$1^{--}\rightarrow 0^{-+}$ meson radiative and pionic transitions and mass splittings

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Radiative and pionic transitions between the ground-state pure $q\bar{q}$ vector and pseudoscalar mesons compatible with observed mass splittings are studied.

The radiative transition between ground-state $q\bar{q}$ mesons is one of the most fundamental processes of hadron physics. In the quark model, it is usually approached in terms of direct magnetic moments of quarks. When this simple model fails to work, inclusion of quark anomalous magnetic moments is also argued. Then however, the intuitive virtue of the model diminishes considerably.

In this Brief Report, we discuss another very viable approach which leads to simple predictions *(expressible in terms of a single parameter)* of the radiative transitions involving pure $q\bar{q}$ ground-state mesons ρ-π, K^*-K, D^*-D, D_s^*-D_s, B_d^*-B_d, ..., explicitly compatible with observed mass splitting patterns. D^* branching ratios in good agreement with experiment and also total widths are predicted. The singlet electromagnetic current is shown to be essential for understanding radiative transitions. We also argue the existence of general selection rules for particular single photon and single pion emission transitions between radially excited and the ground-state $q\bar{q}$ mesons.

Our theoretical framework consists of the quantum-chromodynamics (QCD) Lagrangian with quark mass and electroweak terms and the successful idea of Gell-Mann[1] that the dynamics of observable hadrons is severely constrained by the presence of equal-time commutation relations (ETCR's), derived from the fundamental Lagrangian. For these constraint algebras, we use[2] the ETCR's involving the generators (the vector and axial-vector changes V_α and A_α) of the underlying symmetry groups of QCD. The chiral $SU_F(N)_L\otimes SU_F(N)_R$ charge algebras, $[V_i,V_j]=[A_i,A_j]=if_{ijk}V_k$, and $[V_i,A_j]=if_{ijk}A_k$ are valid,[1] even though the underlying symmetries are broken. Moreover, the *exotic* ETCR's involving $\dot{V}_\alpha$ also hold, i.e., $[\dot{V}_\alpha,V_\beta]=[\dot{V}_\alpha,A_\beta]=0$, where $\dot{V}_\alpha=i[H,V_\alpha]$, where H is the total QCD Hamiltonian and (α,β) is an *exotic* combination[2] of flavor indices.

We write the electromagnetic current $J_\mu^{\rm EM}$ in $SU_F(4)$ $(u,d,s,c$ quark system) as

$$j_\mu^{\rm EM}=j_\mu^3+(1/\sqrt{3})j_\mu^8-(\tfrac{2}{3})^{1/2}j_\mu^{15}+(\sqrt{2}/3)j_\mu^0\ ,$$

while in $SU_F(5)$ (adding the b quark)

$$j_\mu^{\rm EM}=j_\mu^3+(1/\sqrt{3})j_\mu^8-(\tfrac{2}{3})^{1/2}j_\mu^{15}+(\tfrac{2}{5})^{1/2}j_\mu^{24}+[\sqrt{(2/5)}/3]j_\mu^0\ .$$

In general, we write $j_\mu^{\rm EM}=j_\mu^V+j_\mu^S$ where $j_\mu^V\equiv j_\mu^3$ and j_μ^S denotes the isoscalar current. In the presence of the $SU_F(N)$-singlet current j_μ^0, the relevant constraint algebras available are

$$[V_{K^0},j_\mu^{\rm EM}]=[V_{D^0},j_\mu^{\rm EM}]=[V_{D_s^+},j_\mu^V]=[V_{B_d^0},j_\mu^{\rm EM}]=\cdots=0\ , \qquad (1)$$

$$[\dot{V}_{K^0},j_\mu^{\rm EM}]=[\dot{V}_{D^0},j_\mu^{\rm EM}]=[\dot{V}_{D_s^+},j_\mu^V]=[\dot{V}_{B_d^0},j_\mu^{\rm EM}]=\cdots=0\ , \qquad (2)$$

where $V_{K^0}=V_6+iV_7$, $V_{D^0}=V_9-iV_{10}$, $V_{D_s^+}=V_{13}-iV_{14}$, etc.

To extract information from these constraint algebras, we use the notion of asymptotic $SU_F(4)$ symmetry[2] whose key ingredient is the behavior of the charge V_α when acting on a *physical* state with infinite momentum $\mathbf{k}$. Namely, the annihilation operator $a_\alpha(\mathbf{k},\lambda)$ of the *on-mass-shell* physical hadron with momentum $\mathbf{k}$, helicity λ, and physical $SU_F(N)$ index α $(\alpha=\pi,K,D,\ldots)$ is assumed to maintain its linearity [including, however, $SU_F(N)$ particle mixing] under flavor transformations generated by V_α but *only* in the asymptotic limit $\mathbf{k}\rightarrow\infty$. All nonlinear terms vanish like $|\mathbf{k}|^{-(1+\epsilon)}$ $(\epsilon>0)$ as $\mathbf{k}\rightarrow\infty$. (Solvable models generally yield $\epsilon=1$.) Therefore, $a_\alpha(\mathbf{k},\lambda)$ is still *linearly* related to the $SU_F(N)$ *representation* operator $a_j(\mathbf{k},\lambda)$ for $\mathbf{k}\rightarrow\infty$, i.e.,

$$|\alpha,\mathbf{k},\lambda\rangle=\sum_j C_{\alpha j}|j,\mathbf{k},\lambda\rangle,\quad \mathbf{k}\rightarrow\infty\ . \qquad (3)$$

The matrix $C_{\alpha j}$ involves $SU_F(N)$ mixing parameters and hence can be *constrained directly by the constraint algebras without introducing an ad hoc mass matrix.*

We now sandwich one of the ETCR's of Eq. (1), $[V_{K^0},j_\mu^{\rm EM}(0)]=0$ between the states $\langle D_s^+(\mathbf{p}')|$ and $|D^{*+}(\mathbf{p},\lambda=1)\rangle$ with $\mathbf{p}'\rightarrow\infty$ and $\mathbf{p}\rightarrow\infty$. We obtain

$$(D_s^+D^+)\langle D^+(\mathbf{p}')|j_\mu^{\rm EM}|D^{*+}(\mathbf{p})\rangle-\langle D_s^+(\mathbf{p}')|j_\mu^{\rm EM}|D_s^{*+}(\mathbf{p})\rangle(D_s^{*+}D^{*+})+\sum_j (D_s^+D_j^+)\langle D_j^+(\mathbf{p}')|j_\mu^{\rm EM}|D^{*+}(\mathbf{p})\rangle$$
$$-\sum_j \langle D_s^+(\mathbf{p}')|j_\mu^{\rm EM}|D_{sj}^{*+}(\mathbf{p}')\rangle(D_{sj}^{*+}D^{*+})=0\ , \quad (4)$$

where $j=1,2,\ldots,$ denote the first, second $,\ldots,$ radially excited D and D^* states and $(D_s^+D^+)\equiv\langle D^+(\mathbf{p}')|V_{K_0}|D^+(\mathbf{p}')\rangle$ with $\mathbf{p}'\to\infty$, for example. Among the complete set of on-mass-shell intermediate states inserted, only the single-particle 0^{-+} and 1^{--} states survive according to asymptotic symmetry. The third and fourth terms in Eq. (4) express the small leakage—in the asymptotic limit—to the radially excited states. If we *neglect* this effect of intermultiplet mixing we obtain, since $(D_s^+D^+)=(D_s^{*+}D^*)$ in our asymptotic limit,

$$\langle D^+(\mathbf{p}')|j_\mu^{\rm EM}(0)|D^{*+}(\mathbf{p})\rangle=\langle D_s^+(\mathbf{p}')|j_\mu^{\rm EM}(0)|D_s^{*+}(\mathbf{p})\rangle\ ,$$
$$\mathbf{p}\to\infty \text{ and } \mathbf{p}'\to\infty\ . \quad (5)$$

In *exact* flavor symmetry, Eq. (5) holds for *all* values of $\mathbf{p}$ and $\mathbf{p}'$. In *asymptotic* flavor symmetry Eq. (5) holds *only* in our asymptotic limit while also neglecting intermultiplet mixing.

We define the $1^{--}\to0^{-+}\gamma$ coupling constant by

$$\langle P(\mathbf{p}')|j_\mu^{\rm EM}|V(\mathbf{p},\lambda=1)\rangle=g_{VP\gamma}\epsilon_{\mu\nu\rho\sigma}\epsilon^\nu(\mathbf{p},\lambda=1)p^\rho p'^\sigma\ ,$$

where $\epsilon^\nu(\mathbf{p})$ is the polarization four-vector of the 1^{--} meson and we take the limit $\mathbf{p}\to\infty$ and $\mathbf{p}'\to\infty$ but $q^2=(p-p')^2\to0$. Let us take $\mathbf{p}'$ along the Z axis and $\mathbf{p}$ in the Z-X plane (i.e., $p_y=p_y'=0$). Then the implication of the sum rule Eq. (5) is as follows. For $\mu=0$ or 3, a nontrivial sum rule is obtained by using a noncollinear limit[2] (use $p_z=\lambda|\mathbf{p}'|$ and let $\lambda\to1$ and $p_x\to0$), $g_{D^{*+}D^+\gamma}(0)=g_{D_s^{*+}D_s^+\gamma}(0)$. Let us simplify the notation: $\langle D^+\rangle\equiv g_{D^{*+}D^+\gamma}(0)$, $\langle D_s^+\rangle\equiv g_{D_s^{*+}D_s^+\gamma}(0)$, $\langle K^+\rangle\equiv g_{K^{*+}K^+\gamma}$, $\langle\rho^+\pi^+\rangle\equiv g_{\rho^+\pi^+\gamma},\ldots,$ etc.; thus,

$$\langle D^+\rangle=\langle D_s^+\rangle\ . \quad (6)$$

For $\mu=1$ or 2, the collinear limit $(p_x=0)$ yields another nontrivial sum rule:

$$[(D^{*+})^2-(D^+)^2]\langle D^+\rangle=[(D_s^{*+})^2-(D_s^+)^2]\langle D_s^+\rangle\ , \quad (7)$$

where $(D_s^+)^2\equiv$ (mass of $D_s^+)^2$, etc. Equations (6) and (7) require the mass relation $(D^{*+})^2-(D^+)^2=(D_s^{*+})^2-(D_s^+)^2$. Repeating the same procedure between the asymptotic states $\langle K^+|$ and $|\rho^+\rangle$, $\langle D^+|$ and $|\rho^+\rangle$, $\langle D_s^+|$ and $|K^{*+}\rangle$, $\langle B_u^+|$ and $|\rho^+\rangle,\ldots,$ using appropriate ETCR's given by Eq. (1), two very simple predictions emerge:

$$x\equiv\langle\rho^+\pi^+\rangle=\langle K^+\rangle=\langle D^+\rangle=\langle D_s^+\rangle$$
$$=\langle B_u^+\rangle=\langle B_c^+\rangle=\cdots\ , \quad (8)$$

$$(\rho^+)^2-(\pi^+)^2=(K^{*+})^2-(K^+)^2=(D^{*+})^2-(D^+)^2$$
$$=(D_s^{*+})^2-(D_s^+)^2$$
$$=(B_u^{*+})^2-(B_u^+)^2$$
$$=\cdots\ . \quad (9)$$

Consistently, one can also derive the same sum rules Eqs. (8) and (9) by using both algebras, Eqs. (1) and (2), with $\mu=0$.

At the present time, perturbative QCD has not been made to yield in a precise *quantitative* way the experimentally verified relation $(B_u^*)^2-(B_u)^2=(D_s^*)^2-(D_s)^2=(D^*)^2-(D^+)^2\cong0.55$ GeV^2, which involves particles consisting of heavy and light quarks. The relation may however, be understood, qualitatively, in the context of the Schrödinger equation and a long-range confining potential. Then it is found that $M^2(^3S_0)-M^2(^1S_0)\simeq$const. For relations such as $(K^*)^2-K^2=\rho^2-\pi^2$ and $\psi^2-\eta_c^2\simeq(D^*)^2-D^2$ (good to about 20%) which involve pairs of heavy or light quarks, *even the qualitative understanding is not good.*[3] Historically, $(K^*)^2-K^2=\rho^2-\pi^2$ was first noticed in works employing SU(6) symmetry.[4] Actually, the existence of the mass relation Eq. (9), except for $(D_s^{*+})^2-(D_s^+)^2=(D^{*+})^2-(D^+)^2$ derived above, has been known[5] in the present theoretical framework long before experimental confirmation, from the exotic constraint algebras,

$$[V_\alpha,A_{\pi^-}]=[\dot V_\alpha,A_{\pi^-}]=0,\quad \alpha=K^0,D^0,B_d^0,\ldots\ . \quad (10)$$

For example, corresponding to Eq. (4), we obtain, from $[V_{D^0},A_{\pi^-}]=0$ [note that $(D^0\pi^0)\equiv\langle D^0|V_{D^0}|\pi^0(\mathbf{p})\rangle$, etc. with $\mathbf{p}\to\infty$],

$$(D^0\pi^0)\langle\pi^0|A_{\pi^-}|\rho^+(\mathbf{p})\rangle-\langle D^0|A_{\pi^-}|D^{*+}(\mathbf{p})\rangle(D^{*+}\rho^+)+\sum_j(D^0\pi_j^0)\langle\pi_j^0|A_{\pi^-}|\rho^+(\mathbf{p})\rangle$$
$$-\sum_j\langle D^0|A_{\pi^-}|D_j^{*+}(\mathbf{p})\rangle(D_j^{*+}\rho^+)=0\ . \quad (11)$$

If we again neglect, as in Eq. (4), the leakage to the radially excited states, we obtain [since $(\sqrt{2})(D^0\pi^0)=(D^{*+}\rho^+)$]

$$(1/\sqrt{2})\langle \pi^0 | A_{\pi^-} | \rho^+(\mathbf{p})\rangle = \langle D^0 | A_{\pi^-} | D^{*+}(\mathbf{p})\rangle \ .$$

When combined with the sum rule obtained from $[\dot{V}_{D^0}, A_{\pi^-}]=0$ and using the same approximation as before, we find[5] that $(D^{*+})^2-(\rho^+)^2=(D^0)^2-(\pi^0)^2$. In this way the constraint algebras Eq. (10) yield again two simple predictions ($\mathbf{p}\to\infty$),

$$(1/\sqrt{2})\langle \pi^0 | A_{\pi^-} | \rho^+(\mathbf{p})\rangle = -\langle K^0 | A_{\pi^-} | K^{*+}(\mathbf{p})\rangle = \langle D^0 | A_{\pi^-} | (D^{*+})^2\rangle = \cdots \ , \quad (12)$$

$$\rho^2-\pi^2=(K^*)^2-K^2=(D^*)^2-D^2=\cdots \ . \quad (13)$$

Equations (9) and (13) are remarkably consistent with the presence of constraint algebras and asymptotic $SU_F(N)$ symmetry. One can convert the sum rules [Eq. (12)] into sum rules involving pion-emission coupling constants by using PCAC (partial conservation of axial-vector current) in the $\mathbf{p}\to\infty$ limit:

$$\sqrt{1/2}(1/\rho)g_{\rho^+\pi^+\pi^0} = -(1/K^*)g_{K^{*+}K^0\pi^+} = (1/D^*)g_{D^{*+}D^0\pi^+} = \cdots \ . \quad (14)$$

Here a departure from the prediction of exact flavor symmetry is evident (i.e., mass factors arise) unlike the sum rule Eq. (8). The coupling constants g involve only the small *hard-pion* extrapolation $q^2=\pi^2\to 0$.

In addition to Eq. (8), the parametrization of the coupling constants $g_{VP\gamma}$ in broken $SU_F(4)$ compatible with the constraint algebras Eqs. (1) and (2) is

$$\langle K^0\rangle = -2\langle K^+\rangle_3 + x, \quad \langle D^0\rangle = 2\langle K^+\rangle_3 + x \ , \quad \langle D_s^+\rangle_3 = \langle \rho^+\pi^+\rangle = 0 \ , \quad (15)$$

where the subscript 3 refers to $j^3_\mu \equiv j^V_\mu$. We note that in deriving Eqs. (8) and (15) we *do not* assume $SU_I(2)$ symmetry, i.e., $m_u \neq m_d$. We particularly notice that $\langle \rho^+\pi^+\rangle, \langle K^+\rangle, \langle D^+\rangle, \langle D_s^+\rangle, \ldots$ are described solely in terms of x which is given by the asymptotic matrix element of the $SU_F(4)$-singlet current j^0_μ, which involves $b,t,\ldots,$ quarks, whereas $\langle K^0\rangle$ and $\langle D^0\rangle$ involve both x and $\langle K^+\rangle_3$.

We first show that one of the predictions of Eq. (8), $\langle \rho^+\pi^+\rangle = \langle K^+\rangle$ and its well-known mass counterpart $\rho^2-\pi^2=(K^*)^2-K^2$, is in agreement with recent experiment. Indeed we predict that

$$\Gamma_\gamma(\rho^+)/\Gamma_\gamma(K^{*+}) = \{(\rho^2-\pi^2)/[(K^*)^2-K^2](K^*/\rho)\}^3 \times (\langle \rho^+\pi^+\rangle/\langle K^+\rangle)^2 = (K^*/\rho)^3 \ .$$

If we choose[6,7] $\Gamma_\gamma(K^{*+})=(51.1\pm 5.2)$ keV, we predict $\Gamma_\gamma(\rho^+)\simeq 84\pm 9$ keV, while $\Gamma_\gamma(\rho^+)_{\rm expt}$ lies in the range of 60–90 keV. A very recent experiment[8] gives $\Gamma_\gamma(\rho^+)\simeq 81\pm 4\pm 4$ keV.

Now from Eq. (15), we obtain $\langle K^0\rangle/\langle K^+\rangle = 1-2\langle K^+\rangle_3/x$. If we use the $SU_F(3)$ current $j^{\rm EM}_\mu = j^3_\mu + (1/\sqrt{3})j^8_\mu$, then $\langle K^0\rangle/\langle K^+\rangle$ is fixed to the $SU_F(3)$ value of -2. However, since $c,b,\ldots$ quarks exist, $j^{\rm EM}_\mu$ must contain an $SU_F(3)$-singlet current. Indeed, the experimental ratio[6] of $\Gamma_\gamma(K^{*0})/\Gamma_\gamma(K^{*+})$ apparently "knows" this and yields $\langle K^0\rangle/\langle K^+\rangle = \pm(1.5\pm 0.1)$. Our method is therefore subtly different from that of the quark model. Quark masses do not enter into our argument but a correct treatment of the $SU_F(N)$-singlet current becomes crucial. A theoretically natural choice of sign of $\langle K^0\rangle/\langle K^+\rangle$ is found to be $\langle K^0\rangle/\langle K^+\rangle \simeq -(1.5\pm 0.1)$. Then $\langle D^0\rangle/\langle D^+\rangle = 2-\langle K^0\rangle/\langle K^+\rangle \simeq +(3.5\pm 0.1)$. Therefore, once we input the value of $\langle K^0\rangle/\langle K^+\rangle$, we predict that $\Gamma_\gamma(D^{*+})=(K^{*+}/D^{*+})^3\Gamma_\gamma(K^{*+})$ and

$$\Gamma_\gamma(D^{*0})=(K^{*+}/D^{*0})^3(2-\langle K^0\rangle/\langle K^+\rangle)^2\Gamma_\gamma(K^{*+}) \ .$$

Our sum rule, Eq. (12), also allows us to compute the rates of the $D^*\to D\pi$ decays from $\Gamma(K^*\to K\pi)$. In this Brief Report we choose $K^*\to K\pi$ over $\rho\to\pi\pi$ in order to minimize the effect of the finite widths of the 1^{--} mesons. In Tables I and II we display predictions of the branching ratios of D^{*+} and D^{*0} and also their total widths. We emphasize the fact that the results are obtained *solely by asymptotic $SU_F(4)$ rotation* from K^{*+} and K^{*0}. The agreement with experiment[8] is quite good. Only $B(\gamma D^+)$ is perhaps in question but, on the other hand, more precise data are needed. Our results on $D^*\to D\gamma$ are in general similar to those of Brekke and Rosner[9] based on a more phenomenological approach. We also predict that $\Gamma(D_s^{*+}\to D_s^+\gamma)\simeq 3.9$ keV, $\Gamma(B_u^{*+}\to B_u^+\gamma)\simeq 0.24$ keV, and $\Gamma(B_d^{*0}\to B_d^0\gamma)\simeq 0.56$ keV.

We have neglected in Eqs. (5) and (12) the leakage to radially excited states in the asymptotic limit. Although the mixings and various matrix elements of $j^{\rm EM}_\mu$ and A_π

TABLE I. D^{*+} experimental and theoretical branching ratios.

Group	$B(\gamma D^+)$(%)	$B(\pi^0 D^+)$(%)	$B(\pi^+ D^0)$(%)
Mark III[a]	17±5±5	26±2±2	57±4±4
Mark II[a]	22±12	34±7	44±10
SLAC-LBL[a]	8±7	28±9	64±11
Theory[b]	3.4±0.4	31±1	66±1

[a]See Ref. 8.
[b]Theory: $\Gamma(D^{*+}\to\text{all})=133\pm 2$ keV.

TABLE II. D^{*0} experimental and theoretical branching ratios.

Group	$B(\gamma D^0)$(%)	$B(\pi^0 D^0)$(%)
Mark III[a]	37±8±8	63±8±8
Mark II[a]	47±12	53±12
SLAC-LBL[a]	45±15	55±15
JADE[a]	53±9±10	47±9±10
HRS[a]	47±23	
Theory[b]	47±6	53±3

[a]See Ref. 8.
[b]Theory: $\Gamma(D^{*0}\to\text{all})=117\pm 6$ keV.

may arrange themselves to make the effect negligible, we wish to point out a much simpler and attractive possibility which keeps our predictions intact and which already has some support from experiment, namely, that

$$\langle D_j^+(\mathbf{p}')|j_\mu^{\rm EM}|D^*(\mathbf{p})\rangle=\langle D_s^+j(\mathbf{p}')|j_\mu^{\rm EM}|D_s^{*+}(\mathbf{p})\rangle = \cdots = 0\,, \tag{16}$$

$$\langle \pi_j^0|A_{\pi^-}|\rho^+(\mathbf{p})\rangle=\langle D_j^0|A_{\pi^-}|D^*(\mathbf{p})\rangle=\cdots=0\,. \tag{17}$$

Thus, $\rho_j\to\pi\gamma, K_j^*\to K\gamma,\ldots,\rho_j\to\pi\pi, K_j^*\to K\pi,\ldots,$ $(j=1,2,\ldots)$ may be forbidden (or small). We recall that in the present theoretical framework, the sum rules obtained from the constraint algebras, Eqs. (1), (2), and (10), imply[2] that the mass degeneracy $\rho=\omega$ immediately leads to the vanishing of the $\phi\to\pi\gamma$ and $\phi\to\rho\pi$ couplings. In that connection, a recent analysis[10] of $\tau\to\pi\pi\nu_\tau$ is very interesting. It suggests that $\rho(1600)\to 2\pi$ is probably very small. Another (more indirect) experimental indication[6] is that the couplings of $\psi(3685)\to\rho\pi$ and $K^*\bar{K}$ are apparently much smaller than those of $J/\psi\to\rho\pi$ and $K^*\bar{K}$. In any case, the presence of such dynamical selection rules will drastically simplify hadron spectroscopy if they are confirmed.

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