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# Approach to general covariance in string space of BRST string field theory

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We show that Becchi-Rouet-Stora-Tyutin (BRST) quantized string field theory can be made invariant under string-dependent general coordinate transformations $x'^{\mu\sigma_1}=x'^{\mu\sigma_1}[x(\sigma)]$, where $\mu=0,1,\ldots,D-1$ and $0\le\sigma_1\le\pi$. This invariance is achieved by introducing a string-space metric tensor $g_{\mu\sigma_1,\nu\sigma_2}[x(\sigma)]$ and following the usual steps of general relativity, with the following differences: A constraint, which we term "tangent contravariance," must be imposed on the coordinate transformations and metric tensor. This restriction emerges from requiring that $\partial_\sigma x^\mu$ transforms in string space as a contravariant vector. Further stringent constraints on the metric tensor, including Ricci flatness, arise from demanding nilpotence of the BRST charge.

Einstein's principle of general covariance requires that the action of any theory should be invariant under the group of general coordinate transformations of the underlying space-time manifold. The underlying manifold of string field theory is the space of all strings ("loop space" or "string space"), and it appears that the existing Becchi-Rouet-Stora-Tyutin (BRST) formulations of string field theory[1-3] do not respect this principle. The associated limitation of string field theory to a flat background is widely recognized, and a number of proposals have been advanced to overcome this problem.[4-13] The suggestions in this paper have something in common with some of these proposals, but with the following differences. First, we will be concerned with implementing general coordinate invariance in string space, rather than on a background $D$-dimensional space-time.[4,5,13] Second, our focus is on gauging string-dependent coordinate transformations, instead of the more restricted set of string-dependent reparametrizations.[5-8] These objectives will entail introducing a metric string-field tensor $g_{\mu\sigma_1,\nu\sigma_2}[x^\alpha(\sigma)]$ for string space. Finally, the string space metric tensor $g_{\mu\sigma_1,\nu\sigma_2}[x^\alpha(\sigma)]$ is regarded here as an object distinct from the quantized string field $\Phi[x(\sigma)]$ (as opposed to the Kähler geometric approach in Ref. 11), and our framework throughout is BRST quantization.

We begin our discussion with the free bosonic open-string field theory[1-3] whose action is

$$S=\int Dx(\sigma)D\phi(\sigma)\Phi^\dagger[x,\phi]Q\Phi[x,\phi]\ , \tag{1}$$

where $\phi$ is the bosonized ghost.[2] The term "string space" denotes the space of open strings $x^\mu(\sigma)$. The flat background metric $\eta_{\mu\nu}$ enters this action in two ways: implicitly, via the string space measure

$$\int Dx^\mu(\sigma) \tag{2}$$

and explicitly, via expressions such as

$$\int d\sigma\left[-\eta^{\mu\nu}\frac{\delta}{\delta x^\mu}\frac{\delta}{\delta x^\nu}+\eta_{\mu\nu}\partial_\sigma x^\mu\partial_\sigma x^\nu\right] \tag{3}$$

in the BRST charge $Q$. Neither of these terms are generally covariant, even under ordinary coordinate transformations of the background $D$-dimensional space. A simple remedy is to replace $\eta_{\mu\nu}$ by the more general metric tensor $g_{\mu\nu}(x)$, but this is a fairly modest implementation of general covariance, since the arena of string dynamics is the infinite-dimensional string space, rather than the finite-dimensional background space. Moreover, the metric tensor of the background space is only one mode of the closed string field, and it seems unnatural to single it out for special attention in the action. This logic suggests that one should consider metrics defined over strings, rather than points, and try to implement covariance in the full infinite-dimensional string space.

It is useful to introduce the notation for a point on the string

$$x^{\mu\sigma}\equiv x^\mu(\sigma) \tag{4}$$

and treat the pair $(\mu\sigma)$ as a generalized index in string space. The string $x^\mu(\sigma)$ as a whole will be denoted by uppercase $X$, i.e. $x^{\mu\sigma}\in X$. A general coordinate transformation in string space is then denoted by

$$x'^{\mu\sigma}=x'^{\mu\sigma}[X]\ . \tag{5}$$

Next we introduce the metric tensor $g_{\mu\sigma_1,\nu\sigma_2}[X]$ (commas, in our notation, are used to separate index pairs in tensors and do not denote differentiation) which transforms under (5) as

$$g_{\mu\sigma_1,\nu\sigma_2}[X]=\frac{\delta x'^{\rho\sigma_3}}{\delta x^{\mu\sigma_1}}g_{\rho\sigma_3,\tau\sigma_4}[X']\frac{\delta x'^{\tau\sigma_4}}{\delta x^{\nu\sigma_2}}\ , \tag{6}$$

 

where the following summation convention over repeated generalized indices is understood:

$$V_{\mu\sigma} W^{\mu\sigma} \equiv \sum_{\mu} \int_0^{\pi} d\sigma \, V_{\mu\sigma} W^{\mu\sigma} \ . \tag{7}$$

In order to indicate that integration over the continuous index is suppressed, the following overbar notation will be used:

$$V_{\mu\bar{\sigma}} W^{\mu\bar{\sigma}} \equiv \sum_{\mu} V_{\mu\sigma} W^{\mu\sigma} \ . \tag{8}$$

The expressions $V_{\mu\sigma}$ and $W^{\mu\sigma}$ are of course meant to denote covariant and contravariant vectors, respectively, in string space, with the obvious transformation laws

$$\begin{aligned} V_{\mu\sigma}[X'] &= \frac{\delta x^{\rho\sigma_1}}{\delta x'^{\mu\sigma}} V_{\rho\sigma_1}[X] \ , \\ W^{\mu\sigma}[X'] &= \frac{\delta x'^{\mu\sigma}}{\delta x^{\rho\sigma_1}} W^{\rho\sigma_1}[X] \end{aligned} \tag{9}$$

and the inverse of the metric tensor is defined by

$$g^{\mu\sigma_1,\nu\sigma_2} g_{\nu\sigma_2,\rho\sigma_3} = \delta^{\mu}_{\rho} \delta(\sigma_1 - \sigma_3) \ . \tag{10}$$

The connection $\Gamma^{\rho\sigma_3}_{\mu\sigma_1,\nu\sigma_2}$, covariant derivatives $D_{\mu\sigma}$, and the Riemann tensor $R^{\mu\sigma_4}_{\rho\sigma_1,\tau\sigma_2,\nu\sigma_3}$ are all given by the usual formulas of general relativity, with the replacement

$$\begin{aligned} &\mu \to (\mu\sigma) \ , \\ &\frac{\partial}{\partial x^{\mu}} \to \frac{\delta}{\delta x^{\mu\sigma}} \ , \\ &V_{\mu} W^{\mu} \equiv \sum_{\mu} V_{\mu} W^{\mu} \to V_{\mu\sigma} W^{\mu\sigma} \equiv \sum_{\mu} \int d\sigma \, V_{\mu\sigma} W^{\mu\sigma} \ . \end{aligned} \tag{11}$$

Now in string theory the functional derivative of a string field $\delta\Phi/\delta x^{\mu\sigma}$ is a covariant vector, transforming as in Eq. (9), so the expression

$$g^{\mu\sigma_1,\nu\sigma_2} \frac{\delta\Phi}{\delta x^{\mu\sigma_1}} \frac{\delta\Phi}{\delta x^{\nu\sigma_2}} \tag{12}$$

is invariant under general coordinate transformations (5). But the superstring action also contains the term [cf. (3)]

$$\partial x^{\mu\sigma} \equiv \frac{\partial}{\partial\sigma} x^{\mu}(\sigma) \tag{13}$$

and this term does not transform, under arbitrary coordinate transformations, like a contravariant vector. Therefore expressions such as

$$g_{\mu\sigma_1,\nu\sigma_2} \partial x^{\mu\sigma_1} \partial x^{\nu\sigma_2} \tag{14}$$

are not invariant under arbitrary transformations (5). The simplest solution to this problem is to restrict the class of general coordinate transformations to those for which $\partial x^{\mu\sigma}$ *does* transform like a contravariant vector, i.e.,

$$\partial x'^{\mu\sigma_1} = \frac{\delta x'^{\mu\sigma_1}}{\delta x^{\nu\sigma_2}} \partial x^{\nu\sigma_2} \tag{15}$$

and for which the appropriate boundary conditions at $\sigma = 0, \pi$ are preserved. These will be referred to as tangent-contravariant (TC) general coordinate transformations. It is rather trivial to verify from (15) that the inverse of a TC transformation, and product of any two TC transformations, are also TC transformations; i.e., the set of TC transformations is a subgroup of the group of all coordinate transformations (5).

Some examples of TC transformations are in order. It is not hard to see that any coordinate transformation which commutes with arbitrary reparametrizations is a TC transformation, and in fact satisfies the stronger condition

$$\partial x^{\nu\bar{\sigma}_2} \frac{\delta x'^{\mu\sigma_1}}{\delta x^{\nu\bar{\sigma}_2}} = \partial x'^{\mu\sigma_1} \delta(\sigma_1 - \sigma_2) \ . \tag{16}$$

To show this, let $X_f$ denote the reparametrized string, and let $f(\sigma) = \sigma + \epsilon(\sigma)$ be an infinitesimal reparametrization. The condition that the coordinate transformation commutes with arbitrary reparametrizations is that

$$x'^{\mu f(\sigma)}[X] = x'^{\mu\sigma}[X_f] \ . \tag{17}$$

Taylor expanding both sides of (17), and noting that $\epsilon(\sigma)$ is arbitrary, gives condition (16). Therefore, any transformation of the form

$$x'^{\mu\sigma}[X] = F^{\mu}[R[X], x^{\rho\sigma}] \ , \tag{18}$$

where $R[X]$ is any reparametrization-invariant functional of $X$, and $F^{\mu}(R,x)$ is an arbitrary function, is an example of a TC transformation.

A restriction on the set of all possible coordinate transformations, such as Eq. (15), also restricts the set of metrics which can be transformed locally to an inertial frame. Those metrics which can be so transformed will be called "TC metrics." The transformation of the form (5), which takes an arbitrary metric to an inertial frame in the neighborhood of string $X_0$ is

$$\begin{aligned} x'^{a\sigma_1} = {} & x_0'^{a\sigma_1} + e^{a\sigma_1}_{\mu\sigma_2}[X_0](x - x_0)^{\mu\sigma_2} \\ & + \tfrac{1}{2} e^{a\sigma_1}_{\mu\sigma_2}[X_0] \Gamma^{\mu\sigma_2}_{\rho\sigma_3,\tau\sigma_4}[X_0](x - x_0)^{\rho\sigma_3}(x - x_0)^{\tau\sigma_4} \\ & + O((x - x_0)^3) \ , \end{aligned} \tag{19}$$

where

$$g_{\mu\sigma_1,\nu\sigma_2}[X] = e^{a\sigma}_{\mu\sigma_1}[X] \eta_{ab} e^{b\sigma}_{\nu\sigma_2}[X] \ . \tag{20}$$

The condition that (19) is a TC transformation, according to the definition of Eq. (15), is then

$$\begin{aligned} \partial x_0'^{a\sigma_1} + \partial_{\sigma_1} e^{a\sigma_1}_{\rho\sigma}(x - x_0)^{\rho\sigma} &= \partial x_0^{\mu\sigma_2} e^{a\sigma_1}_{\mu\sigma_2} + \partial(x - x_0)^{\mu\sigma_2} e^{a\sigma_1}_{\mu\sigma_2} \\ &\quad + \partial x_0^{\mu\sigma_2} e^{a\sigma_1}_{\lambda\sigma_3} \Gamma^{\lambda\sigma_3}_{\mu\sigma_2,\rho\sigma}(x - x_0)^{\rho\sigma} \ . \end{aligned} \tag{21}$$

This in turn gives us two equations [one from $O(1)$ and from $O(x - x_0)$ in Eq. (21)]. These are

$$\partial x_0^{\mu\sigma_2} e^{a\sigma_1}_{\mu\sigma_2}[X_0] = \partial x_0'^{a\sigma_1} \tag{22}$$

(with $x'$ satisfying appropriate boundary conditions), and

$$D_{\nu\sigma_3}\partial x^{\tau\sigma_1} \equiv \partial x_0^{\mu\sigma_2}\Gamma^{\tau\sigma_1}_{\mu\sigma_2,\nu\sigma_3}[X_0]+\delta^{\tau}_{\nu}\partial_{\sigma_1}\delta(\sigma_1-\sigma_3)$$

$$= e^{\tau\sigma_1}_{a\sigma_2}[X_0]\partial_{\sigma_2}e^{a\sigma_2}_{\nu\sigma_3}[X_0] \; . \tag{23}$$

So the condition that a metric is a TC metric is that there exists a corresponding vielbein satisfying Eq. (23). Of course, any other vielbein related to $e^{a\sigma_1}_{\mu\sigma_2}$ by a local Lorentz transformation

$$e'^{a\sigma}_{\mu\sigma_1}[X]=\Lambda^{a\bar\sigma}_{b\bar\sigma}[X]e^{b\bar\sigma}_{\mu\sigma_1}[X] \tag{24}$$

is also acceptable as the vielbein of a TC metric.

We now propose the following kinetic term for the open-string action, which is invariant under TC general coordinate transformations:

$$S_{\rm cov}=\int Dx(\sigma)D\phi(\sigma)\det{}^{1/2}[g]\Phi^{\dagger}[x,\phi]Q\Phi[x,\phi] \; , \tag{25}$$

where

$$Q=\sum_{m=-\infty}^{\infty} c_m(L^{\rm cov}_{-m}-\delta_{m0}) -\tfrac{1}{2}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}(m-n){:}c_{-m}c_{-n}b_{m+n}{:} \; , \tag{26}$$

$$L_n^{\rm cov}=\tfrac{1}{4}\int_0^{\pi}d\sigma\sum_{\pm}e^{\pm in\sigma}\Big[-e^{\mu\sigma_1}_{a\bar\sigma}\eta^{ab}e^{\nu\sigma_2}_{b\bar\sigma}D_{\mu\sigma_1}\frac{\delta}{\delta x^{\nu\sigma_2}} \pm 2ie^{a\bar\sigma}_{\mu\sigma_1}e^{\nu\sigma_2}_{a\bar\sigma}\partial x^{\mu\sigma_1}\frac{\delta}{\delta x^{\nu\sigma_2}} + e^{a\bar\sigma}_{\mu\sigma_1}\eta_{ab}e^{b\bar\sigma}_{\nu\sigma_2}\partial x^{\mu\sigma_1}\partial x^{\nu\sigma_2}\Big] +\kappa\delta_{n0} \; . \tag{27}$$

The infinite constant $\kappa$ is defined to normal-order $L_0^{\rm cov}$ in an inertial frame and $D_{\mu\sigma}$ is the string space covariant derivative. For a flat metric ($g_{\mu\sigma_1,\nu\sigma_2}[X]=\eta_{\mu\nu}\delta(\sigma_1-\sigma_2)$) Eq. (27) reduces to the usual expression.

From the definition of $L_n^{\rm cov}$ it is clear that $S_{\rm cov}$ is invariant under TC transformations, and also under local Lorentz transformations of the vielbeins Eq. (24). However, $S_{\rm cov}$ only makes sense as a string action in curved string space if the crucial property of nilpotence $Q^2=0$ is also preserved in the critical dimension $D=26$. This amounts to showing that the Virasoro algebra is preserved in curved metrics. But in fact, the Virasoro algebra is *not* preserved in arbitrary metrics and therefore the $Q^2=0$ condition places additional restrictions on the background space of string theory. (This was already noted, in the context of nonlinear $\sigma$ models, in Ref. 14.) It is simplest to derive this restriction by considering the commutator $[L_n^{\rm cov}[X],L_m^{\rm cov}[X]]$ in the neighborhood of a particular string $X_P$, and using string-space Riemann normal coordinates where

$$g_{\mu\sigma_1,\nu\sigma_2}=\eta_{\mu\nu}\delta(\sigma_1-\sigma_2) -\tfrac{1}{3}R_{\mu\sigma_1,\alpha\sigma_3,\nu\sigma_2,\beta\sigma_4}[X_P] \times(x-x_P)^{\alpha\sigma_3}(x-x_P)^{\beta\sigma_4}+\cdots \; . \tag{28}$$

We then find, for the commutator,

$$\begin{aligned}[L_m^{\rm cov},L_n^{\rm cov}]={}&(m-n)L^{\rm cov}_{m+n}+\frac{D}{12}(m^3-m)\delta_{m+n,0}\\ &-\tfrac{1}{4}\int d\sigma_1 d\sigma_2\{[\cos(m\sigma_1)\cos(n\sigma_2)-\cos(m\sigma_2)\cos(n\sigma_1)](e^{\mu\sigma_3}_{a\bar\sigma_1}\eta^{ab}e^{\nu\sigma_4}_{b\bar\sigma_1})(e^{\alpha\sigma_5}_{c\bar\sigma_2}\eta^{cd}e^{\beta\sigma_6}_{d\bar\sigma_2})\\ &\times[R^{\rho\sigma_7}_{\beta\sigma_6,\alpha\sigma_5,\nu\sigma_4}D_{\mu\sigma_3}+R^{\rho\sigma_7}_{\nu\sigma_4,\beta\sigma_6,\mu\sigma_3}D_{\alpha\sigma_5}+\tfrac{1}{2}(D_{\mu\sigma_3}R^{\rho\sigma_7}_{\beta\sigma_6,\alpha\sigma_5,\nu\sigma_4})+\tfrac{1}{2}(D_{\alpha\sigma_5}R^{\rho\sigma_7}_{\nu\sigma_4,\beta\sigma_6,\mu\sigma_3})]\\ &+2[\cos(m\sigma_1)\sin(n\sigma_2)-\sin(m\sigma_2)\cos(n\sigma_1)](e^{\mu\sigma_3}_{a\bar\sigma_1}\eta^{ab}e^{\nu\sigma_4}_{b\bar\sigma_1})(e^{c\bar\sigma_2}_{\alpha\sigma_5}e^{\beta\sigma_6}_{c\bar\sigma_2})\\ &\times\partial x^{\alpha\sigma_5}R^{\rho\sigma_7}_{\nu\sigma_4,\mu\sigma_3,\beta\sigma_6}\}\frac{\delta}{\delta x^{\rho\sigma_7}}+\text{spin-connection terms} \; ,\end{aligned} \tag{29}$$

where the spin-connection terms arise from derivatives acting on vielbeins, e.g.,

$$D_{\mu\sigma_3}(e^{a\bar\sigma}_{\alpha\sigma_1}\eta_{ab}e^{b\bar\sigma}_{\beta\sigma_2})=\omega^{a\bar\sigma}_{\mu\sigma_3,c\sigma_4}e^{c\sigma_4}_{\alpha\sigma_1}\eta_{ab}e^{b\bar\sigma}_{\beta\sigma_2} +e^{a\bar\sigma}_{\alpha\sigma_1}\eta_{ab}\omega^{b\bar\sigma}_{\mu\sigma_3,c\sigma_4}e^{c\sigma_4}_{\beta\sigma_2} \; . \tag{30}$$

The right-hand side of (30) is nonzero in general, despite the antisymmetry of $\omega$, because there is no integration over the $\bar\sigma$ index.

It is crucial that the curvature and spin-connection terms, which spoil the Virasoro algebra, be made to vanish. This is accomplished if

$$D_{\mu\sigma_3}(e^{a\bar\sigma}_{\alpha\sigma_1}\eta_{ab}e^{b\bar\sigma}_{\beta\sigma_2})=0 \tag{31}$$

and

$$e^{\mu\sigma_3}_{a\bar\sigma_1}\eta^{ab}e^{\nu\sigma_4}_{b\bar\sigma_1}R^{\rho\sigma_7}_{\nu\sigma_4,\beta\sigma_6,\mu\sigma_3}=0 \; . \tag{32}$$

Equation (31) is an extremely stringent condition on string space metrics. The only metrics which satisfy this condition are those which can be transformed, under a TC transformation, into the form

$$g_{\mu\sigma_1,\nu\sigma_2}[X]=G_{\mu\nu}(x(\sigma_1))\delta(\sigma_1-\sigma_2) \; . \tag{33}$$

This result is obtained by first noting that (31) implies a "diagonality" condition on the spin connection $\omega^{a\sigma_2}_{\mu\sigma_1,b\sigma_3} \propto \delta(\sigma_2-\sigma_3)$. Then consider vielbeins of the form

$$e^{a\sigma_1}_{\mu\sigma_2}[X]=E^a_\mu(x(\sigma_1))\delta(\sigma_1-\sigma_2)+E^a_\rho(x(\sigma_1))h^{\rho\sigma_1}_{\mu\sigma_2}[X] ,$$

where the loop-dependent part of the vielbein $h^{\rho\sigma_1}_{\mu\sigma_2}$ is infinitesimal. The restrictions on $h[X]$ imposed by diagonality of the spin connection are sufficient to show that the loop dependence of $h_{\mu\sigma_1,\nu\sigma_2}$ must have the form of an infinitesimal coordinate transformation plus a local Lorentz transformation of the vielbein. The loop dependence of the metric can therefore be removed by a TC transformation. For such metrics, Eq. (32) becomes just the usual vacuum Einstein equation

$$R_{\mu\nu}=0 , \tag{34}$$

where $R_{\mu\nu}$ is the Ricci tensor formed from the $D$-dimensional metric $G_{\mu\nu}(x)$.

Given a metric of the form (33), it is still necessary to show that there exist corresponding (loop-dependent) vielbeins $e^{a\sigma_1}_{\mu\sigma_2}$ satisfying Eq. (23). Equation (33) implies that the vielbeins must have the form

$$e^{\tau\sigma_1}_{a\sigma_2}[X]=\delta(\sigma_1-\sigma_2)E^\tau_c(x(\sigma_1))\Lambda^{c\bar\sigma_1}_{a\bar\sigma_1}[X] . \tag{35}$$

Substituting into Eq. (23), we have

$$\partial x^{\mu\sigma}\Gamma^\tau_{\mu\nu}E^b_\tau=\partial_\sigma E^b_\nu+\Lambda^{b\bar\sigma}_{a\bar\sigma}(\partial_\sigma\Lambda^{a\bar\sigma}_{d\bar\sigma})E^d_\nu , \tag{36}$$

where $\Gamma^\tau_{\mu\nu}$ is the affine connection formed from the $D$-dimensional metric $G_{\mu\nu}$. Then, using

$$\partial_\sigma E^b_\nu(x(\sigma))=\partial x^{\mu\sigma}\frac{\partial}{\partial x^\mu}E^b_\nu=\partial x^{\mu\sigma}(\Gamma^\tau_{\mu\nu}E^b_\tau+\omega^b_{\mu c}E^c_\nu) , \tag{37}$$

where $\omega^b_{\mu c}$ is the $D$-dimensional spin-connection formed from $E^a_\mu$, we find

$$\Lambda^{a\bar\sigma}{}_{c\bar\sigma}[X]\eta_{ab}\partial_\sigma\Lambda^{b\bar\sigma}{}_{d\bar\sigma}[X]=-\partial x^{\mu\sigma}\omega_{\mu cd}(x(\sigma)) . \tag{38}$$

The string dependence of $\Lambda^{a\sigma}{}_{b\sigma}[X]$ enters via the factor of $\partial x^{\mu\sigma}$ in (38). It is always possible to find a solution $\Lambda[X]$ of (38); the only constraint which the solution must satisfy is

$$\partial_\sigma[\Lambda^{a\bar\sigma}{}_{c\bar\sigma}[X]\eta_{ab}\Lambda^{b\bar\sigma}{}_{d\bar\sigma}[X]]=\partial_\sigma\eta_{cd}=0 \tag{39}$$

which follows directly from Eq. (38), using the antisymmetry properties of $\omega_{\mu cd}$. Note that the vielbein (35) is string dependent, even if the metric (33) is not, and that the TC transformation to an inertial frame [Eq. (19)] is, in general, a string-dependent transformation.

From one point of view, namely, that of creating a vast extension of the possible background spaces of string theory, Eq. (33) is a disappointment. We have set up a formalism which allows for nontrivial string dependence of the metric, only to find that the nilpotence condition limits us to metrics whose string dependence can be transformed away by a suitable coordinate transformation. However, this limitation on metrics does not collapse general covariance in string space down to general covariance in $D$-dimensional space. There is still the freedom to transform to an inertial frame at any arbitrary string, regardless of the occurrence of self-intersections. As a result of this freedom [which allows us, in particular, to write the metric in the form (28)], the only condition on the curvature tensor is Eq. (32), which in a suitable frame is just the Ricci flatness condition (34).

It appears, then, that our formalism is *not* equivalent to that of summing over world sheets in a curved background space-time. In $\sigma$-model calculations, the Ricci flatness condition is only the one-loop requirement, while in our approach this is the full condition on the curvature. It may be that a theory invariant under general coordinate transformations in string space is incompatible with any simple picture of world sheets embedded in a background $D$-dimensional space time. In any case, the introduction of background curvature in the usual nonlinear $\sigma$ model, and in TC-covariant string field theory, appear to be inequivalent.

The extension of general covariance to interacting string field theory[2,3] is dependent on the details of how the interactions are introduced; we will briefly indicate what seems required for the Witten theory.[2] In Witten's theory the $*$ product and integration involve the overlap of half-strings. For these operations to be general coordinate invariant, it is necessary that the distinction between left and right half-strings be invariant. Let us denote $X=(Y_H;Z_H)$ in the sense that

$$x^{\mu\sigma}=\begin{cases} y^{\mu\sigma}, & 0\le\sigma<\frac{\pi}{2} , \\ z^{\mu(\pi-\sigma)}, & \frac{\pi}{2}<\sigma\le\pi , \end{cases}$$
$$x^{\mu\pi/2}=y^{\mu\pi/2}=z^{\mu\pi/2} \tag{40}$$

and write

$$\begin{aligned} &\Phi[X]=\Phi[Y_H;Z_H] , \\ &\int DX=\int DY_H DZ_H , \\ &\int\Phi=\int DY_H\Phi[Y_H;Y_H] , \\ &\Phi*\Psi[Y_H;Z_H]=\int DW_H\Phi[Y_H,W_H]\Psi[W_H,Z_H] \end{aligned} \tag{41}$$

(reference to ghost degrees of freedom is suppressed; the half-string integration measure can be defined more precisely by discretizing the $\sigma$ variable into $N$ points and taking the $N\to\infty$ limit, cf. Refs. 15 and 16). Then the TC transformations must be further restricted to satisfy

$$x'^{\mu\sigma}=\begin{cases} f^{\mu\sigma}[Y_H], & 0\le\sigma<\frac{\pi}{2} , \\ f^{\mu(\pi-\sigma)}[Z_H], & \frac{\pi}{2}<\sigma\le\pi , \end{cases} \tag{42}$$

where $f^{\mu\sigma}$ is some functional of half-strings and, for consistency,

$$x'^{\mu\pi/2}=f^{\mu\pi/2}(x^{\mu\pi/2}) . \tag{43}$$

Consequently

$$\frac{\delta x'^{\mu\sigma_1}}{\delta x^{\nu\sigma_2}}=0 \quad \text{for } \sigma_{1(2)}>\frac{\pi}{2},\ \sigma_{2(1)}<\frac{\pi}{2} \tag{44}$$

which implies, for the vielbeins and metric, that

$$e^{a\sigma_1}_{\mu\sigma_2}=\begin{cases} e^{a\sigma_1}_{\mu\sigma_2}[Y_H], & \sigma_{1,2}<\frac{\pi}{2}, \\ e^{a\sigma_1}_{\mu\sigma_2}[Z_H], & \sigma_{1,2}>\frac{\pi}{2}, \\ e^{a\pi/2}_{\mu\pi/2}(x^{\rho\pi/2}), & \sigma_{1,2}=\frac{\pi}{2}, \\ 0 & \text{otherwise}, \end{cases}$$

$$g_{\mu\sigma_1,\nu\sigma_2}=\begin{cases} g_{\mu\sigma_1,\nu\sigma_2}[Y_H], & \sigma_{1,2}<\frac{\pi}{2}, \\ g_{\mu\sigma_1,\nu\sigma_2}[Z_H], & \sigma_{1,2}>\frac{\pi}{2}, \\ g_{\mu\pi/2,\nu\pi/2}(x^{\rho\pi/2}), & \sigma_{1,2}=\frac{\pi}{2}, \\ 0 & \text{otherwise}. \end{cases} \tag{45}$$

The Witten theory is then made invariant, under tangent-contravariant transformations, by replacing half-string integration measures $DY_H$ by $DY_H \det^{1/2}[g[Y_H]]$ in the definition of the $*$ product and $\int$, and using the covariantized BRST operator defined in Eqs. (26) and (27) above.

In summary, a fairly straightforward approach to general covariance in string field theory, via the introduction of a string space metric tensor, leads to a new way of introducing Ricci flat background curvature, independent of condensation of any particular mode of the quantized string field.

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[1]W. Siegel, Phys. Lett. **151B**, 391 (1985).
[2]E. Witten, Nucl. Phys. **B268**, 253 (1986).
[3]H. Hata, K. Itoh, H. Kunitomo, and K. Ogawa, Phys. Rev. D **34**, 2360 (1986).
[4]T. Hori, Phys. Lett. B **194**, 487 (1987).
[5]I. Araf'eva and I. Volovich, Trieste Report No. IC/87/61 (unpublished).
[6]K. Bardakci, Nucl. Phys. **B297**, 583 (1988).
[7]M. Kaku, Int. J. Mod. Phys. **A2**, 1 (1987).
[8]I. Bars and S. Yankielowicz, Phys. Lett. B **196**, 329 (1987).
[9]L. Castellani, Phys. Lett. B **206**, 47 (1988).
[10]G. Horowitz, J. Lykken, R. Rohm, and A. Strominger, Phys. Rev. Lett. **57**, 283 (1986).
[11]M. Bowick and S. Rajeev, Nucl. Phys. **B293**, 348 (1987).
[12]S. Das and M. Rubin, in *Lewes String Theory Workshop*, edited by L. Clavelli and A. Halprin (World Scientific, Singapore, 1986).
[13]A. Niemi, Ann. Phys. (N.Y.) **187**, 369 (1988).
[14]T. Banks, D. Nemeschansky, and A. Sen, Nucl. Phys. **B277**, 67 (1986); S. Jain, G. Mandel, and S. Wadia, Phys. Rev. D **35**, 3116 (1987).
[15]M. Srednicki and R. Woodward, Phys. Lett. B **196**, 55 (1987).
[16]J. Bordes and F. Lizzi, Rutherford Report No. RAL-88-035 (unpublished); Phys. Rev. Lett. **61**, 278 (1988).