Making inflation work: Damping of density perturbations due to Planck energy cutoff

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(Received 28 July 1988)

In this paper we propose an alternative method for the computation of classical density perturbations from a quantum field in an inflationary scenario. We compute the power spectrum of density perturbations directly from vacuum fluctuations of the "time-time" component of the energy-momentum tensor. We compute the inhomogeneous part of the correlation function $\langle 0|T^0{}_0(\mathbf{x},t)T^0{}_0(\mathbf{y},t)|0\rangle$ for a massless minimally coupled scalar field in de Sitter space. The Fourier transform of this two-point function leads to the scale-invariant spectrum of perturbations, but is ultraviolet divergent. This expression can be made finite by introducing an (*ad hoc*) small-distance cutoff in the proper length. We argue that this cutoff should be of the order of the Planck length, and show that, in such a case, the density fluctuations have the acceptable magnitude ($\sim 10^{-4}$) for the case of primordial inflation. Thus the inflationary scenario can be made to work without any fine-tuning.

I. INTRODUCTION AND SUMMARY

A Friedmann universe with power-law expansion for the scale factor $[S(t)=t^n,\ n<1]$ fails to explain the origin of galaxies on two major counts. First, it does not have any natural seeds for the origin of density inhomogeneities. Second, the scales on which the inhomogeneities exist today would all have originated outside the "physical horizon" [which is $(\dot{S}/S)^{-1}$] in the early Universe; it is difficult to imagine physical processes which can give rise to such coherence.

An inflationary model can solve both these problems. The quantum fluctuations of the scalar field which drive inflation can provide the seeds for density perturbations. The second difficulty is circumvented because, during the phase of exponential expansion the Hubble radius remains constant, but the proper wavelengths grow exponentially. Thus the galactic scales can originate from inside the horizon at the early epochs.

Given any model for inflation it is therefore possible to compute the spectrum and amplitude of the density perturbations. Such calculations have been done by several people[1-4] with the following result. Inflation leads to a (desirable) "*scale-invariant*" spectrum; but generically the amplitude of perturbation is too large (by a factor 10^5-10^6). This amplitude can be brought down only if the inflationary potential is fine-tuned in a very unnatural way. This makes inflation aesthetically unappealing.

In this paper we suggest a possible way out of this problem. We show that correct values for the amplitude can be obtained if the divergent expressions in field theory are regularized using a cutoff at Planck energy. This key idea has been suggested by one of us (T.P.) in a recent Letter.[5] Here we clarify and elaborate on this idea. In particular, we present the major aspects of this idea without actually relying on any specific model for quantum gravity.

Another issue of concern in inflation is the method by which *classical* density perturbations are computed from *quantum*-mechanical operators. Let us briefly review the conventional approach (as proposed in Refs. 1–4), and, what we believe to be, its unsatisfactory features.

It is natural that if inflation occurs at the grand-unified-theory (GUT) scale or earlier, the driving scalar field should be described by a quantum field theory. A self-consistent treatment would then require that the space-time metric be also quantized. Not having such a theory, one is compelled to describe the system by semiclassical equations which treat gravity classically and matter quantum mechanically. Such a semiclassical description of gravity has a long history and, in a way, formed part of the subject "*quantum field theory in curved space time.*" It is usually believed, at least in the days before the invention of inflation, that the *source for semiclassical gravity is the expectation value of* T_{ik}. According to this viewpoint semiclassical gravity is described by the equations

$$G_{ik}=8\pi\langle\psi|T_{ik}|\psi\rangle\ , \tag{1}$$

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle=\hat{H}|\psi\rangle\ , \tag{2}$$

where $|\psi\rangle$ denotes the quantum state of the field, $\hat{H}$ is the Hamiltonian governing the evolution of ϕ, and G_{ik} stands for $(R_{ij}-\frac{1}{2}g_{ik}R)$. Here (1) is the semiclassical Einstein's equation and (2) is the Schrödinger-picture evolution equation for the quantum state of the field ϕ.

This viewpoint, however, leads to difficulties in the inflationary scenario. It is usual to assume that the quantum field driving the inflation is in the vacuum state in the de Sitter space-time; but the expectation value $\langle 0|T_{ij}(\mathbf{x},t)|0\rangle$ is homogeneous (i.e., independent of $\mathbf{x}$) because of the translational invariance of the vacuum state $|0\rangle$. Thus, we will never get an $\mathbf{x}$-dependent $(\delta\rho/\rho)$ out of this prescription. We must abandon the rule that $\langle 0|T_{ik}|0\rangle$ is the source for semiclassical gravity.

Once we abandon it, we are at a loss to select another unique "source." (The proper approach will be to start with the Wheeler-DeWitt equation in quantum gravity and consider its semiclassical limit. Unfortunately, there are several subtleties involved in this approach.[6]) We need to proceed in a somewhat intuitive manner. The conventional view has been the following.[7]

We define a classical field $\phi_{cl}(\mathbf{x},t)$ as consisting of a homogeneous part and a perturbation:

$$\phi_{cl}(\mathbf{x},t)=\phi_0(t)+\delta\phi(\mathbf{x},t) \ . \tag{3}$$

Since $\phi_0(t)$ cannot be defined as $\langle 0|\phi(\mathbf{x},t)|0\rangle$ (which vanishes), it is defined as the (regularized) rms value:

$$\phi_0(t)\equiv[\langle 0|\phi^2(\mathbf{x},t)|0\rangle]^{1/2} \ . \tag{4}$$

Defining $\delta\phi(\mathbf{x},t)$ is trickier. We first define the power spectrum of the scalar field by

$$P(\mathbf{k},t)=\int d^3\mathbf{x}\, e^{-i\mathbf{k}\cdot\mathbf{x}}\langle 0|\phi(\mathbf{y}+\mathbf{x},t)\phi(\mathbf{y},t)|0\rangle \tag{5}$$

and construct $\delta\phi(\mathbf{x},t)$ as the Fourier transform of $\sqrt{P(\mathbf{x},t)}$. In this manner, $\delta\phi(\mathbf{x},t)$ is made to carry information about the inhomogeneities. This classical field $\phi_{cl}(\mathbf{x},t)$ is then used to construct a classical energy-momentum tensor T_{ik}^{cl} (which is no longer homogeneous). From this T_{ik}^{cl} a nonzero density perturbation can be obtained. This, essentially, is the conventional approach.

It is clear that the information regarding the spatial dependence can be smuggled in only through the expectation values of the two-point functions. But then, it is much more rational and meaningful to use the two-point function made from T_{ik} itself. The quantity $\langle 0|T^0{}_0(\mathbf{x}+\boldsymbol{l},t)T^0{}_0(\mathbf{x},t)|0\rangle$ clearly carries information about the vacuum fluctuations in the energy density. The power spectrum of the density fluctuations is given directly by the Fourier transform of this correlation function:

$$|\rho_{\mathbf{k}}(t)|^2=\int d^3\mathbf{y}\, e^{-i\mathbf{k}\cdot\mathbf{y}}\langle 0|T^0{}_0(\mathbf{x}+\mathbf{y},t)T^0{}_0(\mathbf{x},t)|0\rangle \ . \tag{6}$$

Thus we can calculate our spectrum directly from $\langle 0|T^0{}_0T^0{}_0|0\rangle$. This approach is straightforward and physically meaningful. Moreover it eliminates the need to define an intermediate $\phi_{cl}(\mathbf{x},t)$ in an *ad hoc* manner.

An independent support for the physical relevance of $\langle\psi|T^i{}_k(\mathbf{x}+\boldsymbol{l},t)T^l{}_m(\mathbf{x},t)|\psi\rangle$ comes from the study of the response of a hypothetical particle detector. It turns out that the simplest kind of detector, which attempts to measure the energy density by coupling linearly to $T^i{}_k$ of a quantum field, will respond to the two-point function $\langle 0|T^i{}_kT^l{}_m|0\rangle$ and *not* to the expectation value $\langle T^i{}_k\rangle$. Thus a detector will respond *both* to the mean value of $T^i{}_k$ and to the fluctuations. In particular, even if the mean value of $T^i{}_k$ is zero, as in the vacuum state, the detector will respond to the fluctuations.[8] From an operational point of view, then, the two-point function appears to be a more fundamental measure of $T^i{}_k$ than the expectation value.

The major trouble with our definition (6) is that the expression on the right-hand side is divergent. We feel that one should honestly "own up" this divergence and attempt to regularize it by some physical criterion. This is what we plan to do in the following.

As far as we are aware, this idea of defining $|\rho_{\mathbf{k}}(t)|^2$ directly in terms of $T^i{}_k$ has not been discussed in the literature in *any detail.* Brandenberger mentions this approach briefly in an appendix to one of his papers[9] and notes that the expression is divergent.

In the next section of this paper we will compute the two-point function for a massless minimally coupled scalar field in de Sitter space. We find that unlike the object $\langle 0|T^i{}_k|0\rangle$ which is homogeneous and divergent, the two-point function contains a space-dependent part which is finite.

We then evaluate the Fourier transform of this two-point function to compute $|\rho_{\mathbf{k}}|^2$, and show that the density perturbations have a scale-invariant spectrum at the time of horizon crossing. However, the power spectrum so obtained is ultraviolet divergent and hence nothing can be said about the magnitude of the density contrast until some method of regularization is adopted. We calculate the amplitude of $|\rho_{\mathbf{k}}|^2$ with such a regularization and show that it has the acceptable value. This arises because our cutoff provides an exponential damping for modes with proper wavelengths smaller than Planck length. (This is essentially the idea suggested in Ref. 5.) In the last section we discuss the motivation and the shortcomings of our approach.

II. COMPUTATION OF $\langle 0|T^0{}_0(\mathbf{x}+\boldsymbol{l},t)T^0{}_0(\mathbf{x},t)|0\rangle$ IN DE SITTER SPACE-TIME

In this section we will compute the two-point function $\langle 0|T^0{}_0(\mathbf{x}+\boldsymbol{l})T^0{}_0(\mathbf{x})|0\rangle$ for a massless free scalar field in an external de Sitter universe. (In a realistic inflationary model, the field will not be free; neither will the space-time be eternally de Sitter. However as we will argue in the last section these approximations do not affect the central result appreciably.) We will work with the Lagrangian

$$\mathcal{L}(\phi)=\tfrac{1}{2}g^{ij}\partial_i\phi\partial_k\phi \tag{7}$$

for a quantum scalar field operator $\phi(x)$ in the inflationary metric

$$ds^2=dt^2-S^2(t)|d\mathbf{x}|^2, \quad S(t)=e^{Ht} \ . \tag{8}$$

We quantize the field by expanding the field $\phi(\mathbf{x},t)$ as

$$\hat{\phi}(\mathbf{x},t)=\int\frac{d^3k}{(2\pi)^3}[\hat{a}_{\mathbf{k}}f_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}+\text{H.c.}] \ , \tag{9}$$

where $f_{\mathbf{k}}(t)$ are the properly normalized positive-frequency solutions of the equation

$$\ddot{f}_{\mathbf{k}}+3H\dot{f}_{\mathbf{k}}+\frac{k^2}{S^2(t)}f_{\mathbf{k}}=0 \ . \tag{10}$$

The explicit form for $f_{\mathbf{k}}$ is given by

$$f_{\mathbf{k}}=\frac{H}{\sqrt{2}\,k^{3/2}}(1-i\theta)e^{i\theta}, \quad k=|\mathbf{k}| \ , \tag{11}$$

where $\theta=k/(HS)$. In the $H\to 0$ limit, $f_{\mathbf{k}}$ reduces to the flat-space positive-frequency mode $(2\omega)^{-1/2}e^{-i\omega t}$. This choice of mode decomposition defines the vacuum state

$|0\rangle$, via the relation $a_{\mathbf{k}}|0\rangle=0$. We want to evaluate the correlation function

$$C^{i}{}_{j}{}^{k}{}_{l}(\mathbf{x}+\mathbf{l},\mathbf{x};t)\equiv\langle 0|T^{i}{}_{j}(\mathbf{x}+\mathbf{l},t)T^{k}{}_{l}(\mathbf{x},t)|0\rangle \ , \quad (12)$$

where $T^{i}{}_{k}$ is the energy-momentum tensor for the scalar field:

$$T^{i}{}_{k}=\partial^{i}\phi\partial_{k}\phi-\tfrac{1}{2}\delta^{i}{}_{k}(\partial^{a}\phi\partial_{a}\phi) \ . \quad (13)$$

We introduce a complete set of states $|\psi\rangle$ in (12) so that the correlation function can be written as

$$C^{i}{}_{j}{}^{k}{}_{l}(\mathbf{x}+\mathbf{l},\mathbf{x};t)=\sum_{|\psi\rangle}\langle 0|T^{i}{}_{j}(\mathbf{x}+\mathbf{l},t)|\psi\rangle\langle\psi|T^{k}{}_{l}(\mathbf{x},t)|0\rangle \ . \quad (14)$$

The quantity $\langle 0|T^{a}{}_{b}(\mathbf{y},t)|\psi\rangle$ is nonzero only for the cases when $|\psi\rangle$ is either a vacuum state or a two-particle state. So $C^{i}{}_{j}{}^{k}{}_{l}$ can be expressed as

$$C^{i}{}_{j}{}^{k}{}_{l}(\mathbf{x}+\mathbf{l},\mathbf{x};t)=\langle 0|T^{i}{}_{j}(\mathbf{x}+\mathbf{l},t)|0\rangle\langle 0|T^{k}{}_{l}(\mathbf{x},t)|0\rangle+\sum_{\mathbf{p},\mathbf{q}}\langle 0|T^{i}{}_{j}(\mathbf{x}+\mathbf{l},t)|1_{\mathbf{p}}1_{\mathbf{q}}\rangle\langle 1_{\mathbf{p}}1_{\mathbf{q}}|T^{k}{}_{l}(\mathbf{x},t)|0\rangle \ , \quad (15)$$

where $\mathbf{p}$ and $\mathbf{q}$ are the momenta labeling the two-particle state. Since the vacuum is translationally invariant, the first term on the right-hand side is space independent (though formally divergent). Hence when we evaluate the Fourier transform of $C^{i}{}_{j}{}^{k}{}_{l}$ with respect to $\mathbf{l}$, the first term will give a contribution proportional to $\delta(\mathbf{k})$ where $\mathbf{k}$ is the wave number for the perturbation. However, we are interested only in the $\mathbf{k}\neq 0$ modes. Hence this term will not contribute to $|\rho_{\mathbf{k}}|^2$ in (23), and can be ignored. So all the contribution comes from the second term. We therefore write

$$C^{i}{}_{j}{}^{k}{}_{l}(\mathbf{x}+\mathbf{l},\mathbf{x};t)=\sum_{\mathbf{p},\mathbf{q}}\tau^{i}{}_{j}(\mathbf{x}+\mathbf{l},\mathbf{p},\mathbf{q},t)\tau^{k}{}_{l}{}^{*}(\mathbf{x},\mathbf{p},\mathbf{q},t) \ , \quad (16)$$

where

$$\tau^{i}{}_{j}(\mathbf{x},\mathbf{p},\mathbf{q},t)=\langle 0|T^{i}{}_{j}(\mathbf{x},t)|1_{\mathbf{p}},1_{\mathbf{q}}\rangle \ . \quad (17)$$

To compute τ_{ij} it is easier to first evaluate the quantity $\langle 0|\phi(x)\phi(z)|1_{\mathbf{p}}1_{\mathbf{q}}\rangle$ (where x and z are four-dimensional space-time coordinates). Then $\tau^{i}{}_{j}$ can be obtained as

$$\begin{aligned}\tau^{i}{}_{j}(\mathbf{x},\mathbf{p},\mathbf{q},t)&\equiv\tau^{i}{}_{j}(x,\mathbf{p},\mathbf{q})\\&=\lim_{z\to x}\left[g^{ik}\frac{\partial}{\partial x^{k}}\frac{\partial}{\partial z^{j}}-\frac{1}{2}\delta^{i}{}_{j}g^{ab}\frac{\partial}{\partial x^{a}}\frac{\partial}{\partial z^{b}}\right]\\&\quad\times\langle 0|\phi(x)\phi(z)|1_{p},1_{q}\rangle \ . \end{aligned}\quad (18)$$

After we perform the operations of differentiation we can take the limits of $z\to x$. We have split the operators purely for computational convenience.

We are only interested in the component $\tau^{0}{}_{0}$. Using the mode expansion [Eq. (9)] for the field in the expression for $\tau^{0}{}_{0}$ it can be easily shown to be (see Appendix A)

$$\tau^{0}{}_{0}(\mathbf{x},\mathbf{p},\mathbf{q},t)=\left[\dot f_{\mathbf{p}}\dot f_{\mathbf{q}}-\frac{\mathbf{p}\cdot\mathbf{q}}{S^{2}}f_{\mathbf{p}}f_{\mathbf{q}}\right]e^{i(\mathbf{p}+\mathbf{q})\cdot\mathbf{x}} \ . \quad (19)$$

(Here an overdot signifies differentiation with respect to time and $\mathbf{p}\cdot\mathbf{q}$ denotes a flat-space dot product: $\mathbf{p}\cdot\mathbf{q}=\eta^{\alpha\beta}p_{\alpha}p_{\beta}$, $\alpha=1,2,3$.) Substituting (19) in (16) we find the equal-time correlation function $C(\mathbf{l},t)\equiv C^{0}{}_{0}{}^{0}{}_{0}(\mathbf{x}+\mathbf{l},\mathbf{x};t)$ to be

$$C(\mathbf{l},t)=\int\frac{d^{3}\mathbf{p}}{(2\pi)^{3}}\int\frac{d^{3}\mathbf{q}}{(2\pi)^{3}}e^{i(\mathbf{p}+\mathbf{q})\cdot\mathbf{l}}\left|\dot f_{\mathbf{p}}\dot f_{\mathbf{q}}-\frac{\mathbf{p}\cdot\mathbf{q}}{S^{2}}f_{\mathbf{p}}f_{\mathbf{q}}\right|^{2} \ . \quad (20)$$

Since the explicit form of $f_{\mathbf{p}}(t)$ is known [see Eq. (11)], the expression (20) can be evaluated in a straightforward manner. The details of the calculation are given in Appendix B. The final answer is

$$C(\mathbf{l},t)=\frac{H^{2}}{2\pi^{4}(Sl)^{6}}+\frac{3}{\pi^{4}(Sl)^{8}} \ , \quad (21)$$

where $l=|\mathbf{l}|$. Certain features in this result could have been anticipated from general considerations. From dimensional grounds we know that C should go as $(\text{length})^{-8}$. The only length scales available are H^{-1} and the proper length Sl. Thus any term in C must have the form $(Sl)^{-8}F[H^{-1}/(Sl)]$ where F is a function to be determined. But f and $\dot f$ contain only linear power of H explicitly; therefore we can only have H^{0} or H^{2} explicitly in F. So we expect two terms which go as $(Sl)^{-8}$ and $H^{2}(Sl)^{-6}$. The coefficients, of course, can only be determined by explicit calculation.

In the limit $H\to 0$, S goes to unity and the first term goes to zero and we get the correct flat-space limit. This may be verified by repeating the calculation using the flat-space-time mode functions for the scalar field. (This has been worked out, e.g., in Ref. 8.)

III. POWER SPECTRUM OF THE DENSITY PERTURBATIONS

To calculate the power spectrum of the density perturbation $|\rho_{\mathbf{k}}|^{2}$ we require the Fourier transform of the two-point function in (21). The $\mathbf{k}$ dependence of $|\rho_{\mathbf{k}}|^{2}$ can be decided purely by dimensional grounds. The two terms in (21), on Fourier transform, will give a $(H^{2}k^{3}/S^{6})$ term and a (k^{5}/S^{8})-type term. We are interested in $|\rho_{\mathbf{k}}(t_{\mathbf{k}})|^{2}$ where $t_{\mathbf{k}}$ is the time at which the mode labeled by $\mathbf{k}$ crosses the horizon. At $t=t_{\mathbf{k}}$, $S(t_{\mathbf{k}})\propto k$; therefore, both (k^{3}/S^{6}) and (k^{5}/S^{8}) give terms proportional to $|\mathbf{k}|^{-3}$. *This is precisely the scale-invariant spectrum we are after.*

However, it is not possible to take the Fourier transform of (21) in a straightforward manner. Both the terms in (21) diverge badly near $l=0$; the Fourier transform is ultraviolet divergent. This is a physical divergence which arises in the coincidence limit of the two-point function. We will adopt a procedure for regularizing this expression. This procedure, at the present level of our understanding of quantum gravity, is completely *ad hoc*. However, it leads to very interesting results. In the rest of this section we will work out the consequences of this regularization. The motivation for this procedure is discussed in the next section.

The procedure is the following. We replace the squared proper length $(Sl)^2$ by $(S^2l^2+L^2)$ where L is a length of the order of Planck length. This requirement is equivalent to the following principle: The Planck length is the lower bound to all proper length intervals; proper lengths smaller than the Planck length have no operational significance.

With this prescription, the correlation function is modified to

$$C(l,t)=\frac{H^2}{2\pi^4}\frac{1}{(S^2l^2+L^2)^3}+\frac{3}{\pi^4}\frac{1}{(S^2l^2+L^2)^4}$$
$$=\frac{H^2}{2\pi^4S^6}\frac{1}{[l^2+(L/S)^2]^3}+\frac{3}{\pi^4S^8}\frac{1}{[l^2+(L/S)^2]^4}\;. \tag{22}$$

We can now take a Fourier transform of this expression to get the power spectrum

$$|\rho_{\mathbf{k}}(t)|^2=\int d^3l\, e^{i\mathbf{k}\cdot\mathbf{l}}\left[\frac{H^2}{2\pi^4S^6}\frac{1}{[l^2+(L/S)^2]^3}+\frac{3}{\pi^4S^8}\frac{1}{[l^2+(L/S)^2]^4}\right]. \tag{23}$$

Using standard contour integration techniques we can show that

$$\int\frac{e^{i\mathbf{k}\cdot\mathbf{l}}d^3l}{(l^2+L^2/S^2)^4}=\frac{\pi^2}{24}e^{-kL/S}\left\{3\left[\frac{S}{L}\right]^5\left[1+\frac{kL}{S}+\frac{1}{3}\left[\frac{kL}{S}\right]^2\right]\right\} \tag{24}$$

and

$$\int\frac{e^{i\mathbf{k}\cdot\mathbf{l}}d^3l}{(l^2+L^2/S^2)^3}=\frac{\pi^2}{4}e^{-kL/S}\left[\left[\frac{S}{L}\right]^3\left[1+\frac{kL}{S}\right]\right]. \tag{25}$$

Substituting the value of the above integrals in Eq. (23) we get

$$k^3|\rho_k(t)|^2=\frac{H^8}{8\pi^2}\beta^6e^{-\beta LH}\left[\frac{3}{\beta^3(LH)^5}(1+\beta LH+\tfrac{1}{3}\beta^2L^2H^2)+\frac{1}{\beta^3(LH)^3}(1+\beta LH)\right], \tag{26}$$

where β stands for k/HS. All the $\mathbf{k}$ dependence of the right-hand side has been absorbed in β. This expression gives $|\rho_{\mathbf{k}}(t)|^2$ at some arbitrary instant t. However, we are interested in its value at the time $t=t_{\mathbf{k}}$ when the mode labeled by $\mathbf{k}$ leaves the Hubble horizon. At $t=t_{\mathbf{k}}$, $2\pi k^{-1}S(t_{\mathbf{k}})$ is equal to H^{-1}. Or equivalently $k/(HS)=\beta=2\pi$. At this time the power spectrum is

$$k^{3/2}|\rho_{\mathbf{k}}(t_{\mathbf{k}})|=2\sqrt{2}\,\pi^2H^4e^{-\pi LH}F(LH)\;, \tag{27}$$

where

$$F(x)=\frac{1}{(2\pi x)^{3/2}}\left[(1+2\pi x)\left[1+\frac{3}{x^2}\right]+4\pi^2\right]^{1/2}. \tag{28}$$

Our ultimate aim is to compute density perturbation at the time when the perturbation reenters the Hubble radius in the postinflationary era. This is related to $k^3|\rho_{\mathbf{k}}|^2$ at $t_{\mathbf{k}}$ by Bardeen's gauge-invariant formalism.[4] The density contrast at the time when the mode reenters the Hubble horizon is related to (27) by

$$\left.\frac{k^{3/2}|\delta_k|}{1+w}\right|_{t=t_f}=\left.\frac{k^{3/2}|\delta_k|}{1+w}\right|_{t=t_k}, \tag{29}$$

where t_f is the epoch of reentry and w is the ratio of the pressure to the density and $\delta_{\mathbf{k}}\equiv\rho_{\mathbf{k}}/\rho_0$ where ρ_0 is the average background density. We now use the standard result that

$$1+w(t_k)\approx\frac{\dot\phi_0^2(t_k)}{v_0} \tag{30}$$

and

$$1+w(t_f)\approx\tfrac{4}{3} \tag{31}$$

to get

$$k^{3/2}|\delta_k|_{t=t_f}=\frac{\sqrt{2}}{3\pi}(2\pi)^3e^{-\pi LH}\frac{H^4}{\dot\phi_0^2(t_k)}F(LH)\;. \tag{32}$$

For evaluating $\dot\phi_0^2$ we use the expression (4) in the Bunch-Davies vacuum. It is well known that[7] the regularized value for $\phi_0(t)$ is

$$\phi_0(t)=\frac{H^{3/2}}{2\pi}t^{1/2}\;. \tag{33}$$

At the epoch, $t=t_{\mathbf{k}}$ we have $k/(HS)=2\pi$; hence,

$$\dot\phi_0(t_k)=\frac{H^2}{4\pi}\left[\ln\left[\frac{k}{2\pi H}\right]\right]^{-1/2}. \tag{34}$$

Substituting in (32) we get

$$k^{3/2}|\delta_k|\,|_{t=t_f}=\frac{128\sqrt{2}}{3}\pi^4e^{-\pi LH}F(LH)\ln\left[\frac{k}{2\pi H}\right], \tag{35}$$

where $F(x)$ is defined in (28). This is our final result. For comparison, note that the standard analysis would have given (without any fine-tuning)

$$k^{3/2}|\delta_k|\,|_{t=t_F}=\frac{32\sqrt{2}\,\pi^3}{3\sqrt{1+4\pi^2}}\left[\ln\left[\frac{k}{2\pi H}\right]\right]^{1/2}$$

$$\approx\frac{16\sqrt{2}\,\pi^2}{3}\left[\ln\frac{k}{2\pi H}\right]^{1/2}. \quad (36)$$

Both (35) and (36) show only a weak **k** dependence (logarithmic); this is, of course, a desirable feature. What is most important, however, is that the *amplitude of the fluctuations is damped by an exponential factor in (35).* This can give rise to acceptably low (10^{-4}–10^{-6}) values for the amplitude of the perturbations in our model. *There is no need to resort to any fine-tuning.* [This is a vast improvement over (36) in which the amplitude is about ~ 70, which is too high.] This reduction is essentially due to the damping factor arising from the small distance cutoff which we have imposed.

We will compute the numerical value of this expression when all the parameters in the theory are scaled by Planck length. We take the energy density V_0 during the inflation, to be about $(\epsilon M_P)^4$ where M_P is the Planck mass $(c\hbar/G)^{1/2}=10^{19}$ GeV, and ϵ is of order unity (in other words, the inflation is primordial). We will also take L to be nL_P where $L_P=(G\hbar/c^3)^{1/2}\simeq 10^{-33}$ cm is the Planck length and n is of order unity. With these parametrizations,

$$HL=\left[\frac{8\pi G}{3}V_0 n^2 L_P{}^2\right]^{1/2}=n\epsilon^2\left[\frac{8\pi}{3}\right]^{1/2} \quad (37)$$

so that

$$k^{3/2}|\delta_k|=\frac{128\sqrt{2}\,\pi^4}{3}\exp\left[-n\epsilon^2\left[\frac{8\pi}{3}\right]^{1/2}\right]F\left[n\epsilon^2\left[\frac{8\pi}{3}\right]^{1/2}\right]\ln\left|\frac{k}{2\pi H}\right|. \quad (38)$$

This expression can be as low as 10^{-4}–10^{-6} with ϵ, n remaining of order unity. For example, for $\epsilon=1.1$, $n=1.4$, we get

$$k^{3/2}|\delta_k|\approx 0.6\times 10^{-4}. \quad (39)$$

For $n=1.6$, $\epsilon=1.1$, we get 10^{-5} and for $\epsilon=1.3$, $n=1.2$, we get about 10^{-6}. In the "order-unity range" of 1.2–1.4 or so, for both ϵ and n, we get acceptable range for $|\delta_k|$. *Thus keeping the only two available parameters V_0 and L of the theory at Planck scales, we can produce an acceptable density contrast.* The crucial effect, of course, is from the exponential factor. Note that even for $n=\epsilon=1$, this factor is $\exp(-\sqrt{8\pi^3/3})$ which is about 2×10^{-4}.

In conventional scenarios, primordial inflation with $\epsilon=1$ can be ruled out. This is because, such an inflation will produce too much of gravitational-wave background leading to large anisotropies in $(\Delta T/T)$. In our approach, this problem could be taken care of if we regularize the gravitational-wave amplitude by the same regularization procedure. Recall that in the conventional analysis, the characteristic fluctuations in the gravitational-wave modes are given by [see, for example, Eq. (12) of Ref. 10]

$$(\Delta h)^2_{\mathbf{k}}=\frac{k^3}{(2\pi)^3}\frac{1}{2}\int d^3\mathbf{x}\,e^{-i\mathbf{k}\cdot\mathbf{x}}\langle h_{ij}(\mathbf{x},t)h^{ij}(0,t)\rangle. \quad (40)$$

The two-point function $\langle h_{ij}(\mathbf{x},t)h^{ij}(\mathbf{0},t)\rangle$ will vary as $|\mathbf{x}|^{-2}$ in the conventional model and will be replaced by $(|\mathbf{x}|^2+L^2)$ by our procedure. This will reduce Δh^2 by the same exponential cutoff factor $\exp(-nHL)$, $\simeq 2\times 10^{-4}$ when the waves produced in the inflationary phase leave the horizon. Note that a wavelength which is about 10^{28} cm today ($\simeq H_0{}^{-1}$ today) would have been about $[(T_0/10^{19}\text{ GeV})\times 10^{28}\text{ cm}]=3\times 10^{-4}$ cm at the end of primordial inflation [$T_0=3$ K is the present cosmic microwave background radiation (CMBR) temperature]. Assuming that the inflation is by a factor $Z=10^{29}$ this wavelength would have been about 3×10^{-33} cm at the early stage of inflation. It is therefore necessary to consider quantum gravitational effects on these wave modes. This issue probably requires a more careful analysis, starting from the first principles. We hope to address this issue in a future publication.

IV. DRAWBACKS AND OUTLOOK

In this concluding section we will critically examine the conclusions of this paper and discuss possible objections.

To begin with there are certain, relatively minor, technical objections. We did the calculation for a massless free scalar field in an external de Sitter spacetime. To be realistic we have to introduce some suitably flat, effective potential and take into account effects such as reheating, finite age of the de Sitter space, etc. While this certainly needs to be done, we do not think it will modify the result in any drastic manner. Several calculations have demonstrated the fact that the value of $(\delta\rho/\rho)$ is reasonably independent of these details. Essentially, this value is decided by the Fourier transform of the two-point function of the scalar field $\langle 0|\phi(\mathbf{x},t)\phi(\mathbf{y},t)|0\rangle=G(\mathbf{x},\mathbf{y},t)$. For small $|\mathbf{x}-\mathbf{y}|$, the behavior is always $S^{-2}(t)|\mathbf{x}-\mathbf{y}|^{-2}$. Since our procedure only affects the value near Planck length, the high-**k** behavior will always pick up a factor such as $\exp(-nHL)$. Thus the cutoff will reduce the $(\delta\rho/\rho)$ by a large factor (about 2×10^{-4}) even in a realistic scenario.

The second issue concerns the way in which the expression $\langle 0|T^0{}_0(\mathbf{x}+\mathbf{l},t)T^0{}_0(\mathbf{x},t)|0\rangle$ is evaluated. In calculating this quantity we have dropped two divergent terms. However, this procedure can be fully justified. The first term which we dropped was proportional to

$\langle 0|T^0{}_0(\mathbf{x}+\boldsymbol{l},t)T^0{}_0(\mathbf{x},t)|0\rangle$ [see Eq. (15)]. This quantity is independent of $\mathbf{x},\boldsymbol{l}$ and hence will only produce a $\delta(\mathbf{k})$ contribution on Fourier transforming. In other words, it does not contribute to inhomogeneities. (In fact we could have regularized it with our *ad hoc* procedure and then dropped it, since $\mathbf{k}\neq\mathbf{0}$.) The second term which we dropped was infrared divergent [see Appendix B, Eq. (B14)]. Such infrared divergence of two-point function is well known in de Sitter spacetime (see, e.g., Ref. 11). This term again makes no contribution to the inhomogeneities [as demonstrated in Appendix B, Eq. (B12)]. The procedure we used in calculating various integrals involving $(e^{i\mathbf{p}\cdot\mathbf{x}})$ is standard; in fact, this is how we calculate the Fourier transform of $|\mathbf{x}|^{-1}$ to be $|\mathbf{k}|^{-2}$.

The really important issue, in our opinion, is the following. After calculating $\langle 0|T^0{}_0T^0{}_0|0\rangle$, we find that its Fourier transform is ultraviolet divergent. An *ad hoc* prescription $|x|^2\rightarrow|x|^2+L^2$, is used to get around this problem. Our results depend *crucially* on this prescription; in fact, $(\delta\rho/\rho)_{\mathbf{k}}$ will blow up if L goes to zero. It is necessary to motivate this procedure.

We justify this procedure on the following counts.

(i) To discuss primordial inflation and length scales of the order of 10^{-32} cm, it is necessary to invoke some features from quantum gravity. (Note that a scale of about 1 kpc today would have been about 10^{-32} cm at the beginning of inflation.) One of the features repeatedly attributed to quantum gravity is that it could work as a "universal regulator."[12,13] Our procedure incorporates this feature in a simple manner. As we discussed in Sec. I, the Fourier transform of $\langle 0|T^i{}_kT^l{}_m|0\rangle$ *must* give the correct, physical, power spectrum. If this diverges it is necessary to acknowledge this divergence honestly and try to remedy the situation. We believe we have taken a step in that direction.

(ii) Some toy models of quantum gravity lend support to the view that Planck length should be interpreted as the "zero-point length" in space-time (see Ref. 14). Nonperturbative quantum gravitational corrections to propagators, computed by summing a partial set of Feynman diagrams also support this view (see Ref. 12). [It was, in fact, these considerations which motivated one of us (T.P.) to suggest this damping mechanism. The results in Refs. 12 and 14, however deal with space-time intervals, while we need the proper length on a constant-t surface. The procedure in this paper, in a way, justifies the earlier naive calculation in Ref. 5.] These considerations give additional support for the procedure $|\mathbf{x}|^2\rightarrow|\mathbf{x}|^2+L^2$.

(iii) To study physics at Planck energies, we need a model which incorporates quantum effects of gravity. One of the currently popular ideas is based on strings as fundamental constituents. String theory removes the divergences in conventional field theories essentially because of the nonlocalizability of the strings. This nonlocalizability occurs at about Planck length. Since conventional space-time emerges as a nonperturbative condensate in string theory, we may incorporate the nonlocalizability by adding a "zero-point length" to the proper length. It is conceivable that when $(\delta\rho_k)$ is computed from string theory, modes with proper wavelengths smaller than Planck length will be damped out because of the inherent "size" of the region over which strings cannot be localized.

(iv) We may offer as an *a posteriori* justification for the *ad hoc* procedure, the value of the $(\delta\rho/\rho)$ which we have obtained. Since we get an acceptable value without any fine-tuning, it is probably worth investigating whether the prescription contains some, at present unknown, element of physics.

In this connection, one must make a clear distinction between the *sensitivity* of $(\delta\rho/\rho)$ to the parameters of the theory and the *fine-tuning* of the parameters of the theory. Our expression for $(\delta\rho/\rho)$ is very sensitive to ϵ and n, i.e., small changes in n,ϵ can induce large changes in $(\delta\rho/\rho)$ due to the exponential factor. This, in our opinion, is acceptable. [In a way, this is also inevitable. If all parameters are scaled to Planck length, the "natural value" for $(\delta\rho/\rho)$ will be something like 1, $2\pi^3$, etc. To get a number such as 10^{-6}, it is essential that stronger functions, such as large powers of ϵ and n or exponentials, need to be invoked. These functions will also be sensitive.] What is important is that acceptable $(\delta\rho/\rho)$ was obtained with ϵ and n very close to unity (for example, we *did not* need $\epsilon=5$, $n=20$).

The most important problem which needs to be now addressed is the computation of gravitational-wave perturbations in a rigorous fashion. If the damping mechanism does not work for this case, then the model may not be viable. This issue is under investigation.

APPENDIX A

We will evaluate $\langle 0|\phi(x)\phi(y)|1_{\mathbf{p}}1_{\mathbf{q}}\rangle$ and show how we derive Eq. (19) from Eq. (18). From Eq. (9) we have

$$\phi(\mathbf{y},t)=\phi(y)=\sum_{\mathbf{p}'}[a_{\mathbf{p}'}f_{\mathbf{p}'}(t)e^{i\mathbf{p}'\cdot\mathbf{y}}+\text{H.c.}]\ . \tag{A1}$$

We substitute this in $\langle 0|\phi(x)\phi(y)|1_{\mathbf{p}}1_{\mathbf{q}}\rangle$. The only nonzero term will be the one which arises from $a_{\mathbf{p}}a_{\mathbf{q}}$ acting on $|1_{\mathbf{p}}1_{\mathbf{q}}\rangle$. So we get

$$\langle 0|\phi(x)\phi(y)|1_{\mathbf{p}}1_{\mathbf{q}}\rangle=f_{\mathbf{p}}(t)f_{\mathbf{q}}(t')e^{i\mathbf{p}\cdot\mathbf{x}+i\mathbf{q}\cdot\mathbf{y}}+(\mathbf{p}\leftrightarrow\mathbf{q})\ . \tag{A2}$$

Substituting this in Eq. (18) we get

$$\begin{aligned}\tau^0{}_0(\mathbf{x},\mathbf{p},\mathbf{q},t)&=\tfrac{1}{2}\left[\dot f_{\mathbf{p}}(t)\dot f_{\mathbf{q}}(t)-\frac{1}{S^2}(\mathbf{p}\cdot\mathbf{q})f_{\mathbf{p}}f_{\mathbf{q}}\right]e^{i(\mathbf{p}+\mathbf{q})\cdot\mathbf{x}}+(\mathbf{p}\leftrightarrow\mathbf{q})\\&=\left[\dot f_{\mathbf{p}}(t)\dot f_{\mathbf{q}}(t)-\frac{1}{S^2}(\mathbf{p}\cdot\mathbf{q})f_{\mathbf{p}}f_{\mathbf{q}}\right]e^{i(\mathbf{p}+\mathbf{q})\cdot\mathbf{x}}\ ,\end{aligned} \tag{A3}$$

which is Eq. (19).

APPENDIX B

In this we indicate the main steps in the computation of (21). We begin by expanding (20), getting four terms:

$$C(l,t)=\sum_{i=1}^{4} T_i(l,t) , \tag{B1}$$

where

$$T_1(l,t)=\left[\int \frac{d^3\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot l}|\dot{f}_{\mathbf{p}}(t)|^2\right]^2 , \tag{B2}$$

$$T_2=\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{d^3\mathbf{q}}{(2\pi)^3} e^{i(\mathbf{p}+\mathbf{q})\cdot l}\frac{(\mathbf{p}\cdot\mathbf{q})^2}{S^4}|f_{\mathbf{p}}|^2|f_{\mathbf{q}}|^2 , \tag{B3}$$

$$T_3=-\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{d^3\mathbf{q}}{(2\pi)^3} e^{i(\mathbf{p}+\mathbf{q})\cdot l}\frac{(\mathbf{p}\cdot\mathbf{q})}{S^2}\dot{f}_{\mathbf{p}}\dot{f}_{\mathbf{q}}f^*_{\mathbf{p}}f^*_{\mathbf{q}} , \tag{B4}$$

$$T_4(l,t)=T_3^*(-l,t) . \tag{B5}$$

Using the expression for $f_{\mathbf{p}}(t)$ from Eq. (11) we can express $\dot{f}_{\mathbf{p}}(t)$ in a convenient form as

$$\dot{f}_{\mathbf{p}}=-\frac{(\theta H)^2}{p^{3/2}\sqrt{2}}e^{i\theta} , \tag{B6}$$

where $p=|\mathbf{p}|$ and $\theta=p/(HS)$. The terms T_2, T_3, and T_4 can be expressed, more conveniently as

$$T_2=\frac{1}{S^4}I_{\alpha\beta}I^{\alpha\beta} , \tag{B7}$$

$$T_3=-\frac{1}{S^2}J_\alpha J^\alpha , \tag{B8}$$

$$T_4(l,t)=T_3^*(-l,t) , \tag{B9}$$

where

$$\begin{aligned} I_{\alpha\beta}&=\int \frac{d^3\mathbf{p}}{(2\pi)^3}e^{i\mathbf{p}\cdot l}p_\alpha p_\beta|f_{\mathbf{p}}|^2\\ &=\frac{H^2}{2}\frac{1}{(2\pi)^3}\int \frac{d^3\mathbf{p}}{p^3}p_\alpha p_\beta e^{i\mathbf{p}\cdot l}\\ &\quad+\frac{1}{2S^2}\frac{1}{(2\pi)^3}\int d^3\mathbf{p}\frac{p_\alpha p_\beta}{p}e^{i\mathbf{p}\cdot l} , \end{aligned} \tag{B10}$$

and

$$J_\alpha=\int \frac{d^3p}{(2\pi)^3}\dot{f}_{\mathbf{p}}f^*_{\mathbf{p}}p_\alpha e^{i\mathbf{p}\cdot l} \tag{B11}$$

$$=-\frac{H}{S^2}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{p_\alpha}{p}e^{i\mathbf{p}\cdot l}-\frac{i}{2S^3}\int \frac{d^3p}{(2\pi)^3}p_\alpha e^{i\mathbf{p}\cdot l} . \tag{B12}$$

Here $\alpha=1, 2, 3$, and the repeated index is summed using the flat-space metric, e.g., $J_\alpha J^\alpha=J_1^2+J_2^2+J_3^2$, etc. In arriving at (B10) and (B12) we have used the expression (11) for $f_{\mathbf{p}}$.

To evaluate these expressions we note that $p_\alpha e^{i\mathbf{p}\cdot l}$, etc., can be obtained from $e^{i\mathbf{p}\cdot l}$ by differentiation with respect to l^α. As it stands such procedures are not well defined because of the oscillatory nature of the integrands. However, this can be taken care of with a suitable "$i\epsilon$ prescription"; i.e., we will introduce inside each integral a convergence factor $\exp(-\epsilon|\mathbf{p}|)$. At the end of the calculation we will take the limit of $\epsilon\to 0$. This procedure allows us to calculate in a well-defined manner all the terms, except the first term in $I_{\alpha\beta}$. However in $I_{\alpha\beta}$ the first term does not make any contribution. To see this note that this term can be written (formally) as

$$-\frac{H^2}{2}\frac{1}{(2\pi)^3}\frac{\partial}{\partial l^\alpha}\frac{\partial}{\partial l^\beta}\int \frac{d^3\mathbf{p}}{p^3}e^{i\mathbf{p}\cdot l} . \tag{B13}$$

But the integral in this expression is independent of l:

$$\begin{aligned} Q=\int \frac{d^3\mathbf{p}}{|\mathbf{p}|^3}e^{i\mathbf{p}\cdot l}&=2\pi\int_0^\infty \frac{dp}{p}\int_{-1}^{+1}d\mu\, e^{ipl\mu}\\ &=4\pi\int_0^\infty d\alpha\frac{\sin\alpha}{\alpha^2} . \end{aligned} \tag{B14}$$

Since Q is independent of l its derivative with respect to l^α vanishes. Therefore this term can be dropped. (This same result is obtained if we evaluate this, formally divergent, quantity with a cutoff and then set the cutoff to zero.) Using these facts we can rewrite (B10) and (B12) as

$$\begin{aligned} J_\alpha=&-\frac{H}{2S^2}\frac{1}{(2\pi)^3}\left[-i\frac{\partial}{\partial l^\alpha}\right]\int \frac{d^3p}{|\mathbf{p}|}e^{i\mathbf{p}\cdot l-\epsilon|\mathbf{p}|}\\ &-\frac{i}{2S^3}\frac{1}{(2\pi)^3}\left[-i\frac{\partial}{\partial l^\alpha}\right]\int d^3\mathbf{p}\, e^{i\mathbf{p}\cdot l-\epsilon|\mathbf{p}|} , \end{aligned} \tag{B15}$$

$$I_{\alpha\beta}=\frac{1}{2S^2}\frac{1}{(2\pi)^3}\left[-\frac{\partial}{\partial l^\alpha}\frac{\partial}{\partial l^\beta}\right]\int \frac{d^3\mathbf{p}}{|\mathbf{p}|}e^{i\mathbf{p}\cdot l-\epsilon|\mathbf{p}|} , \tag{B16}$$

and

$$T_1=\left[\frac{1}{2S^4}\frac{1}{(2\pi)^3}\int d^3\mathbf{p}|\mathbf{p}|e^{i\mathbf{p}\cdot l-\epsilon|\mathbf{p}|}\right]^2 . \tag{B17}$$

To evaluate T_1, J_α, $I_{\alpha\beta}$ we need to compute the following integrals:

$$\begin{aligned} A_{(0)}&\equiv\int d^3\mathbf{p}\, e^{i\mathbf{p}\cdot l-\epsilon|\mathbf{p}|}\\ &=\frac{2\pi}{i|l|}\int_0^\infty p\, dp\, e^{-\epsilon p}(e^{ipl}-e^{-ipl}) , \end{aligned} \tag{B18}$$

$$\begin{aligned} A_{(1)}&\equiv\int d^3\mathbf{p}\, e^{i\mathbf{p}\cdot l-\epsilon|\mathbf{p}|}|\mathbf{p}|\\ &=\frac{2\pi}{i|l|}\int_0^\infty p^2dp\, e^{-\epsilon p}(e^{ipl}-e^{-ipl}) , \end{aligned} \tag{B19}$$

$$\begin{aligned} A_{(-1)}&\equiv\int d^3\mathbf{p}\, e^{i\mathbf{p}\cdot l-\epsilon|\mathbf{p}|}|\mathbf{p}|^{-1}\\ &=\frac{2\pi}{i|l|}\int_0^\infty dp\, e^{-\epsilon p}(e^{ipl}-e^{-ipl}) . \end{aligned} \tag{B20}$$

Of these, we actually have to calculate only $A_{(-1)}$; the values of $A_{(0)}, A_{(1)}$ can be found from $A_{(-1)}$ by differentiating with respect to ϵ. By straightforward integration we, therefore, get

$$A_{(-1)}=\frac{4\pi}{l^2+\epsilon^2} \tag{B21}$$

so that

$$A_{(0)}=-\frac{\partial}{\partial\epsilon}A_{(-1)}=\frac{8\pi\epsilon}{(l^2+\epsilon^2)^2} \tag{B22}$$

and

$$A_{(1)}=\frac{\partial^2}{\partial\epsilon^2}A_{(-1)}=-\frac{8\pi(l^2-3\epsilon^2)}{(l^2+\epsilon^2)^2}\ . \tag{B23}$$

We now evaluate J_α, $J_{\alpha\beta}$, etc., by differentiating $A_{(i)}$ with respect to l^α, etc., and substitute these results into (B15) and (B16). This gives

$$T_1=\frac{1}{4\pi^4}\frac{1}{S^8}\frac{(l^2-3\epsilon^2)^2}{(l^2+\epsilon^2)^6}\ , \tag{B24}$$

$$T_2=\frac{1}{4\pi^4}\frac{1}{S^8}\frac{11l^4-2l^2\epsilon^2+3\epsilon^4}{(l^2+\epsilon^2)^6}\ , \tag{B25}$$

$$T_3=\frac{1}{4\pi^4}\frac{H^2}{S^6}\frac{l^2}{(l^2+\epsilon^2)^4}\left[1-\frac{16\epsilon^2}{H^2S^2(l^2+\epsilon^2)^2}-\frac{8i\epsilon}{HS(l^2+\epsilon^2)}\right]\ , \tag{B26}$$

$$T_3(l,t)=T_3^*(-l,t)\ . \tag{B27}$$

This expression, of course, remains finite when the $\epsilon\to 0$ limit is taken. Taking this limit, we get the final answer:

$$C(l,t)=\sum_{i=1}^{4}T_i=\frac{3}{\pi^4(Sl)^8}+\frac{H^2}{2\pi^4(Sl)^6}\ . \tag{B28}$$

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