lambda'}_{\gamma\gamma\to g^*g^*}(\theta)$.

Substituting the expressions of the helicity amplitudes $\mathcal{M}^{\lambda_\gamma\lambda'_\gamma,\lambda\lambda'}$ involved, as given in Appendix A, one gets

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow\eta(1440)}=\pm\frac{64\sqrt{2}}{3\sqrt{\pi}}i\alpha\alpha_s\frac{1}{M^{3/2}}R'_1(0) \tag{8}$$

and therefrom

$$\Gamma(\eta(1440)\to\gamma\gamma)=\frac{512}{9\pi^2}\alpha^2\alpha_s^2\frac{1}{M^4}|R'_1(0)|^2\ . \tag{9}$$

B. $\gamma\gamma \leftrightarrow f_2(1720)$

(i) Assumption $L=0$.

Since no other $J^P=2^+$ glueball candidate seems to exist at lower mass, it is reasonable, *a priori*, to assume $L=0$ for the $f_2(1720)$. Thus $S=2$, while Λ can take the values 0 (corresponding to $\lambda_\gamma,\lambda'_\gamma=\pm,\pm$) or $\pm 2(\lambda_\gamma,\lambda'_\gamma=\pm,\mp)$.

From (3) one derives

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=\frac{1}{4\sqrt{3\pi}}\frac{1}{M^{1/2}}R_0(0)\lim_{\beta\to 0}\int\frac{d(\cos\theta)}{2}\times[(3\cos^2\theta-1)(\mathcal{M}^{\pm\pm,++}+\mathcal{M}^{\pm\pm,--}+2\mathcal{M}^{\pm\pm,00})+3\sin^2\theta(\mathcal{M}^{\pm\pm,+-}+\mathcal{M}^{\pm\pm,-+})+3\sqrt{2}\sin\theta\cos\theta(\mathcal{M}^{\pm\pm,+0}+\mathcal{M}^{\pm\pm,0-}-\mathcal{M}^{\pm\pm,-0}-\mathcal{M}^{\pm\pm,0+})]\ , \tag{10}$$

$$\mathcal{M}^{\mp\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=\frac{\sqrt{2}}{8\sqrt{\pi}}\frac{1}{M^{1/2}}R_0(0)\lim_{\beta\to 0}\int\frac{d(\cos\theta)}{2}[\sin^2\theta(\mathcal{M}^{\mp\pm,++}+\mathcal{M}^{\mp\pm,--}+2\mathcal{M}^{\mp\pm,00})+(1\pm\cos\theta)^2\mathcal{M}^{\mp\pm,-+}+(1\mp\cos\theta)^2\mathcal{M}^{\mp\pm,+-}\pm\sqrt{2}(1\mp\cos\theta)\sin\theta(\mathcal{M}^{\mp\pm,+0}+\mathcal{M}^{\mp\pm,0-})\pm\sqrt{2}(1\pm\cos\theta)\sin\theta(\mathcal{M}^{\mp\pm,-0}+\mathcal{M}^{\mp\pm,0+})]\ . \tag{11}$$

Using again the expressions given in Appendix A, one obtains

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=-\frac{16\sqrt{2}}{3\sqrt{3\pi}}\alpha\alpha_s\frac{1}{M^{1/2}}R_0(0)\ , \tag{12}$$

$$\mathcal{M}^{\mp\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=\frac{64}{9\sqrt{\pi}}(4\ln 2-1-2i\pi)\alpha\alpha_s\frac{1}{M^{1/2}}R_0(0)\ , \tag{13}$$

and therefrom

$$\Gamma(f_2(1720)\to\gamma\gamma)=\frac{1024}{405}\left[1+\frac{1}{4\pi^2}(4\ln 2-1)^2+\frac{3}{32\pi^2}\right]\alpha^2\alpha_s^2\frac{1}{M^2}|R_0(0)|^2\ . \tag{14}$$

It is interesting to notice that the imaginary part of the amplitude given by (13) provides for more than 90% of the total contribution to the decay width.

(ii) Assumption $L=2$, and mixing of $L=0,2$.

For reasons that will become obvious in Sec. IV, we also introduce the assumption $L=2$. In that case, S can take the values 0 or 2.

For $S=0$, one derives, from (6),

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=-\frac{5\sqrt{15}}{\sqrt{2\pi}}\frac{1}{M^{5/2}}R_2''(0)\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2}(3\cos^2\theta-1)(\mathcal{M}^{\pm\pm,++}+\mathcal{M}^{\pm\pm,--}-\mathcal{M}^{\pm\pm,00})\ , \tag{15}$$

$$\mathcal{M}^{\mp\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=-\frac{15\sqrt{5}}{2\sqrt{\pi}}\frac{1}{M^{5/2}}R_2''(0)\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2}\sin^2\theta(\mathcal{M}^{\mp\pm,++}+\mathcal{M}^{\mp\pm,--}-\mathcal{M}^{\mp\pm,00})\ , \tag{16}$$

and therefrom, using the expressions of Appendix A,

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=0\ , \tag{17}$$

$$\mathcal{M}^{\mp\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=\frac{256\sqrt{10}}{3\sqrt{\pi}}\alpha\alpha_s\frac{1}{M^{5/2}}R_2''(0)\ . \tag{18}$$

With $S=2$, on the other hand, one gets, from (6),

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=\frac{5\sqrt{15}}{\sqrt{14\pi}}\frac{1}{M^{5/2}}R_2''(0)\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2}\times\left[(3\cos^2\theta-1)(\mathcal{M}^{\pm\pm,++}+\mathcal{M}^{\pm\pm,--}+2\mathcal{M}^{\pm\pm,00})-3\sin^2\theta(\mathcal{M}^{\pm\pm,+-}+\mathcal{M}^{\pm\pm,-+})+\frac{3}{\sqrt{2}}\sin\theta\cos\theta(\mathcal{M}^{\pm\pm,+0}+\mathcal{M}^{\pm\pm,0-}-\mathcal{M}^{\pm\pm,-0}-\mathcal{M}^{\pm\pm,0+})\right]\ , \tag{19}$$

$$\mathcal{M}^{\mp\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=\frac{15\sqrt{5}}{2\sqrt{7\pi}}\frac{1}{M^{5/2}}R_2''(0)\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2}$$
$$\times[\sin^2\theta(\mathcal{M}^{\mp\pm,++}+\mathcal{M}^{\mp\pm,--}+2\mathcal{M}^{\mp\pm,00})$$
$$-(1\mp\cos\theta)^2\mathcal{M}^{\mp\pm,+-}-(1\pm\cos\theta)^2\mathcal{M}^{\mp\pm,-+}$$
$$\pm\frac{1}{\sqrt{2}}(1\mp\cos\theta)\sin\theta(\mathcal{M}^{\mp\pm,+0}+\mathcal{M}^{\mp\pm,0-})$$
$$\pm\frac{1}{\sqrt{2}}(1\pm\cos\theta)\sin\theta(\mathcal{M}^{\mp\pm,-0}+\mathcal{M}^{\mp\pm,0+})]\ , \tag{20}$$

and therefrom, using again the expressions of Appendix A,

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=\frac{32\sqrt{35}}{\sqrt{3\pi}}\alpha\alpha_s\frac{1}{M^{5/2}}R_2''(0)\ , \tag{21}$$

$$\mathcal{M}^{\mp\pm}_{\gamma\gamma\leftrightarrow f_2(1720)}=\frac{64\sqrt{10}}{9\sqrt{7\pi}}(52\ln 2+11-26i\pi)\times\alpha\alpha_s\frac{1}{M^{5/2}}R_2''(0)\ . \tag{22}$$

Assuming the state $J=2$ to be a mixture of all three L,S states considered, and defining that mixture by $A_{02}|L=0,\ S=2\rangle+A_{20}|L=2,\ S=0\rangle+A_{22}|L=2,\ S=2\rangle$, one then gets

$$\Gamma(f_2(1720)\to\gamma\gamma)=\frac{1024}{405}\frac{\alpha^2\alpha_s^2}{M^2}(\hat{\alpha}+\hat{\beta}+\hat{\gamma}+\hat{\delta}) \tag{23}$$

defining

$$\hat{\alpha}=A_{02}^2|R_0(0)|^2\ ,$$
$$\hat{\beta}=\frac{1690}{7}A_{22}^2\frac{|R_2''(0)|^2}{M^4}\ ,$$
$$\hat{\gamma}=\frac{1}{224\pi^2}\left|2\sqrt{14}(4\ln 2-1)A_{02}R_0(0)+3\sqrt{5}(16\sqrt{7}A_{20}+7\sqrt{6}A_{22})\frac{R_2''(0)}{M^2}\right|^2\ ,$$
$$\hat{\delta}=\frac{1}{224\pi^2}\left|\sqrt{21}A_{02}R_0(0)-4\sqrt{5}(52\ln 2+11)A_{22}\frac{R_2''(0)}{M^2}\right|^2\ .$$

C. $\gamma\gamma\leftrightarrow X(2220)$

The $X(2220)$ has been seen only in one measurement of radiative J/ψ decay performed by the Mark III Collaboration, and its existence was not confirmed in a similar measurement performed by the DM2 Collaboration.[10] Moreover, Mark III was not able to completely determine its spin-parity state, restricting its conclusion to the statement that it should be: J even (the most likely being $J=2$), $P=+1$; finally there seems to be no strong indication that, if it is there, it should be a glueball.[11]

Nevertheless we shall retain the assumption[12] that a glueball may exist at 2.22 GeV with $J^P=0^+$, 2^+, or 4^+. We shall successively consider these three possibilities. For simplicity we shall assume that $L=0$ for $J^P=0^+,2^+$ and $L=2$ for $J^P=4^+$.

(i) Assumption $J=L=S=0$.

From (3) one derives, in that case (noticing: $\Lambda=0$),

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow X(2220)}=\frac{1}{\sqrt{6\pi}}\frac{1}{M^{1/2}}R_0(0)\lim_{\beta\to 0}\int\frac{d(\cos\theta)}{2}\times(\mathcal{M}^{\pm\pm,++}+\mathcal{M}^{\pm\pm,--}-\mathcal{M}^{\pm\pm,00})\ . \tag{24}$$

Using once again the expressions of Appendix A, one gets

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow X(2220)}=-\frac{32}{3\sqrt{3\pi}}\alpha\alpha_s\frac{1}{M^{1/2}}R_0(0) \tag{25}$$

and therefrom

$$\Gamma(X(2220)\to\gamma\gamma)=\frac{64}{27\pi^2}\alpha^2\alpha_s^2\frac{1}{M^2}R_0(0)\ . \tag{26}$$

(ii) Assumption $J=S=2$, $L=0$.

Here the calculation is strictly the same as for the $f_2(1720)$ with $L=0$, therefore the same expression as for the latter [formula (14)] is obtained for the decay width into two photons; only the mass and the wave function at the origin are to be changed.

(iii) Assumption $J=4$, $L=S=2$.

From (6) one gets

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow X(2220)}=-\frac{75\sqrt{6}}{8\sqrt{7\pi}}\frac{1}{M^{5/2}}R_2''(0)\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2}$$
$$\times[\tfrac{1}{5}(3-30\cos^2\theta+35\cos^4\theta)(\mathcal{M}^{\pm\pm,++}+\mathcal{M}^{\pm\pm,--}+2\mathcal{M}^{\pm\pm,00})$$
$$+\sin^2\theta(-1+7\cos^2\theta)(\mathcal{M}^{\pm\pm,+-}+\mathcal{M}^{\pm\pm,-+})$$
$$-\sqrt{2}\sin\theta\cos\theta(-3+7\cos^2\theta)(\mathcal{M}^{\pm\pm,+0}+\mathcal{M}^{\pm\pm,0-}-\mathcal{M}^{\pm\pm,-0}-\mathcal{M}^{\pm\pm,0+})]\ , \tag{27}$$

$$\mathcal{M}^{\mp\pm}_{\gamma\gamma\leftrightarrow X(2220)} = -\frac{15\sqrt{15}}{4\sqrt{7\pi}}\frac{1}{M^{5/2}}R_2''(0)\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2} \times\Big[\sin^2\theta(-1+7\cos^2\theta)(\mathcal{M}^{\mp\pm,++}+\mathcal{M}^{\mp\pm,--}+2\mathcal{M}^{\mp\pm,00}) +(1\mp\cos\theta)^2(1\pm 7\cos\theta+7\cos^2\theta)\mathcal{M}^{\mp\pm,+-} +(1\pm\cos\theta)^2(1\mp 7\cos\theta+7\cos^2\theta)\mathcal{M}^{\mp\pm,-+} \pm\frac{1}{\sqrt{2}}(1\mp\cos\theta)\sin\theta(1\mp 7\cos\theta-14\cos^2\theta)(\mathcal{M}^{\mp\pm,+0}+\mathcal{M}^{\mp\pm,0-}) \pm\frac{1}{\sqrt{2}}(1\pm\cos\theta)\sin\theta(1\pm 7\cos\theta-14\cos^2\theta)(\mathcal{M}^{\mp\pm,-0}+\mathcal{M}^{\mp\pm,0+})\Big]. \tag{28}$$

Using once more the expressions of Appendix A, one obtains

$$\mathcal{M}^{\pm\pm}_{\gamma\gamma\leftrightarrow X(2220)} = -\frac{128}{3\sqrt{21\pi}}\alpha\alpha_s\frac{1}{M^{5/2}}R_2''(0), \tag{29}$$

$$\mathcal{M}^{\mp\pm}_{\gamma\gamma\leftrightarrow X(2220)} = -\frac{128}{63}\frac{11\sqrt{2}}{\sqrt{105\pi}}(4\ln 2+\tfrac{244}{33}-2i\pi) \times\alpha\alpha_s\frac{1}{M^{5/2}}R_2''(0), \tag{30}$$

and therefrom

$$\Gamma(X\to\gamma\gamma) = \frac{2^{13}}{5^3}\frac{11^2}{3^5 7^3}\left[1+\frac{1}{4\pi^2}(4\ln 2+\tfrac{244}{33})^2+\frac{2205}{968\pi^2}\right] \times\alpha^2\alpha_s^2\frac{1}{M^6}|R_2''(0)|^2. \tag{31}$$

IV. NORMALIZATION USING RADIATIVE J/ψ DECAY

For numerical predictions of two-photon decay widths of the gluonium states considered, we still need to normalize our results, i.e., to eliminate the unknown parameter $R_0(0)$ [$R_1'(0)$ or $R_2''(0)$]. This can be done by using measured values for J/ψ decay into a photon plus a gluonium candidate [Fig. 2(a)].

The corresponding helicity amplitudes can indeed be connected, using again formula (1), with the helicity amplitudes of the process $J/\psi\to\gamma g^*g^*$; the latter are computed, in lowest-order QCD, on the basis of the Feynman diagrams of Fig. 2(b). The results of that calculation are given in Appendix B.

We then obtain the ratio $\Gamma(G\to\gamma\gamma)/\Gamma(\psi\to\gamma G)$ as an expression devoid of any adjustable parameter (it does not even depend on α_s) for any gluonium state with specified quantum numbers (J,L,S). We notice that this quantity is (apart from kinematic factors) the inverse of the "stickiness" parameter defined by Chanowitz.[13]

A. $J/\psi\to\gamma\eta(1440)$

In analogy with formula (7) we here get

$$\mathcal{M}^{\lambda_\psi\lambda_\gamma}_{\psi\to\gamma\eta(1440)} = \frac{3i}{\sqrt{\pi}}\frac{1}{M^{3/2}}R_1'(0)\lim_{\beta\to 0}\frac{1}{\beta}\int\frac{d(\cos\theta)}{2} \times(\mathcal{M}^{\lambda_\psi\lambda_\gamma,++}-\mathcal{M}^{\lambda_\psi\lambda_\gamma,--}), \tag{32}$$

where $\lambda_\psi,\lambda_\gamma$ are restricted to the values $+,+$ and $-,-$. Substituting the expressions given in Appendix B for the helicity amplitudes $\mathcal{M}^{\lambda_\psi\lambda_\gamma,\lambda\lambda'}$, one gets

$$\mathcal{M}^{\pm\pm}_{\psi\to\gamma\eta(1440)} = \pm\frac{256i}{3}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi^3}M^{1/2}R_1'(0) \tag{33}$$

and therefrom

$$\Gamma(\psi\to\gamma\eta(1440)) = \frac{8192\pi}{81}\alpha\alpha_s^2\frac{f_\psi^2}{M_\psi^7}\left[1-\frac{M^2}{M_\psi^2}\right]M|R_1'(0)|^2. \tag{34}$$

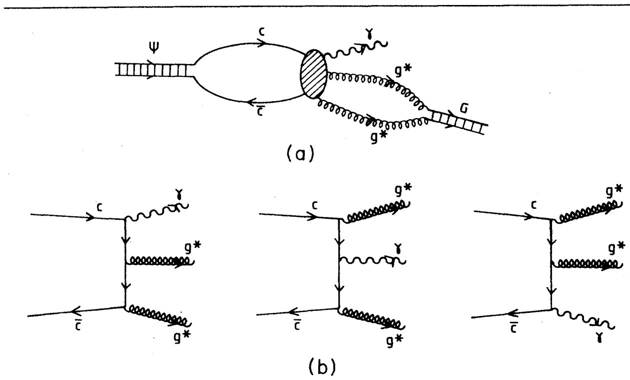

FIG. 2. (a) General representation of the process $\psi\to\gamma G$ in the bound-state model. (b) Lowest-order Feynman graphs for $c\bar{c}\to g^*g^*\gamma$; three additional graphs (giving the same contribution) are derived therefrom by exchanging the gluons.

Comparing formulas (9) and (34), one gets

$$\frac{\Gamma(\eta(1440)\to\gamma\gamma)}{\Gamma(\psi\to\gamma\eta(1440))}=\frac{3\alpha}{16\pi^3}\frac{M_\psi^7}{f_\psi^2 M^5}\frac{1}{1-M^2/M_\psi^2}\ . \tag{35}$$

B. $J/\psi\to\gamma f_2(1720)$

(i) Assumption $L=0$.

Assuming $L=0$, we again apply formulas (10) and (11), substituting $\mathcal{M}^{\lambda_\psi\lambda_\gamma}_{\psi\to\gamma f_2(1720)}$ for $\mathcal{M}^{\lambda_\gamma\lambda_\gamma'}_{\gamma\gamma\leftrightarrow f_2(1720)}$, while the amplitudes on the right-hand side are now defined as $\mathcal{M}^{\lambda_\psi\lambda_\gamma,\lambda\lambda'}$. In addition, we derive, from (3),

$$\mathcal{M}^{0\pm}_{\psi\to\gamma f_2(1720)}=\frac{1}{2\sqrt{2\pi}}\frac{1}{M^{1/2}}R_0(0)\lim_{\beta\to 0}\int\frac{d(\cos\theta)}{2}\times\Bigg\{\pm\sin\theta\cos\theta(\mathcal{M}^{0\pm,++}+\mathcal{M}^{0\pm,--}+2\mathcal{M}^{0\pm,00})$$
$$+\sin\theta[(1\mp\cos\theta)\mathcal{M}^{0\pm,+-}-(1\pm\cos\theta)\mathcal{M}^{0\pm,-+}]$$
$$+\frac{1}{\sqrt{2}}(1\mp\cos\theta)(2\cos\theta\pm1)(\mathcal{M}^{0\pm,+0}+\mathcal{M}^{0\pm,0-})$$
$$+\frac{1}{\sqrt{2}}(1\pm\cos\theta)(2\cos\theta\mp1)(\mathcal{M}^{0\pm,-0}+\mathcal{M}^{0\pm,0+})\Bigg\}\ . \tag{36}$$

Using the expressions of Appendix B, we get

$$\mathcal{M}^{\pm\pm}_{\psi\to\gamma f_2(1720)}=-\frac{128}{9\sqrt{3}}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\frac{1}{M^{1/2}}R_0(0)\ , \tag{37}$$

$$\mathcal{M}^{0\pm}_{\psi\to\gamma f_2(1720)}=-\tfrac{128}{9}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi^2}M^{1/2}R_0(0)\ , \tag{38}$$

$$\mathcal{M}^{\mp\pm}_{\psi\to\gamma f_2(1720)}=-\frac{128\sqrt{2}}{8}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi^3}M^{3/2}R_0(0)\ , \tag{39}$$

and therefrom

$$\Gamma(\psi\to\gamma f_2(1720))=\tfrac{2048}{729}\pi\alpha\alpha_s^2\frac{f_\psi^2}{M_\psi^3}\left[1-\frac{M^2}{M_\psi^2}\right]\times\left[1+3\frac{M^2}{M_\psi^2}+6\frac{M^4}{M_\psi^4}\right]\frac{1}{M}|R_0(0)|^2\ . \tag{40}$$

Comparing formulas (14) and (40), one gets

$$\frac{\Gamma(\gamma\gamma\to f_2(1720))}{\Gamma(\psi\to\gamma f_2(1720))}=\frac{9\alpha}{10\pi}\frac{M_\pi^3}{f_\psi^2 M}\frac{\left[1+\frac{1}{4\pi^2}(4\ln 2-1)^2+\frac{3}{32\pi^2}\right]}{\left[1-\frac{M^2}{M_\psi^2}\right]\left[1+3\frac{M^2}{M_\psi^2}+6\frac{M^4}{M_\psi^{*}}\right]}\ . \tag{41}$$

The problem with the assumption $L=0$ is that the expressions (37)–(39) we get for the helicity amplitudes of the process considered are in contradiction with experimental values obtained by both the Mark III and the DM2 Collaboration. While in both experiments the relative phases between all three amplitudes are confirmed to be compatible with zero, the ratios between these amplitudes are found (averaging between the data of both experiments) to be[10]

$$\frac{\mathcal{M}^{0\pm}}{\mathcal{M}^{\pm\pm}}\simeq-1.2,\quad \frac{\mathcal{M}^{\mp\pm}}{\mathcal{M}^{\pm\pm}}\simeq-1.1\ ,$$

while we obtain, from formulas (37)–(39),

$$\frac{\mathcal{M}^{0\pm}}{\mathcal{M}^{\pm\pm}}\simeq+0.96,\quad \frac{\mathcal{M}^{\mp\pm}}{\mathcal{M}^{\pm\pm}}\simeq+0.76\ .$$

Therefore we are led to try other assumptions, i.e., $L=2$ or a mixing of $L=0$ and 2 (Ref. 14).

(ii) Assumption $L=2$, and mixing of $L=0,2$.

For $S=0$, using formulas (15) and (16) with the adequate substitutions for subscripts and superscripts of the helicity amplitudes, and in addition using the following formula derived from (6):

$$\mathcal{M}^{0\pm}_{\psi\to\gamma f_2(1720)}=\mp\frac{15\sqrt{5}}{\sqrt{\pi}}\frac{1}{M^{5/2}}R_2''(0)\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2}\sin\theta\cos\theta(\mathcal{M}^{0\pm,++}+\mathcal{M}^{0\pm,--}-\mathcal{M}^{0\pm,00}) \tag{42}$$

we get, with the expressions of Appendix B,

$$\mathcal{M}^{0\pm}_{\psi\to\gamma f_2(1720)}=0\ , \tag{43}$$

$$\mathcal{M}^{\mp\pm}_{\psi\to\gamma f_2(1720)}=0\ , \tag{44}$$

$$\mathcal{M}^{\pm\pm}_{\psi\to\gamma f_2(1720)}=-\frac{512\sqrt{10}}{9\sqrt{3}}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\left[1-\frac{M^2}{M_\psi^2}\right]^2\frac{1}{M^{5/2}}R_2''(0)\ . \tag{45}$$

For $S=2$, using formulas (19) and (20) with the adequate substitutions, and in addition the following formula derived from (6):

$$\begin{aligned}\mathcal{M}^{0\pm}_{\psi\to\gamma f_2(1720)}=\mp\frac{15\sqrt{5}}{\sqrt{7\pi}}\frac{1}{M^{5/2}}R_2''(0)\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2}\Big[&\sin\theta\cos\theta(\mathcal{M}^{0\pm,++}+\mathcal{M}^{0\pm,--}+2\mathcal{M}^{0\pm,00})\\&\pm\sin\theta(1\mp\cos\theta)\mathcal{M}^{0\pm,+-}\pm\sin\theta(1\mp\cos\theta)\mathcal{M}^{0\pm,-+}\\&\pm\frac{1}{2\sqrt{2}}(1\mp\cos\theta)(2\cos\theta\pm1)(\mathcal{M}^{0\pm,+0}+\mathcal{M}^{0\pm,0-})\\&\pm\frac{1}{2\sqrt{2}}(1\pm\cos\theta)(2\cos\theta\mp1)(\mathcal{M}^{0\pm,-0}+\mathcal{M}^{0\pm,0+})\Big]\end{aligned} \tag{46}$$

we get, with the expressions of Appendix B,

$$\mathcal{M}^{\pm\pm}_{\psi\to\gamma f_2(1720)}=\frac{128\sqrt{70}}{3\sqrt{3}}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\times\left[1-\frac{4}{21}\left[1-\frac{M^2}{M_\psi^2}\right]^2\right]\frac{1}{M^{5/2}}R_2''(0)\ , \tag{47}$$

$$\mathcal{M}^{0\pm}_{\psi\to\gamma f_2(1720)}=\frac{128\sqrt{70}}{3}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\frac{M}{M_\psi}\times\left[1-\frac{2}{21}\left[1-\frac{M^2}{M_\psi^2}\right]^2\right]\frac{1}{M^{5/2}}R_2''(0)\ , \tag{48}$$

$$\mathcal{M}^{\mp\pm}_{\psi\to\gamma f_2(1720)}=\frac{256\sqrt{35}}{3}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\frac{M^2}{M_\psi^2}\times\left[1+\frac{4}{21}\left[1-\frac{M^2}{M_\psi^2}\right]^2\right]\frac{1}{M^{5/2}}R_2''(0)\ . \tag{49}$$

Assuming, as in Sec. III, that we have a mixture of all three ($L=0$, $S=2$; $L=2$, $S=0$; $L=2$, $S=2$) states considered, with weight coefficients A_{02}, A_{20}, and A_{22}, we get

$$\Gamma(\psi\to\gamma f_2(1720))=\tfrac{2048}{729}\pi\alpha\alpha_s^2\frac{f_\psi^2}{M_\psi^3 M}\left[1-\frac{M^2}{M_\psi^2}\right](\bar{\alpha}+\bar{\beta}+\bar{\gamma}) \tag{50}$$

with

$$\bar{\alpha}=\left|A_{02}R_0(0)-\sqrt{10}\left[4zA_{20}+3\sqrt{7}\left[1-\frac{4z}{21}\right]A_{22}\right]\frac{R_2''(0)}{M^2}\right|^2,$$

$$\bar{\beta}=3\frac{M^2}{M_\psi^2}\left|A_{02}R_0(0)-3\sqrt{70}\left[1-\frac{2z}{21}\right]A_{22}\frac{R_2''(0)}{M^2}\right|^2,$$

$$\bar{\gamma}=6\frac{M^4}{M_\psi^4}\left|A_{02}R_0(0)-3\sqrt{70}\left[1+\frac{4z}{21}\right]A_{22}\frac{R_2''(0)}{M^2}\right|^2,$$

defining

$$z=\left[1-\frac{M^2}{M_\psi^2}\right]^2 .$$

Comparing formulas (26) and (53), one obtains

$$\frac{\Gamma(f_2(1720)\to\gamma\gamma)}{\Gamma(\psi\to\gamma f_2(1720))}=\frac{9\alpha}{10\pi}\frac{M_\psi^3}{f_\psi^2 M}\frac{1}{\left[1-\dfrac{M^2}{M_\psi^2}\right]}\frac{\hat{\alpha}+\hat{\beta}+\hat{\gamma}+\hat{\delta}}{\bar{\alpha}+\bar{\beta}+\bar{\gamma}}\ . \tag{51}$$

Now we can easily check, on the basis of formulas (37)–(39), (43)–(45), and (47)–(49), that there are two simple options that allow us to fit the measured helicity amplitude ratios as quoted above for $J/\psi\to\gamma f_2(1720)$: namely, (i) $A_{02}=0$, $A_{20}/A_{22}\simeq-6.5$ (pure $L=2$) and (ii) $A_{22}=0$, $A_{20}R_2''(0)/[A_{02}M^2R_0(0)]\simeq0.27$ ($L=0,2$ mixing). Actually the first option fits the quoted values very precisely, while the second one fits them up to 10%. For our numerical estimate we shall use both options, making the corresponding substitutions in formula (51).

It is interesting to notice, on the basis of our calculation of helicity amplitudes for $\gamma\gamma\leftrightarrow f_2(1720)$, that in all three cases considered ($L=0$, $L=2$, $L=0,2$ mixing) the $f_2(1720)$, when produced in $\gamma\gamma$ collisions, should be predominantly in a helicity ±2 state.

C. $J/\psi\to\gamma X(2220)$

(i) Assumption $J=L=0$.

Using formula (24), mutatis mutandis, and the expressions of Appendix B, one gets

$$\mathcal{M}^{\pm\pm}_{\psi\to\gamma X(2220)}=-\frac{128\sqrt{2}}{9\sqrt{3}}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\frac{1}{M^{1/2}}R_0(0) \quad (52)$$

and therefrom

$$\Gamma(\psi\to\gamma X(220))=\tfrac{4096}{729}\pi\alpha\alpha_s^2\frac{f_\psi^2}{M_\psi^3}\left[1-\frac{M^2}{M_\psi^2}\right]\frac{1}{M}|R_0(0)|^2 . \quad (53)$$

Comparing formulas (26) and (53), one obtains

$$\frac{\Gamma(X(220)\to\gamma\gamma)}{\Gamma(\psi\to\gamma X(2220))}=\frac{27\alpha}{64\pi^3}\frac{M_\psi^3}{f_\psi^2 M}\frac{1}{(1-M^2/M_\psi^2)} . \quad (54)$$

(ii) Assumption $J=2$, $L=0$.

Here the calculations are the same as for the $f_2(1720)$ with $L=0$, so that we are directly led to the ratio $\Gamma(X(2220)\to\gamma\gamma)/\Gamma(\psi\to\gamma X(2220))$ as given by the right-hand side of (41) with $M=2.22$ GeV.

(iii) Assumption $J=4$, $L=2$.

Using formulas (27) and (28), mutatis mutandis, and in addition the following formula derived from (6):

$$\begin{aligned}\mathcal{M}^{0\pm}_{\psi\to\gamma X(2220)}&=\mp\frac{15\sqrt{30}}{8\sqrt{7\pi}}\frac{1}{M^{5/2}}R_2''(0)\lim_{\beta\to 0}\frac{1}{\beta^2}\int\frac{d(\cos\theta)}{2}\\
&\times\Big[\mp 2\sin\theta\cos\theta(-3+7\cos^2\theta)(\mathcal{M}^{0\pm,++}+\mathcal{M}^{0\pm,--}+2\mathcal{M}^{0\pm,00})\\
&+\sin\theta(1\mp\cos\theta)(1\mp 7\cos\theta-14\cos^2\theta)\mathcal{M}^{0\pm,+-}\\
&-\sin\theta(1\pm\cos\theta)(1\pm 7\cos\theta-14\cos^2\theta)\mathcal{M}^{0\pm,-+}\\
&\mp\frac{1}{2\sqrt{2}}(1\mp\cos\theta)(3\pm 6\cos\theta-21\cos^2\theta\mp 28\cos^3\theta)(\mathcal{M}^{0\pm,+0}+\mathcal{M}^{0\pm,0-})\\
&\pm\frac{1}{2\sqrt{2}}(1\pm\cos\theta)(3\mp 6\cos\theta-21\cos^2\theta\pm 28\cos^3\theta)(\mathcal{M}^{0\pm,-0}+\mathcal{M}^{0\pm,0+})\Big]\end{aligned} \quad (55)$$

one gets, with the expressions of Appendix B,

$$\mathcal{M}^{\pm\pm}_{\psi\to\gamma X(2220)}=+\frac{512\sqrt{2}}{63\sqrt{21}}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\left[1-\frac{M^2}{M_\psi^2}\right]^2\times\frac{1}{M^{5/2}}R_2''(0) , \quad (56)$$

$$\mathcal{M}^{0\pm}_{\psi\to\gamma X(2220)}=-\frac{512\sqrt{5}}{63\sqrt{21}}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\frac{M}{M_\psi}\left[1-\frac{M^2}{M_\psi^2}\right]^2\times\frac{1}{M^{5/2}}R_2''(0) , \quad (57)$$

$$\mathcal{M}^{\mp\pm}_{\psi\to\gamma X(2220)}=+\frac{512\sqrt{5}}{63\sqrt{21}}\pi\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\frac{M^2}{M_\psi^2}\left[1-\frac{M^2}{M_\psi^2}\right]^2\times\frac{1}{M^{5/2}}R_2''(0) , \quad (58)$$

and therefrom

$$\Gamma(\psi\to\gamma X(2220))=\frac{2^{16}}{3^6\times 5\times 7^3}\pi\alpha\alpha_s^2\frac{f_\psi^2}{M_\psi^3}\left[1-\frac{M^2}{M_\psi^2}\right]^5\times\left[1+\frac{5}{2}\frac{M^2}{M_\psi^2}+\frac{5}{2}\frac{M^4}{M_\psi^4}\right]\times\frac{1}{M^5}|R_2''(0)|^2 . \quad (59)$$

Comparing formulas (31) and (59), one gets

$$\frac{\Gamma(X(2220)\to\gamma\gamma)}{\Gamma(\psi\to\gamma X(2220))}=\frac{363\alpha}{200\pi}\frac{M_\psi^3}{f_\psi^2 M}\frac{1+\frac{1}{4\pi^2}\left[4\ln 2+\frac{244}{33}\right]^2+\frac{2205}{968\pi^2}}{\left[1-\frac{M^2}{M_\psi^2}\right]^5\left[1+\frac{5}{2}\frac{M^2}{M_\psi^2}+\frac{5}{2}\frac{M^4}{M_\psi^4}\right]} . \quad (60)$$

V. NUMERICAL RESULTS, AND COMPARISON WITH PRESENT EXPERIMENTAL LIMITS

Regarding our experimental knowledge of radiative J/ψ decay giving rise to potential gluonium states, the values measured are those of $\Gamma(J/\psi\to\gamma G)B(G\to xy\cdots)$, where $xy\cdots$ are the final-state particles observed together with the photon. We shall use those values in order to directly determine predicted values for $\Gamma(\gamma\gamma\to G)B(G\to xy\cdots)$.

We take the following experimental values (to the extent that they have been measured by several experimental groups, we take world averages) (Refs. 10 and 15):

$$\Gamma(\psi\to\gamma\eta(1440))B(\eta(1440)\to K\bar{K}\pi)=277\text{ eV},$$

$$\Gamma(\psi\to\gamma\eta(1440))B(\eta(1440)\to\rho\rho)=95\text{ eV},$$

$$\Gamma(\psi\to\gamma\eta(1440))B(\eta(1440)\to\omega\omega)=19\text{ eV},$$

$$\Gamma(\psi\to\gamma\eta(1440))B(\eta(1440)\to\gamma\rho^0)=7\text{ eV},$$

$$\Gamma(\psi\to\gamma f_2(1720))B(f_2(1720)\to K\bar{K})=60\text{ eV},$$

$$\Gamma(\psi\to\gamma f_2(1720))B(f_2(1720)\to\eta\eta)=16\text{ eV},$$

$$\Gamma(\psi\to\gamma f_2(1720))B(f_2(1720)\to\pi\pi)=13\text{ eV},$$

$$\Gamma(\psi\to\gamma X(2220))B(X(2220)\to K\bar{K})=6\text{ eV}.$$

Therefrom, using formulas (35), (41), (51), (54), and (60), we get the predictions for $\Gamma(\gamma\gamma\to G)B(G\to xy\cdots)$ as listed in Table I; these predictions are compared with the corresponding lowest measured upper limits, wherever available.[4] It is seen from Table I that, wherever experimental upper limits exist, our predictions tend to be largely below those limits.

VI. CONCLUSION

From our results (see Table I), it appears that 2-γ decay widths of gluonium states tend, on the average, to be significantly smaller than the corresponding decay widths of most quarkonia decaying into two photons, as far as they have been measured. Let us mention that, in general, larger theoretical values were obtained for $\Gamma(G\to\gamma\gamma)$ by other authors, using different theoretical models.[16]

It should be emphasized, on the other hand, that we have assumed those gluonium states to be pure, i.e., without any admixture of $q\bar{q}$ states. Since the ratio $\Gamma(X\to\gamma\gamma)/\Gamma(\psi\to\gamma X)$ is much larger for $X=q\bar{q}$ than for gg, any significant admixture of $q\bar{q}$ would have considerably increased our predicted values.

In spite of the smallness of our predictions, we conclude from our study that there is some reasonable hope of observing the production of the main glueball candidates, i.e., the $\eta(1440)$ and the $f_2(1720)$, in $\gamma\gamma$ collisions in near future, provided the $\gamma\gamma$ luminosities of e^+e^- machines are increased, with respect to present conditions, by about 1 order of magnitude.[17]

Till now the absence of such an observation has been regarded as a confirmation of the glueball interpretation of those particles. It would obviously be preferable to have a positive experimental evidence in order to support that interpretation. At the same time, such an evidence would provide a check of the quark-box mechanism here considered, which would be important, in our opinion, from the point of view of testing perturbative QCD.

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APPENDIX A: HELICITY AMPLITUDES FOR $\gamma\gamma\to g^*g^*$

Defining the amplitudes as $\mathcal{M}^{\lambda_\gamma\lambda'_\gamma\lambda\lambda'}$ ($\lambda_\gamma,\lambda'_\gamma$ being the helicities of the photons and λ,λ' those of the gluons), the following expressions have been derived for them from

TABLE I. Comparison of our predictions for the widths of various decay processes $\Gamma(G\to\gamma\gamma)$ times the branching ratios $B(G\to xy\cdots)$ with presently available lowest measured upper limits (from Ref. 4). In the second row, for $f_2(1720)$, $L=2$ refers to the mixture of $|L=2, S=0\rangle$ and $|L=2, S=2\rangle$, while $L=m$ refers to the mixture of $|L=0, S=2\rangle$ and $|L=2, S=0\rangle$, both mixtures being defined in such a way that they fit the measured helicity amplitude ratios of the processes $J/\psi\to\gamma f_2(1720)$ (see Sec. IV).

Mode	J	Our prediction (eV) L	$\Gamma\times B$	Lowest measured upper limit (eV)
$\Gamma(\eta(1440)\to\gamma\gamma)B(\eta(1440)\to K\bar{K}\pi)$	0	1	90	1600
$\Gamma(\eta(1440)\to\gamma\gamma)B(\eta(1440)\to\rho\rho)$	0	1	30	
$\Gamma(\eta(1440)\to\gamma\gamma)B(\eta(1440)\to\omega\omega)$	0	1	6	
$\Gamma(\eta(1440)\to\gamma\gamma)B(\eta(1440)\to\gamma\rho^0)$	0	1	2	200
		0	20	
$\Gamma(f_2(1720)\to\gamma\gamma)B(f_2(1720)\to K\bar{K})$	2	2	85	200
		m	95	
		0	5	
$\Gamma(f_2(1720)\to\gamma\gamma)B(f_2(1720)\to\eta\eta)$	2	2	22	
		m	25	
		0	4	
$\Gamma(f_2(1720)\to\gamma\gamma)B(f_2(1720)\to\pi\pi)$	2	2	18	255
		m	20	
	0	0	0.2	
$\Gamma(X(2220)\to\gamma\gamma)B(X(2220)\to K\bar{K})$	2	0	1.2	1000
	4	2	200	

Ref. 7, taking account of color, including only the contribution of u,d,s quarks in the quark box and neglecting their mass, and limiting the expansion into powers of β (the gluon c.m. velocity) to terms in β^0, β, and β^2:

$$\mathcal{M}^{++,\pm\pm} = -3C[1+\cos^2\theta \mp 2\beta\sin\theta + \beta^2(-2+7\cos^2\theta\sin^2\theta)] ,$$

$$\mathcal{M}^{++,\pm\mp} = -3C\sin^2\theta[1+\beta^2(4-7\cos^2\theta)] ,$$

$$\mathcal{M}^{++,\pm 0} = \mp 3\sqrt{2}C\sin\theta\cos\theta[1\pm\beta+\beta^2(\tfrac{9}{2}-7\cos^2\theta)] ,$$

$$\mathcal{M}^{++,0\pm} = \pm 3\sqrt{2}C\sin\theta\cos\theta[1\pm\beta+\beta^2(\tfrac{9}{2}-7\cos^2\theta)] ,$$

$$\mathcal{M}^{++,00} = +6C\sin^2\theta(1-7\beta^2\cos^2\theta) ,$$

$$\mathcal{M}^{+-,\pm\pm} = 2C\sin^2\theta\left\{x+\frac{\beta^2}{30}[6x(-8+11\cos^2\theta) - 197+29\cos^2\theta]\right\} ,$$

$$\mathcal{M}^{+-,\pm\mp} = 2C(1\pm\cos\theta) \times\left\{x(1\pm\cos\theta)+\frac{\beta^2}{30} \times[6x(-2\mp 8\cos\theta+5\cos^2\theta\pm 11\cos^3\theta) -53\mp 117\cos\theta+165\cos^2\theta \pm 29\cos^3\theta]\right\} ,$$

$$\mathcal{M}^{+-,\pm 0} = -2\sqrt{2}C\sin\theta(1\pm\cos\theta) \times\left\{x+\frac{\beta^2}{30}[3x(-13\mp 6\cos\theta+22\cos^2\theta) -121\pm 68\cos\theta+29\cos^2\theta]\right\} ,$$

$$\mathcal{M}^{+-,0\pm} = -2\sqrt{2}C\sin\theta(1\mp\cos\theta) \times\left\{x+\frac{\beta^2}{30}[3x(-13\pm 6\cos\theta+22\cos^2\theta) -121\mp 68\cos\theta+29\cos^2\theta]\right\} ,$$

$$\mathcal{M}^{+-,00} = 4C\sin^2\theta\left\{x+\frac{\beta^2}{30}[6x(-8+11\cos^2\theta) -17+29\cos^2\theta]\right\} .$$

Here θ is the c.m. scattering angle; we have defined

$$C=\frac{8\sqrt{2}}{9}\alpha\alpha_s \quad\text{and}\quad x=4\ln 2-1-2i\pi .$$

The remaining helicity amplitudes are given by the relation

$$\mathcal{M}^{-\lambda_\gamma-\lambda'_\gamma,-\lambda-\lambda'} = (-)^{\lambda+\lambda'}\mathcal{M}^{\lambda_\gamma\lambda'_\gamma,\lambda\lambda'} .$$

APPENDIX B: HELICITY AMPLITUDES FOR $J/\psi\to\gamma g^* g^*$

Defining the amplitudes as $\mathcal{M}^{\lambda_\psi\lambda_\gamma,\lambda\lambda'}$, the following expressions have been obtained for them by a calculation of the diagrams of Fig. 2(b), treating the J/ψ as a $c\bar{c}$ bound state ($M_c = M_\psi/2$), and taking account of color (it is an easy task to expand these expressions into powers of β, up to order 2):

$$\mathcal{M}^{++,\pm\pm} = -K_\theta[(1+\cos^2\theta)(1-\beta^2)\mp 2y\beta(1\mp\beta)^2] ,$$

$$\mathcal{M}^{++,\pm\mp} = -K_\theta\sin^2\theta(1+\beta^2) ,$$

$$\mathcal{M}^{++,\pm 0} = \mp\sqrt{2}K_\theta\sin\theta[\cos\theta-(1-y)\beta\mp y\beta^2]\sqrt{1-\beta^2} ,$$

$$\mathcal{M}^{++,0\pm} = \pm\sqrt{2}K_\theta\sin\theta[\cos\theta+(1-y)\beta\pm y\beta^2]\sqrt{1-\beta^2} ,$$

$$\mathcal{M}^{++,00} = 2K_\theta\sin^2\theta(1-\beta^2) ,$$

$$\mathcal{M}^{0+,\pm\pm} = -\sqrt{2}K_\theta\sqrt{y}\,\sin\theta\cos\theta(1-\beta^2) ,$$

$$\mathcal{M}^{0+,\pm\mp} = \mp\sqrt{2}K_\theta\sqrt{y}\,\sin\theta(1\mp\cos\theta)(1+\beta^2) ,$$

$$\mathcal{M}^{0+,\pm 0} = \mp K_\theta\sqrt{y}\,(1\mp\cos\theta)[1\pm 2\cos\theta\mp(1-y)\beta-y\beta^2] \times\sqrt{1-\beta^2} ,$$

$$\mathcal{M}^{0+,0\pm} = \pm K_\theta\sqrt{y}\,(1\pm\cos\theta)[1\mp 2\cos\theta\mp(1-y)\beta-y\beta^2] \times\sqrt{1-\beta^2} ,$$

$$\mathcal{M}^{0+,00} = -2\sqrt{2}K_\theta\sqrt{y}\,\sin\theta\cos\theta(1-\beta^2) ,$$

$$\mathcal{M}^{-+,\pm\pm} = -K_\theta y\sin^2\theta(1-\beta^2) ,$$

$$\mathcal{M}^{-+,\pm\mp} = -K_\theta y(1\mp\cos\theta)^2(1+\beta^2) ,$$

$$\mathcal{M}^{-+,\pm 0} = -\sqrt{2}K_\theta y\sin\theta(1\mp\cos\theta)\sqrt{1-\beta^2} ,$$

$$\mathcal{M}^{-+,0\pm} = -\sqrt{2}K_\theta y\sin\theta(1\pm\cos\theta)\sqrt{1-\beta^2} ,$$

$$\mathcal{M}^{-+,00} = -2K_\theta y\sin^2\theta(1-\beta^2) .$$

Here θ is the emission angle of the gluons in their c.m. frame with respect to the $\psi\gamma$ axis; we have defined $y=M^2/M_\psi^2$ and

$$K_\theta = \tfrac{64}{9}\pi^{3/2}\sqrt{\alpha}\alpha_s\frac{f_\psi}{M_\psi}\frac{1}{[(1+y\beta^2)^2-(1-y)^2\beta^2\cos^2\theta]} ,$$

where $f_\psi \simeq 0.27$ GeV is the decay constant of the J/ψ [it is related to its radial wave function at the origin by $f_\psi=\sqrt{3/2\pi M_\psi}R_0^\psi(0)$]. The remaining amplitudes are given by the relation

$$\mathcal{M}^{-\lambda_\psi-\lambda_\gamma,-\lambda-\lambda'} = -(-)^{\lambda_\psi+\lambda+\lambda'}\mathcal{M}^{\lambda_\psi\lambda_\gamma,\lambda\lambda'} .$$

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[7]See, e.g., D. M. Brink and G. R. Satchler, *Angular Momentum* (Clarendon, Oxford, 1962).

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[9]B. de Tollis, Nuovo Cimento **32**, 757 (1964); **35**, 1182 (1965).

[10]See K. Königsmann, Phys. Rep. **139**, 243 (1986).

[11]Actually the most plausible assumption for the $X(2220)$ seems to be a quarkonium ($s\bar{s}, J^P=2^+$) state, which might be a recurrence (with $L=3$) of the $f_2'(1525)$. This assumption seems to be confirmed by the values quoted for helicity amplitude ratios in the Mark III measurement; see G. Eigen, Report No. CALT-68-1483 (unpublished). Indeed, with the assignment $J=2$, the $\mathcal{M}^{\mp\pm}$ amplitude appears to be suppressed, just as for the $f_2'(1525)$ and contrary to what is found for the $f_2(1720)$.

[12]B. F. L. Ward, Phys. Rev. D **31**, 2849 (1985).

[13]See M. S. Chanowitz, in *Proceedings of the VIth International Workshop on Photon-Photon Collisions,* Lake Tahoe, 1984, edited by R. L. Lander (World Scientific, Singapore, 1985), p. 95; the author emphasizes the importance of that parameter for testing the glueball nature of a particle.

[14]See B. A. Li, Q. X. Shen, and K. F. Liu, Phys. Rev. D **35**, 1070 (1987); those authors also introduced a mixing of L,S states, but retained ony the second option here considered.

[15]Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **B170**, 1 (1986).

[16]In particular for $\Gamma(\eta(1440)\rightarrow\gamma\gamma)$, considerably higher values have been predicted by a number of authors: C. Rosenzweig, A. Salomone, and J. Schecter, Phys. Rev. D **24**, 2545 (1981); T. Teshima and S. Oneda, *ibid.* **29**, 2067 (1984); M. Franck and P. J. O'Donnel, Phys. Lett. **144B**, 45 (1984); Phys. Rev. D **32**, 1739 (1985); N. N. Achasov and G. N. Shestakov, Phys. Lett. **156B**, 434 (1985); H. E. Haber and J. Perrier, Phys. Rev. D **32**, 2961 (1985); F. Caruso, E. Predazzi, A. C. B. Antunes, and I. Tiomno, Z. Phys. C **30**, 493 (1986); T. Barnes (Ref. 4), p. 25.

[17]Future e^+e^- colliders with an energy of ≈ 10 GeV and a design luminosity of $\approx 10^{33}$ cm^{-2}/s, presently planned in order to be used mainly as B-meson factories, but also in other fields such as two-photon physics, would be particularly well fit for the kind of study here considered. See Report No. PR-88-09 of the Paul-Scherrer Institute, CH-5234 Villigen, Switzerland (unpublished); in particular Sec. II-11.