Photon polarization in charged-pion radiative decay

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We discuss the dependence of the photon polarization in the radiative pion decay $\pi^+ \to e^+ \nu_e \gamma$ on the parameter $\gamma = F^A/F^V$, the ratio of the axial-vector to vector pion form factor for the decay, and show that the measurement of the outgoing photon polarization provides an additional information in determining a unique value of γ.

I. INTRODUCTION

In spite of its small branching ratio (10^{-8}), the radiative pion decay $\pi^+ \to e^+ \nu_e \gamma$ is of interest because it contains important information on the structure of the pion. Furthermore, with the introduction of pion factories such as LAMPF, SIN, and TRIUMF, the radiative pion decay can be studied with enough statistics and hence has received renewed experimental attention.

In the radiative-pion-decay process, there are two contributions to the amplitude: (i) inner bremsstrahlung and (ii) the structure-dependent contribution which depends on the axial-vector form factor F^A and the vector form factor F^V (see Fig. 1). The vector form factor is related through the conserved-vector-current (CVC) hypothesis to the form factor in $\pi^0 \to \gamma\gamma$ (Ref. 1). The various theoretical models[2-8] predict the value of the axial-vector form factor or its ratio to the vector form factor, $\gamma = F^A/F^V$, and it can be extracted from experiments. [The latest experimental results are given from LAMPF (Ref. 9) and from SIN (Refs. 10 and 11) and earlier experimental results as well as general discussion on theoretical models are reviewed in Refs. 12 and 13.]

In the electronic decay mode, structure-dependent terms are of the same order of magnitude as inner bremsstrahlung terms while the muonic decay mode is dominated by inner bremsstrahlung terms. To suppress bremsstrahlung, most of the experiments[9-14] are performed with the opening angle ($\theta_{e\gamma}$) near 180°. Then the decay rate is dominated by the term proportional to $(1+\gamma)^2$, which shows the quadratic ambiguity in the determination of γ. Even though recent experimental analyses[9-11] as well as theoretical model calculations[6-8] favor the positive γ value, the negative sign of γ is not completely ruled out.

The purpose of this paper is to show that the experimental measurement of the outgoing photon polarization in the electronic decay mode gives additional information on the parameter γ. Thus, the measurements of the outgoing photon polarization and the decay rate determine the size of γ uniquely.

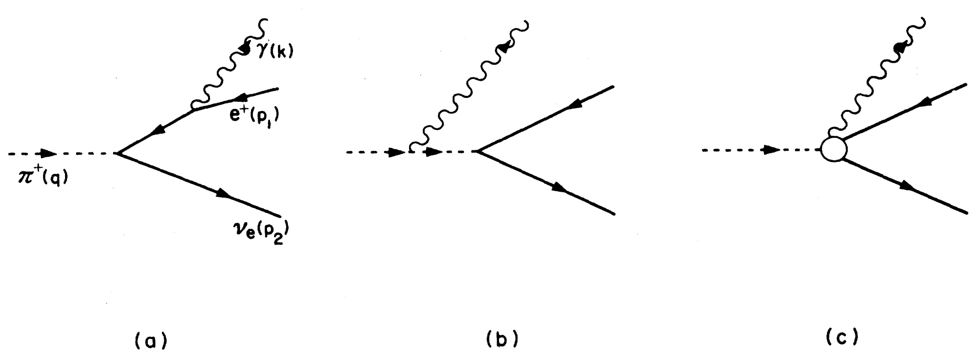

FIG. 1. Diagrams contributing to the radiative decay $\pi^+ \to e^+ \nu_e \gamma$: (a) and (b) represent inner-bremsstrahlung contributions, and (c) represents the structure-dependent contribution governed by the vector form factor F^V and the axial-vector form factor F^A, respectively.

II. PHOTON POLARIZATION

The transition amplitude of the pion radiative decay $\pi^+ \to e^+ \nu_e \gamma$ is described as

$$M = M_{\rm IB} + M_{\rm SD} \ , \tag{1}$$

where the inner-bremsstrahlung term $M_{\rm IB}$ and the structure-dependent term $M_{\rm SD}$ are given, following the convention of Ref. 12, as

$$M_{\rm IB} = -ie\frac{G_F f_\pi}{2^{1/2}} m_e m_\pi \cos\theta_C \bar{u}_\nu(p_2) \times \left[\frac{2p_1\cdot\epsilon^* + \epsilon^* k\!\!\!/}{2k\cdot p_1} - \frac{q\cdot\epsilon^*}{k\cdot q}\right](1-\gamma_5)v_e(p_1) \ , \tag{2}$$

$$M_{\rm SD} = \frac{eG_F\cos\theta_C}{2^{1/2}m_\pi}\bar{u}_\nu(p_2)\gamma_\mu(1+\gamma_5)v_e(p_1) \times [F^V\epsilon^{\mu\alpha\beta\sigma}\epsilon^*{}_\alpha q_\beta k_\sigma + iF^A(k\cdot q\,\epsilon^{*\mu} - q\cdot\epsilon^* k^\mu)] \ . \tag{3}$$

In Eqs. (2) and (3), q, p_1, p_2, and k are momenta of pion, positron, neutrino, and photon, respectively. Also ϵ^μ is the photon polarization vector, θ_C the Cabibbo angle, G_F the Fermi constant, and f_π the pion decay constant.

The method[15] in treating the real photon polarization can be applied here to obtain the polarization of the outgoing photon beam. The real photon with momentum $\mathbf{k}$ can be described by

$$\epsilon(\lambda) = \frac{1}{2^{1/2}}(-\lambda\hat{\mathbf{a}} - i\hat{\mathbf{k}}\times\hat{\mathbf{a}}) \ , \tag{4}$$

where λ is $+1$ (-1) for right- (left-) handed circular polarization, $\hat{\mathbf{k}}$ is the unit vector along $\mathbf{k}$, and $\hat{\mathbf{a}}$ is an arbitrary unit vector perpendicular to $\mathbf{k}$. In the pion rest frame the following relation[16] can be used,

$$\epsilon^{\mu\alpha\beta\sigma}\epsilon^*_\alpha q_\beta k_\sigma = i\lambda\epsilon^{*\mu}k\cdot q \ , \tag{5}$$

and Eq. (3) is simplified to

$$M_{\rm SD} = \frac{eG_F\cos\theta_c}{2^{1/2}m_\pi}\bar{u}_\nu(p_2)\gamma_\mu(1+\gamma_5)v_e(p_1) \times [i(F^A + \lambda F^V)k\cdot q\epsilon^{*\mu} - iF^A q\cdot\epsilon^* k^\mu] \ . \tag{6}$$

The photon density matrix can be specified by Stokes parameters ξ_i $(i=1,2,3)$ which are contained in the photon density matrix in the helicity basis

$$\rho_{\lambda\lambda'} = \frac{1}{2}\left[\delta_{\lambda\lambda'} + \frac{i}{2}\xi_1(\lambda-\lambda') + \tfrac{1}{2}\xi_2(\lambda+\lambda') + \tfrac{1}{2}\xi_3(\lambda\lambda'-1)\right] , \tag{7}$$

where ξ_3 is the degree of linear polarization with respect to the $\hat{\mathbf{a}}$ direction and the $\hat{\mathbf{k}}\times\hat{\mathbf{a}}$ direction, ξ_1 is the degree of linear polarization with respect to the two orthogonal axes oriented at 45° to the $\hat{\mathbf{a}}$ direction, and ξ_2 is the degree of circular polarization. Since one can obtain the following relation from Eq. (4),

$$\epsilon^i(\lambda')\epsilon^j(\lambda)^* = \frac{1}{2}\left[(\delta^{ij} - \hat{k}^i\hat{k}^j)\delta_{\lambda\lambda'} - \frac{i}{2}(\lambda+\lambda')\epsilon^{ijk}\hat{k}^k - \frac{i}{2}(\lambda'-\lambda)[\hat{a}^i(\hat{\mathbf{k}}\times\hat{\mathbf{a}})^j + \hat{a}^j(\hat{\mathbf{k}}\times\hat{\mathbf{a}})^i] + \tfrac{1}{2}(\lambda\lambda'-1)[\hat{a}^i\hat{a}^j - (\hat{\mathbf{k}}\times\hat{\mathbf{a}})^i(\hat{\mathbf{k}}\times\hat{\mathbf{a}})^j]\right] , \tag{8}$$

the photon density matrix of the radiative pion decay can be obtained explicitly from the absolute square of the transition amplitude after replacing $\epsilon^\mu(k,\lambda')\epsilon^\nu(k,\lambda)^*$ by Eq. (8).

If one chooses $\hat{\mathbf{a}}$ for simplicity as

$$\hat{\mathbf{a}} = \frac{\mathbf{p}_1\times\mathbf{k}}{|\mathbf{p}_1\times\mathbf{k}|} \ , \tag{9}$$

the decay rate becomes, in the pion rest frame,

$$\begin{aligned}\frac{d^2\Gamma}{dx\,dy} = \frac{\alpha}{4\pi}\Gamma(\pi\to e\nu)\Big(& I_B(x,y)\delta_{\lambda\lambda'} + \tfrac{1}{2}(\lambda+\lambda')I_C(x,y) + \tfrac{1}{2}(\lambda\lambda'-1)I_L(x,y) \\ & + (F^V/f_\pi)\{\delta_{\lambda\lambda'}[F(x,y)(1+\gamma) + G(x,y)(1-\gamma)] \\ & \quad + \tfrac{1}{2}(\lambda+\lambda')[F(x,y)(1+\gamma) - G(x,y)(1-\gamma)] - \tfrac{1}{2}(\lambda\lambda'-1)2\gamma F(x,y)\} \\ & + (m_\pi F^V/2m_e f_\pi)^2\{\delta_{\lambda\lambda'}[S^+(x,y)(1+\gamma)^2 + S^-(x,y)(1-\gamma)^2] \\ & \quad + \tfrac{1}{2}(\lambda+\lambda')[S^+(x,y)(1+\gamma)^2 - S^-(x,y)(1-\gamma)^2] + \tfrac{1}{2}(\lambda\lambda'-1)S_L(1-\gamma^2)\}\Big) \ ,\end{aligned} \tag{10}$$

where $\Gamma(\pi\to e\nu)$ is the decay rate for $\pi\to e\nu$ and x,y are defined in the pion rest frame as

$$x=2E_\gamma/m_\pi\ , \tag{11a}$$

$$y=2E_e/m_\pi\ . \tag{11b}$$

The functions of x and y in Eq. (10) are defined as

$$I_B(x,y)=\frac{(1-y)[1+(1-x)^2]}{x^2(x+y-1)}\ , \tag{12a}$$

$$I_C(x,y)=-\frac{(1-y)(2-x)}{x(x+y-1)}\ , \tag{12b}$$

$$I_L(x,y)=-2\frac{(1-y)(1-x)}{x^2(x+y-1)}\ , \tag{12c}$$

$$F(x,y)=-\frac{(1-x)(1-y)}{x}\ , \tag{13a}$$

$$G(x,y)=\frac{1-y}{x}\left[1-x+\frac{x^2}{x+y-1}\right]\ , \tag{13b}$$

$$S^+(x,y)=(1-x)(x+y-1)^2\ , \tag{14a}$$

$$S^-(x,y)=(1-x)(1-y)^2\ , \tag{14b}$$

$$S_L(x,y)=-2(1-x)(1-y)(x+y-1)\ . \tag{14c}$$

Equations (12)–(14) represent the contributions of inner bremsstrahlung, interference between inner-bremsstrahlung and structure-dependent terms, and structure dependence to the radiative-pion-decay process, respectively. Also the coefficients of $\delta_{\lambda\lambda'}$, $\frac{1}{2}(\lambda\lambda'-1)$, and $\frac{1}{2}(\lambda+\lambda')$ in Eq. (10) describe the radiative-pion-decay rate when the final photon polarization is not detected, the contribution of linear polarization, and that of circular polarization, respectively. Explicitly the Stokes parameters of the final photon in $\pi^+\to e^+\nu_e\gamma$ become

$$\xi_1=0\ , \tag{15a}$$

$$\xi_2=\frac{B}{A}\ , \tag{15b}$$

$$\xi_3=\frac{C}{A}\ , \tag{15c}$$

where A,B,C are defined as

$$A=\mathrm{I}_B+(F^V/f_\pi)[F(1+\gamma)+G(1-\gamma)] \\ +(m_\pi F^V/2m_e f_\pi)^2[S^+(1+\gamma)^2+S^-(1-\gamma)^2]\ , \tag{16a}$$

$$B=\mathrm{I}_C+(F^V/f_\pi)[F(1+\gamma)-G(1-\gamma)] \\ +(m_\pi F^V/2m_e f_\pi)^2[S^+(1+\gamma)^2-S^-(1-\gamma)^2]\ , \tag{16b}$$

$$C=I_L-2(F^V/f_\pi)\gamma F+(m_\pi F^V/2m_e f_\pi)^2 S_L(1-\gamma^2)\ . \tag{16c}$$

Since the opening angle between positron and photon momentum, $\theta_{e\gamma}$, in the pion decay process with $m_e\simeq 0$ satisfies

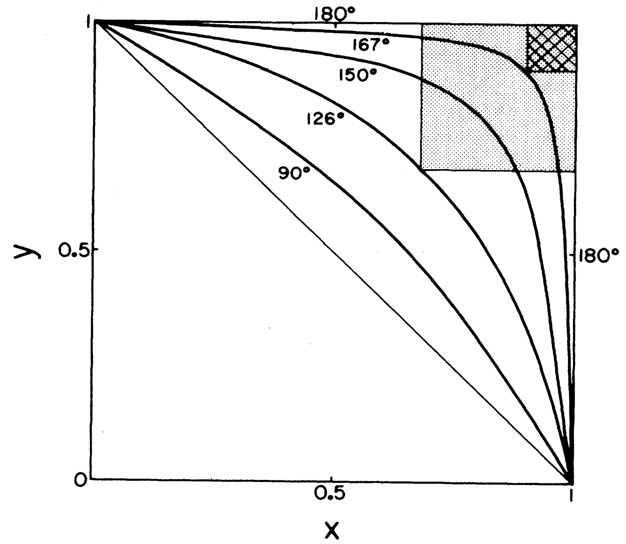

FIG. 2. The kinematically allowed region is bounded by the lines $x=1$, $y=1$, and $x+y=1$. The shaded and cross-hatched regions are for the minimum energy E_0 of 48 and 62.8 MeV, respectively.

$$\cos\theta_{e\gamma}=1-2\frac{1}{xy}(x+y-1)\ , \tag{17}$$

the kinematically allowed regions for x and y for each given values of $\theta_{e\gamma}$ are shown in Fig. 2.

III. DISCUSSIONS

The experiments are usually set up to detect the decay rate at the large opening angle $\theta_{e\gamma}\approx 180°$. This is because, for the large opening angle and large-x and -y region, inner-bremsstrahlung and interference contributions are relatively small[17] compared to the structure-dependent term.

One can see the structure-dependent contribution from Eqs. (10), (14), and (15) by considering $(m_\pi F^V/2m_e f_\pi)^2$ terms only. Since S^+, S^- are positive and S_L is negative for all kinematically allowed regions, the sign of the linear polarization of the outgoing photon beam, ξ_3, depends on the value of γ. If $|\gamma|$ is greater than 1, ξ_3 is positive while, if $|\gamma|$ is smaller than 1, then ξ_3 is negative. (If γ is $+1$, $\xi_3=0$ and $\xi_2=1$, i.e., the outgoing photon is purely right-hand-circular polarized. On the other hand, if γ is -1, $\xi_3=0$ and $\xi_2=-1$, i.e., the outgoing photon is purely left-hand-circular polarized.)

Now consider both structure-dependent and inner bremsstrahlung contributions. If one considers the positron and photon energy larger than a certain energy E_0, which corresponds to the cuts of x at x_0 and y at y_0, respectively, and intergrates Eq. (10) over x and y, the decay rate becomes

$$\Gamma=\frac{\alpha}{4\pi}\Gamma(\pi\to e\nu)A'[\delta_{\lambda\lambda'}+\tfrac{1}{2}(\lambda+\lambda')\xi_2'+\tfrac{1}{2}(\lambda\lambda'-1)\xi_3']\ , \tag{18}$$

where A' is related to the decay rate when the final photon polarization is not measured, and ξ_2' and ξ_3' are the Stokes parameters after they are integrated over x and y. Using the explicit value of $F^V=0.0265$ and $f_\pi=0.945$ (Ref. 12), one obtains the decay rate as shown in Figs. 3(a) and 3(b), and ξ_2' and ξ_3' are shown in Figs. 4(a) and 4(b). In Figs. 3(a) and 4(a), the lowest energy E_0 is 48 MeV which corresponds to $x_0=y_0=0.688$ and in Figs. 3(b) and 4(b), E_0 is 62.8 MeV which corresponds to $x_0=y_0=0.9$.

From Fig. 3, it is clear that the decay rate $\Gamma(\pi\to e\nu\gamma)$ is symmetric around $\gamma=-1$. This gives the quadratic ambiguity in determining the value of γ. On the other hand, the polarizations of outgoing photon beam, ξ_2' and ξ_3', resolve the ambiguity as shown in Fig. 4 and it is much clearer in ξ_3'.

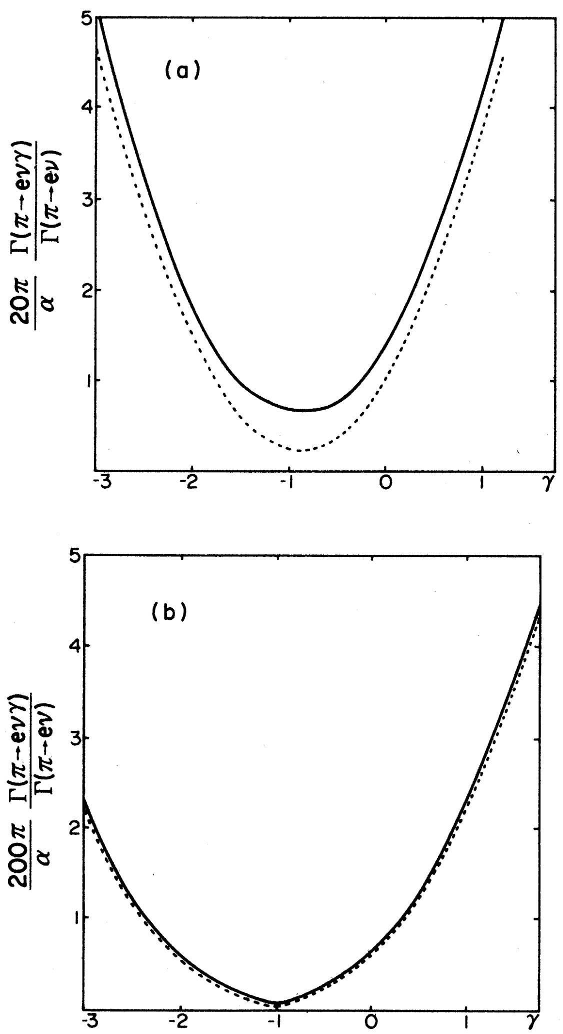

FIG. 3. The ratio of $\Gamma(\pi\to e\nu\gamma)/\Gamma(\pi\to e\nu)$ vs $\gamma=F^A/F^V$ for (a) $x_0=y_0=0.688$ and (b) $x_0=y_0=0.9$. The solid line is for inner-bremsstrahlung and structure-dependent contributions and the dotted line is for the structure-dependent contribution only.

Recent experimental data analyzed with a maximum-likelihood method, as well as theoretical predictions, favor positive values of γ. That is, the latest experimental results are $\gamma=0.25\pm0.12$ (LAMPF) (Ref. 9), 0.52 ± 0.06 (SIN I) (Ref. 10), and 0.7 ± 0.5 (SIN II) (Ref. 11) while some recent theoretical predictions indicate the values $\gamma=0.45$ (Ref. 6), 0.67 ± 0.04 (Ref. 7), and 0.42 ± 0.2 (Ref. 8). Positive values of γ which are less than 1 imply negative values for ξ_3' and positive values for ξ_2' as can be seen from Figs. 4(a) and 4(b). On the other hand, if the quadratic ambiguity of γ from the decay process as shown in Fig. 3 is considered, the negative solutions of γ corresponding to the above positive values of γ becomes less than -2, and each of them predicts a positive value for ξ_3'.

One can see also that, even though the structure dependence is dominant near $\theta_{e\gamma}=180°$, one should consider the contribution of inner bremsstrahlung in order to determine the value of γ when experiments are done at

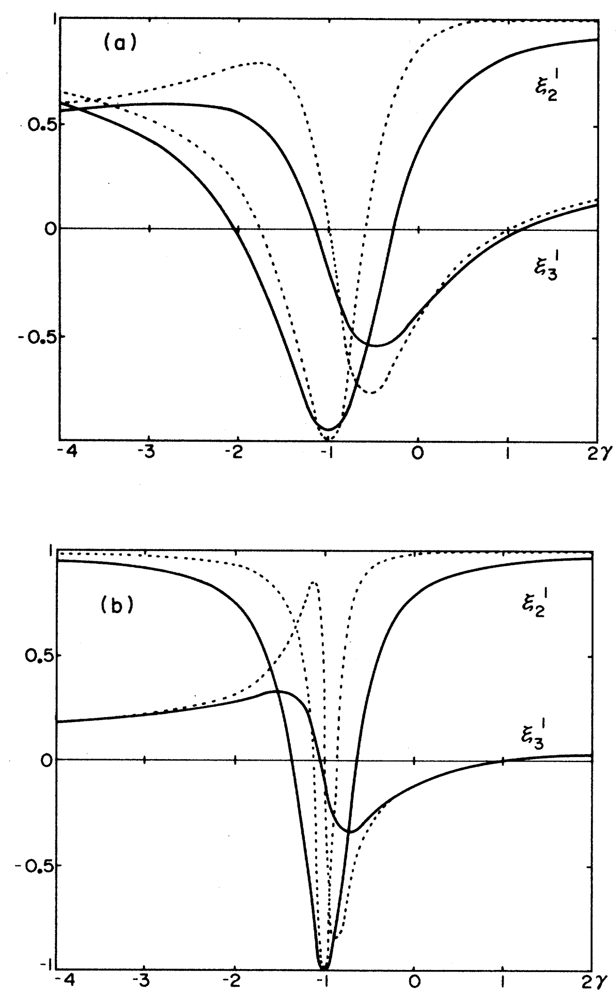

FIG. 4. ξ_2' and ξ_3' vs γ. The symbols are the same as those employed in Fig. 3.

the angle away from 180°.

It should be noted that, if one puts $\lambda'=\lambda$ in Eq. (10), Eq. (11) of Ref. 18 is obtained except for a relative sign difference[19] in the interference terms. Also, if the time-reversal invariance does not hold in the process, structure-dependent form factors can be complex and one obtains a nonzero value of ξ_1 instead of Eq. (15a):

$$\xi_1=\frac{D}{A}\,, \tag{19}$$

where D is defined as

$$D=2\,\mathrm{Im}(F^V/f_\pi)F+2(m_\pi/2m_e)^2\mathrm{Im}(F^VF^{A*}/f_\pi^2)S_L \tag{20}$$

and A is the real part of Eq. (16a). The formulas for ξ_2 and ξ_3 in Eqs. (15a) and (15b) remain the same, aside from the fact that B and C now should be taken as the real parts of the expressions given in Eqs. (16b) and (16c), respectively.

Therefore, one can determine the sign as well as the size of γ uniquely from the measurement of the outgoing photon polarization and decay rate in the charged pion radiative decay.

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[1]V. G. Vaks and B. L. Ioffe, Nuovo Cimento **10**, 342 (1958); H. W. Atherton *et al.*, Phys. Lett. **158B**, 81 (1985).

[2]M. Moreno and J. Pestieau, Phys. Rev. D **13**, 175 (1976); N. Paver and M. D. Scadron, Nuovo Cimento **78A**, 159 (1983); L. Ametller, C. Ayala, and A. Bramon, Phys. Rev. D **29**, 916 (1984); C. Y. Lee, *ibid.* **32**, 658 (1985).

[3]M. Moreno, Phys. Rev. D **16**, 720 (1977).

[4]N. F. Nasrallah, N. A. Papadopoulos, and K. Schilcher, Phys. Lett. **113B**, 61 (1982).

[5]V. N. Pervushkin and M. K. Volkov, Phys. Lett. **58B**, 74 (1975); T. Das, V. Mathur, and S. Okubo, Phys. Rev. Lett. **19**, 859 (1967).

[6]B. R. Holstein, Phys. Rev. D **33**, 3316 (1986).

[7]C. A. Dominguez and J. Solá, Phys. Lett. B **208**, 131 (1988).

[8]M. V. Margvelashvili, Phys. Lett. B **215**, 763 (1988).

[9]L. E. Piilonen *et al.*, Phys. Rev. Lett. **57**, 1402 (1986).

[10]A. Bay *et al.*, Phys. Lett. B **174**, 445 (1986).

[11]S. Egli *et al.*, Phys. Lett. B **175**, 97 (1986).

[12]D. A. Bryman, P. Depommier, and C. Leroy, Phys. Rep. **88**, 151 (1982).

[13]F. Scheck, *Leptons, Hadrons, and Nuclei* (North-Holland, Amsterdam, 1983).

[14]P. Depommier, J. Heintze, C. Rubbia, and V. Soergel, Phys. Lett. **7**, 285 (1963); A. Stetz *et al.*, Nucl. Phys. **B138**, 285 (1979).

[15]Jinhyuk Choi, S. Y. Choi, S. H. Ie, H. S. Song, and R. H. Good, Jr., Phys. Rev. D **36**, 1320 (1987); Jinhyuk Choi, H. S. Song, and R. H. Good, Jr., J. Korean Phys. Soc. **20**, 399 (1987); Jinhyuk Choi, H. S. Kim, S. B. Kim, and H. S. Song, Phys. Rev. D **38**, 40 (1988).

[16]This relation was given in Ref. 12, but it is valid only in the pion rest frame.

[17]S. A. Bludman and J. A. Young, Phys. Rev. **118**, 602 (1960).

[18]F. Scheck and R. Tegen, Nucl. Phys. **A364**, 209 (1981).

[19]An indication that F^V and f_π have equal signs is discussed in Ref. 11 and references therein.