Colliding superposed waves in the Einstein-Maxwell theory

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(Received 26 September 1988; revised manuscript received 22 December 1988)

We reformulate the initial data on the characteristic surface for colliding waves in the Einstein-Maxwell theory. This approach takes into account the superposition principle for gravitational and electromagnetic waves. Finding exact solutions for colliding superposed waves happens to be a rather challenging problem.

I. INTRODUCTION

Khan and Penrose[1] and Szekeres[2] gave the first exact solutions that describe colliding parallel (collinearly) polarized gravitational waves in general relativity. Bell and Szekeres[3] extended the problem of colliding pure gravitational waves to the case of pure electromagnetic (em) waves. The em degree of freedom in the latter case naturally creates conformal curvature on the null boundaries, whereas the interaction region becomes conformally flat. From the uniqueness property of the non-null Einstein-Maxwell (EM) solutions, the Bell-Szekeres (BS) solution is transformable to the Bertotti-Robinson solution.[4] All the solutions presented by Khan and Penrose (KP), Szekeres, and Bell and Szekeres (BS) represent colliding waves with single polarization, and naturally the next step was to search for colliding waves with the second polarization. This task was accomplished first, in the realm of pure impulsive gravitational waves, by the Nutku-Halil[5,6] (NH) solution. Generalization of the BS solution to the cross polarized case took a relatively longer period.[7] Shortly after the publication of the NH solution we attempted to extend the same procedure to the EM theory.[8] We were well aware, however, that solutions obtained by imitation of the stationary axisymmetrical EM fields could serve no more than as solutions for the sake of solutions and that they do not represent superposed waves. More specifically, in the black-hole solutions, one can talk about a charged hole and study its coupled EM fields, but in colliding gravitational waves (CGW's) the waves are not charged and therefore introduction of em waves must follow certain rules. The same criticism applies to gravity coupled with other fields, such as a perfect fluid and scalar fields.

Our principal aim in this paper is to concentrate on this particular point and reformulate the problem of CGW's under the light of a superposition principle in the EM theory.

The first published paper on colliding waves in the EM theory appeared in 1985 by Chandrasekhar and Xanthopoulos[9] (CX). A series of solutions followed subsequently with new features.[10–12] The timelike nature of the emerging space-time singularity and the formation of a horizon prior to the singularity are the distinctive features worth mentioning. These solutions were obtained by employing the NH-type Ernst potential within the context of EM theory. Among these solutions, the more interesting ones are the ones that admit gravitational and em limits independently. This is the least requirement (although not sufficient) for a proper superposition principle that leads to the formulation of colliding wave packets in general relativity. Collision of waves with single plane-wave fronts are known to inherit the singularity structure from the pure gravitational waves. Does a singularity arise in colliding wave packets formed from properly superposed wave fronts? Present knowledge that has been acquired does not suffice to answer this question positively.

In Sec. II we review the method for solving EM equations used so far. In Sec. III we describe the various superposed wave forms on the initial characteristic surface and this is followed by the conclusion in Sec. IV.

II. METHOD FOR SOLVING EM EQUATIONS

The most essential equations to the EM theory consist of the symmetrical pair of Ernst equations[13]

$$\begin{aligned}(\xi\bar{\xi}+\eta\bar{\eta}-1)\nabla^2\xi&=2\nabla\xi(\bar{\xi}\nabla\xi+\bar{\eta}\nabla\eta)\ ,\\(\xi\bar{\xi}+\eta\bar{\eta}-1)\nabla^2\eta&=2\nabla\eta(\bar{\xi}\nabla\xi+\bar{\eta}\nabla\eta)\ ,\end{aligned}\tag{1}$$

where ξ and η represent the gravitational and electromagnetic (em) complex potentials, respectively. This pair of equations can be parameterized alternatively by introducing new potentials Z and H in accordance with

$$Z=\frac{1+\xi}{1-\xi},\quad H=\frac{\eta}{1-\xi}\ ,\tag{2}$$

which transform the above pair of equations into

$$\begin{aligned}(\mathrm{Re}Z-|H|^2)\nabla^2Z&=(\nabla Z)^2-2\bar{H}\nabla Z\cdot\nabla H\ ,\\(\mathrm{Re}Z-|H|^2)\nabla^2H&=\nabla H\cdot\nabla Z-2\bar{H}(\nabla H)^2\ .\end{aligned}\tag{3}$$

Next, two auxiliary real potentials Ψ and Φ are introduced for convenience through the relation

$$Z=\Psi+|H|^2-i\Phi\ .\tag{4}$$

To specify the problem suitably for the description of CGW's, we have to define on which coordinates the operators ∇ and ∇^2 act. The choice of coordinates is rather important and plays a significant role in obtaining new solutions. Null coordinates u and v form a useful set in the formulation of the problem, whereas oblate (prolate) types of coordinates proved convenient in solving the equations. The coordinates

$$\tau=u\sqrt{1-v^2}+v\sqrt{1-u^2}, \quad \sigma=u\sqrt{1-v^2}-v\sqrt{1-u^2} \tag{5}$$

were defined in the pure gravitational problem by NH. We had shown also that by replacing $u \to u^{n_1}$ and $v \to v^{n_2}$ with (n_1,n_2) arbitrary real parameters, new solutions can be obtained.[14] Similarly, in the generalization of the BS solution to the noncollinear polarization case, we introduced conveniently the coordinates[7]

$$\tau=\sin(au+bv), \quad \sigma=\sin(au-bv) \ . \tag{6}$$

Without loss of generality, we can fix the constants $a=1=b$ and observe that the sets (5) and (6) are related by the replacements $u\to\sin u$ and $v\to\sin v$. One important point that aids in choosing the (τ,σ) set is the fact that in passing from linear to cross polarized waves, one of the metric function, namely U, that appears in the Szekeres metric below is kept unchanged in the (τ,σ) coordinates as (in the next section we shall discuss this point more)

$$e^{-U}=\sqrt{1-\tau^2}\sqrt{1-\sigma^2} \ . \tag{7}$$

The space-time line element in the null coordinates (u,v) was given first by Szekeres,

$$ds^2=2e^{-M}du\,dv-e^{-U}(e^{V}\cosh W\,dx^2+e^{-V}\cosh W\,dy^2-2\sinh W\,dx\,dy) \tag{8}$$

in which all metric functions depend on u and v alone. The metric of CX employs (τ,σ) coordinates directly and their line element is quoted as

$$ds^2=e^{\nu+\mu_3}\sqrt{\Delta}\left[\frac{d\tau^2}{1-\tau^2}-\frac{d\sigma^2}{1-\sigma^2}\right] -\sqrt{\Delta\delta}\left[\chi\,dx^2+\frac{1}{\chi}(dy-q_2dx)^2\right] , \tag{9}$$

where $\Delta=1-\tau^2$, $\delta=1-\sigma^2$, and metric functions depend on (τ,σ) alone. We note also that CX's notation is (η,μ) in place of our (τ,σ) here.

The base manifold on which the differential operators of the Ernst equations act is given by

$$ds_0^2=\frac{d\tau^2}{1-\tau^2}-\frac{d\sigma^2}{1-\sigma^2}+(1-\tau^2)(1-\sigma^2)d\phi^2 , \tag{10}$$

where ϕ is considered to be a Killing coordinate. Once a set (ξ,η), or (Z,H), of solutions to the Ernst equations is known, the metric function χ is given by

$$\chi=\frac{\sqrt{\Delta\delta}}{\Psi} , \tag{11}$$

where Ψ is obtained from (4). What remains now is to integrate q_2 and $\nu+\mu_3$ from the following coupled equations:

$$q_{2,\tau}=\frac{\delta}{\Psi^2}(\Phi_\sigma-2\,\mathrm{Im}H\bar{H}_\sigma) , \quad q_{2,\sigma}=\frac{\Delta}{\Psi^2}(\Phi_\tau-2\,\mathrm{Im}H\bar{H}_\tau) , \tag{12}$$

$$\frac{-\sigma}{1-\sigma^2}(\nu+\mu_3)_\tau-\frac{\tau}{1-\tau^2}(\nu+\mu_3)_\sigma =\frac{1}{\chi^2}(\chi_\tau\chi_\sigma+q_{2,\tau}q_{2,\sigma})+\frac{2\chi}{\sqrt{\Delta\delta}}(H_\tau\bar{H}_\sigma+\bar{H}_\tau H_\sigma) ,$$

$$2\tau(\nu+\mu_3)_\tau+2\sigma(\nu+\mu_3)_\sigma =\frac{3}{1-\tau^2}+\frac{1}{1-\sigma^2}-\frac{4}{\sqrt{\Delta\delta}}(\Delta H_\tau\bar{H}_\tau+\delta H_\sigma\bar{H}_\sigma) -\frac{1}{\chi^2}[\Delta(\chi_\tau^2+q_{2,\tau}^2)+\delta(\chi_\sigma^2+q_{2,\sigma}^2)] . \tag{13}$$

The usual trend in solving the pair of Ernst equations (1) or (2) has been to make a suitable choice for ξ,η in such a way that the pair reduces to a single vacuum Ernst equation. One such possible choice is provided by

$$\xi=\beta\xi_0 , \quad \eta=\sqrt{1-\beta^2}\xi_0 \quad (\beta=\text{real constant} , \ 0\le|\beta|\le 1) \tag{14}$$

which reduces the Ernst equations (1) to

$$(\xi_0\bar{\xi}_0-1)\nabla^2\xi_0=2\bar{\xi}_0(\nabla\xi_0)^2 , \tag{15}$$

whose solution is readily available. It is well known that it admits the NH type of solution

$$\xi_0=p\tau\mp iq\sigma \quad (p^2+q^2=1) . \tag{16}$$

The straightforward integration of the quadrature equations [for the $(-)$ choice of sign] results in the following metric functions:

$$e^{\nu+\mu_3}=\frac{(1-\beta p\tau)^2+q^2\beta^2\sigma^2}{\sqrt{1-\tau^2}} , \quad q_2=\frac{p(1+\beta^2)(1-\tau^2)+2\beta\tau q^2(1-\sigma^2)}{q(1-p^2\tau^2-q^2\sigma^2)} , \quad \chi=\frac{\sqrt{\Delta\delta}}{1-p^2\tau^2-q^2\sigma^2}[(1-\beta p\tau)^2+q^2\beta^2\sigma^2] . \tag{17}$$

This solution has the feature that in the limits $\beta=0$ and 1 it reduces to em and gravitational limits, respectively. We have analyzed this solution independently[15] from CX and obtained the Weyl curvature components to satisfy

$$9\Psi_2^2=\Psi_0\Psi_4 , \tag{18}$$

i.e., the space-time belongs to a particular type-D class. This solution (and all the others obtained by CX) has the property that em and gravitational waves overlap on the initial characteristic surface. In the next section we shall show that this property does not imply that the different types of waves were superposed in the initial data.

III. COLLISION OF SUPERPOSED WAVES IN GENERAL RELATIVITY

On account of being a highly nonlinear theory, superposition of waves in general relativity works only in a particular coordinate system. This coordinate system is known to be the harmonic, or Brinkmann,[16] system, in which the line element can be expressed by

$$ds^2=2\,dU\,dV-dX^2-dY^2-2H(U,X,Y)dU^2\ , \tag{19}$$

where $H(U,X,Y)$ contains the only available information that provides a nonflat metric. The Riemann tensor components of this line element are nontrivial only in the direction of motion and for this reason the metric represents longitudinal waves. The type of the waves (i.e., gravitational, em, scalar, etc.) can be characterized by specifying the function $H(U,X,Y)$. In the following we consider a number of particular cases.

A. Superposition of impulsive gravitational wave with a shock em wave

The choice for $H(U,X,Y)$ in this case is given by

$$H(U,X,Y)=\tfrac{1}{2}A_0(Y^2-X^2)\delta(U)-\tfrac{1}{2}B_0(X^2+Y^2)\theta(U)\ , \tag{20}$$

where A_0 and B_0 are amplitude constants and $\delta(U)$ and $\theta(U)$ represent the usual Dirac delta function and the unit step function, respectively. We would like to add that we restrict ourselves exclusively to the case of linearly polarized waves. To extend the discussion to cover also the cross-polarized waves, it suffices to include additional terms in $H(U,X,Y)$. It is evident that the first term represents the impulsive gravitational wave, while the second stands for the shock em wave, and by superposition, we mean their addition in this sense. Once we define superposition, we have to seek a new coordinate system in which we can discuss the collision of such superposed waves. The harmonic coordinate system (U,V,X,Y) is not a good choice for the discussion of collision. A useful coordinate system (u,v,x,y), in which collision of waves can suitably be formulated is known as the Rosen form.[17] The line element in the Rosen form that represents linearly polarized waves is given by

$$ds^2=2\,du\,dv-F^2(u)dx^2-G^2(u)dy^2\ , \tag{21}$$

where F and G depend only on the null coordinate u. Let us add that the nontrivial Riemann tensor components occur in the x,y directions, therefore, this form of the metric represents transverse waves. The transformation that brings our line element (19) into the Rosen form (21) is given by

$$\begin{aligned}&U=u,\quad X=xF\ ,\\ &V=v+\tfrac{1}{2}(x^2FF_u+y^2GG_u),\quad Y=yG\ .\end{aligned} \tag{22}$$

Direct substitution of these coordinates into (19) gives us the conditions that the Rosen metric functions F and G must satisfy. These are the differential equations

$$\begin{aligned}&F_{uu}=-[A_0\delta(u)+B_0\theta(u)]F\ ,\\ &G_{uu}=[A_0\delta(u)-B_0\theta(u)]G\ ,\end{aligned} \tag{23}$$

whose solutions are

$$\begin{aligned}&F(u)=\cos[au\theta(u)]-\frac{a_0}{a}\sin[au\theta(u)]\ ,\\ &G(u)=\cos[au\theta(u)]+\frac{a_0}{a}\sin[au\theta(u)]\ ,\end{aligned} \tag{24}$$

where the constants are related by

$$a_0=A_0,\quad a^2=B_0\ .$$

We can check that our line element represents the correct initial data for superposed impulse gravitational and the shock em waves. For this purpose, we observe that in the limit $a\to 0$ (and let $a_0=1$) we obtain

$$F(u)=1-u\theta(u),\quad G(u)=1+u\theta(u) \tag{25}$$

which are the incoming (region-II) metric functions for the KP solution for the impulsive waves. Similarly, in the limit $a_0\to 0$ we obtain

$$F(u)=G(u)=\cos[au\theta(u)] \tag{26}$$

which are the incoming (region-II) metric functions for the BS solution for the em shocks.

The nonvanishing Newman-Penrose quantities for the line element are

$$\Psi_4=A_0\delta(u),\quad \Phi_{22}=B_0\theta(u)\ . \tag{27}$$

Solutions for colliding EM waves that are obtained so far by the method of the previous section have, in region II, the general behaviors

$$\begin{aligned}&\mathrm{Re}\Psi_4=\mathrm{const}\times\delta(u)+P(u)\theta(u)\ ,\\ &\mathrm{Im}\Psi_4=\mathrm{const}\times\delta(u)+Q(u)\delta(u)\ ,\\ &\Phi_{22}=R(u)\theta(u)\ ,\end{aligned} \tag{28}$$

where P, Q, and R are functions of u. By an inverse transformation to the harmonic coordinates such a metric cannot be reduced to a simple superposition of gravitational and em waves. (Note that due to the cross polarization, it gives $\mathrm{Im}\Psi_4\neq 0$, in the latter case.)

Interchanging now u and v, we can write the incoming Rosen line element for region III:

$$\begin{aligned}ds^2=2\,du\,dv&-\left\{\cos[bv\theta(v)]-\frac{b_0}{b}\sin[bv\theta(v)]\right\}^2dx^2\\ &-\left\{\cos[bv\theta(v)]+\frac{b_0}{b}\sin[bv\theta(v)]\right\}^2dy^2\ ,\end{aligned} \tag{29}$$

where the constants (b_0,b) are similar to the constants (a_0,a) of region II. What remains now is to find the interaction region (region IV) such that for $u<0$ we shall recover the line element (29). None of the solutions obtained by CX has the simultaneous boundary conditions of KP and BS. The reason for this is connected with the improper choice of the (τ,σ) coordinates. One may choose the gravitational coordinates (5) so that one may reduce to the KP limit without the BS limit. The choice of em coordinates (6) gives us an opposite situation, namely, BS limit without the KP limit. It is therefore important to discover a set of (τ,σ) coordinates which will play the dual role simultaneously. Finding of such coordinates goes as follows.

The Rosen form of region II is given in Szekeres form by

$$ds^2=2\,du\,dv-e^{-U}(e^{V}dx^2+e^{-V}dy^2)\ , \tag{30}$$

where

$$e^{-U}=FG,\quad e^{V}=\frac{F}{G}\ . \tag{31}$$

e^{-U} is given explicitly by

$$\begin{aligned}e^{-U}&=\cos^2[au\,\theta(u)]-\frac{a_0^2}{a^2}\sin^2[au\,\theta(u)]\\&=1-\left[1+\frac{a_0^2}{a^2}\right]\sin^2[au\,\theta(u)]\ .\end{aligned} \tag{32}$$

In region III we have the same expression with $u\rightarrow v$, $a_0\rightarrow b_0$, and $a\rightarrow b$. We can extrapolate the metric function e^{-U} into the interaction region without much effort, such that it satisfies the equation

$$(e^{-U})_{uv}=0\ . \tag{33}$$

We obtain

$$\begin{aligned}e^{-U}=1&-\left[1+\frac{a_0^2}{a^2}\right]\sin^2[au\,\theta(u)]\\&-\left[1+\frac{b_0^2}{b^2}\right]\sin^2[bv\,\theta(v)]\ ,\end{aligned} \tag{34}$$

which is required to be expressed in the coordinates (τ,σ) by

$$e^{-U}=\sqrt{1-\tau^2}\sqrt{1-\sigma^2}\ .$$

Such a possible (τ,σ) set is given by

$$\begin{aligned}\tau&=\frac{1}{p}\sin au\left[1-\left[\frac{\sin bv}{p'}\right]^2\right]^{1/2}+\frac{1}{p'}\sin bv\left[1-\left[\frac{\sin au}{p}\right]^2\right]^{1/2}\ ,\\ \sigma&=\frac{1}{p}\sin au\left[1-\left[\frac{\sin bv}{p'}\right]^2\right]^{1/2}-\frac{1}{p'}\sin bv\left[1-\left[\frac{\sin au}{p}\right]^2\right]^{1/2}\ ,\end{aligned} \tag{35}$$

where

$$\frac{1}{p}=\left[1+\frac{a_0^2}{a^2}\right]^{1/2},\quad \frac{1}{p'}=\left[1+\frac{b_0^2}{b^2}\right]^{1/2}\ . \tag{36}$$

It can be checked that in the simultaneous limits $a\rightarrow 0$ and $b\rightarrow 0$ this set of (τ,σ) reduces to (5), whereas to obtain (6), it suffices to set $a_0\rightarrow 0$ and $b_0\rightarrow 0$.

The em vector potential for the present problem can be chosen by

$$A_\mu=A(u,v)\delta_\mu^x\ , \tag{37}$$

such that the em null tetrad components are expressed by

$$\begin{aligned}\phi_2&=-\frac{1}{\sqrt{2}}A_u e^{(U-V)/2}\ ,\\ \phi_0&=\frac{1}{\sqrt{2}}A_v e^{(U-V)/2}\ .\end{aligned} \tag{38}$$

The basic EM equations to be solved for A and V are

$$\left[e^{-V}\left[\frac{1-\tau^2}{1-\sigma^2}\right]^{1/2}A_\tau\right]_\tau-\left[e^{-V}\left[\frac{1-\sigma^2}{1-\tau^2}\right]^{1/2}A_\sigma\right]_\sigma=0\ , \tag{39}$$

$$\begin{aligned}&[(1-\tau^2)V_\tau]_\tau-[(1-\sigma^2)V_\sigma]_\sigma\\&\quad=-ke^{-V}\left[\left[\frac{1-\tau^2}{1-\sigma^2}\right]^{1/2}A_\tau^2-\left[\frac{1-\sigma^2}{1-\tau^2}\right]^{1/2}A_\sigma^2\right]\ .\end{aligned} \tag{40}$$

The BS solution corresponds to

$$e^{V}=\left[\frac{1-\sigma^2}{1-\tau^2}\right]^{1/2},\quad A=\sqrt{2/k}\,\sigma\ , \tag{41}$$

while the KP solution is given by

$$e^{V}=\frac{1+\tau}{1-\tau},\quad A=0\ . \tag{42}$$

It would be rather interesting to find the solution that describes both KP and BS solutions simultaneously.

Finally we want to discuss a particular case of the problem that we have just formulated. Let us suppress the em field from region II and the gravitational field from region III. The problem reduces then to the case of collision of an impulse gravitational wave with an em shock wave. This problem is relatively much simpler and its solution is available in the literature.[18] It is given in terms of the Szekeres metric functions by

$$\begin{aligned}e^{V}&=\frac{1+a_0u}{1-a_0u}\ ,\\ A&=\sqrt{2/k}\,\sin bv\,e^{V/2}\ ,\\ e^{-M}&=\cos bv\left[\frac{1-a_0^2u^2}{\cos^2 bv-a_0^2u^2}\right]^{1/2}\ .\end{aligned} \tag{43}$$

This problem has been solved without reference to the (τ,σ) coordinates, but once we attempt to extend the waves to the cross polarized cases we realize the difficulty in working with the null coordinates.

B. Superposition of impulsive gravitational waves

In analogy with the superposition of an impulsive gravitational and a shock em wave, we can superpose also two or more gravitational impulse waves. For this purpose we choose the $H(U,X,Y)$ function by

$$H(U,X,Y)=(X^2-Y^2)[a_0\delta(U)+b_0\delta(U-U_1)]\ , \tag{44}$$

where a_0 and b_0 are amplitude constants and U_1 shows the location of the successive wave front. The metric is transformed into the Rosen form (21) provided F and G satisfy in this case

$$F_{uu}=[a_0\delta(u)+b_0\delta(u-u_1)]F\ ,\qquad G_{uu}=-[a_0\delta(u)+b_0\delta(u-u_1)]G\ . \tag{45}$$

Solutions for F and G are given by

$$\begin{aligned}F(u)&=1+a_0u\,\theta(u)\\&\quad+b_0(1+a_0u_1)(u-u_1)\theta(u-u_1)\ ,\\G(u)&=1-a_0u\,\theta(u)\\&\quad-b_0(1-a_0u_1)(u-u_1)\theta(u-u_1)\ .\end{aligned} \tag{46}$$

In order to find a useful pair of (τ,σ) coordinates, we manipulate the expression $e^{-U}=FG$, in such a way that it is expressed in the form (7). We observe, after simple calculation that, if the wave front is chosen to be located at $u_1=(2/a_0b_0)^{1/2}$, then the metric function e^{-U} is expressed by

$$e^{-U}=1-a_0^2u^2\theta(u)-b_0^2(u-u_1)^2\theta(u-u_1)\ , \tag{47}$$

i.e., in the KP form. We have a similar expression in region III given by

$$e^{-U}=1-c_0^2v^2\theta(v)-d_0^2(v-v_1)^2\theta(v-v_1)\ , \tag{48}$$

with different constants c_0 and d_0. To simplify it more, we make the choices for the constants

$$a_0=1=c_0,\quad b_0=\frac{2}{u_1^2},\quad d_0=\frac{2}{v_1^2}\ , \tag{49}$$

such that u_1 and v_1 remain the only free parameters in the problem. We define now the suitable (τ,σ) coordinates for the problem by

$$\tau=\eta\sqrt{1-\mu^2}+\mu\sqrt{1-\eta^2}\ ,\qquad \sigma=\eta\sqrt{1-\mu^2}-\mu\sqrt{1-\eta^2}\ , \tag{50}$$

where

$$\eta^2=u^2+\frac{4}{u_1^2}(u-u_1)^2,\quad \mu^2=v^2+\frac{4}{v_1^2}(v-v_1)^2\ . \tag{51}$$

For $u<u_1$ and $v<v_1$ these coordinates reduce to the ones (5), introduced by NH. It would be rather interesting to see an exact solution for the collision problem of superposed impulsive gravitational waves as described in this section. The weak-field approximation of this problem has already been considered by Szekeres.[2]

We want to point out also that we can superpose the shock gravitational waves in a similar manner. This amounts to consider the $H(U,X,Y)$ in the form

$$H=-\tfrac{1}{2}(X^2-Y^2)[b_0\theta(U)+b_1\theta(U-U_1)]\ . \tag{52}$$

However, since the collision problem of single shocks has not been solved yet, it will be inconvenient to discuss the collision of their superposition. By the particular choices,

$$b_0=-b_1=\frac{1}{u_1}$$

and in the limit $u_1\to 0$, solution of this problem must reduce to the solution of KP.

C. Superposition of shock em waves

Another example for colliding superposed waves in EM theory is when the $H(U,X,Y)$ function is given by[19]

$$H(U,X,Y)=\tfrac{1}{2}(X^2+Y^2)\sum_{i=1}^{m}A_i\theta(U-U_i)\ , \tag{53}$$

which represents the superposition of an arbitrary number of em shock waves. The constants A_i and U_i stand for the amplitude and location of the ith shock, respectively. The metric is transformed into the Rosen form as described in Ref. 19. The Riemann tensor for this metric gives a series of decoupled δ functions implying that no interaction occurs between the different shocks. As it has been shown in Ref. 19, this model is a soluble one—in fact, the only solved model for colliding superposed waves so far—and the solution amounts to the replacements

$$au\,\theta(u)\to\sum_{i=1}^{m}a_i(u-u_i)\theta(u-u_i)\ ,\qquad bv\,\theta(v)\to\sum_{i=1}^{m}b_i(v-v_i)\theta(v-v_i)\ , \tag{54}$$

in the BS solution.

IV. CONCLUSION

A single plane wave is an idealized concept that may hardly exist in nature. The occurrence of singularities due to their focusing also is attributed to the perfectly planar property of the plane waves.[20] It is therefore more realistic to consider such plane waves in succession, so that we can handle the sum as a wave packet. In this paper we have formulated the initial characteristic data for the collision problem of such superposed waves. Finding an exact solution, however, remains challenging and we expect that this new approach will guide the researchers in the field of CGW's.

ACKNOWLEDGMENTS

I would like to thank my former colleagues Y. Nutku and M. Gurses for collaboration that resulted in the superposition law for waves in CGW's.

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