Steepest-descent contours in the path-integral approach to quantum cosmology. I. The de Sitter minisuperspace model

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(Received 28 November 1988)

We consider the issue of finding a convergent contour of integration in the path-integral representation of the wave function for a simple exactly soluble model, the de Sitter minisuperspace model. Following a suggestion of Hartle, we look for the steepest-descent contour in the space of complex four-metrics. We determine all the possible contours which give solutions to the Wheeler-DeWitt equation or Green's functions of the Wheeler-DeWitt operator. We attempt to apply the boundary-condition proposal of Hartle and Hawking. We find that the proposal does not fix the solution uniquely because, although the initial point of the paths is fixed, the contour is not. We find a contour which represents the Vilenkin wave function and discuss the differences between the Hartle-Hawking and Vilenkin wave functions. We also discuss some of the implications of integrating over complex metrics. One consequence is that the signature of the metric is not respected, even at the semiclassical level.

I. INTRODUCTION

Many modern approaches to quantum gravity involve a Euclidean functional integral of the form

$$\int_C \mathcal{D}g_{\mu\nu} \exp(-I[g_{\mu\nu}]) \ , \qquad (1.1)$$

where I is the Euclidean version of the Einstein-Hilbert action:

$$I = -\int d^4x \, g^{1/2}(R - 2\Lambda) - 2\int d^3x \, h^{1/2} K \ . \qquad (1.2)$$

There are at least five things that need to be specified before (1.1) may be regarded as properly and uniquely defined. These are (i) gauge-fixing terms, (ii) a regularization scheme, (iii) a measure, (iv) boundary conditions, and (v) a contour of integration. This last ingredient is highly nontrivial in quantum gravity because the Einstein-Hilbert action has a special feature that distinguishes it from most matter-field actions—it is not positive definite. As a consequence, the path integral (1.1) will not converge if taken over real Euclidean metrics.

A way around this difficulty was proposed by Gibbons, Hawking, and Perry.[1] They observed that it is the conformal part of the Euclidean four-metric which is responsible for the Euclidean action becoming negative. They therefore suggested that the integral over all metrics be split up into a sum over conformal equivalence classes and a sum over conformal factors within each class, where the representant is chosen in such a way that $R = 4\Lambda$. They claimed that the integral could then be made convergent if the integration over conformal factors was rotated to lie parallel to the imaginary axis.

There have, however, been objections to this proposal. First of all, it is not obvious that the proposed split into conformal equivalence classes can always be carried out in the manner claimed—it seems to miss out on some metrics.[2] Second, the proposal does not work when the metric is coupled to nonconformally invariant manner—the positivity of the Euclidean matter action is not preserved by the conformal rotation.[2] And finally, the proposal is very difficult to implement in concrete examples.

In an attempt to sidestep these difficulties, an alternative proposal was made by Hartle. He suggested that the path integral (1.1) be taken along the steepest-descent path in the space of complex four-metrics.[3] In this approach, one does not necessarily take a Euclidean or Lorentzian path-integral expression to be the starting point. Rather, $g_{\mu\nu}$ is regarded as complex and the integration is taken along the contour along which the real part of the action increases most rapidly. Such a contour is not unique, as we shall see in the example below.

In this paper, we will apply this idea to a simple minisuperspace model in which the path integral can be evaluated exactly. The model is the de Sitter minisuperspace model, in which one restricts the metric to be of the Robertson-Walker type, described by a single scale factor, and the action is taken to be the Einstein-Hilbert action with cosmological term. This model is to quantum cosmology what the simple harmonic oscillator is to basic quantum mechanics—it is the simplest nontrivial exactly soluble model. Our primary aim is to determine the contours that yield a convergent path integral. We will then go on to study the implementation of the boundary-condition proposals of Hartle and Hawking,[4] and of Vilenkin.[5] This is of interest because a careful discussion of exactly how the Hartle-Hawking proposal is implemented in the path integral has never been given. In addition, the Vilenkin proposal is normally expressed as a statement about the behavior of the wave function in superspace. Although a path-integral version of this proposal has been suggested,[6] its consequences have not been worked out in detail. It would be useful for the purposes of comparing the proposals to have a path-integral expression encapsulating both the Hartle-Hawking and Vilenkin proposals.

There are a number of issues concerning a more precise definition of the above proposal which we will not go into here. For example, we do not fully address the issue of what it means to integrate along a path of steepest descent in a multidimensional integral. In our model, it will turn out that all the integrations but one (the lapse integration) are essentially trivial, so the definition of steepest descents is not a pressing issue. These and other issues concerning the contour are discussed in Ref. 7.

The general path-integral construction for minisuperspace models, including all the ghost and gauge-fixing terms, and a discussion of the measure and operator-ordering problem, was given in Ref. 8. It was found that the minisuperspace propagator, in the gauge $\dot{N}=0$, where N is the lapse function, reduces to an expression of the form

$$G(q''|q')=\int dT\langle q'',T|q',0\rangle \ , \tag{1.3}$$

where T is proportional to N and $\langle q'',T|q',0\rangle$ is the usual (Euclidean) quantum-mechanical propagator:

$$\langle q'',T|q',0\rangle=\int \mathcal{D}p\,\mathcal{D}q\,\exp(-I[p,q]) \ . \tag{1.4}$$

Operating on (1.3) with $\hat{H}''$, the Hamiltonian operator at q'', and using the fact that (1.4) satisfies the Euclidean Schrödinger equation, one obtains

$$\hat{H}''G=-\int dT\frac{\partial}{\partial T}\langle q'',T|q',0\rangle$$
$$=-[\langle q'',T|q',0\rangle]_{T_1}^{T_2} \ , \tag{1.5}$$

where T_1 and T_2 are the beginning and end points of the T contour. If T is taken to have an infinite range, then the right-hand side of (1.5) is zero, assuming that $\langle q'',T|q',0\rangle$ goes to zero at the end points, and G is then a solution to the Wheeler-DeWitt equation. If T is taken to have a half-infinite range, $T_1=0$, then the right-hand side of (1.5) is a delta function, and G is a Green's function of the Wheeler-DeWitt operator, again assuming certain convergence properties. This was discussed in Ref. 8, where the convergence properties were assumed. In the simple model of this paper, we shall show explicitly that contours can be found for which the assumed convergence properties hold. We are mainly interested in solutions to the Wheeler-DeWitt equation, but we will briefly discuss the Green's functions also. We do not wish to suggest, however, that one of these objects is more relevant than the other. The point of view that the Green's functions are the objects of interest has been put forward by Teitelboim.[9] A further possibility for the T contour, which seems to us to be an attractive one, is to take it to be a closed loop about the origin. This yields a solution to the Wheeler-DeWitt equation.

In Secs. II and III we consider the path integral for the amplitude (1.3), with fixed initial q and fixed initial p. All the p and q integrations are essentially trivial (i.e., Gaussian), leaving the T integral as the only nontrivial one. We list the possible contours and evaluate the path integral along these contours. In Sec. IV we attempt to apply the boundary-condition proposal of Hartle and Hawking, which is that one integrate over compact four-metrics. We find that it does not fix the solution uniquely in that although it fixes the end points, it does not fix the contour. We also find a contour for the Vilenkin wave function, and discuss the difference between the two wave functions. In Sec. V we discuss some of the consequences of a complex contour. The main consequence is that the signature of the metric is not respected. This means that the path integral can receive contributions from saddle points at which the action has an unexpected sign. We summarize and conclude in Sec. VI.

II. THE DE SITTER MINISUPERSPACE MODEL

The metric of the de Sitter minisuperspace model may be taken to be

$$ds^2=\frac{N^2(\tau)}{q(\tau)}d\tau^2+q(\tau)d\Omega_3^2 \ , \tag{2.1}$$

where $d\Omega_3^2$ is the metric on the unit three-sphere. The factor of q dividing N^2 turns out to simplify the algebra (it makes the Hamiltonian quadratic). The Einstein-Hilbert action with cosmological constant for this metric is

$$I[q(\tau)]=\tfrac{1}{2}\int_{\tau'}^{\tau''}d\tau N\left[-\frac{\dot{q}^2}{4N^2}+\lambda q-1\right] \ , \tag{2.2}$$

where λ is the (rescaled) cosmological constant. This is the action appropriate to fixed initial and final q. The extremizing solution is a four-sphere of radius $\lambda^{-1/2}$, i.e., the Euclidean section of de Sitter space.

The Hamiltonian form of the Euclidean action is

$$I[p(\tau),q(\tau)]=\int_{\tau'}^{\tau''}d\tau(p\dot{q}-NH) \ , \tag{2.3}$$

where

$$H=\tfrac{1}{2}(-4p^2-\lambda q+1) \ . \tag{2.4}$$

The classical field equations are (taking $N=1$)

$$\dot{p}=\frac{\lambda}{2}, \quad \dot{q}=-4p \ . \tag{2.5}$$

In addition, the Lagrange multiplier N enforces the Hamiltonian constraint, $H=0$.

As mentioned in the Introduction, the explicit path-integral construction for this model in the gauge $\dot{N}=0$, for fixed initial and final q, is given by

$$G(q''|q')=\int dT\int \mathcal{D}p\,\mathcal{D}q\,\exp\{-I[p(\tau),q(\tau)]\} \ , \tag{2.6}$$

where $T=N(\tau''-\tau')$ (Ref. 8). Our aim is to determine the contours for which (2.6) converges.

We are also interested in the path integral between fixed initial p and final q,

$$\tilde{G}(q''|p')=\int dT\int \mathcal{D}p\,\mathcal{D}q\,\exp\{-\tilde{I}[p(\tau),q(\tau)]\} \tag{2.7}$$

and this is what we shall consider first, since it is simpler. In (2.7), $\tilde{I}$ is the Hamiltonian form of the Euclidean action appropriate to fixed initial p and final q:

$$\tilde{I}=\int_0^T d\tau(p\dot{q}-H)+p'q' \ , \tag{2.8}$$

where p',q' are the initial values of p,q, and we have absorbed N into the definition of τ. It has been argued that the propagation amplitude from fixed initial p is the appropriate object to consider when implementing Hartle-Hawking boundary conditions, in more general models.[10] For the model considered here, however, either (2.6) or (2.7) is suitable. (2.6) will be discussed in the next section.

The contour of integration in (2.7) runs through the infinite-dimensional space with coordinates $(T,p(\tau),q(\tau))$, subject to the boundary conditions

$$p(0)=p', \quad q(T)=q'' \tag{2.9}$$

with $p(T)$ and $q(0)$ free. It can be studied most effectively by separating out those integrations for which the contour is essentially trivial (i.e., Gaussian). It turns out that this is most easily achieved by performing the following shift of the variables of integration: let

$$q(\tau)=\bar{q}(\tau)+Q(\tau), \quad p(\tau)=\bar{p}(\tau)+P(\tau) , \tag{2.10}$$

where $\bar{q}(\tau)$ and $\bar{p}(\tau)$ are the solution to the field equations (2.5), satisfying the boundary conditions (2.9), but *they do not satisfy the constraint equation* $H=0$. That is, they are saddle points of the p,q functional integral but they are not saddle points of the T integral. The explicit forms for $\bar{q}(\tau)$ and $\bar{p}(\tau)$ are

$$\begin{aligned}\bar{q}(\tau)&=-\lambda(\tau^2-T^2)-4p'(\tau-T)+q'' ,\\ \bar{p}(\tau)&=\frac{\lambda\tau}{2}+p' .\end{aligned} \tag{2.11}$$

In terms of the new variables (2.10), the action is

$$\tilde{I}=\tilde{I}_0(q'',T|p',0)+\tilde{I}_2[Q(\tau),P(\tau)] , \tag{2.12}$$

where $\tilde{I}_0$ is the action of the solution (2.11),

$$\tilde{I}_0=\frac{\lambda^2T^3}{6}+\lambda p'T^2+\left[\frac{\lambda q''-1}{2}+2p'^2\right]T+q''p' \tag{2.13}$$

and

$$\tilde{I}_2=\int_0^T d\tau(P\dot{Q}+2P^2) . \tag{2.14}$$

The path integral (2.7) may now be written

$$\tilde{G}(q''|p')=\int dT\exp(-\tilde{I}_0)\int \mathcal{D}P\,\mathcal{D}Q\exp(-\tilde{I}_2) . \tag{2.15}$$

Consider the functional integral over P and Q. It may be defined by a time-slicing procedure and the measure is just the Liouville measure $dP\,dQ$ on each time slice.[8] It follows from (2.9) and (2.10) that the boundary conditions are that P is zero on the initial slice and integrated on the final slice, and Q is integrated on the initial slice and zero on the final slice. There are therefore an equal number of P and Q integrations. This path integral can be evaluated exactly, since it is so simple. However, the result can clearly depend only on T, and this dependence can be determined by a simple scaling argument. Let $\tau\to\tau/T$, $P\to T^{-1/2}P$, and $Q\to T^{1/2}Q$. Then all dependence on T drops out of the action $\tilde{I}_2$. Furthermore, the measure is unchanged by this transformation since there are an equal number of P and Q integrations. The functional integration over P and Q is therefore just equal to a constant, independent of T. There is of course the question of choosing contours for P and Q. We will return to this point in the final section.

The path integral (2.15) is now just a single ordinary integration

$$\tilde{G}(q''|p')=\int dT\exp[-\tilde{I}_0(q'',T|p',0)] \tag{2.16}$$

and we can proceed to the steepest-descent analysis. Consider first the case $\lambda q''<1$. There are two saddle points, at

$$T=T_\pm=\frac{1}{\lambda}[-2p'\pm(1-\lambda q'')^{1/2}] \tag{2.17}$$

at which $\tilde{I}_0$ takes the values

$$\tilde{I}_0=\mp\frac{1}{3\lambda}(1-\lambda q'')^{3/2}+\frac{1}{\lambda}\left[p'-\frac{4p'^3}{3}\right] . \tag{2.18}$$

These saddle points correspond to solutions of the field equations (2.5) *and* the constraint, $H=0$.

Because we are considering the propagator with fixed initial p, the geometric interpretation of the saddle points is a little more subtle that the case of fixed initial q. There are two possible geometric pictures, if $|p'|<\frac{1}{2}$. These are that the boundary three-spheres are connected by sections of four-sphere which either include or exclude the equator. The former possibility is realized at T_+ when $p'<0$ and at T_- when $p'>0$, whilst the latter possibility is realized at T_+ when $p'>0$, and at T_- when $p'<0$. If $|p'|>\frac{1}{2}$ there is no obvious geometric picture.

The steepest-descent paths are the paths for which $\text{Im}(\tilde{I}_0)$ is constant. The paths which pass through the saddle points are shown in Fig. 1.

Consider now the case $\lambda q''>1$. The saddle points are now at

$$T=\frac{1}{\lambda}[-2p'\pm i(\lambda q''-1)^{1/2}] \tag{2.19}$$

with action

$$\tilde{I}_0=\pm\frac{i}{3\lambda}(\lambda q''-1)^{3/2}+\frac{1}{\lambda}\left[p'-\frac{4p'^3}{3}\right] . \tag{2.20}$$

The steepest-descent paths passing through these points are shown in Fig. 2.

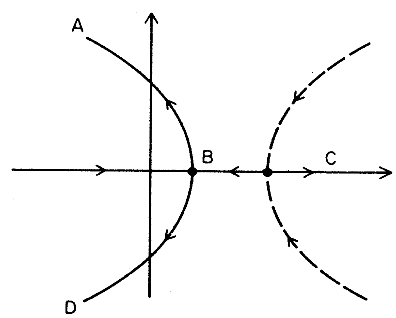

FIG. 1. The steepest-descent paths in the complex T plane for $\tilde{G}(q''|p')$ in the case $\lambda q''<1$. The arrows point downhill.

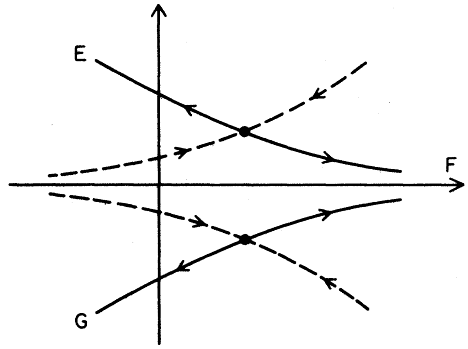

FIG. 2. The case $\lambda q''>1$.

The integral can be evaluated exactly in terms of Airy functions.[11] Let $T=\tilde{T}-2p'/\lambda$. Then (2.16) becomes

$$\tilde{G}(q''|p')=\exp\left[\frac{1}{\lambda}\left[\frac{4p'^3}{3}-p'\right]\right]\times\int d\tilde{T}\exp\left[\frac{-\lambda^2\tilde{T}^3}{6}+\frac{(1-\lambda q'')\tilde{T}}{2}\right]. \quad (2.21)$$

In Fig. 1 there are two distinct contours along which the integral will converge. The first possibility is to take the contour *ABD*. This corresponds to the contour *EFG* in Fig. 2. It may be distorted into the contour $\mathrm{Re}(\tilde{T})=0$, and using (2.21), one obtains

$$\tilde{G}(q''|p')=\exp\left[\frac{1}{\lambda}\left[\frac{4p'^3}{3}-p'\right]\right]\mathrm{Ai}\left[\frac{1-\lambda q''}{(2\lambda)^{2/3}}\right], \quad (2.22)$$

where we have ignored overall constant factors. (2.22) has the asymptotic forms

$$\tilde{G}(q''|p')=\exp\left[\frac{1}{\lambda}\left[\frac{4p'^3}{3}-p'\right]\right]\times\exp\left[\frac{-1}{3\lambda}(1-\lambda q'')^{3/2}\right]\quad \text{for } \lambda q''<1, \quad (2.23)$$

$$\tilde{G}(q''|p')=\exp\left[\frac{1}{\lambda}\left[\frac{4p'^3}{3}-p'\right]\right]\times\cos\left[\frac{1}{3\lambda}(1-\lambda q'')^{3/2}-\frac{\pi}{4}\right]\quad \text{for } \lambda q''>1. \quad (2.24)$$

The second possibility is to take the contour *ABC*. It corresponds to the contour *EF* in Fig. 2. It may be distorted into the contour in which $\tilde{T}$ runs from $+i\infty$ to 0 and then from 0 to ∞. Using (2.21), one thus obtains

$$\tilde{G}(q''|p')=\exp\left[\frac{1}{\lambda}\left[\frac{4p'^3}{3}-p'\right]\right]\times\left[\mathrm{Ai}\left[\frac{1-\lambda q''}{(2\lambda)^{2/3}}\right]+i\,\mathrm{Bi}\left[\frac{1-\lambda q''}{(2\lambda)^{2/3}}\right]\right]. \quad (2.25)$$

(2.25) has the asymptotic forms

$$\tilde{G}(q''|p')=\exp\left[\frac{1}{\lambda}\left[\frac{4p'^3}{3}-p'\right]\right]\times\exp\left[\frac{1}{3\lambda}(1-\lambda q'')^{3/2}\right]\quad \text{for } \lambda q''<1, \quad (2.26)$$

$$\tilde{G}(q''|p')=\exp\left[\frac{1}{\lambda}\left[\frac{4p'^3}{3}-p'\right]\right]\times\exp\left[\frac{-i}{3\lambda}(1-\lambda q'')^{3/2}\right]\quad \text{for } \lambda q''>1. \quad (2.27)$$

The contour *DBC* will of course yield a third solution, the complex conjugate of (2.25), but this is not linearly independent of the first two.

Using (2.21), for example, one may verify that $\tilde{G}(q''|p')$ is an exact solution to the Wheeler-DeWitt equation at q'':

$$\hat{H}''\tilde{G}=\frac{1}{2}\left[4\frac{d^2}{dq''^2}-\lambda q''+1\right]\tilde{G}=0, \quad (2.28)$$

where $\hat{H}''$ is the Hamiltonian (2.4) with the substitution $p\to -d/dq$. There is no factor of i because these are Euclidean momenta. Had we done everything from the outset in a Lorentzian framework, we would have Lorentzian momenta p_L, say, and the classical Hamiltonian (2.4) would have the opposite relative sign between the potential and kinetic terms. We would then have made the substitution $p_L\to -i(d/dq)$ and the resulting Wheeler-DeWitt equation would coincide precisely with (2.28). We should point out, however, that not all authors agree on this issue (see Ref. 9, for example). Note that the operator ordering in (2.28) is the one corresponding to our choice of the Liouville measure in (2.15). $\tilde{G}$ is also a solution to the Wheeler-DeWitt equation at p' in the momentum representation:

$$\hat{H}'\tilde{G}=\frac{1}{2}\left[-4p'^2-\lambda\frac{d}{dp'}+1\right]\tilde{G}=0, \quad (2.29)$$

where $\hat{H}'$ is the Hamiltonian (2.4) with the substitution $q\to d/dp$.

III. FIXED INITIAL *Q*

We now consider the propagator between fixed values of q, (2.6). This may be calculated by following the steps used in the previous section; thus we begin by shifting to the solution of the field equations (2.5) (again without enforcing the constraint equation) subject to the boundary conditions

$$q(T)=q'', \quad q(0)=q' . \tag{3.1}$$

The solution is

$$\bar{q}(\tau)=-\lambda\tau^2+\left[\frac{q''-q'}{T}+\lambda T\right]\tau+q' ,$$
$$\bar{p}(\tau)=\frac{\lambda\tau}{2}-\frac{1}{4}\left[\frac{q''-q'}{T}+\lambda T\right] . \tag{3.2}$$

Following the same steps as in Sec. II, the path integral (2.6) reduces to

$$G(q''|q')=\int\frac{dT}{T^{1/2}}\exp[-I_0(q'',T|q',0)] , \tag{3.3}$$

where I_0 is the action of the solution (3.2), and is given by

$$I_0=\frac{\lambda^2T^3}{24}+\left[\frac{\lambda(q''+q')}{4}-\frac{1}{2}\right]T-\frac{(q''-q')^2}{8T} . \tag{3.4}$$

The integral (3.3) has four saddle points, and there are four different possibilities for their location, depending on whether each of $\lambda q'$ and $\lambda q''$ are greater or less than one. We are ultimately interested in the case $q'=0$ so we will restrict attention to the two cases $\lambda q'<\lambda q''<1$ and $\lambda q'<1<\lambda q''$.

Consider first the case $\lambda q'<\lambda q''<1$. Two of the saddle points are at real positive values of T,

$$T=\frac{1}{\lambda}[(1-\lambda q')^{1/2}\pm(1-\lambda q'')^{1/2}] \tag{3.5}$$

with negative action,

$$I_0=-\frac{1}{3\lambda}[(1-\lambda q')^{3/2}\pm(1-\lambda q'')^{3/2}] . \tag{3.6}$$

The other two saddle points are at real negative values of T,

$$T=-\frac{1}{\lambda}[(1-\lambda q')^{1/2}\pm(1-\lambda q'')^{1/2}] \tag{3.7}$$

with positive action,

$$I_0=+\frac{1}{3\lambda}[(1-\lambda q')^{3/2}\pm(1-\lambda q'')^{3/2}] . \tag{3.8}$$

The steepest-descent contours passing through these four saddle points are shown in Fig. 3.

In the case $\lambda q'<1<\lambda q''$, two of the saddle points are at

$$T=\frac{1}{\lambda}[(1-\lambda q')^{1/2}\pm i(\lambda q''-1)^{1/2}] \tag{3.9}$$

with action

$$I_0=-\frac{1}{3\lambda}[(1-\lambda q')^{3/2}\mp i(\lambda q''-1)^{3/2}] . \tag{3.10}$$

The other two saddle points are at

$$T=-\frac{1}{\lambda}[(1-\lambda q')^{1/2}\pm i(\lambda q''-1)^{1/2}] \tag{3.11}$$

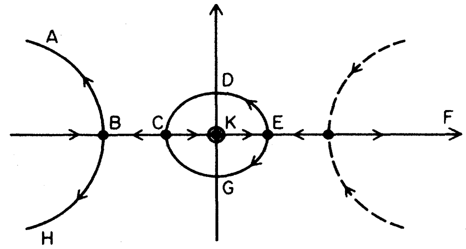

FIG. 3. The steepest-descent paths in the complex T plane for $G(q''|q')$ the case $\lambda q'<\lambda q''<1$.

with action

$$I_0=+\frac{1}{3\lambda}[(1-\lambda q')^{3/2}\mp i(\lambda q''-1)^{3/2}] . \tag{3.12}$$

The steepest-descent contours passing through these saddle points are shown in Fig. 4.

The saddle points (3.5) correspond to solutions to the Einstein equations representing a section of four-sphere of radius $\lambda^{-1/2}$ interpolating between two three-spheres. The plus/minus signs correspond, respectively, to the situation in which the section of four-sphere does/does not include the equator. The saddle points (3.7) are the same as (3.5) but with the sign of T reversed. Their significance will be discussed later. The saddle points (3.9) and (3.11) correspond to some kind of "complex" geometry consisting of a section of four-sphere matched onto a section of de Sitter space.

Consider now the possible contours in Figs. 3 and 4. The exponent in (3.3) is single-valued. However, because of the factor of $T^{-1/2}$ in the measure, the whole integrand is not. This means that a branch cut should be included in the complex T plane and due care be taken when choosing the contours. There are six contours for which the cut is essentially irrelevant. In Fig. 3 these are *ABH*, *AEH*, *ADF*, *AGF*, *HGF*, and *HDF*. These correspond in Fig. 4, respectively, to *IKR*, *INR*, *ILN*, *IKQPN*, *RPN*, and *RKJLN*. One contour for which the cut is relevant is that which runs just below the positive real axis *FE*, round $T=0$ and then back out just above the positive real axis EF, with the cut taken along the positive real axis.

The integral (3.3) along these contours is not one that appears in standard tables. However, it may be evaluated as follows. It is easily verified that (3.3) satisfies the

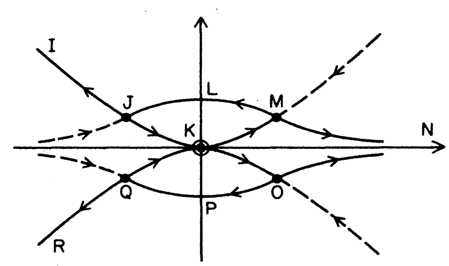

FIG. 4. The case $\lambda q'<1<\lambda q''$.

TABLE I. This table shows the result of evaluating the path integral exactly along certain infinite contours, yielding solutions to the Wheeler-DeWitt equation, and their asymptotic forms. We have used the notation $z=(1-\lambda q)(2\lambda)^{-2/3}$, $I(q)=(1/3\lambda)(1-\lambda q)^{3/2}$, and $S(q)=(1/3\lambda)(\lambda q-1)^{3/2}-\pi/4$. Each contour is labeled by the letters appearing in Fig. 3 and the contour into which it is distorted in Fig. 4 is shown in parentheses. Only the contributions from saddle points which are global maxima are shown in the asymptotic forms.

Contour	$\lambda^{1/3}G(q''\|q')$	$\lambda q'<\lambda q''<1$	$\lambda q'<1<\lambda q''$
ABH (*IKR*)	$\mathrm{Ai}(z'')\mathrm{Ai}(z')$	$e^{-I(q')}e^{-I(q'')}$	$e^{-I(q')}\cos S(q'')$
AEH (*INR*)	$\mathrm{Ai}(z'')\mathrm{Bi}(z')+\mathrm{Ai}(z')\mathrm{Bi}(z'')$	$e^{I(q')}e^{-I(q'')}$	$e^{I(q')}\cos S(q'')$
ADF (*ILN*)	$[\mathrm{Ai}(z'')+i\,\mathrm{Bi}(z'')][\mathrm{Ai}(z')+i\,\mathrm{Bi}(z')]$	$e^{I(q')}e^{I(q'')}$	$e^{I(q')}e^{-iS(q'')}$

Wheeler-DeWitt equation (2.28), with respect to both q' and q''. The solutions to (2.28) are Airy functions. (3.3) is symmetric in q' and q'' so it is equal to sums of symmetric products of Airy functions. That is,

$$G(q''|q')=a\,\mathrm{Ai}(z'')\mathrm{Ai}(z')+b\,\mathrm{Bi}(z'')\mathrm{Bi}(z') +c[\mathrm{Ai}(z'')\mathrm{Bi}(z')+\mathrm{Bi}(z'')\mathrm{Ai}(z')]\ , \quad (3.13)$$

where $z=(1-\lambda q)(2\lambda)^{-2/3}$ and a, b, and c are coefficients to be determined. By writing the q's in terms of z in (3.4), one first sees that a, b, and c depend on λ by an overall prefactor, $\lambda^{-1/3}$. One may then set $z''=z'=z$ and expand both sides of (3.13) as a Taylor series about $z=0$, to quadratic order in z. The terms that occur on the left-hand side, involving G and it's derivatives at $z=0$, may be evaluated using (3.3), in terms of the gamma function. a, b, and c may then be determined by matching the series on either side of (3.13). The results of a selection of contours and their asymptotic forms, are shown in Table I. The contours are labeled by the letters appearing in Fig. 3, with the contour into which it is deformed in Fig. 4 shown in parentheses. Only the contributions from the saddle points which are global maxima are shown in the asymptotic forms.

In addition to these solutions to the Wheeler-DeWitt equation, there are also Green's functions of the Wheeler-DeWitt operator, obtained by integrating over a half-infinite contour which begins at the essential singularity at $T=0$, leaving it in the direction $\mathrm{Re}(T)<0$. There are two independent ones, given by the contours *KI* and *KR* in Fig. 4, with corresponding contours in Fig. 3. The integral along these contours can be evaluated using methods similar to that above. The results are shown in Table II.

Finally, one may have thought that one could use the closed contour *CDEGC*. Indeed, the possibility of using a closed contour was mentioned in the Introduction. It has the attraction that convergence is not an issue to be investigated, in contrast with the other contours. Moreover, since it is generally the case that quantum-mechanical propagator (1.4) has an essential singularity at $T=0$, one might have thought that it will always give a nontrivial result. The model considered here, however, gives a trivial result for this contour. The reason is that, as noted above, there is a branch point at $T=0$. It has the consequence that the integrand will not return to it's original value if T goes once around the origin, so the contour *CDEGC* will not yield a solution to the Wheeler-DeWitt equation. Of course, one could always take a contour which goes twice around the origin, but this would yield zero. So the closed contour is no use in this model.

The closed contour may be of value in models for which the minisuperspace is more than one dimensional, however. For an n-dimensional minisuperspace, the prefactor in the quantum-mechanical propagator (1.4) behaves like $T^{-n/2}$ for small T, so the closed contour may give nontrivial results for even-dimensional minisuperspace models. We hope to investigate this possibility in future publications.

IV. BOUNDARY CONDITIONS

We have studied the possible contours in the path-integral representations of $G(q''|q')$ and $\tilde{G}(q''|p')$ and their correspondence to solutions of the Wheeler-DeWitt equation and Green functions of the Wheeler-DeWitt operator, for this simple model. We now attempt to apply certain boundary-condition proposals for the wave function of the Universe, to determine the extent to which they fix the solution uniquely. The boundary-condition proposals we shall consider are those of Hartle and Hawking[4] and of Vilenkin.[5]

The first point to note is that the initial values alone do not uniquely fix the form of the propagator. One also has to specify a contour. The solution is uniquely determined only after one has specified two pieces of information—the initial values and the contour of integration. This will generally be true of all minisuperspace models.

TABLE II. This table shows the result of evaluating the path integral exactly along certain half-infinite contour, yielding Green's functions of the Wheeler-DeWitt operator. Here, $z_>$ ($z_<$) is the greater (lesser) of z'' and z'. Likewise $q_>$ and $q_<$. Note that $z_>=(1-\lambda q_<)(2\lambda)^{-2/3}$, and vice versa.

Contour	$\lambda^{1/3}G(q''\|q')$	$(\lambda q_>)<1$	$(\lambda q_>)>1$
ABK (*IJK*)	$\mathrm{Ai}(z_>)[\mathrm{Ai}(z_<)+i\,\mathrm{Bi}(z_<)]$	$e^{-I(q_<)}e^{I(q_>)}$	$e^{-I(q_<)}e^{-iS(q_>)}$
HBK (*RQK*)	$\mathrm{Ai}(z_>)[\mathrm{Ai}(z_<)-i\,\mathrm{Bi}(z_<)]$	$e^{-I(q_<)}e^{I(q_>)}$	$e^{-I(q_<)}e^{+iS(q_>)}$

A. The Hartle-Hawking proposal

Consider first the boundary-condition proposal of Hartle and Hawking.[4] Their proposal is that the wave function be defined by a path integral over compact four-metrics which match a prescribed three-metric on a three-surface. In the de Sitter minisuperspace model, this means that one calculates $G(q''|q')$ with $q'=0$, and the saddle points then correspond to more than half or less than half of a four-sphere. From the above, one can see immediately that this does not fix the solution uniquely, however, because it does not obviously fix the contour.

In their original paper, Hartle and Hawking claimed that the contour of integration passed only through the saddle point corresponding to less than half a four-sphere, implying that the wave function is exponentially growing for small q. Many subsequent authors have taken this to be the defining feature of the Hartle-Hawking proposal. We have seen, however, that there are many possible contours for this simple model, not all of which pass through the saddle point in question (the point E in Fig. 3), and even those that do pass through it do not always receive their dominant contribution from this point. The contour is not fixed unless one puts in some extra information, which Hartle and Hawking did not obviously do. We therefore wish to question the reasoning which led to their claim above.

Hartle and Hawking gave two arguments which were designed to show that the dominant contribution comes from the saddle point corresponding to less than half a four-sphere. The first involved considering the functional integral over the scale factor a, with the action written in terms of conformal time η:

$$I=\tfrac{1}{2}\int d\eta(-a'^2-a^2+\lambda a^4) . \tag{4.1}$$

They argued that the a contour would have to be rotated to lie parallel to the imaginary axis for the path integral to converge. They then argued that although the more-than-half a four-sphere saddle point has more negative action than the less-than-half saddle point, it is the latter that would dominate because of the conformal rotation. They did not, however, give the contours explicitly; nor did they fix the gauge and consider the integral over the lapse. In this paper we have given what we believe to be a more careful discussion of the possible contours, and we have seen that the lapse integral plays a crucial role in determining which saddle points contribute.

Their second argument involved writing the wave function as an inverse Fourier or Laplace transform. In our approach, one would expect to be able to obtain such an expression in a deductive fashion from the path integral (2.6), by integrating out everything except the p integral on the final slice, $p=p''$ say. We will return to this point in the final section. There would be two saddle points corresponding, respectively, to more than or less than half a four-sphere. Hartle and Hawking assumed that the p contour has to be parallel to the imaginary axis, to the right of any singularities. From this they deduced that it can be distorted to pass only through the less-than-half saddle point, but not the other one. Our criticism of this argument is that it is not obvious why one should choose that particular contour as the starting point. There will be other contours for which the integral converges, which also yield solutions, but pass through the other saddle point.

To summarize the previous two paragraphs: we feel that the original arguments given by Hartle and Hawking are not correct as they stand, first because they have implicitly made a choice of contour, but without motivating that choice, and second, because they did not consider a properly defined path integral. The Hartle-Hawking proposal therefore uniquely specifies the wave function only after a choice of contour has been made.

One could now ask whether or not there exists a way of fixing the contour so as to obtain the result they claimed to obtain. Consider Fig. 3. The saddle point at E is the one corresponding to less than half a four-sphere. The idea, then, is to propose a rule which singles out a contour which is dominated by the saddle point at E. One possible step in this direction is to demand that the result be real, which was always claimed to be one of the properties of the Hartle-Hawking wave function. This does not fix it uniquely, however. There are three contours which give a real result: namely, ABH, AEH, and the contour which runs along the positive real axis on either side of the cut.

We mentioned above that the closed contour has certain attractions, but is trivial in this model. It could be, however, that it provides a good way of specifying the Hartle-Hawking wave function for even-dimensional minisuperspace models. We have little evidence to support this claim at the time of writing but hope to report on this possibility in future publications (see, however, the models in Refs. 3, 12, and 13 in which nontrivial closed contours were found).

Another possible approach to the question of imposing Hartle-Hawking boundary conditions is to fix the initial momenta. As mentioned in Sec. II it has been argued that this may be what one has to do in more general models.[10] The idea is that one chooses the initial momentum p' so that it corresponds, via the Hamiltonian constraint, to zero initial q. This means that $p'=\pm\frac{1}{2}$ in this model. It is not obvious how one decides between the plus and minus signs, but one way might be to demand that in the Euclidean region, the final surface is to the "future" of the initial surface at the saddle point. This means that we need $T_{\pm}>0$ in (2.17), which implies that $p'=-\frac{1}{2}$. Let us also demand that the result be real. This uniquely fixes the contour to be ABD in Fig. 1, and the path integral gives the result (2.22), with $p'=-\frac{1}{2}$. This is indeed the wave function Hartle and Hawking claimed to obtain.

Whilst this may be the most appropriate way to define the Hartle-Hawking wave function, one can object to it for the following reason. Fixing the initial momentum in the manner described above corresponds to zero three-geometry only at the semiclassical level. In the full path integral, because the initial p is fixed, one integrates over all initial q. This means that most of the geometries in the sum do not close, which appears to depart rather radically from the original proposal of Hartle and Hawking.

So these are the possible approaches to imposing Hartle-Hawking boundary conditions. They are both problematic, and at the moment we regard the issue of defining the Hartle-Hawking proposal precisely as still open.

B. The Vilenkin proposal

Consider next the proposal of Vilenkin. His proposal is normally given as a statement about the behavior of the wave function in minisuperspace. It is that the wave function should consist of outgoing modes at singular boundaries of minisuperspace.[5] In this model, it implies that the wave function should be of the form e^{-iS} in the oscillatory region, with no components proportional to e^{+iS}. It is easy to see how this is achieved in the path-integral representation. One chooses the contour in (3.3) such that the dominant contribution comes from either the saddle point at M in Fig. 4, or the one at J. There are two infinite contours which are dominated by M: namely, ILN and $RKJLN$. These differ only in their subdominant contributions. There is just one contour which is dominated by the saddle point at J: namely, the half-infinite contour KI, which yields a Green's function. Whether the Vilenkin proposal should yield a solution or a Green's function is not clear, so we shall entertain both possibilities.

To decide between these possible contours, it is useful to consider an alternative definition of the Vilenkin wave function, which is not obviously equivalent to the proposal above. It is that it be defined by a path integral over Lorentzian metrics starting at $q=0$ (Ref. 6). This suggests that in our model, the T contour is taken to be one which may be distorted to lie along the imaginary axis. Both the infinite contour RKI and the half-infinite contour KI may be so distorted in Fig. 4. However, only the half-infinite one yields the asymptotic form e^{-iS}; thus only if one takes the half-infinite contour do the two versions of the proposal agree, in this model. A possible path-integral statement of the Vilenkin proposal is therefore that it be defined by a path integral over Lorentzian metrics with positive lapse.

With this definition of the Vilenkin wave function, one finds that it has the form

$$\Psi_V(q)=\exp\left[-\frac{1}{3\lambda}\right]\exp\left[-\frac{i}{3\lambda}(\lambda q-1)^{3/2}\right] \qquad (4.2)$$

in the oscillatory region. Contrast this with the explicit form of the Hartle-Hawking wave function, obtained using the contour suggested above for $\tilde{G}(q''|p')$ with $p'=-\frac{1}{2}$. In the oscillatory region, it has the form

$$\Psi_{\rm HH}=\exp\left[\frac{1}{3\lambda}\right]\cos\left[\frac{1}{3\lambda}(\lambda q-1)^{3/2}-\frac{\pi}{4}\right]. \qquad (4.3)$$

There are two key differences between (4.2) and (4.3). The first is that the Vilenkin wave function consists of a single WKB component, e^{-iS}, whilst the Hartle-Hawking wave function is a sum of two such components. It is normally argued that these two components have negligible interference and may therefore be considered separately.[14]

The second, and more important difference, is the sign in the prefactor, $\exp(\pm 1/3\lambda)$. This is important for the following reason. One is normally interested in a more complicated minisuperspace model consisting of a Robertson-Walker model coupled to a homogeneous scalar field ϕ, with potential $V(\phi)$ (Ref. 5). In regions where the potential is approximately flat, the ϕ dependence of the wave function is negligible and the minisuperspace model is well approximated by the de Sitter model described here, with $\lambda=V(\phi)$. The wave functions for the scalar field model are peaked about sets of classical solutions with Hamilton-Jacobi function $S=\pm(\lambda q-1)^{3/2}/3\lambda$. Each member of the set may be labeled by its initial value of ϕ. The prefactor $\exp[\pm 1/3V(\phi)]$ then gives a measure on this set of initial values, and hence on the set of classical solutions. Clearly, the behavior of this measure is going to depend rather crucially on which sign one takes, i.e., on whether one uses the Hartle-Hawking (+) or Vilenkin (−) wave functions. The consequences of each sign have been thoroughly discussed by Vilenkin[5] and will not be pursued here. We will however, discuss the origin of these signs.

The factor $\exp(1/3\lambda)$ in the Hartle-Hawking wave function has the interpretation as the amplitude to propagate from $q=0$ to $q=\lambda^{-1/2}$, since $-1/3\lambda$ is the action of half a four-sphere. The factor with the opposite sign in the Vilenkin wave function has essentially the same interpretation, except that it comes from the saddle point with the opposite value of T. These saddle points are discussed in the next section.

To discuss these issues more thoroughly, one needs to include the dependence on the scalar field, which we have certainly not done here. We hope to address this in a future publication.

V. CONSEQUENCES OF A COMPLEX CONTOUR

The fact that the contour is over complex metrics has a number of interesting consequences. First, let us discuss the significance of the saddle points (3.7). Recall that they are the same as the saddle points (3.5), but with the sign of T reversed, so they have positive Euclidean action. This is rather surprising because one would expect that the only solution to the Euclidean Einstein equations in this model are sections of four-sphere, which always have negative action. On the face of it there seems to be no way the extremum value of the action can be anything but negative. The explanation is as follows. These unexpected saddle points have arisen because we have allowed the four-metric to be complex in the contour of integration. The action (1.2) involves a square root $(\det g_{\mu\nu})^{1/2}$ and by convention, one takes the positive sign. This square root causes no problem if one deals only with real Euclidean metrics. However, if, as here, one lets the metric become complex, then the contour may go around the branch point at $\det g_{\mu\nu}=0$ and end up on the second Riemann sheet, thus changing the sign of the action. So for every extremum with positive T there will be another solution with T replaced by $-T$, and equal action but of opposite sign. Hence the extra unexpected saddle points, (3.7). A similar phenomenon was observed in the Regge calculus model studied by Hartle.[3]

These considerations mean that the signature of the metric is not respected in the path integral, in this approach. Some readers may find this unacceptable. We find it an inevitable consequence of our choice of complex contour, which in turn is necessary in order that the path integral converges.

One might object to having saddle points at $T<0$ on the grounds that if a coupling to matter was included, the positivity of the matter action would be destroyed. Whilst this is certainly true, it is not necessarily a fatal objection. When considering the Euclidean path integral for matter alone, one may integrate over real field configurations to obtain convergence. On coupling to gravity, however, since we need to integrate over complex metrics to ensure convergence of the gravitational part of the path integral, the matter action will become complex. One would therefore expect to have to integrate over complex matter fields also, to ensure convergence of the matter part of the path integral. So if the matter action is real and negative at a saddle point of the T integral for which $T<0$, one could still obtain convergence of the matter action by integrating over complex matter-field configurations.

A related point concerns the signature of the three-metric. The physically relevant range of the scale factor squared q is $q>0$, that is $\det h_{ij}>0$. One can ask whether or not this condition should be required of the paths summed over in the path integral. The technical difficulty involved in imposing this condition was discussed in Ref. 8 (see also Ref. 15 for an example in which the condition $q>0$ was imposed in the path integral). Our attitude towards this should be clear from the above—it is that one should not attempt to impose such a condition. This is again a consequence of our choice of complex contour.

A further related point concerns the range of integration of the lapse function, i.e., of T. Teitelboim, who considered Lorentzian path integrals, has argued that the range of T should not be infinite because the four-metric becomes degenerate as the contour passes through $T=0$. Furthermore, he advocates that the range be taken to be from zero to infinity. This involves the notion of some kind of causality condition in superspace—that the final geometry should lie to the "future" of the initial geometry.[9] It has the consequence that G is a Green's function of the Wheeler-DeWitt operator, as pointed out earlier. Our considerations here shed new light on his arguments. Because we are allowing a complex contour, T may have an infinite range but need not pass through $T=0$. Essentially it avoids passing through $T=0$ by going around it in the complex plane. It is also interesting to note that the T contour is uniquely specified by Teitelboim's causality condition, in contrast with the ambiguities encountered in the previous section. One might reasonably interpret his condition in our approach as implying that the T contour be that which may be distorted into one running from up the positive imaginary axis. This is the contour KI in Fig. 4, for example. It is therefore very similar to the Vilenkin proposal, discussed above. Teitelboim, however, was considering the propagator between arbitrary three-geometries, whereas Vilenkin was interested in the special case of tunneling from zero initial geometry.

Finally, we make some remarks on the semiclassical approximation in quantum gravity. A commonly used approximation to the path integral in quantum gravity is to take it to be a discrete sum of terms of the form $\sum_k e^{-I_k}$, where the I_k are the actions of solutions to the Einstein equations. This approximation can be incorrect in at least two ways. First of all, it depends on the contour of integration. One ought to show that there exists a contour which passes through all the saddle points represented in the sum. We have seen in the simple model considered here, for example, that a typical contour will not pass through all the saddle points. Second, since a complex contour appears to be necessary to ensure convergence, there can be complex saddle points. This means that the I_k in the above should not be just the actions of real Euclidean solutions, as is usually taken to be the case, but the actions of all complex solutions. These considerations could be relevant for the mechanism recently proposed by Coleman for the vanishing of the cosmological constant, which uses the semiclassical approximation to the path integral in an essential way.[16]

VI. SUMMARY AND DISCUSSION

We have studied the steepest-descent contours in the path integral for the de Sitter minisuperspace model, and evaluated the path integral exactly. By choosing different contours, we generated different solutions to the Wheeler-DeWitt equation. We attempted to apply the boundary-condition proposal of Hartle and Hawking to the path integral. This proposal fixes the initial point q' in the propagator $G(q''|q')$ but it does not fix the solution uniquely because it does not fix the contour. We tentatively suggested that a possible contour for the Hartle-Hawking proposal, applicable only to even-dimensional models, is to take the lapse contour to be a closed contour about the origin. We also considered the possibility that the Hartle-Hawking wave function be defined to be $\tilde{G}(q''|p')$ with p' corresponding to zero three-geometry. We found a contour for the Vilenkin proposal and contrasted the Hartle-Hawking and Vilenkin wave functions. Finally, we discussed some of the consequences of a complex contour. The main consequence is that the signature of the metric is not respected in the sum over paths. This means that there can be saddle points of the path integral which have neither Lorentzian nor Euclidean signature.

In this paper we studied the contours in a minisuperspace model, in which the metric was severely restricted, but the path integral was still a functional integral over $q(\tau)$. Seemingly more restricted models have recently been studied. Hartle[3] and Louko and Tuckey[12] studied the contours in Regge calculus models. There, the geometry is a Regge lattice which is specified by a *finite* number of edge lengths. The functional integral in these models reduces to a finite number of ordinary integrations over the edge lengths. A related model, something of a hybrid between the Regge models and the one studied in this paper, involved summing over geometries which were precisely sections of four-sphere, allowing only the four-sphere radius to vary.[13] The functional integral thus reduced to an ordinary integral over the radius. These exceedingly restricted models might reasonably be referred to as "microsuperspace" models.

In some ways the model studied here is little better than the microsuperspace models described above. This is because we have really only specified the contour of integration for one variable T, but not the remaining ones—P and Q in Sec. II. Although we believe the heuristic evaluation of the P and Q integrals is correct, we did not say precisely what the contours are. At this point one comes up against the issue of defining steepest descents for functional integrals. It is not difficult to find contours for P and Q—in each integration T is merely regarded as a complex parameter—but it is not obvious what would happen if the integrations were done in a different order. An important consequence of the P and Q integrations, however, was that it brought in the factor $T^{-1/2}$ in the measure, thereby eliminating the possibility of a closed contour. This is perhaps something one should watch out for when studying microsuperspace models.

For similar reasons, we have not studied the relationship between the $\tilde{G}(q''|p')$ and $G(q''|q')$. One would expect that they are related by some kind of Fourier transform, but one cannot necessarily assume this because G and $\tilde{G}$ are not square integrable. However, one ought to be able to *derive* such a relationship by starting with the expression (2.6) for $G(q''|q')$ and then carrying out all the integrations except the p integration on the initial slice. If one knew the contours for p and q on each slice, one would thereby deduce the appropriate contours to take in the Fourier transform. One might have thought that the relationship between $\tilde{G}$ and G is straightforward, but this is not so, as may be seen by comparing the obtained forms for $\tilde{G}$ and G. $\tilde{G}(q''|p')$ is a product of a function of p' with a function of q''. On Fourier transforming, one would thus deduce that $G(q''|q')$ is a product of a function of q' with a function of q''. We have seen, however, that this is not the case—it is a *sum* of such products. This suggests that the relationship between the T contours in $\tilde{G}$ and in G is not so obvious. We hope to return to this point in a future publication.

The continuation of the investigation described here may be found in papers II and III of this series.[13,17]

Note added in proof. After completion of this work, we became aware of an independent proposal of Linde which yields a wave function essentially the same as the Vilenkin wave function Zh. Eksp. Teor. Fiz. **87**, 369 (1984) (A. Linde, [Sov. Phys. JETP **60**, 211 (1984)]; Lett. Nuovo Cimento **39**, 401 (1984); Rep. Prog. Phys. **47**, 925 (1984)). Linde seems to regard a purely Lorentzian path integral as his starting point. Because the usual Wick rotation to Euclidean time leads to a minus sign in front of the kinetic term for the scale factor, he proposed that the Wick rotation should be performed in the "wrong" direction. In our approach, it seems reasonable to interpret this proposal as implying ythat one should take the T contour in Figs. 3 and 4 to be the distortion into the region $\mathrm{Re}(T)<0$ of the contour running up the positive imaginary axis. This is essentially the same as the way we defined the Vilenkin wave function in Sec. IV, and yields a wave function proportional to $\exp(-1/3\lambda)$ in the semiclassical approximation. We are grateful to Andrei Linde for bringing his proposal to our attention.

ACKNOWLEDGMENTS

We are grateful to Jim Hartle, Mike Turner, and Frank Wilczek, the organizers of the ITP workshop on Cosmology and Microphysics, at which most of this work was carried out. We would like to thank Alex Vilenkin for informative discussions on his proposal and Jim Hartle for many useful conversations on contours of integration. This research was supported in part by the National Science Foundation under Grant No. PHY82-17853, supplemented by funds from the National Aeronautics and Space Administration, at the University of California at Santa Barbara. J.L. was supported in part by the University of Helsinki and the Academy of Finland.

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