PHYSICAL REVIEW D VOLUME 39, NUMBER 8 15 APRIL 1989

Fractional spin in the gauged O(3) σ model

Taejin Lee, Chekuri N. Rao, and K. S. Viswanathan
Department of Physics, Simon Fraser University, Burnaby, British Columbia, Canada V5A 1S6
(Received 19 December 1988)

The soliton sector of the O(3) nonlinear σ model coupled to an Abelian gauge field with a topological mass is quantized semiclassically. We evaluate the spin of the soliton with unit topological charge and find that it is $\theta/2\pi$, independent of the topological mass parameter to all orders. Gauging the σ model thus does not affect the fractional spin of the soliton.

I. INTRODUCTION

The O(3) nonlinear σ model or its equivalent, the CP^1 model in $2+1$ space-time dimensions, may have an important application in the phenomenon of high-T_c superconductivity. Since the configuration space of the model, a space of continuous maps from the two-dimensional space S^2 to the field manifold S^2, has a nontrivial homotopy $\pi_2(S^2)=\mathbb{Z}$, the model admits solitons[1,2] which are classified by their homotopy classes. As discussed by Wilczek and Zee[3] and subsequently by others,[4–7] the intriguing feature of the model is that the soliton acquires fractional spin and exotic statistics[3–8] through a topological term called the Hopf invariant in the action. The Hopf invariant can be represented in terms of an auxiliary gauge field by the familiar Chern-Simons term of the Abelian gauge theory[9–13] which has been widely discussed in connection with the fractional spin and exotic statistics of charged particles (nonsoliton sector) in $(2+1)$-dimensional theories. Dzyaloshinskii, Polyakov, and Wiegmann[14] (see also Refs. 15 and 16) recently proposed a possible mechanism for high-T_c superconductivity: the solitons of the O(3) σ model, being fermions with half-integer spin when a specific value is chosen for the coefficient of the Hopf invariant, bind electrons (holes) and become charged bosons. Thus they may play the role of the neutral fermions in the study of the Hubbard model by Anderson.[17]

In this paper we discuss the nonlinear σ model in which the kinetic term for the gauge field is included in the action, thus the gauge field becomes an independent dynamical field. Such an inclusion is not unreasonable, considering that the kinetic term for the gauge field[18] can be generated in the effective action, even if it is not contained in the classical action. Aside from its potential relevance in high-T_c superconductivity, this model is interesting in its own right and deserves a detailed study. Karabali and Murthy[19] have recently studied this model and found that the inclusion of the kinetic term does not affect the fractional spin of the soliton up to fifth order in $1/m$, when m, the topological mass of the gauge field, is sufficiently large. They conjectured that the fractional spin may be independent of m to all orders and further speculated a phase transition in the fractional part of the spin when $1/m$ becomes comparable to the size of the soliton. In this paper we provide a definitive answer: The fractional angular momentum is not modified by inclusion of the kinetic term for the gauge field, i.e., the independence on m persists to all orders [not just to $O(1/m^5)$ as in Ref. 19] as long as $m\neq 0$. Therefore, the main features of the soliton, i.e., the fractional spin and exotic statistics, remain unchanged and previous discussions for the O(3) σ model in the context of high-T_c superconductivity may be also applicable to the gauged model without modification.

II. THE MODEL

The nonlinear σ model coupled to an Abelian gauge field with a topological mass is described by the action

$$S=\int d^3x(\mathcal{L}_\sigma+\mathcal{L}_g)\ ,$$
$$\mathcal{L}_\sigma=\frac{1}{2f}\partial_\mu \mathbf{n}\cdot\partial^\mu \mathbf{n}+eA_\mu J^\mu\ , \tag{1}$$
$$\mathcal{L}_g=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{m}{4}\epsilon_{\mu\nu\lambda}A^\mu F^{\nu\lambda}\ ,$$

where $\mathbf{n}=(n^1,n^2,n^3)$, $\mathbf{n}\cdot\mathbf{n}=1$, and J_μ is the topological current given by

$$J_\mu=\frac{1}{8\pi}\epsilon_{\mu\nu\lambda}\mathbf{n}\cdot(\partial^\nu\mathbf{n}\times\partial^\lambda\mathbf{n})\ . \tag{2}$$

The coefficients in front of the Chern-Simons term, m, and the interaction term, e, are dimensionful due to the inclusion of kinetic term for the gauge fields; m has the dimension of mass and e has the dimension of $m^{1/2}$.

If the kinetic term were absent, the gauge field can be completely integrated out in the path integral to yield the nonlocal Hopf invariant in the effective action

$$S_H=\theta\int d^3x\ A_\mu J^\mu,\quad \theta=\frac{e^2}{2m}\ , \tag{3}$$

where A_μ is defined in terms of $\mathbf{n}$ through the equation

$$J_\mu=\epsilon_{\mu\nu\lambda}\partial^\nu A^\lambda\ . \tag{4}$$

In order to quantize the model semiclassically we must find the equations for the soliton. The equations of motion following from the action are

$$\partial_\mu F^{\mu\nu}-m\epsilon^{\nu\mu\lambda}\partial_\mu A_\lambda+eJ^\nu=0\ , \tag{5a}$$

$$-\frac{1}{f}\partial^2\mathbf{n}+\frac{1}{f}\mathbf{n}(\mathbf{n}\cdot\partial^2\mathbf{n})+\frac{e}{4\pi}\epsilon^{\nu\mu\lambda}\mathbf{n}\times\partial_\lambda\mathbf{n}\partial_\nu A_\mu=0\ . \tag{5b}$$

We look for a static radially symmetric soliton solution. Since the soliton is characterized by the same topological charge

$$Q=\frac{1}{8\pi}\int d^2x\ \epsilon^{ij}\mathbf{n}\cdot(\partial_i\mathbf{n}\times\partial_j\mathbf{n}) \tag{6}$$

as in the usual nonlinear O(3) σ model, we take the following ansatz for $\mathbf{n}$:

$$n^1=\cos\phi\sin g(r),\quad n^2=\sin\phi\sin g(r)\ ,\qquad n^3=\cos g(r)\ , \tag{7}$$

where (r,ϕ) are the polar coordinates. With the boundary conditions for $g(r)$,

$$g(0)=0\quad\text{and}\quad g(\infty)=\pi\ , \tag{8}$$

the ansatz Eq. (7) corresponds to a soliton with $Q=1$. It is convenient to choose the Coulomb gauge for the static solution $\partial_i A^i=0$. Then A^i can be written as

$$A^i=\epsilon^{ij}\partial_j\varphi\ . \tag{9}$$

To respect the spherical symmetry for $\mathbf{n}$, we take

$$A^0=A^0(r),\quad \varphi=\varphi(r)\ . \tag{10}$$

For the ansatz Eqs. (7)–(10) we have

$$J^0=\frac{1}{4\pi r}\sin g(r)\frac{d}{dr}g(r),\quad J^i=0\ , \tag{11}$$

and the equations of motion are reduced to

$$\partial_i(\nabla^2\varphi-mA_0)=0\ , \tag{12a}$$

$$\nabla^2(A_0-m\varphi)=eJ_0\ , \tag{12b}$$

$$\frac{1}{f}\left[g''+\frac{1}{r}g'-\frac{1}{2r^2}\sin 2g\right]-\frac{e}{4\pi r}(\sin g)A_0'=0\ , \tag{12c}$$

where a prime denotes derivation with respect to r.

Choosing the constant of integration to be zero, we can write A_0 in terms of J_0 from Eqs. (12a) and (12b):

$$A_0=e\int G(\mathbf{x}-\mathbf{x}')J_0(\mathbf{x}')d^2x'\ , \tag{13a}$$

where

$$(\nabla^2-\mu^2)G(\mathbf{x}-\mathbf{x}')=\delta^2(\mathbf{x}-\mathbf{x}')\ . \tag{13b}$$

The above equations (12a)–(12c) define the static solution of the model. In passing we note that the soliton with $Q=1$ of the usual nonlinear σ model is described by the ansatz in Eq. (7) with

$$g(r)=\cos^{-1}\left[\frac{r^2-\lambda^2}{r^2+\lambda^2}\right]\ , \tag{14}$$

which satisfies

$$g''+\frac{1}{r}g'-\frac{1}{2r^2}\sin 2g=0\ . \tag{15}$$

λ in Eq. (14) is an arbitrary scale parameter of the soliton. We now turn to the semiclassical quantization of the soliton sector with $Q=1$.

III. QUANTIZATION

We expand the field variables $\mathbf{n}$ and A_μ around the soliton solution and apply the canonical Hamiltonian method.[2,20] Since our primary concern is the induced spin of the soliton, we consider a U(1) family of configurations characterized by a single collective coordinate $\alpha(t)$ corresponding to the zero mode of rotation:[7]

$$n^1=\cos[\phi+\alpha(t)]\sin g(r),\quad n^2=\sin[\phi+\alpha(t)]\sin g(r)\ ,\qquad n^3=\cos g(r)\ . \tag{16}$$

We will, however, take into account the full degrees of freedom for the gauge fields. We write $A_\mu=A_\mu^{\rm cl}+A_\mu^q$, where $A_\mu^{\rm cl}$ is the gauge part of the classical soliton solution. Replacing the fields $\mathbf{n}$ and A_μ as above in the action and dropping the superscript of q in A_μ^q, the action can be rewritten as

$$\begin{aligned}
S&=\int d^3x(\mathcal{L}_g+\mathcal{L}_\alpha+\mathcal{L}_{\rm int})-M_s\ ,\\
\mathcal{L}_g&=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{m}{4}\epsilon_{\mu\nu\lambda}A^\mu F^{\nu\lambda}\ ,\\
\mathcal{L}_{\rm int}&=\frac{e}{4\pi r}(\cos g)'\epsilon^{ij}x_iA_j\dot\alpha\ ,\\
L_\alpha&=\int\mathcal{L}_\alpha d^2x=\frac{\kappa}{2}\dot\alpha^2+\rho\dot\alpha\ ,
\end{aligned} \tag{17}$$

where

$$\kappa=\frac{1}{f}\int d^2x\sin^2 g,\quad \rho=-\frac{e}{4\pi}\int d^2x(\cos g)'\varphi'\ ,\qquad \dot\alpha=\frac{d\alpha}{dt},\quad g'=\frac{dg}{dr}\ . \tag{18}$$

The soliton mass M_s is given by

$$M_s=\frac{1}{2}\int d^2x\left[\frac{1}{f}\left[(g')^2+\frac{1}{r^2}\sin 2g\right]+\frac{e}{4\pi r}(\cos g)'A_0^{\rm cl}\right]\ . \tag{19}$$

Equations of motion (12a)–(12c) have been used to simplify Eq. (17).

The canonical Hamiltonian procedure starts by defining momenta conjugate to A_μ and α:

$$\Pi_\mu=\frac{\delta L}{\delta\dot A^\mu},\quad \beta=\frac{\partial L}{\partial\dot\alpha}\ . \tag{20}$$

We obtain a primary constraint $\chi_1=\Pi_0=0$, and momenta conjugate to A^i and α as

$$\Pi_i=E_i+\frac{m}{2}\epsilon_{ij}A^j,\quad \beta=\kappa\dot\alpha+\rho+\gamma(t)\ , \tag{21}$$

where

$$\gamma(t)=\int d^2x\frac{e}{4\pi r}(\cos g)'\epsilon^{ij}x_iA_j \tag{22}$$

and $E_i = F_{i0}$. A Legendre transformation yields the Hamiltonian

$$H = \int d^2x (\Pi_i \dot{A}^i + \beta\dot{\alpha} - \mathcal{L}) = H_\alpha + \int d^2x\, \mathcal{H}_g \ , \tag{23}$$

where

$$H_\alpha = \frac{1}{2\kappa}[\beta - \rho - \gamma(t)]^2 \ ,$$
$$\mathcal{H}_g = \frac{1}{2}\left[\Pi_i + \frac{m}{2}\epsilon_{ij}A^j\right]^2 + \tfrac{1}{2}B^2 \ , \tag{24}$$
$$B = \epsilon_{ij}\partial^i A^j \ .$$

A secondary constraint χ_2 is generated by requiring that the primary constraint $\chi_1 = 0$ be consistently imposed:

$$0 = [\Pi_0, H]_{\rm PB} = \partial_i \Pi^i + \frac{m}{2}B \equiv \chi_2 \ , \tag{25}$$

where the Poisson brackets $[\ ]_{\rm PB}$ is defined by

$$[C,D]_{\rm PB} = \frac{\partial C}{\partial\alpha}\frac{\partial D}{\partial\beta} - \frac{\partial D}{\partial\alpha}\frac{\partial C}{\partial\beta} + \int d^2x \left[\frac{\delta C}{\delta A^\mu}\frac{\delta D}{\delta \Pi_\mu} - \frac{\delta D}{\delta A^\mu}\frac{\delta C}{\delta \Pi_\mu}\right] .$$

After some algebra we find that these constraints are of first class and that no more constraints are generated by requiring consistency:

$$[\chi_1, \chi_2]_{\rm PB} = 0, \quad [\chi_2, H]_{\rm PB} = 0 \ . \tag{26}$$

This is in contrast with the work of Karabali and Murthy.[19] They obtain an effective action by integrating out the gauge fields and then make an expansion in powers of $(1/m)$. The resultant effective action involves higher-time-derivative terms and an infinite sequence of second-class constraints arises upon imposing the canonical quantization scheme. We note that the constraints χ_1 and χ_2 are those of the Abelian gauge theory with the Chern-Simons term[10] and we have, upon quantization,

$$[\Pi_\mu(\mathbf{x},t), A_\nu(\mathbf{x}',t)] = -i\eta_{\mu\nu}\delta(\mathbf{x}-\mathbf{x}'), \quad [\beta(t), \alpha(t)] = -i \ .$$

IV. FRACTIONAL SPIN

We next evaluate the angular momentum of the soliton and identify the induced spin. The gauge-invariant energy-momentum tensor is given by

$$T_{\mu\nu} = -F_\mu{}^\lambda F_{\nu\lambda} + \tfrac{1}{2}\eta_{\mu\nu}F_{\lambda\sigma}F^{\lambda\sigma} + \frac{1}{f}\partial_\mu \mathbf{n}\cdot\partial_\nu\mathbf{n} - \frac{1}{2f}\eta_{\mu\nu}\partial_\lambda\mathbf{n}\cdot\partial^\lambda\mathbf{n} \tag{27}$$

and the angular momentum is defined by

$$M = \epsilon_{ij}\int d^2x\, x^i T^{0j} \ . \tag{28}$$

In Eq. (27), $T_{\mu\nu}$ is the total energy-momentum tensor defined by the response of the system to variations in the metric tensor $g_{\mu\nu}$. To identify the spin we need to express the energy-momentum tensor and the angular-momentum operator in terms of canonical variables defined in the previous section. Substituting Eq. (16) for $\mathbf{n}$ and replacing A_μ by $A_\mu^{\rm cl} + A_\mu$, we find the following expression for M:

$$M = -\int d^2x\, x^k \left[(\nabla^2\varphi + B)E_k - \frac{1}{m}(\partial_k\nabla^2\varphi)B\right] + \beta - \gamma(t) - \rho + \int d^2x\, x^k(\partial_k A_0^{\rm cl})\nabla^2\varphi \ , \tag{29}$$

where

$$E_k = \Pi_k + \frac{m}{2}\epsilon_{kl}A^l \tag{30}$$

and ρ and $\gamma(t)$ are defined in Eqs. (18) and (22). The terms linear in electromagnetic field operators can, however, be shown to vanish: In establishing this result we make use of the classical equations of motion [Eqs. (12a)–(12c)], the Gauss-law constraint

$$\partial_i \Pi^i + \frac{m}{2}B = 0 \ , \tag{31}$$

and the fact that $\varphi = \varphi(r)$, a function of the radial variable only. Thus the angular-momentum operator separates into three terms: the electromagnetic contribution, the soliton canonical angular momentum β, and the θ term. We find

$$M = -\int d^2x (\mathbf{x}\cdot\mathbf{E})B + \beta - \rho + \int d^2x\, x^k(\partial_k A_0^{\rm cl})\nabla^2\varphi \ . \tag{32}$$

The last two terms in Eq. (32) can be simplified by using the equations of motion [Eqs. (12a)–(12c)] written in the form

$$\varphi = em\int d^2x' G(\mathbf{x}-\mathbf{x}')J^0(r') \ , \tag{33}$$

where $J^0 = (1/4\pi r)\,\mathrm{sing}(r)g(r)'$. The Green's function G is given by

$$G(\mathbf{x}-\mathbf{x}') = \int \frac{d^2k}{(2\pi)^2}\frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}{k^2(k^2+m^2)} = -\frac{1}{2\pi m^2}[\ln(m|\mathbf{x}-\mathbf{x}'|) + K_0(m|\mathbf{x}-\mathbf{x}'|)] \ . \tag{34}$$

In Eq. (34), K_ν is the modified Bessel function. Using Eqs. (33) and (34), we can rewrite Eq. (32) as

$$M = -\int d^2x(\mathbf{x}\cdot\mathbf{E})B + \beta + \frac{\theta}{2\pi}\int d^2x\, d^2x' J^0(r) \times [1 - m|\mathbf{x}-\mathbf{x}'|K_1(m|\mathbf{x}-\mathbf{x}'|)]J^0(r') \ , \tag{35}$$

where $\theta = e^2/2m$. Equation (35) expresses the angular momenta of the electromagnetic field and the soliton angular momentum which consists of β (conjugate to α) and the θ term.

We discuss only the eigenvalues of the angular momentum of the soliton sector. Defining

$$M_{\text{soliton}}=\beta+M_{\text{fr}} , \tag{36}$$

where

$$M_{\text{fr}}=\frac{\theta}{2\pi}\int d^2x\, d^2x' J^0(r) \times[1-m|\mathbf{x}-\mathbf{x}'|K_1(m|\mathbf{x}-\mathbf{x}'|)]J^0(r') . \tag{37}$$

Now eigenvalues of β are integers, since β is $(1/i)\partial/\partial\alpha$ in the coordinate representation, whose eigenfunctions are $e^{in\alpha}$, $n\in\mathbb{Z}$. The fractional part of the angular momentum has the property that $M_{\text{fr}}=0$, for $m=0$. This can be easily arrived at from Eqs. (32) and (12a)-(12c). We now show a rather surprising result that, for $m\neq0$,

$$M_{\text{fr}}=\frac{\theta}{2\pi}Q^2 \quad (m\neq0,\ Q=1) . \tag{38}$$

There are two distinct ways to show that M_{fr} is independent of m, $m\neq0$ and both methods shed light into the nature of the θ term. For $m\neq0$, one can perform the angular integrations in Eq. (37) utilizing the fact that the topological current $J^0(r)$ is a function of only the radial coordinate. One finds that

$$\begin{aligned}M_{\text{fr}}^r&=\int d^2x\, d^2x' J^0(r)[m|\mathbf{x}-\mathbf{x}'|K_1(m|\mathbf{x}-\mathbf{x}'|)]J^0(r')\\&=-4\pi\left[\int_0^\infty r\,dr\,J^0(r)mrK_1(mr)\int_0^r r'dr'J^0(r')I_0(mr')-\int_0^\infty r\,dr\,J^0(r)mrI_1(mr)\int_r^\infty r'dr'J^0(r')K_0(mr')\right] .\end{aligned} \tag{39}$$

Now it is possible to show that Eq. (39) is a total derivative. Solving for A_0 from Eqs. (12a) and (12b) in the form

$$A_0=\frac{e}{(2\pi)^2}\left[\int_0^r r'dr'J^0(r')K_0(mr')I_0(mr')+\int_r^\infty r'dr'J^0(r')I_0(mr')K_0(mr')\right] \tag{40}$$

and substituting it into Eq. (12c), we find that

$$\begin{aligned}&\int_0^\infty r\,dr\frac{1}{f}\left[rg'g''+(g')^2-\frac{g'}{2r}\sin2g\right]\\&\quad=\frac{e^2}{2\pi}\left[\int_0^\infty r\,dr\,J^0(r)mrK_1(mr)\int_0^r r'dr'J^0(r')I_0(mr')-\int_0^\infty r\,dr\,J^0(r)mrI_1(mr)\int_r^\infty r'dr'J^0(r')K_0(mr')\right] .\end{aligned} \tag{41}$$

It is easy to see that the left-hand side (LHS) of Eq. (41) is a total derivative and vanishes due to the boundary conditions $g(0)=0$, $g(\infty)=\pi$, and so does $M_{\text{fr}}^r=0$. Hence our conclusion that the spin of the soliton is $(\theta/2\pi)Q^2$ to all orders in m (for $m\neq0$). The fractional part of the soliton spin is thus unaffected by the inclusion of the gauge field kinetic term. We have shown that the $Q=1$ soliton of the nonlinear O(3) σ model coupled to a U(1) gauge field with the topological mass term in $2+1$ dimensions has a fractional spin whose value is strictly $\theta/2\pi$.

We can reach the same conclusion by another way. Note that the model contains three dimensionful parameters, namely f, m, and e, whose dimensions are, in mass units, -1, 1, and $-\frac{1}{2}$, respectively. We may choose f to set the scale unit and treat $\overline{m}=mf$ and $\overline{e}=e\sqrt{f}$ as two independent dimensionless parameters of the model. Then the fractional spin can be written as

$$M_{\text{fr}}=\frac{\theta}{2\pi}\int d^2z\, d^2z' \overline{J}^0(z) \times[1-|\mathbf{z}-\mathbf{z}'|K_1(|\mathbf{z}-\mathbf{z}'|)]\overline{J}^0(z') , \tag{42}$$

where we define a dimensionless variable $\mathbf{z}$ by $\mathbf{z}=m\mathbf{x}$, $z=|\mathbf{z}|$ and dimensionless quantities by

$$\theta=\frac{\overline{e}^2}{2\overline{m}} ,$$

$$\overline{J}^0(z)=J^0(zm^{-1})m^{-2}=\frac{1}{4\pi z}(\sin\overline{g})\overline{g}' ,$$

$$\overline{g}(z)=g(zm^{-1}) .$$

The equation determining $\overline{g}(z)$ can be obtained from Eq. (12c):

$$\left[\overline{g}''+\frac{1}{z}\overline{g}'-\frac{1}{2z^2}\sin2\overline{g}\right]-\frac{\overline{e}}{4\pi z}(\sin\overline{g})\left[\frac{1}{\nabla_z^2-1}\overline{J}^0\right]'=0 . \tag{43}$$

A simple observation can be made: Eq. (43) contains only $\overline{e}$. This implies that $\overline{g}$ and $\overline{J}^0$ do not depend on $\overline{m}$, so $2\pi M_{\text{fr}}/\theta=\lim_{\overline{m}\to\infty}2\pi M_{\text{fr}}/\theta$. The evaluation of M_{fr} in the limit $\overline{m}\to\infty$ can be conveniently carried out by making use of an alternative expression of M_{fr}:

$$M_{\text{fr}}=\frac{\theta}{2\pi}\int d^2y\, d^2y' \tilde{J}^0(y) \times[1-\overline{m}|\mathbf{y}-\mathbf{y}'|K_1(\overline{m}|\mathbf{y}-\mathbf{y}'|)\tilde{J}^0(y') , \tag{44}$$

where $\mathbf{y}$ is a dimensionless variable defined by $\mathbf{y}=\mathbf{x}f^{-1}$, $y=|\mathbf{y}|$ and

$$\tilde{J}^0(z)=J^0(fy)f^2=\frac{1}{4\pi y}(\sin\tilde{g})\tilde{g}' ,$$

$$\tilde{g}(z)=g(fy) .$$

We can draw the same conclusion with

$$K_1(x)=\left[\frac{\pi}{2x}\right]^{1/2}e^{-x}[1+O(X^{-1})], \quad x\gg1 .$$

V. CONCLUSIONS

We have discussed the (2+1)-dimensional O(3) σ model coupled to an Abelian gauge field with a topological mass and have evaluated the fractional part of the spin of the soliton with unit topological charge. This model without the gauge kinetic term has been discussed by many authors. In the absence of the kinetic term this model is equivalent to the O(3) σ model with the Hopf invariant which is the third homotopy of the map **n** from the space-time S^3 to the field manifold S^2. The Hopf invariant is also related to the topological quantity called the linking number of the soliton trajectory.[3,15,21] In such a case the fractional spin of the soliton can be calculated in various ways[3–7] and it is found to be $\theta/2\pi$. If the kinetic term for the gauge field is introduced in the action, the gauge field becomes dynamical and Eq. (4) is no longer valid. Karabali and Murthy[19] have studied the model in a formal $1/m$ expansion and found no correction to the soliton spin up to order $1/m^5$. However, the procedure involves an infinite sequence of second-class constraints and they were unable to determine the exact dependence on $1/m$. In this paper we have carried out a semiclassical quantization of the model treating the nonvanishing classical gauge fields in the soliton sector as background gauge fields. The method results in only two first-class constraints. We have shown that no corrections arise to the spin of the soliton which remains at $\theta/2\pi$ for $m\neq0$. For $m=0$, we find the fractional part of the spin vanishes.

We conclude this paper with a simple physical argument which also leads us to the main result of our work. One may determine the spin and statistics as follows. Consider a pair of solitons located at a great distance apart. Rotating them adiabatically through π, one may find a phase factor e^{is} acquired by the wave function (of the pair of solitons) and, hence, the induced spin of the soliton M_{fr}, $e^{is}=\exp(i2\pi M_{\mathrm{fr}})$. In evaluating the phase factor for the adiabatic process described above, we may be able to take a static limit where higher-derivative terms in space and time can be suppressed. The kinetic term for the gauge fields can be neglected in such a limit, compared with the Chern-Simons term in the action Eq. (1). Thus the kinetic term may not affect the phase factor, hence the spin of the soliton. It can be understood in this context that the spin of the soliton in the gauged model is also related to the topological quantity such as the linking number of the O(3) σ model with the Hopf invariant only. However, the evaluation of the Hopf integral corresponding to the aforementioned process has never been performed in the literature due to a technical difficulty: In Ref. 3 we can find an evaluation of the Hopf integral corresponding to a sequence of processes, creating a soliton-antisoliton pair rotating the soliton through 2π, then annihilation.

ACKNOWLEDGMENTS

This work has been supported in part by an operating grant from the Natural Sciences and Engineering Research Council of Canada. One of us (T.L.) thanks A. N. Redlich for conversations on recent work on high-T_c superconductivity and also thanks X. G. Wen and A. Zee for sending him their recent works.

[1] A. A. Belavin and A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 503 (1975) [JETP Lett. **22**, 245 (1975)].

[2] For review see, R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982).

[3] F. Wilczek and A. Zee, Phys. Rev. Lett. **51**, 2250 (1983); D. Ravenel and A. Zee, Commun. Math. Phys. **98**, 239 (1985).

[4] Y. S. Wu and A. Zee, Phys. Lett. **147B**, 325 (1984).

[5] A. M. Din and W. J. Zakrzewski, Phys. Lett. **146B**, 341 (1984).

[6] T. Jaroszewicz, Phys. Lett. **146B**, 337 (1984); **159B**, 299 (1985).

[7] M. Bowick, D. Karabali, and L. C. R. Wijewardhana, Nucl. Phys. **B271**, 417 (1986).

[8] F. Wilczek, Phys. Rev. Lett. **49**, 957 (1982); Y. S. Wu, *ibid.* **52**, 2103 (1984); **53**, 111 (1984).

[9] J. F. Schonfeld, Nucl. Phys. **B185**, 157 (1981).

[10] S. Deser, R. Jackiw, and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982); Ann. Phys. (N.Y.) **140**, 372 (1982).

[11] R. Blankenbecler and D. Boyanovsky, Phys. Rev. D **34**, 612 (1986).

[12] R. Jackiw, Phys. Rev. D **29**, 2375 (1984).

[13] G. W. Semenoff, Phys. Rev. Lett. **61**, 517 (1988).

[14] I. E. Dzyaloshinskii, A. M. Polyakov, and P. W. Wiegmann, Phys. Lett. A **127**, 112 (1988).

[15] A. M. Polyakov, J. Mod. Phys. Lett. **A3**, 325 (1988).

[16] P. B. Wiegmann, Phys. Rev. Lett. **60**, 821 (1988).

[17] P. W. Anderson, Science **B235**, 1196 (1987); G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580 (1988); Z. Zou and P. W. Anderson, *ibid.* **37**, 627 (1988).

[18] The kinetic term was also considered in recent works; X. G. Wen and A. Zee, ITP Santa Barbara Report No. NSF-ITP-88-145, 1988 (unpublished); J. B. Marston, Phys. Rev. Lett. **61**, 1914 (1988).

[19] D. Karabali and G. Murthy, Phys. Rev. D **35**, 1522 (1987).

[20] N. H. Christ and T. D. Lee, Phys. Rev. D **12**, 1606 (1975).

[21] C.-H. Tze, J. Mod. Phys. **A3**, 1959 (1988); C.-H. Tze and Soonkeon Nam, Phys. Lett. B **210**, 76 (1988); S. Deser and A. N. Redlich, Phys. Rev. Lett. **61**, 1541 (1988).