Gluon poles from Feynman graphs, and gauge-invariant amplitudes

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With the object of defining a physically measurable quark-gluon-plasma frequency, two SU(3) gauge-invariant amplitudes, the photon propagator, and the S matrix for quark-quark scattering are studied. It is shown through explicit calculation that, at the two-loop level (α_s^3) in a general gauge, no subset of diagrams exists in these amplitudes, out of the entire set of Feynman diagrams available, with the property of yielding a gauge-invariant sum while probing the inherent non-Abelian nature of the theory. This implies that formulations of non-Abelian linear response theory based solely on gauge-dependent two-point functions or the electric field propagators are incomplete and unphysical. It also suggests that the Schwinger-Dyson equation for a properly defined, gauge-invariant, quark-gluon-plasma frequency contains an infinite number of proper skeleton graphs.

I. INTRODUCTION

Recently, there has been much interest in calculating the collective modes of a quark-gluon plasma.[1–3]. These authors have approached the problem by applying linear response theory to the electric field propagator,[4] a formalism effective for the QED case, in which one first perturbatively calculates the response of the system to a pulsed external electric field, and then converts this perturbative result into a pole-producing denominator using a nonperturbative, Schwinger-Dyson-type sum. (See Ref. 5 though, for a different approach, using the effective action and the background-field method.) The generalization of this technique to the QCD case is however ambiguous, first because the electric field is not gauge invariant in SU(3) and it is therefore unclear what a pole in such a propagator physically means. In an analogous and familiar case, the poles appearing in the free gluon propagator in the transverse (or "physical") gauges[6] (for which the axial gauge is a particular limit) have no particular physical meaning. They are clearly an artifact of the gauge choice and in fact lead to an added complication when using these gauges. One must similarly suspect ascribing physical meaning to poles appearing in the electric field propagator of QCD. Second, even with the first point not withstanding, it is by no means clear whether the method of obtaining such a pole, that is, by a resummation of terms arising from the iterative expansion of a finite set of proper skeleton graphs (which of course one can formally always do), is itself a gauge-invariant procedure. Feynman graphs in general have complicated gauge dependences which are then canceled by delicate interplays with other graphs when calculating a physical quantity. The above use of linear response theory is thus suspicious both in what and in how one is calculating when considering the case of QCD.

In order to circumvent the first objection, consider the photon propagator. It has the desirable property of not only being SU(3) gauge invariant, but is also a physical, external probe of the plasma. One imagines scattering photons off of the plasma and observing their products. These photons need not be on mass shell, as they can be thought of either being generated by electron-electron scattering ($Q^2 \leq 0$), or by electron-positron annihilation ($Q^2 \geq 0$). Using the optical theorem, the photon propagator can be related to these events by cutting the fermion lines in a fashion appropriate to the particular process one is considering. Whether such experiments are or are not realizable in the laboratory because of the difficulties of sustaining and confining such a plasma is irrelevant, for they can nevertheless be meaningfully theoretically discussed. In such experiments, one expects the photon propagator to acquire a dependence on the collective modes of the system (were this not the case, one might seriously wonder about the meaning of discussing such modes if they are not available to external observation), albeit, perhaps in only a perturbatively small and smoothly dependent fashion (i.e., the modes would not in general appear as resonances). We therefore expect the collective modes of the system to appear in the photon propagator in a manner suggested by Figs. 1(a) and 1(b), where the blobs indicate some plasma-frequency dependence appearing in the gluon denominator in a fashion as yet to be determined.

The purpose of this paper is to show, through explicit calculation, that there is no one-to-one correlation between such a plasma frequency dependence in the photon propagator and an iterative expansion of a finite set of proper skeleton graphs explicitly containing the non-Abelian nature of the theory. It will be shown that the SU(3) gauge invariance of the photon propagator is achieved to order α_s^3 in perturbative QCD by a summation over all graphs contributing at that order, no subset of which, containing the non-Abelian vertices of the theory, can be isolated and found to be invariant amongst itself. It is conjectured this behavior persists to all orders. The alternatives would then be that to calculate an observable collective mode (in the photon propagator) either one must sum all the graphs at any particular order or, a new scheme, dissecting and reorganizing Feynman

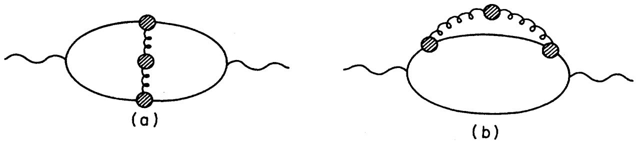

FIG. 1. The fashion in which an external photon propagator is expected to depend on the collective modes of the quark-gluon plasma.

graphs in some new fashion, must be invented. (See, for example, the works of Cornwall and co-workers[7,8] and Nadkarni[9] in which one such scheme, the "gauge-invariant propagator method," is proposed.)

Section II discusses how we can simplify the analysis of the photon propagator by showing that for our purposes we need only study the S matrix of quark-quark scattering (with on-shell quarks) at zero temperature. It concludes by demonstrating the patterns of cancellations that exist between the graphs at the one-loop level which ensure the gauge independence of the renormalized S matrix. Section III extends these simple results to the two-loop level, where it is shown that gauge independence inextricably involves all the graphs appearing at that level. Section IV presents the conclusions of the work.

II. THE S MATRIX

Although the photon propagator is the amplitude of physical interest, it is not the simplest amplitude we can study for our purposes. Consider the S matrix for quark-quark scattering. One observes that by placing the internal quark lines on shell, this, S-matrix element is contained within the phase space of the photon propagator. It also has the attractive feature that, on a graphical level, the gauge invariance of the scattering amplitude is more readily obtained. This can be seen by working in a general covariant gauge for the gluon propagator at zero temperature,

$$G_{ab}^{\mu\nu}(q) = -i\frac{\delta_{ab}}{q^2+i\epsilon}\left[g^{\mu\nu}-(1-\zeta)\frac{q^\mu q^\nu}{q^2}\right], \tag{1}$$

and considering the lowest-order QCD graphs for scattering and for the photon propagator, shown in Figs. 2 and 3, respectively. Whereas Fig. 2 is in itself gauge invariant, one needs all three graphs of Fig. 3 to preserve this property in the photon propagator. We can thus simplify our analysis somewhat, by first studying the graphs appearing in the S matrix for quark-quark scattering.

A further simplification can be made by observing that, whereas we are interested in QCD at finite temperature, our analysis must first apply at zero temperature. This can be easily seen by using the real-time formalism[10] for the gluon and quark propagators in which the temperature dependence of these Green's functions is explicitly separated from the zero-temperature piece. Hence a minimal requirement for our analysis to work at finite temperature is that it must first work at zero temperature. We have thus now restricted the problem of finding a graphical representation for the physical poles appearing in Fig. 1 at finite temperature, to first finding a similar such representation for the poles appearing in the S matrix at zero temperature.

Let us now examine quark-quark scattering at the one-loop level. The relevant graphs are shown in Figs. 4 and 5. Working again in a general covariant gauge [Eq. (1)], we wish to trace the graphical interplays present at this order that result in the elimination of all of the $(1-\zeta)$ dependence. The results of these graphs at $O(1-\zeta)$ for the case of massless fermions are[11]

$$\begin{aligned}
&(1-\zeta)\delta^{ab}[u(p_2)\gamma^\mu\bar{u}(p_1)][u(p_4)\gamma^\nu\bar{u}(p_3)]\\
&\quad\times\Bigg[-N\left[\int d^nk\frac{2k^\mu k^\nu+q^2g^{\mu\nu}}{k^4(q-k)^2}-\frac{1}{q^2}\int d^nk\frac{k^\mu k^\nu+g^{\mu\nu}(q^2-k^2)}{k^4}\right] && \text{[Fig. 4(a)]}\\
&\quad\quad -N\left[\frac{1}{q^2}\int d^nk\frac{k^\mu k^\nu-g^{\mu\nu}k^2}{k^4}\right] && \text{[Fig. 4(b)]}\\
&\quad\quad +2N\left[\int d^nk\frac{k^\mu k^\nu+q^2g^{\mu\nu}}{k^4(q-k)^2}-\frac{1}{q^2}\int d^nk\frac{g^{\mu\nu}q^2}{k^4}\right] && \text{[Figs. 4(c)+4(d)]}\\
&\quad\quad -N\left[\int d^nk\frac{q^2g^{\mu\nu}}{k^4(q-k)^2}\right] && \text{[Figs. 4(e)+4(f)]}\\
&\quad\quad +\frac{1}{N}\left[\frac{1}{q^2}\int d^nk\frac{g^{\mu\nu}q^2}{k^4}\right] && \text{[Figs. 5(a)+5(b)]}\\
&\quad\quad +\left[\frac{N^2-1}{2N}\right]\left[\int d^nk\frac{4g^{\mu\nu}}{k^2(p_4-k)^2}\right]\Bigg], && \text{[Fig. 5(c)]}
\end{aligned}$$

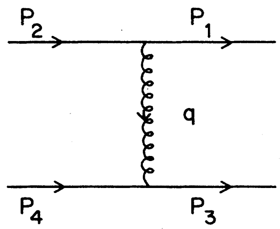

FIG. 2. The lowest-order QCD graph for quark-quark scattering.

where $N=3$ for SU(3) and n is $(4+\epsilon)$ in dimensional regularization. These integrals add to yield a total dependence at $O(1-\zeta)$ of

$$(1-\zeta)\delta^{ab}[u(p_2)\gamma^{\mu}\bar{u}(p_1)][u(p_4)\gamma^{\nu}\bar{u}(p_3)]$$
$$\times\left[-\left[N-\frac{1}{N}\right]\left[\int d^n k\frac{g^{\mu\nu}}{k^4}+2\int d^n k\frac{g^{\mu\nu}}{k^2(p_4-k)^2}\right]\right].$$

Since we have massless fermions $(p_4^2=0)$, we can make the replacement

$$\int d^n k\frac{g^{\mu\nu}}{k^4}\Longrightarrow\int d^n k\frac{g^{\mu\nu}}{k^2(p_4-k)^2},$$

so our result becomes

$$(1-\zeta)\delta^{ab}[u(p_2)\gamma^{\mu}\bar{u}(p_1)][u(p_4)\gamma^{\nu}\bar{u}(p_3)]$$
$$\times\left[+\left[N-\frac{1}{N}\right]\int d^n k\frac{g^{\mu\nu}}{k^2(p_4-k)^2}\right]. \quad (2)$$

However, we still need to renormalize our initial and outgoing states to obtain the S matrix according to the prescription

$$S=\frac{G^n_{\text{conn}}}{(\sqrt{Z})^n}.$$

We thus must subtract the two graphs in Fig. 6. Their contributions exactly cancel our result in Eq. (2), yielding altogether a gauge-invariant result.

The point of this simple exercise has been to observe the patterns of cancellations existing between the graphs present in the S matrix at the one-loop level, with the eventual hope of finding some subset of graphs that could

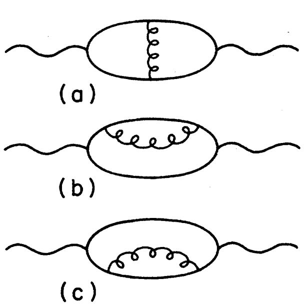

FIG. 3. The lowest-order QCD corrections to the photon propagator. Wavy lines are photons, curly lines are gluons.

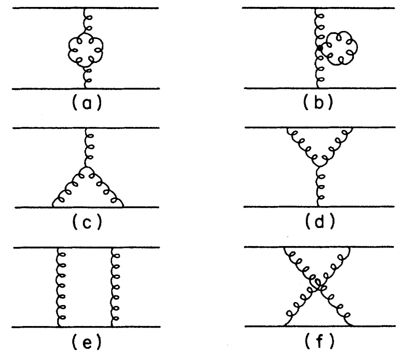

FIG. 4. Some of the one-loop scattering graphs entering the S matrix for quark-quark scattering.

be iterated in a Schwinger-Dyson-type fashion to yield a gauge-invariant result. The results at this point for such a program are already discouraging. It would appear that all the graphs, as a body, must be retained before achieving a physical result. We notice however that the q dependent integrals are contained and canceled within a smaller set of graphs: Figs. 4(a), 4(c), 4(d), and 4(e). (It is in fact only these graphs that have nonzero value when evaluating the integrals using dimensional regularization. They have been explicitly kept here in order to mimic as much as possible the more complicated case of the photon propagator in which, for example, the quark self-energy graphs are then not zero.) We might wonder if the interplay between this smaller subset of graphs does not reflect some pattern apparent at higher orders, which is only masked at this level because of the simplicity of the graphs involved. Unfortunately this is not the case, as the next section, examining the graphs at the two-loop level, will show.

One last remark however, must first be made. The graphs shown in Figs. 4 and 5 are not the full set of graphs contributing at this order. Two last graphs remain, namely, the fermion loop and the ghost loop corrections to the gluon propagator, Fig. 7. These graphs do not have any explicit, $(1-\zeta)$, gauge dependence. The ghost loop though is clearly gauge dependent, as one could have just as well chosen a gauge without ghosts. On the other hand, the fermion loop graph is gauge invariant and can be iterated to yield a gauge-invariant pole. This last point indicates why linear-response theory works in QED, where one at the lowest level is effectively

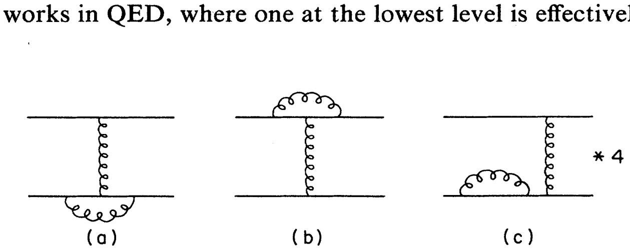

FIG. 5. The remaining one-loop scattering graphs with explicit $(1-\zeta)$ gauge dependence.

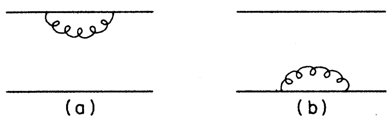

FIG. 6. The wave-function renormalizations which must be included in the S matrix at the one-loop level.

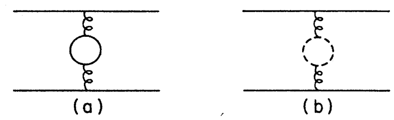

FIG. 7. Two graphs with no explicit $(1-\xi)$ dependence. Dashed lines are ghosts.

concerned with exactly only such fermion loop graphs. In the case of QCD, one could again formally add these graphs to apparently obtain a physical pole; however, this would now be an incorrect result. At each level in perturbation theory there are competing graphs, some much larger than the fermion loop graph [such as the gluon loop graph, Fig. 4(a)], which one would then be ignoring. This would be erroneous, as the physics is clearly dominated by the non-Abelian character of the theory, which thus must be included to obtain a physical result.

III. TWO-LOOP GRAPHS

We will now examine the gauge dependence of the two-loop graphs appearing in the S matrix for quark-quark scattering at zero temperature. For simplicity we will ignore in this section all integrals that do not depend on q. That is, all integrals that either have no external momenta, as in Fig. 4(b), or are quark wave-function renormalizations, as in Fig. 5(c), will be dropped. Specifically, we will trace how the $O(1-\xi)$, q-dependent integrals embedded in the iterated two-gluon-loop graph, Fig. 8(a) (which is highly amenable to a Schwinger-Dyson-type sum) is canceled by other graphs.

As a guide, we observe that the topological structure of the two-gluon-loop graph inherently decouples the two-loop momenta, k and l, while coupling each independently to q. Since the renormalized S matrix is gauge independent for all dimensions $n \le 4$ when regularized using dimensional regularization,[12] the gauge independence of graph 8(a) can only be eliminated by graphs containing the same generic, decoupled integrals. Also, by similar reasoning, only those graphs proportional to N^2 in SU(N) need be considered.

We start our investigation with the graphs in which the decoupling of the loop momenta occurs naturally, due to topology, Figs. 8 and 9. In each of these sets of graphs there are large interplays by which most, but not all, of the gauge dependence is canceled. To see this last fact, the explicit expressions for each of the graphs in the two figures will be listed. The $O(1-\xi)$ dependence of the graphs in Fig. 8 is

$$
\begin{aligned}
&(1-\xi)iN^2\delta^{ab}[u(p_2)\gamma^{\mu}\bar{u}(p_1)][u(p_4)\gamma^{\rho}\bar{u}(p_3)]q^{-6}\\
&\times\left[\int d^nk\frac{2k^{\mu}k^{\nu}+q^2g^{\mu\nu}}{k^4(k-q)^2}\int d^nl\frac{10l^{\nu}l^{\rho}+4q^2g^{\nu\rho}}{l^2(q-1)^2}\right. && \text{[Fig. 8(a)]}\\
&\quad-\int d^nk\frac{k^{\mu}k^{\nu}+q^2g^{\mu\nu}}{k^4(k-q)^2}\int d^nl\frac{5l^{\nu}l^{\rho}+2q^2g^{\nu\rho}}{l^2(q-1)^2}\\
&\quad-\int d^nk\frac{2q^2k^{\mu}k^{\nu}+q^4g^{\mu\nu}}{k^4(k-q)^2}\left[\int d^nl\frac{g^{\nu\rho}}{l^2(q-l)^2}-2\int d^nl\frac{l^{\nu}l^{\rho}}{l^2(q-l)^2(p_4-l)^2}\right] && \text{[Fig. 8(b)]}\\
&\quad-\int d^nk\frac{k^{\mu}k^{\nu}+q^2g^{\mu\nu}}{k^4(k-q)^2}\int d^nl\frac{5l^{\nu}l^{\rho}+2q^2g^{\nu\rho}}{l^2(q-l)^2}\\
&\quad-\int d^nk\frac{2q^2k^{\mu}k^{\nu}+q^4g^{\mu\nu}}{k^4(k-q)^2}\left[\int d^nl\frac{g^{\nu\rho}}{l^2(q-l)^2}-2\int d^nl\frac{l^{\nu}l^{\rho}}{l^2(q-l)^2(p_2+l)^2}\right] && \text{[Fig. 8(c)]}\\
&\quad+\int d^nk\frac{q^2k^{\mu}k^{\nu}+q^4g^{\mu\nu}}{k^4(k-q)^2}\left[\int d^nl\frac{g^{\nu\rho}}{l^2(q-l)^2}-2\int d^nl\frac{l^{\nu}l^{\rho}}{l^2(q-l)^2(p_4-l)^2}\right]\\
&\quad+\int d^nk\frac{q^2k^{\mu}k^{\nu}+q^4g^{\mu\nu}}{k^4(k-q)^2}\left[\int d^nl\frac{g^{\nu\rho}}{l^2(q-l)^2}-2\int d^nl\frac{l^{\nu}l^{\rho}}{l^2(q-l)^2(p_2+l)^2}\right] && \text{[Fig. 8(d)]}\\
&\quad-2\int d^nk\frac{2k^{\mu}k^{\nu}+q^2g^{\mu\nu}}{k^2(k-q)^2}\int d^nl\frac{l^{\nu}l^{\rho}}{l^2(q-l)^2} && \text{[Figs. 8(e)+8(f)]}\\
&\quad\left.+2\int d^nk\frac{k^{\mu}k^{\nu}+q^2g^{\mu\nu}}{k^4(k-q)^2}\int d^nl\frac{l^{\nu}l^{\rho}}{l^2(q-l)^2}\right]. && \text{[Figs. 8(g)+8(h)]}
\end{aligned}
$$

This can be added and reduced into scalar integrals to yield, as a final result for Fig. 8,

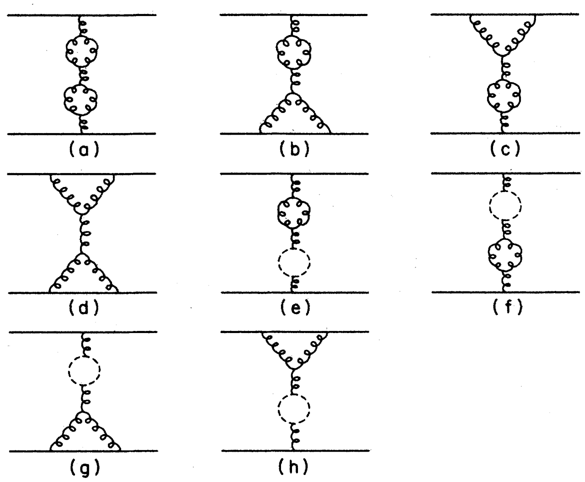

FIG. 8. All the N^2-dependent, topologically decoupling graphs that exist at the two-loop level in the S matrix which do not contain any four-point vertices.

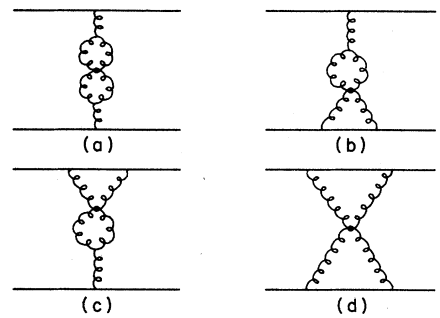

FIG. 9. All the N^2-dependent, topologically decoupling graphs that exist at the two-loop level in the S matrix which do contain four-point vertices.

$$(1-\zeta)iN^2\delta^{ab}[u(p_2)\gamma^\mu\bar{u}(p_1)][u(p_4)\gamma^\rho\bar{u}(p_2)]q^{-4} \times\left[+g^{\mu\rho}\frac{3}{8}\left[\int d^nk\frac{1}{k^2(q-k)^2}-\frac{q^2}{2}\int d^nk\frac{1}{k^4(k-q)^2}\right]\int d^nl\frac{1}{l^2(q-l)^2}\right]. \quad (3)$$

Again, it is emphasized that this is the result for these graphs only when restricting to the particular form that both integrals depend on q. There are other terms present in which this is not the case and that have been ignored. In the case of the S matrix these additional terms can be set to zero (as is normally done in dimensional regularization), however, as discussed in Sec. II, the corresponding terms in the photon propagator are not zero and must be retained. For example, graph 8(b) also contains terms in which the d^nl integration contains only a p_4 dependence. Had the results of merely tracing the terms where both integrals depended on q not been sufficiently convincing, we would then need to return to these ignored terms and trace how their gauge dependence is subsequently canceled. For example, the contributions of these terms must communicate to the gauge-dependent pieces of Fig. 10(a), which in turn communicates to the gauge-dependent pieces of Figs. 10(b) and 10(c), and so on. We will see, however, that the results from merely isolating the q-dependent integrals are unambiguous, and so we will continue dropping all other terms.

The $O(1-\zeta)$ dependence of the graphs in Fig. 9 is

$$\begin{aligned}
&(1-\zeta)iN^2\delta^{ab}q^{-4}\\
&\times\Bigg\{-[u(p_2)\gamma^\mu\bar{u}(p_1)][u(p_4)\gamma^\rho\bar{u}(p_3)]\frac{9}{2}\int d^nk\,d^nl\frac{[k^\mu k^\rho+\frac{1}{2}g^{\mu\rho}(q^2+k^2)]}{k^4(k-q)^2l^2(q-l)^2} && [\text{Fig. 9(a)}]\\
&\quad+[u(p_2)\gamma^\mu\bar{u}(p_1)][u(p_4)\gamma^\rho\bar{u}(p_3)]\frac{9}{8}\int d^nk\,d^nl\frac{g^{\mu\rho}(q^2+k^2)}{k^4(k-q)^2l^2(q-l)^2}\\
&\quad+[u(p_2)\gamma^\mu\bar{u}(p_1)]\frac{3}{4}\int d^nk\,d^nl\frac{u(p_4)\gamma^\lambda(\not p_4-\not l)\gamma^\rho\bar{u}(p_3)[q^2(k^\lambda g^{\mu\rho}-k^\rho g^{\mu\lambda})+k^\mu(k^\rho q^\lambda-k^\lambda q^\rho)]}{k^4(k-q)^2l^2(q-l)^2(p_4-l)^2} && [\text{Fig. 9(b)}]\\
&\quad+[u(p_2)\gamma^\mu\bar{u}(p_1)][u(p_4)\gamma^\rho\bar{u}(p_3)]\frac{9}{8}\int d^nk\,d^nl\frac{g^{\mu\rho}(q^2+k^2)}{k^4(k-q)^2l^2(q-l)^2}\\
&\quad-[u(p_4)\gamma^\mu\bar{u}(p_3)]\frac{3}{4}\int d^nk\,d^nl\frac{u(p_2)\gamma^\lambda(\not p_2+\not l)\gamma^\rho\bar{u}(p_1)[q^2(k^\lambda g^{\mu\rho}-k^\rho g^{\mu\lambda})+k^\mu(k^\rho q^\lambda-k^\lambda q^\rho)]}{k^4(k-q)^2l^2(q-l)^2(p_2+l)^2} && [\text{Fig. 9(c)}]\\
&\quad-[u(p_2)\gamma^\mu\bar{u}(p_1)]\frac{3}{4}\int d^nk\,d^nl\frac{u(p_4)\gamma^\lambda(\not p_4-\not l)\gamma^\rho\bar{u}(p_3)q^2(k^\lambda g^{\mu\rho}-k^\rho g^{\mu\lambda})}{k^4(k-q)^2l^2(q-l)^2(p_4-l)^2}\\
&\quad+[u(p_4)\gamma^\mu\bar{u}(p_3)]\frac{3}{4}\int d^nk\,d^nl\frac{u(p_2)\gamma^\lambda(\not p_2+\not l)\gamma^\rho\bar{u}(p_1)q^2(k^\lambda g^{\mu\rho}-k^\rho g^{\mu\lambda})}{k^4(k-q)^2l^2(q-l)^2(p_2+l)^2}\Bigg\}. && [\text{Fig. 9(d)}]
\end{aligned}$$

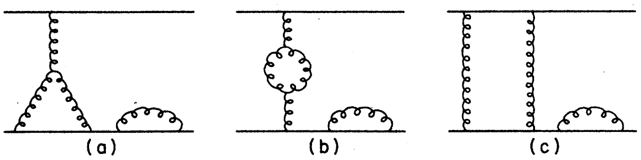

FIG. 10. Graphs not being considered but whose gauge dependence does communicate with the graphs in Fig. 8.

Again converting into scalar integrals, these terms add to yield

$$(1-\xi)iN^2\delta^{ab}[u(p_2)\gamma^\mu\bar{u}(p_1)][u(p_4)\gamma^\rho\bar{u}(p_3)]q^{-4}\times\left[-g^{\mu\rho}\frac{5}{4}\left[\int d^nk\frac{1}{k^2(q-k)^2}-\frac{q^2}{2}\int d^nk\frac{1}{k^4(k-q)^2}\right]\times\int d^nl\frac{1}{l^2(q-l)^2}\right] \quad (4)$$

as the total $O(1-\xi)$ gauge-dependent expression for the graphs in Fig. 9.

Comparing Eqs. (3) and (4), we see that the sum of all the graphs in Figs. 8 and 9 still does not yield a gauge-invariant expression. Where is the remaining gauge dependence? Recall that Figs. 8 and 9 only represent the graphs that are proportional to N^2 and which by their topology alone, result in decoupled, q-dependent integrals. But this is not the only means of producing such terms. Consider the graphs in Fig. 11. These highly complicated, topologically interweaved graphs, contain terms in the desired form. The numerator contractions conspire to yield terms in which the middle gluon propagators, labeled $(k-l)$ in the figure, are effectively pinched. This occurs because any integral with at least one k and one l variable in the numerator can be found to contain pieces in the desired, decoupled form. For example the integral

$$\int d^nk\, d^nl\frac{k^\lambda l^\rho}{k^4(k-q)^2(k-l)^2l^2(q-l)^2}$$

can be shown to contain the expression

$$\frac{1}{6}\int d^nk\, d^nl\frac{1}{k^4(k-q)^2l^2(q-l)^2}\left[\frac{q^\lambda q^\rho}{q^2}-g^{\lambda\rho}\right].$$

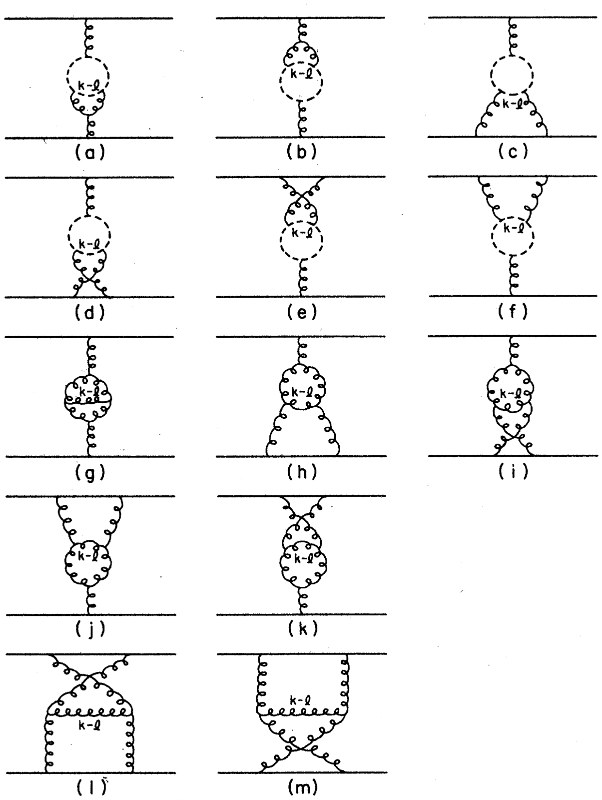

FIG. 11. The N^2-dependent, nontopologically decoupling graphs with the required q^2-dependent integrals, that exist at the two-loop level in the S matrix.

At this point our analysis may end. Having initially wished to trace the manner in which the gauge dependence of merely the iterated, two-gluon loop, propagator correction graph, Fig. 8(a), is canceled in the S matrix, we find that we have been compelled to discuss not only the entire set of graphs in Figs. 8 and 9, but also the monstrously complicated graphs of Fig. 11. These latter graphs inextricably link us with the entire plethora of remaining graphs entering the S matrix at this order, a link we had hoped to avoid. Thus, any program that limits these graphs to an isolated subset with the eventual purpose of extracting a physical result is doomed to failure.

IV. CONCLUSIONS

The collective modes of a quark-gluon plasma, if they exist, must be susceptible to observation by an external, physical probe. The photon propagator is one such probe. It has been shown that in the photon propagator, no subset of the graphs that appear at the first, second, or third order in α_s can be isolated to form a gauge-invariant result. It is conjectured this behavior persists to all orders, i.e., that the gauge dependence of merely one non-Abelian graph ties us to the entire body of non-Abelian graphs entering the photon propagator at that order. A collective mode of the plasma could then only appear in this amplitude as a resummation of an iterative expansion of an infinite number of proper skeleton graphs.

The conclusions of this analysis have serious implications for the present results being reported by Kajantie and his co-workers using linear response theory.[1,2] When applied to QCD, their approach asserts that the plasma frequency can be calculated solely by considering the lowest-order, gluon propagator corrections [indicated in Figs. 4(a), 4(b), and (7)] as calculated in the temporal axial gauge. However, as the Landau gauge is a particular limit in the set of all covariant gauges, so is the temporal axial gauge a particular limit of another set of gauges, namely, the general class of all transverse gauges:[6]

$$G_{ab}^{\mu\nu}(q)=\frac{\delta_{ab}}{q^2+i\epsilon}\left[g^{\mu\nu}-\frac{q^\mu c^\nu+q^\nu c^\mu}{qc}+(1-\eta)\frac{q^\mu q^\nu c^2}{(qc)^2}\right],$$

where c^μ is some constant vector with $c^2\neq 0$. All such gauges are ghostless. The temporal axial gauge is the particular limit that c^μ is purely timelike and that $\eta=0$. One could now repeat the analysis of Secs. II and III, now tracing the cancellation of all terms proportional to $(1-\eta)$, but that would be unnecessary. It is clear that the results of this approach, which explicitly depend on the $(1-\eta)$ term, will not appear in any way in the photon propagator. The results are incomplete and unphysical.

Although only one matrix element has been considered, it is believed that the results of this paper apply to any other gauge-invariant amplitude one might construct to probe the plasma. The photon propagator is perhaps the simplest of all such physical probes. If this is the case, then it is hard to imagine how in a more complicated amplitude the Feynman graphs entering that amplitude should nevertheless conspire to yield simpler results. It is thus conjectured that the Schwinger-Dyson equation for any collective mode must receive contributions from an infinite number of proper skeleton graphs.

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