Critical line and dilaton in scale-invariant QED

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We have found a novel spontaneous-chiral-symmetry-breaking solution to the ladder Schwinger-Dyson equation for QED plus a chiral-invariant four-fermion interaction. The critical line is explicitly obtained in the plane of two coupling constants of gauge and four-fermion interactions. The existence of a dilaton pole has also been examined on the full critical line.

There has recently been renewed interest in the phase structure of QED in the context of the possible existence of a nontrivial ultraviolet fixed point which will describe a sensible interacting gauge theory free of the fear of Landau's ghost.[1] Actually, based on the spontaneous-chiral-symmetry-breaking (CSB) solution to the ladder Schwinger-Dyson (SD) equation which was revealed by Maskawa and Nakajima[2] and subsequently reexamined by Fukuda and Kugo[3] for the coupling constant $\alpha \equiv e^2/4\pi$ larger than a certain critical coupling constant $\alpha_c = \pi/3$, Miransky and others[4,5] proposed that this critical coupling constant should be regarded as precisely the nontrivial ultraviolet fixed point mentioned above.

Furthermore the dynamics of this fixed-point theory was found by one of the authors (K.Y.), Bando and Matumoto[6] to be able to resolve the long-standing notorious problem of excessive flavor-changing neutral currents (FCNC's) in technicolor (TC) theories.[7] Actually, this dynamical model (scale-invariant TC model) seems to be the only TC that survives the FCNC's syndrome.[8]

Then the problem is whether or not these new features of scale-invariant QED based on the ladder SD equation persist beyond the ladder (quenched planar) approximation. Quite recently, Monte Carlo studies of lattice QED (Ref. 9) have indeed signaled the existence of the nontrivial ultraviolet fixed point, both in noncompact and compact versions including dynamical fermion loops, which strongly suggest that much of the above result may not be a mere artifact of the ladder approximation.

Thus it is extremely interesting to analyze the dynamical issues of the possible nontrivial ultraviolet fixed point in QED and other asymptotically *nonfree* gauge theories, which may provide us with a completely new basis for building unified theories.

One such problem is the dilaton, a Nambu-Goldstone boson associated with the spontaneous breakdown of the expected scale invariance at the fixed point. Bardeen and co-workers[5] unsuccessfully attempted to observe the dilaton pole in the $J^{PC}=0^{++}$ channel of the fermion-antifermion scattering amplitude within the framework of the ladder SD equation for QED now including a chiral-invariant four-fermion interaction. They actually solved the SD equation only for $\alpha > \alpha_c$, which is not enough to conclude the absence of the dilaton in this model, however. If the dilaton exists in such a system, the scale-invariant TC model predicts a technidilaton[6] whose phenomenological signatures would be outstanding in TeV physics.[10]

In this paper we shall present a full set of spontaneous CSB solutions $\Sigma(p^2)$ to the ladder SD equation for QED plus the chiral-invariant four-fermion interaction for $\alpha \le \alpha_c$ as well as $\alpha > \alpha_c$, by which we discover the full critical line in the whole two-dimensional parameter space of the gauge and the four-fermion coupling constants. This is indeed the line where the fixed point, if any, exists. Then we look for the dilaton pole on this line to identify the scale-invariant fixed point. We find no dilaton pole on the full critical line.

Following Bardeen and co-workers,[5] we shall start with the ladder SD equation for the dynamical fermion mass $\Sigma(p^2)$, which is written in Euclidean space in the Landau gauge as

$$\Sigma(x) = \frac{g}{\Lambda^2}\int_{\epsilon^2}^{\Lambda^2} dy \frac{y\Sigma(y)}{y+\Sigma(y)^2} + 3\lambda \int_{\epsilon^2}^{\Lambda^2} dy \frac{\Sigma(y)}{y+\Sigma(y)^2}\left[\frac{y}{x}\theta(x-y)+\theta(y-x)\right], \tag{1}$$

where $g \equiv G\Lambda^2/4\pi^2$ and $\lambda \equiv \alpha/4\pi$, with G being the coupling constant of the chiral-invariant four-fermion interaction $(G/2)[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]$, and Λ and ϵ are, respectively, the ultraviolet and infrared cutoffs.

We solve Eq. (1) both analytically and numerically in the whole parameter space (λ, g). Analytical solutions are obtained by the bifurcation technique:[11] we convert[3] (1) into a differential equation whose general solutions are given by

$$\Sigma(x) = \begin{cases} Ax^{-1/2+\sigma} + Bx^{-1/2-\sigma} \quad (0<\lambda<\lambda_c)\,, & (2)\\ x^{-1/2}(C+D\ln x) \quad (\lambda=\lambda_c)\,, & (3)\\ Ex^{-1/2+i\rho}+Fx^{-1/2-i\rho} \quad (\lambda>\lambda_c)\,, & (4)\end{cases}$$

where $\sigma \equiv \frac{1}{2}\sqrt{1-\lambda/\lambda_c}$, $\rho \equiv \frac{1}{2}\sqrt{\lambda/\lambda_c - 1}$, and $A, B, \ldots, F$ are the constants to be determined by the infrared and ultraviolet boundary conditions. A spontaneous CSB solution exists only when both boundary conditions are satisfied, which determines a single line in the

(λ,g) plane for fixed Λ/ϵ:

$$g=[(\tfrac{1}{2}+\sigma)^2-(\tfrac{1}{2}-\sigma)^2(\epsilon^2/\Lambda^2)^{2\sigma}]/[1-(\epsilon^2/\Lambda^2)^{2\sigma}] \quad (0<\lambda<\lambda_c)\ , \tag{5}$$

$$g=\tfrac{1}{4}\{1+4/[\ln(\Lambda^2/\epsilon^2)]\} \quad (\lambda=\lambda_c)\ , \tag{6}$$

$$\ln(\Lambda^2/\epsilon^2)=\{n\pi+\arctan[\rho/(g-\tfrac{1}{4}+\rho^2)]\}/\rho \quad (\lambda>\lambda_c)\ , \tag{7}$$

which separates the spontaneously broken and unbroken phases of the chiral symmetry.[12] This is the generalization of the Miransky's scaling[4] in pure QED ($g=0$, $\lambda>\lambda_c$). While Eq. (7) is essentially the same as the result of Bardeen and co-workers,[5] (5) and (6) are indeed novel solutions which can exist only when the four-fermion coupling is included ($g\neq 0$). The existence of such spontaneous CSB solutions for $\lambda\le\lambda_c$ was overlooked in Ref. 5.

Then taking the limit $\Lambda/\epsilon\to\infty$ of Eqs. (5)–(7), we find the critical line

$$g=\tfrac{1}{4}(1+\sqrt{1-\lambda/\lambda_c})^2 \quad \text{for } \lambda\le\lambda_c\ , \tag{8}$$

as well as the previous result,[5] $g<\frac{1}{4}(\lambda=\lambda_c)$. This is our main result. In fact, the critical line is the ingredient essential to the study of the phase structure: the fixed point must lie on the critical line.[13]

We also obtained numerical solutions for the full nonlinear gap equation (1), which is depicted in Fig. 1, in agreement with the analytical solutions. The point $(\lambda,g)=(0,1)$ corresponds to the critical coupling of the Nambu–Jona-Lasinio model and the point $(\frac{1}{12},0)$ to the critical gauge coupling obtained earlier.[2–4]

We now look for the ultraviolet fixed point where the dilaton pole is expected to exist on the critical line. Actually, for $\epsilon=0$, Eq. (1) is invariant under the scale transformation $\Sigma(p^2)\to\kappa\Sigma(p^2/\kappa^2)$ if the change of the cutoff

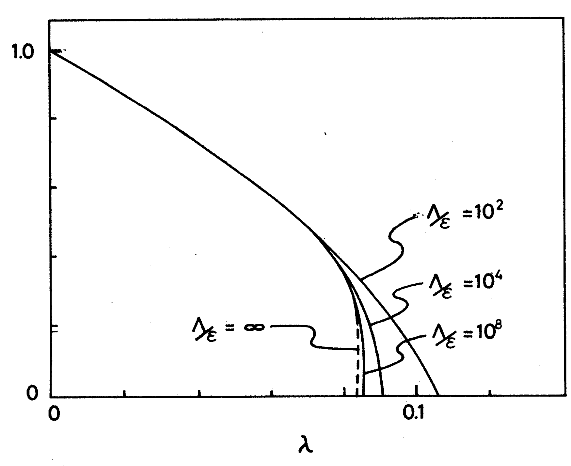

FIG. 1. "Critical lines" for various values of cutoff. Results for finite cutoff are obtained by numerically solving the ladder SD equation (1) in its nonlinear form. In the infinite cutoff limit these lines approach the full critical line, solid line (8), and dashed line (for $g<\frac{1}{4}$).

$\Lambda\to\kappa\Lambda$ is simultaneously performed as would be trivial in the $\Lambda\to\infty$ limit. In the case of $\epsilon=0$, the solution of SD equation (1) takes the form

$$\Sigma(p^2)=e^t u(t+t_0),\quad t\equiv\ln p,\quad t_0\equiv-\ln\Sigma(0)\ , \tag{9}$$

where $\Sigma(0)$ plays the role of ϵ in the above.[14] The expression of the renormalized scalar denominator of the fermion-antifermion scattering amplitude at zero momentum is given in Ref. 5,

$$D_S^R(0)\equiv[1+GB_S^0(0)]Z_S^2/G\ , \tag{10}$$

where $B_S^0(0)$ is the scalar bubble function at zero momentum defined by

$$B_S^0(0)\equiv -i(2\pi)^{-4}\int d^4p\,\mathrm{Tr}\{[\not p-\Sigma(p^2)]^{-1}\Gamma_S^0(p,p)\times[\not p-\Sigma(p^2)]^{-1}\}\ , \tag{11}$$

and Z_S is the renormalization constant of the scalar vertex function $\Gamma_S^0(p,p)$:

$$\Gamma_S^0(p,p)=-e^{(t+t_0)}u'(t+t_0)/Z_S\ , \tag{12}$$

$$Z_S=-\tfrac{1}{2}e^{(t_\Lambda+t_0)}[u''(t_\Lambda+t_0)+3u'(t_\Lambda+t_0)],\quad t_\Lambda\equiv\ln\Lambda\ . \tag{13}$$

The exact expression for $B_S^0(0)$ may be obtained:[15]

$$B_S^0(0)=(\lambda_c/\lambda)(\Lambda^2/\pi^2)[u''(t_\Lambda+t_0)+u'(t_\Lambda+t_0)]\times[u''(t_\Lambda+t_0)+3u'(t_\Lambda+t_0)]^{-1}\ , \tag{14}$$

which yields[15]

$$D_S^R(0)=\frac{1}{2\pi^2\Sigma(0)^2}\left[1+\frac{6\lambda}{g}\,\frac{g+3\lambda}{1+[\Sigma(\Lambda^2)/\Lambda]^2}\times S(\lambda,g;\Lambda)[\Lambda\Sigma(\Lambda^2)]^2\right]\ , \tag{15}$$

$$S(\lambda,g;\Lambda)\equiv\frac{1}{1+[\Sigma(\Lambda^2)/\Lambda]^2}-\frac{g}{(g+3\lambda)^2}\ . \tag{16}$$

The solutions (2)–(4) (more explicitly Refs. 12 and 14) are substituted into (15) and (16), resulting in the vanishing $S(\lambda,g;\Lambda)$ and diverging $[\Sigma(\Lambda^2)/\Lambda]^2$ on the critical line ($\Lambda\to\infty$), both of which precisely cancel each other to yield the finite scalar denominator:

$$D_S^R(0)=\frac{1}{2\pi^2}[1+6\lambda(1+3\lambda/g)]\Sigma(0)^2\neq 0\ .$$

No dilaton pole exists in accord with the conclusion of Bardeen and co-workers,[5] but now on the whole critical line.

Bardeen and co-workers identified the point $(\lambda,g)=(\frac{1}{12},\frac{1}{4})$ with an "ultraviolet fixed point," but failed to observe the dilaton pole there. Actually they solved the SD equation only in the strong-coupling phase ($\lambda>\lambda_c$) and approached this point in a peculiar direction, $g=\frac{1}{4}$ and $\lambda\downarrow\lambda_c$, which is not *a priori* justified. Correct direction should be identified to be consistent with the renormalization-group flow[15] [requiring the scaling law of $\Sigma(p^2)$ near the critical line, we obtain the upward vertical direction as the renormalization-group flow]. Here we have solved the SD equation in the whole region of the (λ,g) plane, and found that the dilaton pole does not exist not only on the particular point $(\lambda,g)=(\frac{1}{12},\frac{1}{4})$ but also on the whole critical line, no matter which direction we may take to approach the critical line. Our result of no dilaton pole agrees with the general argument based on the vacuum energy.[16]

In conclusion we have found the full critical line of QED plus four-fermion interaction in the quenched planar approximation. Although we found no scale-invariant fixed point signaled by the dilaton pole, our critical line is certainly the first step to reveal the nontrivial phase structure of this system, i.e., renormalization-group flow, fixed points, anomalous dimension, etc.

Note added. After submitting the original version of our manuscript, we were informed that the same critical line (8) was also obtained independently by T. Appelquist, M. Soldate, T. Takeuchi, and L. C. R. Wijewardhana [Yale University Report No. YCTP-P19-88, 1988 (unpublished)]. The same conclusion of the dilaton pole as that in our revised version has also been obtained by T. Nonoyama, T. B. Suzuki, and K. Yamawaki [Nagoya University Report No. DPNU-89-09, 1989 (unpublished)], C. N. Leung, S. T. Love, and W. A. Bardeen [Purdue University/Fermilab Report No. PURD-TH-89-01/FERMILAB-PUB-89/22-T (unpublished)], and by V. P. Gusynin and V. A. Miransky (private communication). The importance of our critical line has recently been emphasized by V. A. Miransky and K. Yamawaki [Mod. Phys. Lett. **A4**, 129 (1989)], which demonstrated a very large anomalous dimension $(2>\gamma_m>1)$ on the critical line $(0<\lambda<\lambda_c)$.

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[7]Suppressing FCNC's by the large anomalous dimension in the TC with an ultraviolet fixed point was first considered by B. Holdom, Phys. Rev. D **24**, 1441 (1981), and subsequently by H. Georgi and S. L. Glashow, Phys. Rev. Lett. **47**, 1511 (1981), and by K. Yamawaki and T. Yokota, Nucl. Phys. **B223**, 144 (1983). All of them, however, simply assumed *ad hoc* the existence of a nontrivial ultraviolet fixed point and a large anomalous dimension, in sharp contrast to Ref. 6 which explicitly demonstrated the existence of such a TC theory with a concrete value of the anomalous dimension $(\gamma_m=1)$ in the framework of the ladder SD equation. The suppression mechanism similar to Ref. 6 has also been considered by T. Akiba and T. Yanagida, Phys. Lett. **169B**, 432 (1986) within the *cutoff version* (Refs. 2 and 3) of the ladder SD equation.

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[11]The SD equation (1) has a trivial solution $\Sigma(x)\equiv 0$ for any λ and g. Hence, in order to show the existence of spontaneous CSB, it is sufficient to consider the bifurcation solution from the trivial one, which satisfies the equation

$$\Sigma(x)=\frac{g}{\Lambda^2}\int_{\epsilon^2}^{\Lambda^2}dy\,\Sigma(y)+3\lambda\int_{\epsilon^2}^{\Lambda^2}dy\,\Sigma(y)\left[\frac{1}{x}\theta(x-y)+\frac{1}{y}\theta(y-x)\right].$$

For details, see D. Atkinson and P. W. Johnson, J. Math. Phys. **28**, 2488 (1987); K.-I. Kondo and Y. Kikukawa, Nagoya University Report No. DPNU-88-20 (unpublished).

[12]Equation (5) is obtained by equating the ratio A/B obtained from each boundary conditions, and (6) and (7) are obtained in the same way. The explicit forms of these solutions read as follows. (i) For $0<\lambda<\lambda_c$,

$$\Sigma(x)=\frac{\sqrt{1-(2\sigma)^2}}{2\sigma}\Sigma(\epsilon^2)\left[\frac{x}{\epsilon^2}\right]^{-1/2}\sinh\left[\sigma\ln\frac{x}{\epsilon^2}-i\theta\right],$$

(ii) For $\lambda=\lambda_c$,

$$\Sigma(x)=\tfrac{1}{2}\Sigma(\epsilon^2)\left[\frac{x}{\epsilon^2}\right]^{-1/2}\left[\ln\frac{x}{\epsilon^2}+2\right].$$

(iii) For $\lambda>\lambda_c$,

$$\Sigma(x)=\frac{\sqrt{1+(2\rho)^2}}{2\rho}\Sigma(\epsilon^2)\left[\frac{x}{\epsilon^2}\right]^{-1/2}\sin\left[\rho\ln\frac{x}{\epsilon^2}+\theta\right],$$

where $\theta\equiv\arctan(2\rho)=i\,\mathrm{arctanh}(2\sigma)$.

[13]K-I. Aoki, talk at the Annual Meeting of the Japan Physical Society, Koriyama, 1988 (unpublished), criticized the concep-

tual flaw of the arguments of Ref. 5 concerning the fixed point, based on the exact renormalization-group analysis [K. G. Wilson and J. Kogut, Phys. Rep. **12C**, 75 (1973); J. Polchinski, Nucl. Phys. **B231**, 269 (1984)]. He actually obtained nearby edge behavior of the critical line based not on the SD equation but on the *perturbation* with respect to either the gauge or the four-fermion coupling constant, and further made a *conjecture* on the possible form of the full critical line to which the gross structure of ours happened to be similar except for an essential difference concerning the shape of the line at $\lambda \simeq \lambda_c$ due mainly to the difference of approximations, perturbation versus ladder SD equation. (Aoki's talk can be found in a recent paper [Report No. RIFP-758, 1988 (unpublished)], which however does not include the above perturbation result, but does instead numerical analysis of the SD equation newly performed after our work on the critical line.)

[14]Explicit solutions in this case take the same form as those given in Ref. 12 except for the replacement $x/\epsilon^2 \to x/\Sigma(0)^2$ and $\Sigma(\epsilon^2) \to \Sigma(0)$.

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