Spectra of the transitions $\Upsilon(nS)\to\Upsilon(mS)\pi^+\pi^-$

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The spectra of the transitions $\Upsilon(nS)\to\Upsilon(mS)\pi^+\pi^-$ are discussed, taking account of multipole-expansion corrections as well as of final-state $\pi^+\pi^-$ and $\Upsilon\pi$ interactions. The inclusion of a four-quark resonance in the $\Upsilon\pi$ channel, as suggested by Voloshin, together with $\pi^+\pi^-$ final-state interactions, may explain the presence of two peaks in the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum. Further tests of this model are proposed.

I. INTRODUCTION

The spectra and widths of several $\pi^+\pi^-$ transitions in the Υ system have been measured with considerable accuracy, by both the ARGUS and CLEO Collaborations.[1-4] These transitions, as well as the analogous decay $\psi'\to\psi\pi^+\pi^-$ (Ref. 5), are for the most part described well using the multipole expansion in QCD in conjunction with potential models, with the conversion of gluons into pions being described using soft-pion theorems [PCAC (partial conservation of axial-vector current), current algebra].[6-11]

One anomaly concerns the transition $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$. In early data[2] the spectrum for this transition appeared to be rather flat, rather than strongly peaked at high energies, as is predicted by the calculations based on soft-pion theorems. More recent CLEO data[1] suggested that the spectrum may have an unusual double-peaked shape. This is confirmed by very recent CLEO data,[4] with three times the statistics of the data of Ref. 1, and the observed spectrum cannot be fitted using any of the known models.

In this paper we consider various modifications of the standard multipole-plus-current-algebra model which preserve its essential features. From the observed spectra we know that these modifications should have a larger effect on the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ decay than on the $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$ decay. Two types of corrections can be envisioned that do not completely destroy the soft-pion framework: (1) corrections to the heavy-quark–gluon vertex (multipole-expansion corrections) and (2) final-state interactions.

We first discuss the multipole-expansion corrections, and argue that the largest potential correction is from higher terms in the expansion in kr, where r is the size of the quarkonium state and k the magnitude of a gluon three-momentum. This correction is largest for the decay $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ but is insufficient to explain the observed spectrum.

We turn next to final-state interactions, which may occur between either the two pions or between the final Υ and one of the pions. Constraints from data on $\pi^+\pi^-$ processes preclude any low-mass narrow resonance structure in the $\pi^+\pi^-$ final state.[12] While the effect of the $\pi^+\pi^-$ interaction is negligible for most of the observed transitions,[6] we find that it significantly alters the shape of the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum, although it cannot by itself explain the double peak in the spectrum.

Finally we discuss the suggestion of Voloshin[13] that the shape of the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum is due to the presence of a narrow four-quark isovector state, coupled to the $\Upsilon\pi$ final state, whose mass lies between that of the $\Upsilon(2S)$ and that of the $\Upsilon(3S)$. Such a state would greatly enhance the low-energy part of the dipion spectrum in the decay $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$, without having much effect on the $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum. Moreover, we show that inclusion of the $\pi^+\pi^-$ interaction can give a double-peaked shape, rather than a single peak at low $\pi^+\pi^-$ invariant mass. We suggest some further ways to test this model.

The outline of this paper is as follows. In Sec. II we review the soft-pion approach for calculating these decays, emphasizing that within the framework the contribution to the spectrum due to emission of the pions in a state having a nonzero angular momentum is negligible. In Sec. III we consider multipole-expansion corrections. In Sec. IV we discuss the effect of final-state interactions of the pions. In Sec. V we explore the possibility of structure in the $\Upsilon\pi$ channel. Section VI contains our conclusions.

II. SOFT-PION CALCULATION OF HADRONIC DECAYS

To obtain the hadronic amplitude for the decay of an $\Upsilon(nS)$ state, we follow the soft-pion method.[6-11] We pay particular attention to the results of Voloshin and Zakharov[9] and of Novikov and Shifman,[10] according to which the transition rates in question are amenable to what is virtually a "first-principles" calculation in QCD.

To lowest order in the multipole expansion, the interaction Hamiltonian is simply $\mathcal{H}_{\rm int}=-\frac{1}{2}g\mathbf{r}\cdot\mathbf{E}^a\xi^a$, where $\mathbf{E}^a$ is the color-electric field ($a=1,\ldots,8$ is a color index), ξ^a is an operator which changes a color-singlet state into a color-octet state, and $g^2=4\pi\alpha_s$. Accordingly, the process we are interested in first occurs at second order in perturbation theory; the intermediate state must have the quark and antiquark in a color octet (together with a soft gluon to make an overall color singlet).

It is pertinent to say something about the nature of these intermediate states. We know that in the limit of extremely heavy quark mass the interaction Hamiltonian for a color-octet quark-antiquark pair is $\mathcal{H}_8 = +2\alpha_s/3r$, and so the intermediate states comprise a continuum of Coulombic states (plus a soft gluon). We shall avoid constructing explicit models of these states in the case at hand, where confinement must be taken into account, but shall accept the arguments of Ref. 8 that these states should lie in a narrow energy band near the b-flavor threshold, i.e., just below the $\Upsilon(4S)$ at 10.50 GeV.

The amplitude for the transition $\Upsilon(nS) \to \Upsilon(mS)\pi^+\pi^-$ is

$$A = \tfrac{2}{3}\langle mS | r_i G_{8,P} r_j | nS \rangle \langle \pi^+\pi^- | \pi\alpha_s E_i^a E_j^a | 0 \rangle \quad (2.1)$$

($i,j = 1,2,3$ are three-vector indices), where $G_{8,P}$ is a Green's function formed from states in which the quark-antiquark pair are in a color octet and in a P wave:

$$G_{8,P} = \sum_{\text{octet}} \frac{|8,P\rangle\langle 8,P|}{M - M_{8,P}} , \quad (2.2)$$

where M is the mass of the initial quarkonium state and $M_{8,P}$ that of the intermediate color-octet state; we neglect the kinetic energy due to recoil of the octet state, as well as the energy of the emitted gluon. Only P-wave octet states contribute since the interaction Hamiltonian goes like r_i. Equation (2.1) has been widely used; we shall comment on its derivation in Sec. III. The hadronic matrix element, in the absence of final-state interactions, has been calculated[9,10] in terms of the corresponding matrix element of the QCD energy-momentum tensor, with the result (see also Ref. 14)

$$\langle \pi^+\pi^- | \pi\alpha_s E_i^a E_j^a | 0 \rangle_{\text{soft-pion}} = \frac{1}{3}\left[\frac{2\pi^2}{9} + O(\alpha_s)\right] q^2\delta_{ij} - \frac{\pi\alpha_s\rho^G}{4}(p_i p_j - q_i q_j) , \quad (2.3)$$

where the coupling α_s and the glue fraction of the pion's momentum ρ^G are to be evaluated at the scale of the inverse radius of the quarkonium system. (For the Υ we expect[14] $\alpha_s \approx 0.4$, $\rho^G \approx \frac{1}{2}$.) Here we define $q = p_1 + p_2$ and $p = p_1 - p_2$, where $p_1 = (\epsilon_1, \mathbf{k}_1)$, $p_2 = (\epsilon_2, \mathbf{k}_2)$ are the four-momenta of the pions; the corresponding three-momenta will be denoted by $\mathbf{q}, \mathbf{p}$. The fact that the leading term in (2.3) goes like q^2, i.e., the existence of an "Adler zero," is responsible for the typical strong peak in the observed spectra at large $\pi^+\pi^-$ invariant mass.

Novikov and Shifman[10] worked out the dipion spectrum implied by (2.3), showing in particular that the D-wave spectrum is small but calculable. We shall reproduce their result, fixing overall constant factors and improving the approximations made in the D-wave formula. We begin by splitting (2.3) into S- and D-wave parts. In covariant notation, the wave function of a D-wave pion pair has the tensor structure $p_\mu p_\nu - \frac{1}{3}(q_\mu q_\nu - q^2 g_{\mu\nu})$, since in the rest frame of the pion pair this goes over to $p_i p_j - \frac{1}{3}\mathbf{p}^2\delta_{ij}$, while for the S wave we will allow two tensor structures, $g_{\mu\nu}q^2$ and $q_\mu q_\nu - q^2 g_{\mu\nu}$, since in the rest frame both of these go over to δ_{ij}.

Splitting the amplitude into S- and D-wave parts, we get

$$\langle (\pi\pi)_S | \pi\alpha_s E_i^a E_j^a | 0 \rangle_{\text{soft-pion}} = \left[\frac{2\pi^2}{27} q^2\delta_{ij} + \frac{\pi\alpha_s\rho^G}{6}(q_i q_j + q^2\delta_{ij})\right] , \quad (2.4)$$

$$\langle (\pi\pi)_D | \pi\alpha_s E_i^a E_j^a | 0 \rangle_{\text{soft-pion}} = \frac{\pi\alpha_s\rho^G}{4}\left[p_i p_j - \tfrac{1}{3}(q_i q_j + q^2\delta_{ij})\left[1 - \frac{4m_\pi^2}{q^2}\right]\right] . \quad (2.5)$$

The leading term in the S-wave amplitude (2.4), which has no factor α_s, has its origin[9,10] in the trace of the gluonic part of the QCD energy-momentum tensor $\theta^G_{\mu\mu}$; this term is enhanced due to the well-known trace anomaly in QCD. The D-wave amplitude receives no contribution from the trace, and so is smaller by a factor of α_s than the S-wave amplitude. The $O(\alpha_s)$ corrections to the leading term in (2.3) were dropped since there are other $O(\alpha_s)$ corrections of the same form that are not calculated in the present approach, and following Novikov and Shifman[10] we have dropped them altogether. The term of the same order proportional to $q_i q_j + q^2\delta_{ij}$ ($q_\mu q_\nu - q^2 g_{\mu\nu}$ in covariant notation) was kept since it has a different structure than the leading term and might significantly affect the shape of the spectrum. In the second term of the D-wave amplitude we have followed Novikov and Shifman in inserting an *ad hoc* threshold factor to guarantee that the D-wave amplitude vanishes at threshold.

Using the double-dipole quarkonium matrix element[14]

$$\langle (m\,{}^3S_1)_j | r_k G_8 r_l | n({}^3S_1)_i \rangle = \frac{I}{3}\delta_{ij}\delta_{kl} , \quad (2.6)$$

where $I = \langle mS | r G_8 r | nS \rangle$, we obtain the S- and D-wave amplitudes

$$A_i^S = \frac{4\pi^2 I \epsilon_i}{81}\left[q^2 + \frac{3\alpha_s\rho^G}{4\pi}(\mathbf{q}^2 + 3q^2)\right] , \quad (2.7)$$

$$A_i^D = \frac{\pi\alpha_s\rho^G I \epsilon_i}{18}\left[\mathbf{p}^2 - \tfrac{1}{3}(\mathbf{q}^2 + 3q^2)\left[1 - \frac{4m_\pi^2}{q^2}\right]\right] , \quad (2.8)$$

where ϵ_i is a polarization vector for the final quarkonium state; the summation over final polarizations is done according to $\sum\epsilon_i\epsilon_j=\delta_{ij}$. The decay rates for two-pion emission are obtained after squaring (2.7) and (2.8) and doing the phase-space integrations. For the S wave we obtain

$$\frac{d\Gamma_S}{dM_{\pi\pi}}=\frac{\pi|I|^2M_{\pi\pi}}{13122M^3}K(M^2_{\pi\pi}+K_1)^2 \times(M^2_{\pi\pi}-4m^2_\pi)^{1/2}(E'^2-M'^2)^{1/2} , \quad (2.9)$$

where

$$K=M^2+M'^2-M^2_{\pi\pi} ,$$

$$K_1=(3\alpha_s\rho^G/4\pi)\{[(M^2-M'^2+M^2_{\pi\pi})/2M]^2+2M^2_{\pi\pi}\} ,$$

M and M' are the masses, respectively, of the initial and final quarkonium states, and $E'=(M^2-M'^2-M^2_{\pi\pi})/2M_{\pi\pi}$ is the energy of the final quarkonium state in the $\pi^+\pi^-$ rest frame.

For the D wave we get

$$\frac{d\Gamma_D}{dM_{\pi\pi}}=\frac{(\alpha_s\rho^G)^2|I|^2M_{\pi\pi}}{20736\pi M^7}K \times[\tfrac{1}{5}(y^5_{\max}-y^5_{\min})-\tfrac{1}{2}a(y^4_{\max}-y^4_{\min}) +\tfrac{1}{3}(a^2+2b)(y^3_{\max}-y^3_{\min})-ab(y^2_{\max}-y^2_{\min})+b^2(y_{\max}-y_{\min})] , \quad (2.10)$$

where

$$y_{\max,\min}=(E'+M_{\pi\pi}/2)^2-[\tfrac{1}{2}(M^2_{\pi\pi}-4m^2_\pi)^{1/2}\mp(E'^2-M'^2)^{1/2}]^2$$

and where

$$a=K+2m^2_\pi,\quad b=\tfrac{1}{4}\{a^2-\tfrac{1}{3}[(K-2M^2)^2-4M^2_{\pi\pi}M^2](1-4m^2_\pi/M^2_{\pi\pi})\} .$$

The corresponding spectra for the transitions $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ are shown in Fig. 1. The spectrum for D-wave pions does show a peak at lower energy than for the S-wave pions but the rate for the D wave is much too small (less than 0.01% of the total rate, using the values of α_s and ρ^G mentioned above) to explain the peak at low energy in the spectrum, even if the spectrum were to be enhanced by the presence of a hypothetical D wave $\pi^+\pi^-$ resonance.[15]

The extreme smallness of the D-wave contribution is due not only to the fact that the corresponding rate goes like α_s^2, but also because the amplitude (2.8) vanishes in the heavy-quarkonium limit; i.e., there is a kinematic suppresion as well. The D-wave contributions from the soft-pion amplitude will be ignored from now on in our discussion. Moreover, in the case of S-wave pions the $O(\alpha_s)$ correction term in (2.4) has only a negligible effect on the shape of the spectrum and will be omitted.

The experimentalists have searched for evidence that the pions in the decay $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ are being produced with nonzero angular momentum, so far with negative results.[4] The extreme smallness of the D-wave component in the soft-pion calculation makes it evident that the presence of any appreciable component having nonzero angular momentum would definitely indicate that some nonmultipole process is present.

It is remarkable that it has been possible to fix the overall constants in the formulas (2.9) and (2.10), so that it is possible to make an estimate of the total rates, given a model for the quarkonium matrix element I. In the approximation[16] in which the nonlocality of the Green's function $G_{8,P}$ is neglected, so that $I=G_0\langle mS|r^2|nS\rangle$, with G_0 a constant, one can use potential-model wave functions to compute the remaining overlap integral. In this approximation $G_0\approx1/[M_{8,P}-M(nS)]\approx2\ \text{GeV}^{-1}$, using the estimate of the intermediate-state mass mentioned above. This approximation was studied in Ref. 14, using the potential model of Ref. 17; from the observed value of the $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$ rate a value $G_0=4.4\ \text{GeV}^{-1}$ was extracted, probably indicating that some important effects have been omitted in this model, but also showing the apparent overall correctness of the soft-pion approach for this decay.

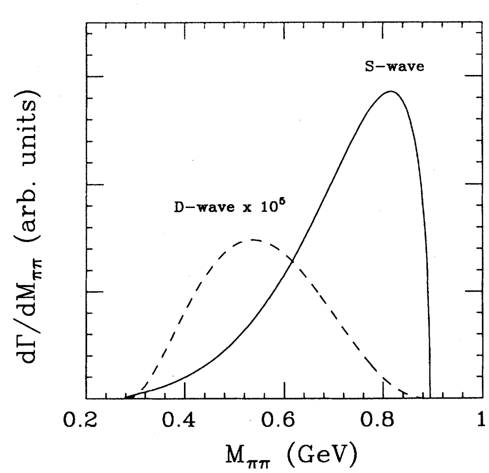

FIG. 1. Soft-pion predictions for $\pi^+\pi^-$ emitted in an S wave and in a D wave.

III. MULTIPOLE-EXPANSION CORRECTIONS

Now we consider possible modifications of the quarkonium part of the amplitude (2.1). Novikov and Shifman[10] suggested that the multipole expansion may break down near threshold. To get an idea of the magnitude of the omitted corrections we shall review the derivation of (2.1). This derivation is almost entirely analogous to the case of electromagnetic transitions.[18] However, some special features arise in connection with the presence of the soft-pion part of the amplitude, and we would like to make clear all of the terms that have been omitted in the standard formula (2.1).

Since we have decided to ignore $O(\alpha_s)$ corrections to the soft-pion amplitude (2.3), we shall similarly restrict ourselves to first order in α_s, in the quarkonium matrix element. One begins with the Hamiltonian of a quark and an antiquark in an external gluonic field:

$$\mathcal{H}_{\rm int}=gA_0^a(\mathbf{r}_1)t_1^a+gA_0^a(\mathbf{r}_2)t_2^a+O(v^2/c^2)\ , \tag{3.1}$$

where $t_1^a=\frac{1}{2}\lambda_1^a$ and $t_2^a=-\frac{1}{2}\lambda_2^a$, with λ_1^a and λ_2^a being Gell-Mann matrices. The nonleading terms in (4.1) are $O(v^2/c^2)$ corrections, i.e., are suppressed by powers of the quark mass. These lead to corrections to (2.1) of the same order, and it is reasonable to neglect them since in the Υ system $v/c\sim\frac{1}{10}$.

The remaining correction will come from the expansion in kr where $k=|\mathbf{k}|$ is the magnitude of a gluon three-momentum and r is the radius of the quarkonium system. (Recall that the multipole expansion is based on the requirement $kr<1$.) As a numerical estimate, k will have a maximum value, near threshold, of $k_{\rm max}\approx\frac{1}{2}\Delta$, where Δ is the difference in mass between the initial and final quarkonium states, and r can be estimated as the average radius $\langle r\rangle$ of the Υ states in a potential model. (The kinematic regime where the dipole approximation breaks down is for small dipion mass, since small dipion mass is correlated with large three-momentum of one or both pions.)

A typical potential model[17] gives $\langle r\rangle=2.3\ {\rm GeV}^{-1}$ for the $\Upsilon(2S)$ and $\langle r\rangle=3.5\ {\rm GeV}^{-1}$ for the $\Upsilon(3S)$. Using the masses $M(1S)=9.46$, $M(2S)=10.02$, $M(3S)=10.355$ GeV, this means that $k_{\rm max}\langle r\rangle\sim1.5$ for the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ decay, suggesting that this multipole-expansion correction may be very important for this decay and of less importance for the others. Moreover the correction would be largest near threshold where the three-momentum is greatest.

To derive the explicit form of the corresponding correction, we assume the gluonic field has spatial dependence $A_0^a(\mathbf{r}_1)\simeq A_0^a(0)e^{i\mathbf{k}\cdot\mathbf{r}_1}$, $A_0^a(\mathbf{r}_2)\simeq A_0^a(0)e^{i\mathbf{k}\cdot\mathbf{r}_2}$, where $\mathbf{k}$ is the three-momentum of the emitted gluon. For equal masses the coordinates are given in terms of the relative coordinate $\mathbf{r}$ by $\mathbf{r}_1=-\mathbf{r}_2=\frac{1}{2}\mathbf{r}$. Making these substitutions, the Hamiltonian is

$$\mathcal{H}_{\rm int}=g\cos(\tfrac{1}{2}\mathbf{k}\cdot\mathbf{r})A_0^a(0)t^a+ig\sin(\tfrac{1}{2}\mathbf{k}\cdot\mathbf{r})A_0^a(0)\xi^a\ , \tag{3.2}$$

where $t^a=t_1^a+t_2^a$ and $\xi^a=t_1^a-t_2^a$. The operator t^a annihilates a color-singlet state, however, so since we are working only to order $\alpha_s\sim g^2$, we discard the term proportional to t^a, which cannot contribute to this order. Then

$$\begin{aligned}\mathcal{H}_{\rm int}&=ig\sin(\tfrac{1}{2}\mathbf{k}\cdot\mathbf{r})A_0^a(0)\xi^a\\&=\frac{g}{2}\sum_{l={\rm odd}}(2l+1)i^lj_l(\tfrac{1}{2}kr)P_l(\mathbf{k}\cdot\mathbf{r}/kr)A_0^a(0)\xi^a\\&\approx-\frac{3igj_1(\frac{1}{2}kr)}{2kr}\mathbf{k}\cdot\mathbf{r}A_0^a(0)\xi^a+\cdots\ ,\end{aligned} \tag{3.3}$$

where j_l are spherical Bessel functions and P_l are Legendre polynomials, and we see that only P-,F-, . . . , wave states contribute to the sum over intermediate states.

If we assume that only P waves contribute, i.e., we take just the first term in the equation above, then the angular integrations leave $j_1(\frac{1}{2}kr)$ in the remaining integrands. Then $ik_iA_0^a(0)/2=E_i^a$ and we recover the interaction $\mathcal{H}_{\rm int}=-\frac{1}{2}g\mathbf{r}\cdot\mathbf{E}^a\xi^a$ which was used to derive (2.1). Just as in (2.1), the required hadronic amplitude is $\langle\pi^+\pi^-|\pi\alpha_sE_i^aE_j^a|0\rangle$, for which the soft-pion approach supplies the expression (2.3).

For the next term in (3.3), corresponding to F-wave octet states, each matrix element would include some terms contributing additional powers of k_i, so that we would require an amplitude such as $\langle\pi^+\pi^-|\partial_iE_j^a\partial_kE_l^a|0\rangle$. Such a term is impossible to obtain in the soft-pion approach which only permits calculation of amplitudes quadratic in the momenta. Thus, the necessity of using the soft-pion approach restricts us to using the first, i.e., dipole, term in (3.3). However, it is likely that the overlap of the color-singlet S waves with the color-octet F (and higher) waves is very small, due to the centrifugal barrier.

We conclude that, as far as the interaction Hamiltonian is concerned, we are probably not omitting any significant effects if we restrict ourselves to the first term in (3.3), i.e., the "double-dipole" approximation (2.1), but with the radius r replaced by a Bessel function:

$$r\to\frac{6}{k}j_1(\tfrac{1}{2}kr)\approx r-\frac{k^2r^3}{40}+\cdots\ . \tag{3.4}$$

The resulting momentum dependence will mostly affect the spectrum near threshold, where k is large.

Thus, although Ref. 10 claimed that the multipole expansion breaks down near threshold, we see that under reasonable assumptions about the intermediate-state wave functions it is accurate to use the following form for the quarkonium part of amplitude of the transition $\Upsilon(nS)\to\Upsilon(mS)\pi^+\pi^-$:

$$\sum_{\rm octet}\frac{\langle 1S|j_1(\frac{1}{2}k_2r)/k_2|8,P\rangle\langle 8,P|j_1(\frac{1}{2}k_1r)/k_1|nS\rangle}{M-M_{8,P}-\epsilon_1}+(1\leftrightarrow2)\ , \tag{3.5}$$

where we have taken account of the fact that the intermediate state in (2.2) in fact contains a gluon, so that a gluon energy must appear in the denominator. Our point is that the multipole expansion itself does not present an intractable problem; the difficulty is our relative ignorance of the intermediate-state spectrum and wave functions.

The expansion (3.4) suggests that a reasonable scenario for the transition $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ might be as follows. The overlap integrals of r between, say, the $\Upsilon(3S)$ and the intermediate states could turn out to be unexpectedly small, perhaps due to the zeros in the $3S$ wave function. This would cause the total rate to be small, as is observed, and the correction term in (3.4) might dominate, perhaps enhancing the spectrum near threshold (depending, e.g., on the sign of the overlap of r^3 between the initial and intermediate states).

We have found, however, that the $M_{\pi\pi}$ dependence resulting from (3.4) is almost certainly too weak to counteract the effect of the Adler zero. The Bessel function simply does not vary sufficiently rapidly over the ranges of k and r relevant to the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ decay. In particular, we studied the effect of the various momentum factors in the amplitude (3.5) in the approximation, used in Sec. II to estimate the total rates, in which the nonlocality of the Green's function is neglected, and found that the shape of the spectrum was modified by at most a few percent. This is reasonable, since, using the estimate for kr made above, we find that the second term in the expansion in (3.4) amounts to a 5% correction for the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ decay. In principle, one might be able to contrive an explicit model for the intermediate state wave functions that enhance the decay rate more strongly near threshold than our simple model, but we have been unable to do so. In the absence of a really convincing model for the intermediate states, we shall ignore the multipole-expansion corrections in the remainder of this paper, but it should be borne in mind that these will be most important at low $M_{\pi\pi}$.

IV. $\pi^+\pi^-$ FINAL-STATE INTERACTIONS

Now we turn to the effect of the interaction of the pions in the final state. Since we are neglecting the $O(\alpha_s)$ correction in (2.3) the amplitude is

$$A=\frac{2I}{9}\langle\pi^+\pi^-|\pi\alpha_s\mathbf{E}^a\cdot\mathbf{E}^a|0\rangle\ , \tag{4.1}$$

where the hadronic amplitude is modified as follows:

$$\langle\pi^+\pi^-|\pi\alpha_s\mathbf{E}^a\cdot\mathbf{E}^a|0\rangle =F(q^2)\langle\pi^+\pi^-|\pi\alpha_s\mathbf{E}^a\cdot\mathbf{E}^a|0\rangle_{\text{soft-pion}}\ . \tag{4.2}$$

Here we shall assume that the form factor $F(q^2)$ is the same as for the pion-pion scattering amplitude:

$$T(q^2)=F(q^2)t(q^2)\ , \tag{4.3}$$

where $t(q^2)=(8\pi f_\pi^2)^{-1}q^2$ is the current-algebra amplitude[19] ($f_\pi=0.094$ GeV is the pion decay constant) and $T(q^2)$ is the amplitude with final-state interactions taken into account, to be determined using phase-shift data. Here we are concerned only with the isoscalar, S-wave channel; this is obvious since (4.1) comes from the amplitude $\langle\pi^+\pi^-|\theta^G_{\mu\mu}|0\rangle$. The data on low-energy $\pi^+\pi^-$ interactions rule out[12] the possibility of any new resonances as the cause of the peculiar $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum; nevertheless, the pions will scatter elastically off each other and this may affect the shape of the $\pi^+\pi^-$ spectrum.

Data on the S-wave, isoscalar phase shift δ essentially fix $T(q^2)$, given an appropriate parametrization that obeys general requirements such as unitarity and analyticity. We shall adopt the parametrization of Mennessier,[20] since it has a relatively simple analytic form. Since we only need the phase shift at energies less than about 1 GeV, we shall use the three-parameter form, analogous to "model A" in Ref. 19: namely,

$$T(q^2)=\frac{g^2_{\sigma\pi\pi}+\lambda(m^2_\sigma-q^2)}{(m^2_\sigma-q^2)[1-\lambda\zeta(q^2)]-g^2_{\sigma\pi\pi}\zeta(q^2)}\ , \tag{4.4}$$

where λ is the pion self-coupling, $g_{\sigma\pi\pi}$ is the coupling of the pions to a fictitious "sigma" resonance of bare mass m_σ, and the function $\zeta(q^2)$ is

$$\zeta(q^2)=\frac{2}{\pi}\left[1-\left[1-\frac{4m^2_\pi}{q^2}\right]^{1/2}\times\left[\ln\frac{\sqrt{q^2}+(q^2-4m^2_\pi)^{1/2}}{2m_\pi}-\frac{i\pi}{2}\right]\right]\ . \tag{4.5}$$

Equation (4.3) was derived simply by writing down a theory with a ϕ^4 pion-pion coupling and a $\sigma\pi\pi$ coupling, computing one-loop diagrams, and summing leading logarithms in the usual way. Since we know such a theory to be unitary the resulting amplitude (4.3) will obviously satisfy our unitarity requirements. This procedure can be generalized by including an arbitrary number of resonances; for example, by including a second resonance as well as couplings to kaons[19] one can model the rise in the phase shift at energies above ~ 1 GeV. Our suggestion is then to use the same postulated interactions to "tie together the pion lines" in the Feynman diagram corresponding to the quarkonium decay, with the result (4.1).

We have obtained a fit (Fig. 2) to selected phase-shift data[20,21] using the values $\lambda=-0.73$, $g=0.64$ GeV, $m_\sigma=0.71$ GeV. Obtaining the form factor $F(q^2)$ through (4.2), we have, from (4.1),

$$\langle\pi^+\pi^-|\pi\alpha_s\mathbf{E}^a\cdot\mathbf{E}^a|0\rangle=\frac{16\pi^3f^2_\pi}{9}T(q^2)\ . \tag{4.6}$$

The spectra can be computed as before, and the results are shown in Fig. 3. Shown are the exclusive CLEO data.[3,4] The final-state interactions have only a small effect on the $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum. The effect on the transition $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ is somewhat larger, but certainly not large enough to give a peak at low $M_{\pi\pi}$. The total rate for the decay $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$ is increased by a factor ~ 2.6, corresponding to a constant $G_0\approx 1.7\ \text{GeV}^{-1}$, in excellent

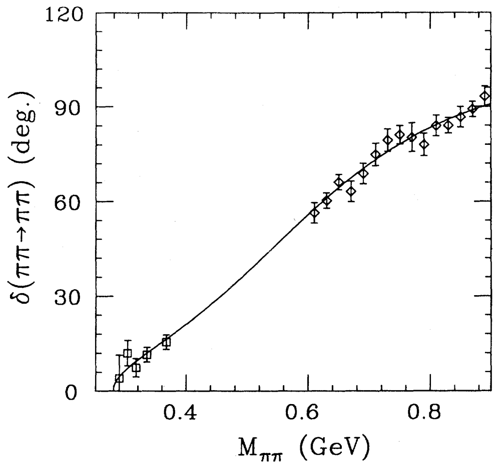

FIG. 2. Fit to S-wave, isoscalar, phase-shift data from Ref. 21 (diamonds) and Ref. 22 (squares). The curve is the phase shift derived from our Eq. (4.4).

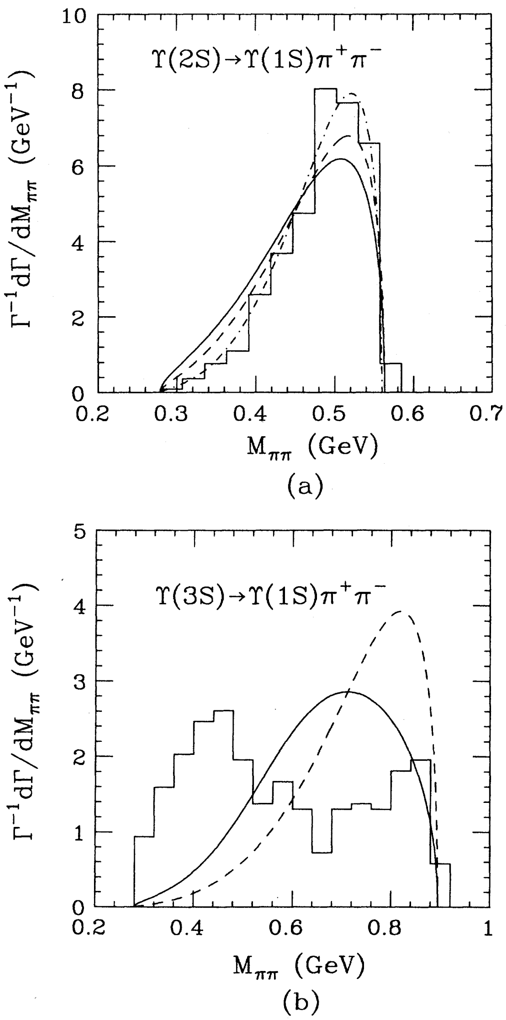

FIG. 3. Spectra in the soft-pion model without (dashed curve) and with (solid curve) final-state interaction of the pions: (a) $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$, (b) $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$. The dot-dashed line in (a) shows the effect of a phenomenological threshold factor (see text) with parameter $C=2.2$.

agreement with the estimate made in Sec. II. As might be expected, though, the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ rate is increased by an even larger factor ~ 6, making even worse the problem of the small observed total rate for this decay.

We mention here that to fit the $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$ data in this model it is necessary to incorporate an additional correction factor $1-Cm_\pi^2/q^2$ that mostly affects the shape of the spectrum near threshold. Figure 3(a) shows the effect of taking $C=2.2$. There are several theoretical models for the values of parameter C (see the discussion in Ref. 11), but none of them are really convincing. Hence, in what follows we shall omit this phenomenological factor and just look for qualitative agreement with the observed spectra.

To avoid confusion, we mention that our model for the $\pi^+\pi^-$ interaction has been selected for its simplicity, in order to see the magnitude of the effect on the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum, and is by no means meant to be definitive. In particular, the sigma resonance is just a means of parametrizing the amplitudes (4.1) and (4.2), and is not being put forth as a new physical state. The problem of determining the resonance content of the $J=0$, $I=0$ channel is a subtle one, and has recently been discussed in detail by Au, Morgan, and Pennington.[23] In principle, any model of pion physics which supplies an amplitude $T(q^2)$ can be used in (4.5).

V. $\Upsilon\pi$ FINAL-STATE INTERACTIONS

Next we turn to the other possibility for final-state interaction: namely, the interaction between the Υ and one of the outgoing pions. This has been discussed by Voloshin,[13] who suggested that the shape of the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum is due to the existence of one or more resonances in the $\Upsilon\pi$ channel. Let us assume for simplicity that there is only one resonance and give it a mass $M_{\rm res}$ and a width Γ. We mention that, although the $\pi^+\pi^-$ pair is in an S wave, each pion may be in either an S or a P wave. The squared amplitude has the form

$$|A|^2\sim\left|\frac{T(q^2)/q^2}{M-(M_{\rm res}-i\Gamma/2)-\epsilon_1}+(1\leftrightarrow 2)\right|^2 f(\epsilon_1,\epsilon_2)\;, \tag{5.1}$$

where ϵ_1,ϵ_2 are the pion energies. If the pions are emitted individually in S waves, then $f(\epsilon_1,\epsilon_2)=(\epsilon_1\epsilon_2)^2$; if they are emitted individually in P waves, then $f(\epsilon_1,\epsilon_2)=(\epsilon_1^2-m_\pi^2)(\epsilon_2^2-m_\pi^2)$ (Ref. 13). In distinction from Voloshin's treatment, we have included the effect of the $\pi^+\pi^-$ interaction, which is always present.

If the resonance mass is much greater than the mass of either quarkonium state, then the matrix element is approximately constant and the decay spectrum reduces to the shape given by the standard soft-pion result plus $\pi^+\pi^-$ final-state interactions. However, in the case $M-M_{\rm res}>0$ interesting structure can appear. If the real part of either denominator can actually vanish, then the decay $\Upsilon(3S)\to(\pi+\text{resonance})$ can occur; the signal for this decay could be a monochromatic pion if the resonance were sufficiently narrow and if the $\pi^+\pi^-$ final-state interactions did not distort the dipion spectrum much.

The presumed nonobservation of monochromatic pions restricts the resonance to be heavier than $M(3S)-M_{\text{res}}\approx 10.21$ GeV. [In any event, the resonance could be produced by monochromatic pion emission from the $\Upsilon(4S)$ state; Voloshin estimates that the corresponding branching ratio is 0.05%.] However, if the resonance is only slightly above this threshold its effect will still be seen as the structure in the Dalitz plot involving the variable $M^2_{\Upsilon\pi}$. The effect of the resonance is seen only indirectly in the distribution with respect to $M_{\pi\pi}$ that is usually plotted. In addition, final-state $\pi^+\pi^-$ interactions tend to smooth out the effects of a resonance in the dipion mass spectrum.

We can generate a dipion spectrum that peaks at low energy via this mechanism if we assume that the resonance is light and narrow, i.e., with a width of tens of MeV similar to that of the $b\bar{b}$ states below flavor threshold. As an example, in Fig. 4 we show the dipion spectrum in the case of S-wave pion emission, with the parameter choices $M_{\text{res}}=10.213$ GeV, $\Gamma=10$ MeV; if we also include final-state effects we get a double-peaked spectrum that qualitatively resembles the observed $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum. The $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum is not greatly modified by including the $\Upsilon\pi$ resonance. We have not made an exhaustive search of parameters of a possible new resonance; this mass and width are chosen strictly for purposes of illustration. The low-energy peak occurs at slightly higher energy than in the data. (There is no way we can model the sharp rise of the data at low $M_{\pi\pi}$ and maintain a vestige of the soft-pion picture.)

The $\Upsilon(3S)\to\Upsilon(2S)\pi^+\pi^-$ spectrum does not differ much from the prediction of the standard model (Fig. 5). The data on this transition are of rather low statistics, due to the small phase space, but the most recent data[4] suggest that this transition may also have a peculiar spectrum. If this is so, then the standard picture is ruled out for this decay, since the remaining corrections (e.g., from the multipole expansion) are guaranteed to be negligible for this decay.

The resonance shows up much more sharply in the spectrum with respect to the invariant mass of the Υ and one pion, which we plot in Fig. 6. As the resonance broadens, or as M_{res} increases, its effect on both the dipion spectrum and on the $\Upsilon\pi$ spectrum fade away. For example, taking $\Gamma=40$ MeV produces an essentially flat spectrum in $M^2_{\Upsilon\pi}$ and a $M_{\pi\pi}$ spectrum again peaked at large dipion mass. Thus besides providing a second, high-energy, peak, the $\pi^+\pi^-$ interaction restricts the ranges of values of M_{res} and Γ in Voloshin's model that can result in a low-energy peak. Unfortunately, the low-energy peak in this model appears at slightly higher $M_{\pi\pi}$ than in the observed spectrum, so that a quantitative fit to the data is not possible; this could be due neglected corrections that are most important near threshold.

The possibility of a resonance in the $\Upsilon\pi$ channel is easily testable by looking at the distribution of events in

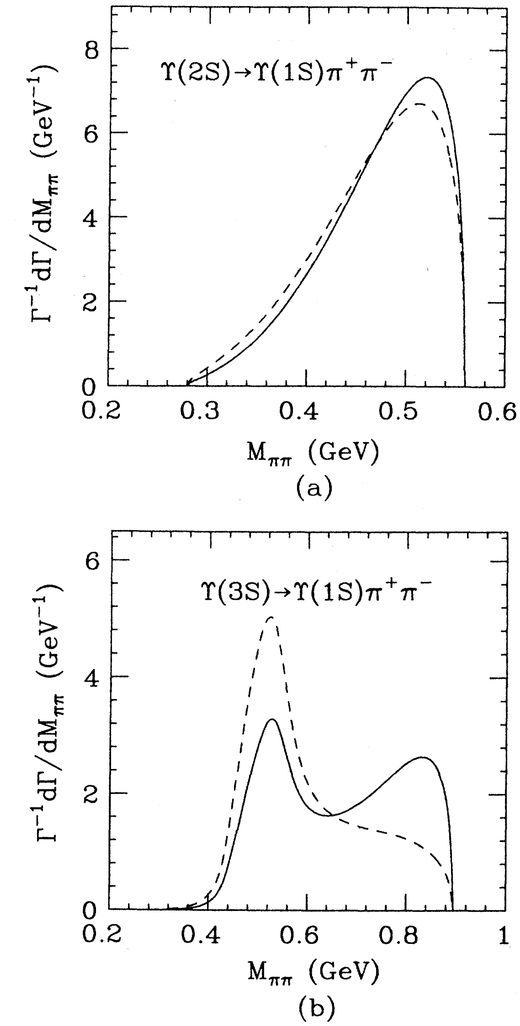

FIG. 4. Spectra in the case of an $\Upsilon\pi$ resonance at $M_{\text{res}}=10.213$ GeV and width 10 MeV, without (dashed curve) and with (solid curve) final-state interactions: (a) $\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-$, (b) $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$.

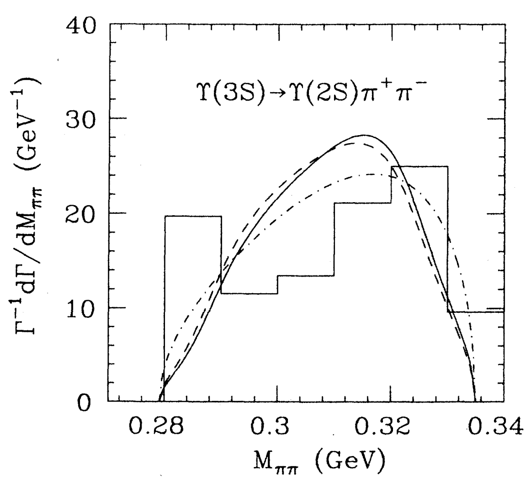

FIG. 5. Spectrum for $\Upsilon(3S)\to\Upsilon(2S)\pi^+\pi^-$, in the case of the resonance, without (dashed curve) and with (solid curve) final-state interactions; the dot-dashed line is the standard soft-pion result.

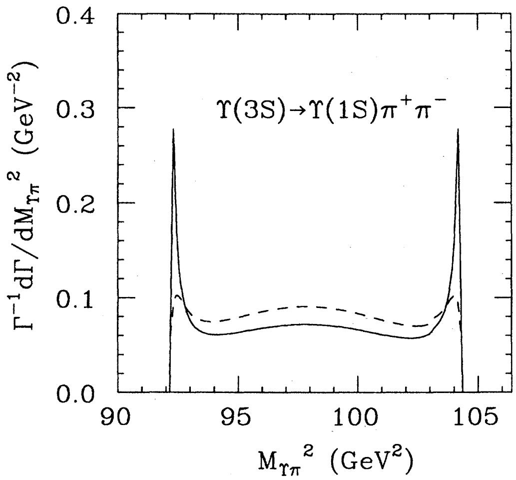

FIG. 6. Distribution in $M^2_{\Upsilon\pi}$ in the case of the $\Upsilon\pi$ resonance at $M_{\rm res}=10.213$ GeV. Solid line: $\Gamma=10$ MeV; dashed line: $\Gamma=40$ MeV.

$M^2_{\Upsilon\pi}$, especially for low $M^2_{\pi\pi}$. This is shown in Fig. 6. Equivalently one could look at the single-pion spectrum,[13] that is, $d\Gamma/d\epsilon_\pi$. (If recoil is neglected then $M^2_{\Upsilon\pi}\approx M^2_\Upsilon+m^2_\pi+2M_\Upsilon\epsilon_\pi$.) The case shown in Fig. 6 is an extreme one; the model we used for the $\pi^+\pi^-$ interaction forced us to take a very narrow width for the resonance. The observation of a statistically significant peaking at the upper and lower ends of the $M^2_{\Upsilon\pi}$ spectrum would be a test for the presence of an $\Upsilon\pi$ resonance, independent of our particular model.

VI. DISCUSSION

We have explored various modifications of the standard gluon-multipole-plus-current-algebra picture for decays of Υ excited states. Pion final-state interactions alone cannot be the source of the discrepancy between theory and experiment. The only alternative, within the standard framework, that can account for the observed dipion spectrum in the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ transition is nontrivial behavior in the $\Upsilon\pi$ channel due to the presence of a four-quark, isovector state[13] with mass close to $M(3S)-m_\pi\approx 10.21$ GeV. We have shown that a second, high-energy peak appears when the $\pi^+\pi^-$ interaction is taken into account. Besides explaining the $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$ spectrum, the existence of such a state would be of great intrinsic interest. It would be the first unambiguous observation of a four-quark, $q\bar{q}q\bar{q}$ state. On the other hand, if no structure is observed in the $\Upsilon\pi$ channel (i.e., if the corresponding distribution is found to be consistent with phase space) then the multipole-soft-pion picture will have to be abandoned for this decay.

It may in fact be the case that the multipole amplitude is overwhelmed by some nonmultipole mechanism for the decay $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$, as in the model of Lipkin and Tuan.[14] These authors propose, as an alternative to Υ decay by sequential gluon emission, the decay path $\Upsilon\to B\bar{B}\to B^*\bar{B}\pi\to B\bar{B}\pi\pi\to\Upsilon\pi\pi$. They essentially ignore all soft-pion results so their quarkonium amplitude does not contain the Adler zero. Any such model, though, will remain unsatisfying unless it gives a quantitative explanation of why the multipole couplings are dominated by some other mechanism only for the decay $\Upsilon(3S)\to\Upsilon(1S)\pi^+\pi^-$.

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