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Cross section of monopole-induced Skyrmion decay

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The inclusive process of monopole catalysis of proton decay ($M+p\rightarrow M+e^{+}+$pions) is analyzed within the chiral-soliton (Skyrmion) approach proposed by Callan and Witten. Monopole-induced baryon decay is described in terms of an effective radial (1+1)-dimensional bosonic action for a coupled scalar chiral kink field (baryon) and scalar sine-Gordon field (lepton). We construct the initial data appropriate to a Skyrmion impinging on the monopole with fixed impact parameter and velocity β and integrate numerically the classical field equations in terms of a spatially discretized version of the action. The results show that baryon-number violation at the monopole suffers practically no suppression at the classical level. In particular, there is no sign of a crossover, in terms of some critical impact parameter, from a positron to a proton in the final state. The numerical calculations predict a time scale for baryon-number violation which supports a catalysis cross section of standard size $\sigma\simeq 1$ mb$/\beta$.

I. INTRODUCTION

One of the most fascinating implications of grand unification is the ability of the monopoles of grand unified theories to catalyze certain baryon-number-violating processes with reaction cross sections eventually attaining strong-interaction scales, $\sigma_{\rm cat}(M+P\rightarrow M+e^{+}+\text{pions})\sim 1/M_N^2$ ($M_N=$nucleon mass). This is the so-called Callan-Rubakov effect. Since the initial pioneering papers of Rubakov[1] and Callan,[2] to which one should perhaps associate some precursor works,[3] considerable literature has developed on this subject. The motivations were to justify the initial approximations,[4] to clarify the physical interpretation of the effect,[5] and to explore various phenomenological issues[6] (the grand unification scheme, the observed monopole flux, . . .) This list of references is by no means exhaustive; fortunately, several comprehensive reviews exist on monopole physics.[7]

The possibility of unsuppressed monopole-induced baryon decay is linked to the subtle physics of the monopole-fermion system. It now appears clearly that this involves three basic aspects. One is the existence of a lowest angular momentum wave in which the fermions suffer no centrifugal barrier. The second is the peculiar boundary conditions at the monopole location which couple quarks of different species (color, flavor, chirality) with each other and with leptons. This constraint on the fermion field amplitudes is, however, imposed at the very short distances corresponding to the monopole core $[r_X\simeq M_X^{-1}\simeq(10^{-15}\ \text{GeV})^{-1}]$ and represents therefore only a necessary condition for baryon- or lepton-charge-violating scattering to proceed unsuppressed by small-coupling-constant effects (X-boson and Higgs-meson exchange, instanton tunneling). Finally, the third aspect is the coupling via the axial anomaly of fermions to the dyonic degrees of freedom of the monopole associated with the long-range (electro, chromo, and weak) gauge fields. This interaction leads, through the well-understood mechanisms of (1+1)-dimensional field theories, to a spontaneous breakdown of the global (baryon, lepton, chirality) symmetries, which is reflected by the presence of multifermionic condensates in the monopole sector. The physical picture is that a group of initial quarks and leptons propagating in the lowest angular momentum wave is able to penetrate unsuppressed right through to the monopole core, thanks to the absence of a centrifugal barrier and to the screening by multifermion condensates of the monopole dyonic charges. The passage through the monopole core permits then a transfer of charges between fermions which conserves the gauged charges, via dyonic charge leakage, but violates the global charges. Thus, the outgoing fermions may evolve to a final state with changed global charges.

Experimentally, the catalysis effect is linked to one other important aspect of grand unification: namely, the primordial monopole flux ϕ_M. The existing catalysis data give experimental information which involves both ϕ_M and $\sigma_{\rm cat}$ (Refs. 7 and 8). The situation at present is characterized by the existence of a large gap between the terrestrial and astrophysical (neutron stars) upper bounds in the product $\phi_M\sigma_{\rm cat}$; roughly speaking, $(\phi_M\sigma_{\rm cat}\beta/\text{mb})\leq 7\times10^{-14}$ and 10^{-22} s^{-1}sr^{-1}, where β is the proton-monopole relative velocity. The Parker limit, $\phi_M\leq 10^{-16}(M_{\rm mon}/10^{17}\ \text{GeV})$ cm^{-2}s^{-1}sr^{-1} ($M_{\rm mon}$ =monopole mass), and some of the existing experimental limits[8] $\phi_M\leq 2.5\times10^{-15}$ and 6×10^{-12} cm^{-2}s^{-1}sr^{-1}, are compatible with $\sigma_{\rm cat}\simeq X/\beta$ mb where, roughly, $X\simeq 10^{-2}-10^{+2}$ and $10^{-10}-10^{-6}$, respectively. Thus, the continued lack of observation of catalysis manifestations and the persistence of the above gap could eventually lead one[9] to doubt the detectability of the Callan-Rubakov effect.

Returning to the theoretical discussion, one of the major unknowns at present seems to be the interference effect of the strong interactions: namely, quark confinement. In order to have $\sigma_{\rm cat}\sim M_N^{-2}$, rather than M_X^{-2} or M_W^{-2}, it is necessary that the anomaly-induced interactions which screen the monopole long-range charges persist down to the hadronic (Fermi) scale. Although the actual realization of such a situation appears rather plausible, it has never been convincingly established and so one lacks an uncontroversial estimate of the size of the catalysis cross section. Nevertheless, this possibility has often been invoked to set qualitative upper-limit estimates, such as $\sigma_{\rm cat}\beta \simeq 1$ mb, where the dependence on velocity is appropriate to exothermic reactions at threshold, or $\sigma_{\rm cat}\beta^2 \simeq \pi/2M_N^2$, appropriate to the free-fall cross section[10] or also half the unitarity limit.

To reach a sharper estimate, Craigie[7] and Bernreuther and Craigie[11] have considered the effect of folding the effective Rubakov multifermionic interaction with the momentum distribution of nonrelativistic constituent quarks inside the nucleon. Ma and Tang[12] have also discussed the reduction effect on the free-fall cross section due to quark confinement inside a bag. It seems, however, that these quark-model descriptions truncate somehow the fermion-monopole dynamics: firstly, through an implicit assumption on the range of the multifermion interaction; secondly, because of the absence of reference to the boundary conditions on the fermion field amplitudes.

Both of these difficulties are avoided to some extent in the current-algebra picture. Here, the nonconservation of the global quantum numbers in the monopole sector is associated with the Wess-Zumino consistency conditions for the anomalous commutator of the vector and axial-vector currents and is realized explicitly via the anomalous effective interaction in the QCD Lagrangian. The approach deals directly with hadrons and makes explicit use of the hadron-lepton interactions induced by coupling to the dyonic degrees of freedom as well as the boundary conditions at the monopole core. Thus Nair[13] has considered the case of point nucleons and established the catalysis effect, via the familiar cluster argument, in terms of the excitation of a mixed nucleon-electron condensate around the monopole. Craigie, Nahm, and Rubakov[14] showed that the color gauge dynamics of the quarks in the field of a monopole amounts to a radial $(1+1)$-dimensional chromodynamics (QCD_2) with a distance-dependent coupling constant, which hence allows for a dual nonlinear σ-model description. Of the various existing current-algebra approaches, one which ranks best in terms of predictive power and simplicity of implementation is perhaps the chiral-soliton (Skyrmion) approach of Callan and Witten.[15] The baryon-lepton-monopole system is described here in terms of a radial $(1+1)$-dimensional field theory for two real scalar fields coupled through a boundary condition at the monopole. The catalysis effect is realized as a solution to the classical field equations with initial conditions appropriate to a Skyrmion impinging on the monopole with fixed impact parameter b and velocity β, which evolves in time, for b less than some critical impact $b_{\rm crit}$, to an outgoing positron and mesonic radiation. In their paper,[15] Callan and Witten exposed the basic elements of the Skyrmion approach but did not really test its workability. Our purpose in this paper is precisely to fill in this gap in the hope of reaching a semiquantitative estimate of the catalysis cross section.

In Sec. II we recall in large outline the Callan-Witten approach. A slightly different presentation from theirs is followed which hopefully should cast some further light on the problem. The generalization to the three-flavor case and to the case of effective Lagrangians incorporating the vector mesons is discussed in Appendix B. In Sec. III we discuss the construction of the initial data for the radial kink field in terms of the Skyrmion profile. In Sec. IV we consider a discretized version of the classical field theory as a preparation to the numerical integration of the field equations whose results are presented in Sec. V. The interpretation and discussion of the results are offered in Sec. VI and the conclusions in Sec. VII.

II. THE CHIRAL-SOLITON APPROACH

Let us first recall some of the characteristic properties of the grand unification monopoles.[7] The existing candidates for grand-unified-theory (GUT) groups satisfy with relative ease the necessary conditions for the appearance of topological 't Hooft-Polyakov-type monopoles: nontrivial $\pi_2(H_n/H_{n-1})=\pi_1(H_{n-1})/\pi_1(H_n)$, Dirac quantization condition $e^{i2\pi Q_M}=1$, quantum stability conditions,.... Here $H_n \rightarrow H_{n-1}$ represents a generic step in the hierarchy of Higgs symmetry breakings from the GUT group G to the standard-model group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_W \times \mathrm{U}(1)$ and Q_M is the monopole magnetic charge generator in a singular (Dirac string) gauge. Monopoles are expected with masses and sizes from GUT scales $M_{\rm mon} \simeq M_X/\alpha$, $r_{\rm mon} \simeq M_X^{-1}$ to electroweak scales M_W/α, M_W^{-1} and with magnetic charges belonging to the diagonal long-range gauge symmetries, generally given by linear combinations of the commuting generators of the standard-model group, $Q_M = aQ_{\rm em} + bY_C + cT_W^3$. Each monopole solution admits a tower of dyonic states characterized by the electric charges associated with the diagonal generators $Q_{\rm em}$, T_C^3, Y_C, T_W^3 with typical Coulomb energy mass splittings, from $M_X\alpha$ to $M_W\alpha$.

The conservation constraints on the various gauged charges, which must balance so as not to excite large field barriers at the monopole, and the boundary conditions on the fermion amplitudes at the monopole core put severe constraints on the allowed quark-lepton processes which violate the global quantum numbers, baryon and lepton charge, chirality,.... One generally ignores here the weak charge, based on the assumption that T_W^3 loses meaning as a result of the spontaneous breakdown of the electroweak symmetry. This situation applies in principle to distances above the electroweak scale ($r \gtrsim M_W^{-1}$) but could, more optimistically, persist down to core distances M_X^{-1} to the extent that, in the monopole sector, the electroweak Higgs-field parameters are modified by coupling with the GUT Higgs field so as to retain their vacuum value down to M_X^{-1}. An alternative, somewhat unconventional, possibility could be that the electroweak Higgs field acquires a GUT scale expectation value, which would then induce large fermion mass barrier terms.[5]

For the color gauge interactions, it seems reasonable to assume that the color symmetry remains unbroken in the monopole sector. The confinement of color at $r \gtrsim \Lambda_{\rm QCD}^{-1}$

in color-singlet configurations should then be relevant only at the hadronization stage of the initial- and final-quark states and have little effect on the quark-lepton subprocesses. An assumption of this kind is of course necessary if one wishes to develop an effective Lagrangian description of the monopole-proton system which assumes confinement of quarks inside hadrons. One may therefore limit consideration to the sole hadronic and electromagnetic degrees of freedom and, in the spirit of the Skyrme model, describe the dynamics in terms of an effective mesonic chiral Lagrangian.

Let us then associate the chiral field $U(x)\in SU(N_F)_L \times SU(N_F)_R/SU(N_F)_V$ to the nucleon, the Dirac field $\psi(x)$ to the electron, the electromagnetic field potential $A_\mu(x)$ to the photon, and consider the following $U(1)_{em}$ gauged action for the combined system:

$$\Gamma(U,\psi,A_\mu)=\Gamma^{NA}(U,A_\mu)+\Gamma^{AN}(U,A_\mu)+\Gamma^{e}(\psi,A_\mu)+\Gamma^{\gamma}(A_\mu)\ , \tag{1}$$

$$\Gamma^{NA}(U,A_\mu)=\int d^4x\left[\frac{F_\pi^2}{16}\left[-\mathrm{Tr}(\tilde L_\mu \tilde L^{\mu})+\frac{M_\pi^2}{\hat m}\mathrm{Tr}[M_q(U+U^\dagger-2)]\right]+\frac{1}{32e_s^2}\mathrm{Tr}[\tilde L_\mu,\tilde L_\nu]^2+\frac{1}{32f_s^2}\mathrm{Tr}\{\tilde L_\mu,\tilde L_\nu\}^2+\cdots\right]\ , \tag{2}$$

$$\Gamma^{AN}(u,A_\mu)=\frac{iN_c}{240\pi^2}\int d^5y\,\mathrm{Tr}(L^5)-\frac{iN_c}{48\pi^2}\int d^4x\,\mathrm{Tr}[eA(L^3+R^3)-2e^2dA\,A(L+R)-e^2U\,dA\,U^\dagger AL-e^2U^\dagger dA\,UAR]\ , \tag{3}$$

$$\Gamma^{e}(\psi,A_\mu)+\Gamma^{\gamma}(A_\mu)=\int d^4x\,\bar\psi(i\not{D}-m)\psi-\int d^4x\tfrac14 F_{\mu\nu}F^{\mu\nu}\ , \tag{4}$$

where F_π, M_π are the pion decay and mass parameters, $M_q=\mathrm{diag}(m_u,m_d,m_s)$ is the quark mass matrix, $\hat m=(m_u+m_d)/2$, $N_c=3$ is the number of colors, $e=|e|$ is the electron charge ($\alpha=e^2/4\pi\simeq\frac{1}{137}$). Two quartic interaction terms of $O((\partial_\mu U)^4)$, with coupling constants e_s^{-2} and f_s^{-2}, have been included in the nonanomalous (NA) action and the anomalous action (AN) is the gauged Wess-Zumino-Witten action. Our space-time metric has signature ($\mu=0,1,2,3$) $(+---)$ and we use the notation

$$L_\mu=\partial_\mu UU^\dagger,\quad R_\mu=U^\dagger\partial_\mu U,\quad \tilde L_\mu=D_\mu UU^\dagger,\quad \tilde R_\mu=U^\dagger D_\mu U,\quad DU=\partial U+e[A,U],\quad D_\mu\psi=\partial_\mu\psi-ieA_\mu\psi\ ,$$
$$AL^3=\epsilon^{\mu\nu\alpha\beta}\left[A_\mu\frac{Q}{i}\right]L_\nu L_\alpha L_\beta,\quad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,\quad A=\frac{Q}{i}A_\mu dx^\mu,\quad Q=\mathrm{diag}(\tfrac23,-\tfrac13,-\tfrac13),\quad L^5=\epsilon^{ijklm}L_iL_jL_kL_lL_m\ , \tag{5}$$

where ϵ denotes the antisymmetric invariant tensor normalized as $\epsilon_{0123}=+1$.

The above action allows for the following conserved gauge-invariant electric and baryonic currents:

$$J_\mu^Q(x)=ie\frac{F_\pi^2}{8}\mathrm{Tr}[Q(\tilde L_\mu+\tilde R_\mu)]-\frac{ie}{8}\mathrm{Tr}\left\{\left[\frac{1}{e_s^2}[Q,\tilde L^{\nu}][\tilde L_\mu,\tilde L_\nu]+\frac{1}{f_s^2}\{Q,\tilde L^{\nu}\}\{\tilde L_\mu,\tilde L_\nu\}\right]+(\tilde L\leftrightarrow\tilde R)+\cdots\right\}+\frac{eN_c}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\{Q(R_\nu R_\alpha R_\beta+L_\nu L_\alpha L_\beta)+4ie[(\partial_\nu A_\alpha)G_\beta-\tfrac12 A_\nu(\partial_\alpha G_\beta)]\}-e\bar\psi\gamma_\mu\psi\ , \tag{6}$$

$$J^{B\mu}(x)=\frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\{R_\nu R_\alpha R_\beta+3ie\,\partial_\nu[A_\alpha Q(L_\beta+R_\beta)]+3e^2\partial_\nu[(UQU^\dagger Q-Q^2)A_\alpha A_\beta]\}\ , \tag{7}$$

where $G_\beta=Q^2(L_\beta+R_\beta)+\frac12(UQU^\dagger QL_\beta+U^\dagger QUQR_\beta)$. [As written, the anomalous parity electric and baryonic currents coincide with those of Ref. 16 but deviate slightly from those adopted in Ref. 15 in that the last terms inside the curly brackets in our Eqs. (6) and (7) are absent in Ref. 15.]

Since the GUT monopoles typically carry very large masses and have very small sizes, one may treat them to an excellent approximation as localized in space, static in time, and pointlike. Thus, the monopole sector is characterized by the em potential

$$A_\mu(x)=A_\mu^M(\mathbf{x})+a_\mu(x)\ , \tag{8}$$

$$A^{M\varphi}(\mathbf{x})=\begin{cases}-g_D\dfrac{1+\cos\theta}{r\sin\theta} & (S)\ ,\\ +g_D\dfrac{1-\cos\theta}{r\sin\theta} & (N)\ ,\end{cases} \tag{9}$$

where $a_\mu(x)$ represents the quantum dyonic component while the classical component $A_\mu^M(\mathbf{x})$ is recognized as the potential [in the south (S) and north (N) hemispheres, with Dirac string along $z>0$ and $z<0$] for a point Dirac monopole localized at the origin with magnetic charge g_D (magnetic field strength $B^r=g_D/r^2$). The use of curved spherical coordinates r,θ,φ is very useful here and some helpful formulas are catalogued in Appendix A. Let us recall briefly that, in the mathematical representation of monopoles, three-dimensional space is replaced by the manifold $S^2 \times R_+$ (three-sphere with origin excluded), where S^2 (with the north and south hemispheres as the covering patches) serves as a base manifold for a U(1) principal fiber bundle with a connection given by the gauge field $A_\mu(x)$, sections given by the matter fields $U(x)$, $\psi(x)$, and a transition function $g(\varphi)=e^{+i(2eg_D)Q\varphi}$. The transition function $g(\varphi)$ which flips the Dirac string from the $z>0$ to the $z<0$ half-axes and relates thereby the south-hemisphere field sections to the north ones acts as the vector gauge transformation $[g_L(x)=g_R(x)=g(\varphi)]$:

$$[A^M(\mathbf{x})]_S \to [A^M(\mathbf{x})]_N = g(\varphi)\left[[A^M(\mathbf{x})]_S+\frac{1}{e}d\right]g(\varphi)^\dagger ,$$
$$U(x)\to g(\varphi)U(x)g(\varphi)^\dagger, \quad \psi(x)\to g(\varphi)\psi(x) . \tag{10}$$

Thus, the condition that the matter fields are well defined upon encircling the string (2π jump in φ) yields the Dirac quantization relation $2eg_D=\pm$integer. The same string invisibility condition also implies the important continuity constraint

$$g(\varphi)U(r,\theta=0,\varphi)g(\varphi)^\dagger = U(r,\theta=0,\varphi=0) , \tag{11}$$

which states that the north-hemisphere chiral field corresponding to our south-hemisphere field $U(x)$ must be continuous and hence, φ independent, along the line $\theta=0$. This statement essentially rephrases the compatibility condition for the existence of monopole solutions: namely, that one must be able to extend the fields from the sphere at infinity S^2 to all space, given by the ball B^3 ($\partial B^3=S^2$), without encountering a singularity.

Two other remarkable properties of monopoles are the existence of the generalized angular momentum $\mathbf{J}=\mathbf{L}+\mathbf{\Sigma}/2-2eg_D\hat{\mathbf{x}}$ and the fact that $\mathbf{J}$ allows for a lowest angular momentum $j_{\min}=|q|-\frac{1}{2}$ ($q\equiv eg_D$) with a vanishing centrifugal barrier. The presence of a centrifugal barrier for the higher partial waves implies a suppression of the fermionic fields in the corresponding waves at the origin. Since the penetration to the monopole core is a necessary condition for the violation of the global charges, it follows that the corresponding processes are essentially dominated by the minimal $j_{\min}$ wave. We shall deal exclusively in the following with the lowest magnetic charge (fundamental monopole) case, $2eg_D=2q=-1$; hence, $j_{\min}=0$, where the choice of sign follows from our convention of denoting e as the absolute value of the electron charge.

The chiral-field configuration which describes the proton-monopole system in the $j_{\min}=0$ wave and satisfies the continuity condition, Eq. (11), may be most directly found by considering a choice of radial configuration for $U(x)$ which commutes with Q. By specializing to the two-flavor case, $U(x)=e^{2i\tau^a\pi_a(x)/F_\pi}\in SU(2)$, where $Q=\frac{1}{2}(\tau_3+\frac{1}{3})$ and τ_i are the Pauli matrix generators, one[15] is led to the ansatz $U_K(r,t)=e^{if(r,t)\tau_3}$ where $f(r,t)$ is defined mod(2π).

The quantum em field in the monopole sector must correspond to a gauge transformation of global type associated with motions which leave the em charge invariant and is spherically symmetric in space. This leads one to the familiar parametrization of the dyonic fluctuations $a_\mu(x)=-\epsilon_{\mu\nu}\partial^\nu\phi_a(r,t)$, where we use notation appropriate to a (1+1)-dimensional radially reduced space-time ($\mu=t,r$) with a metric $g_{tt}=-g_{rr}=+1$ and an antisymmetric tensor ϵ normalized as $\epsilon_{tr}=+1$. In components, $a^t=\partial_r\phi_a(r,t)$, $a^r=-\partial_t\phi_a(r,t)$, and the radial electric field strength is $E^r=F^{rt}=\Box\Phi_a(r,t)$. Note, however, that the relativistic covariance at our reduced radial level is only formal. The chiral action simplifies to

$$\Gamma^{AN}+\Gamma^{NA}=\int\int dr\,dt\,4\pi r^2\left[\frac{F_\pi^2}{8}[(\partial_\mu f)^2+2M_\pi^2(\cos f-1)]+\frac{1}{4f_s^2}(\partial_\mu f)^4+\frac{e}{2\pi}\epsilon^{\mu\nu}a_\mu\partial_\nu f\right] . \tag{12}$$

The corresponding $j_{\min}$ wave projection for the electron field is standard. For the Abelian monopole case at hand, one writes the Dirac spinor in the chirality basis

$$\left[\gamma_0=\begin{bmatrix}0&1\\1&0\end{bmatrix},\ \gamma^m=\begin{bmatrix}0&-\sigma^m\\\sigma^m&0\end{bmatrix},\ \gamma_5=-\gamma_0\gamma^1\gamma^2\gamma^3=\begin{bmatrix}+1&\\&-1\end{bmatrix}\right]$$

as

$$\psi(x)=\frac{1}{\sqrt{4\pi}r}\begin{bmatrix}\chi_+(r,t)\eta_{j_{\min}m}(\hat{\mathbf{x}})\\\chi_-(r,t)\eta_{j_{\min}m}(\hat{\mathbf{x}})\end{bmatrix} , \tag{13}$$

where the index $\pm$ stands for the chirality and $\eta_{jm}(\hat{\mathbf{x}})$ denotes the spinor monopole harmonics, eigenstates of $\mathbf{J}^2$, J_z in the normalization

$$\int\frac{d\Omega_{\hat{\mathbf{x}}}}{4\pi}\eta_{jm}^*(\hat{\mathbf{x}})\eta_{jm}(\hat{\mathbf{x}})=1 .$$

We recall[7] here the result that the $j_{\min}$ spinor harmonic is an isolated nondegenerate zero mode of the Dirac operator on the sphere S^2 $[D_\Omega\eta_{j_{\min}m}(\hat{\mathbf{x}})=0]$ and an eigenstate of the S^2 chirality operator $\sigma^r \equiv \sigma \cdot \hat{\mathbf{x}}$ and that for $j_{\min} \equiv |q| - \frac{1}{2} = 0$ one has $\sigma^r \eta_0(\hat{\mathbf{x}}) = (q/|q|)\eta_0(\hat{\mathbf{x}})$. The use of (13) and of the above em field parametrization in the Dirac Lagrangian reduces it to the formal (1+1)-dimensional Dirac Lagrangian:

$$\bar{\psi}(i\not{D} - m)\psi = \frac{1}{4\pi r^2}\left[\chi_+^* \left[i\mathcal{D}_t\chi_+ + i\frac{q}{|q|}\mathcal{D}_r\chi_+ - m\chi_-\right] + \chi_-^*\left[i\mathcal{D}_t\chi_- - i\frac{q}{|q|}\mathcal{D}_r\chi_- - m\chi_+\right]\right] = \frac{1}{4\pi r^2}\bar{\chi}(i\bar{\gamma}^\mu\mathcal{D}_\mu - m)\chi \quad [\mathcal{D}_\mu = (\partial_\mu - iea_\mu)] , \tag{14}$$

where the last equation is written in the Dirac spinor notation

$$\chi(r,t) = \begin{bmatrix} \chi_+(r,t) \\ \chi_-(r,t) \end{bmatrix} ,$$

$$\bar{\gamma}^0 = \sigma^1, \quad \bar{\gamma}^1 = -i\frac{q}{|q|}\sigma^2, \quad \bar{\gamma}_5 = \bar{\gamma}_0\bar{\gamma}^1 = \frac{q}{|q|}\sigma^3 .$$

The $j_{\min} = 0$ case is characterized by the existence of a one-to-one correspondence between chirality and the sign of the electric charge (or, more specifically, the sign of $q = eg_D$). First, note that in our sign conventions the $\binom{L}{R}$ chirality eigenvalues of $\gamma_5 = \pm 1$ correspond to the helicity eigenvalues $h = \sigma \cdot \hat{\mathbf{p}} = \mp 1$ and that $h^{\text{out}} = \sigma^r$, $h^{\text{in}} = -\sigma^r$. The important restriction $\sigma^r = q/|q|$ shows then that fermion-monopole scattering must necessarily flip helicity and chirality ($h^{\text{in}} = -q$, $h^{\text{out}} = q$) with e_R^-, e_L^+ allowed to propagate as in states and e_L^-, e_R^+ as out states. Also, the condition for a well-defined Hermitian extension of the Hamiltonian associated to (14) requires the boundary condition

$$\frac{\chi_+(0)}{\chi_-(0)} \equiv \frac{\chi_L(0)}{\chi_R(0)} = e^{i(\theta - \pi/2)} ,$$

where θ is the CP-nonconserving θ-vacuum angle.

The bosonized version[17] associates with the spinor $\chi(r,t)$ a real scalar field $\phi(r,t)$ via the correspondence

$$\begin{bmatrix} \chi_+(r,t) \\ i\chi_-(r,t) \end{bmatrix} = \left[\frac{C\mu}{2\pi}\right] \times :\exp\left[-i\sqrt{\pi}\int_0^r dr'[\dot{\phi}(r',t) \pm \phi'(r',t)]\right]:$$

and the familiar rules

$$\bar{\chi}\bar{\gamma}_\mu\chi \rightarrow -\frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial^\nu\phi, \quad \bar{\chi}\bar{\gamma}_\mu\bar{\gamma}_5\chi \rightarrow -\frac{1}{\sqrt{\pi}}\partial_\mu\phi ,$$

$$\bar{\chi}i\not{\partial}\chi \rightarrow \tfrac{1}{2}(\partial_\mu\phi)^2, \quad -m\bar{\chi}\chi \rightarrow \frac{Cm\mu}{\pi}:\cos(2\sqrt{\pi}\phi): ,$$

where C is a numerical constant ($C \simeq e^{0.577}$) accompanying the renormalization mass floating scale μ, necessary for the regularization of infinities in defining the normal product (no pairing of fields inside : :). The boundary condition is $\phi(0)/2\sqrt{\pi} = \theta$. For convenience, we shall consider the rescaling $\phi(r,t) \rightarrow \phi(r,t)/2\sqrt{\pi}$ so as to let the vacuum values of $\phi(r,t)$ jump by 2π discrete steps, and use the notation $C\mu m/\pi \equiv M^2/4\pi$. Recall the familiar variational estimate for the mass parameter, $M \simeq \mu$.

The complete reduced Lagrangian reads

$$\Gamma_{\text{eff}}(f,\phi,\phi_a) = \int\int dr\,dt\left[4\pi r^2\left[\frac{F_\pi^2}{8}[(\partial_\mu f)^2 + 2M_\pi^2(\cos f - 1)] + \frac{1}{4f_s^2}(\partial_\mu f)^4\right] + \frac{1}{8\pi}(\partial_\mu\phi)^2 + \frac{M^2}{4\pi}(\cos\phi - 1) + \frac{e}{2\pi}\partial^\mu\phi_a\partial_\mu(f - \phi) + 2\pi r^2(\Box\phi_a)^2\right] , \tag{15}$$

where we have subtracted away the vacuum terms [for $f \rightarrow 0$ and $\phi \rightarrow 0 \bmod(2\pi)$] as well as the monopole contribution to the energy $2\pi r^2(g_D/r^2)^2$. The electric, baryonic, and leptonic currents and charges associated with the effective action (15) are

$$\begin{aligned} J_\mu^Q(r,t) &= -\frac{e}{2\pi}\epsilon_{\mu\nu}\partial^\nu[f(r,t) - \phi(r,t)], \quad Q(t) = e[\Delta f(t) - \Delta\phi(t)] , \\ J_\mu^B(r,t) &= -\frac{1}{2\pi}\epsilon_{\mu\nu}\partial^\nu f(r,t), \quad B(t) = \Delta f(t) , \\ J_\mu^L(r,t) &= -\frac{1}{2\pi}\epsilon_{\mu\nu}\partial^\nu\phi(r,t), \quad L(t) = \Delta\phi(t) . \end{aligned} \tag{16}$$

The baryonic, leptonic, and electric charges (B, L, Q) are determined by the differences between the field values at the inner and outer boundaries, $\Delta f(t) \equiv [f(\infty,t) - f(0,t)]/2\pi$, $\Delta\phi(t) \equiv [\phi(\infty,t) - \phi(0,t)]/2\pi$, which must jump by integer values in order for B, L, and Q to be properly quantized. The assignment for the proton-electron system of $B(P) = 1$, $L(e^-) = +1, \ldots$ implies that $Q = (B - L)$. The apparent topological conservation of both the electric and baryonic and leptonic currents is only formal since charge can leak through the origin. The conservation of Q is necessary for an unbroken em gauge invariance and leads to the boundary condition $\Delta\phi(t) - \Delta f(t) = f(0,t) - \phi(0,t) = 0$. On the other hand, B and L are determined by $f(0,t)$ and $\phi(0,t)$ and there is no corresponding gauge symmetry to invoke for their possible conservation. However, the difference $B - L$ is conserved in our case, consistently with the situation prevailing in the standard GUT groups [SU(5),SO(10), . . .]. Let us also note that for the alternative choice $2eg_D = +1$, corresponding in our notation to the antimonopole case, the same relations as (16) hold except for changes of the signs.

The discussion of $\Gamma_{\rm eff}(f,\phi,\phi_a)$, Eq. (15), is greatly simplified if one is allowed to integrate by parts the matter-radiation interaction term and drop the boundary terms. This results in the replacement

$$\frac{e}{2\pi}\partial^\nu\phi_a\partial_\nu(f-\phi)\to-\frac{e}{2\pi}\Box\phi_a(f-\phi)\ .$$

The only dependence on ϕ_a then is through $\Box\phi_a$, which becomes an auxiliary field variable that can be eliminated exactly by use of the field equations. The result for the effective action

$$\Gamma_{\rm eff}(f,\phi)=\int\int dr\,dt\left[4\pi r^2\left[\frac{F_\pi^2}{8}(\dot f^{\,2}-f'^2)+\frac{F_\pi^2M_\pi^2}{4}(\cos f-1)+\frac{1}{4f_s^2}(\dot f^{\,2}-f'^2)^2\right]\right.$$
$$\left.+\frac{1}{8\pi}(\dot\phi^2-\phi'^2)+\frac{M^2}{4\pi}(\cos\phi-1)-\frac{e^2}{32\pi^3r^2}(f-\phi)^2\right] \tag{17}$$

is in essentially the standard form quoted by Callan and Witten.[15] The fields f and ϕ are coupled through a Coulomb energy term whose singularity at $r=0$ implies $f(0,t)-\phi(0,t)=0$, which therefore automatically takes care of the charge conservation. To trace out the analogies with the alternative derivation of Eqs. (15) and (17), we observe that $\epsilon^{\mu\nu}\partial_\mu\phi_a\partial_\nu\phi$ corresponds to the familiar $U_A(1)$ anomaly-induced interaction and that the elimination of $\Box\phi_a$ is the counterpart of the familiar em axial gauge fixing. On the other hand, it does not appear possible to transform our Callan effective action (15) to an analog of the Rubakov effective action, in the form obtained by Nair[13] or some generalization of it. For this to be the case, it should be necessary to find a transformation of the field variables by which one cancels away the nondiagonal coupling $\partial_\mu\phi_a\partial^\mu(f-\phi)$ so as to obtain noninteracting fields f, ϕ, and ϕ_a, in the limit of vanishing M^2 and M_π^2 mass terms. While the shift $\phi(r,t)\to\phi(r,t)+2e\phi_a(r,t)$ satisfies this purpose for $\phi(r,t)$ there is no corresponding result for $f(r,t)$, even by invoking the general transformation $f(r,t)\to p(r)f(r,t)+q(r)\phi(r,t)$.

The proton-electron dynamics in the monopole field is governed at the classical level by the field equations

$$-\ddot f+f''+\frac{2}{r}f'-M_\pi^2\sin f+\frac{4}{f_s^2F_\pi^2r^2}\left[-\frac{d}{dt}[r^2\dot f(\dot f^{\,2}-f'^2)]+\frac{d}{dr}[r^2f'(\dot f^{\,2}-f'^2)]\right]-\frac{e^2}{16\pi^4F_\pi^2r^4}(f-\phi)=0\ ,$$
$$-\ddot\phi+\phi''-M^2\sin\phi+\frac{e^2}{4\pi^2r^2}(f-\phi)=0\ . \tag{18}$$

The associated energy-momentum densities

$$\mathcal{E}(r,t)=\left[F_\pi^2\frac{\pi r^2}{2}(\dot f^{\,2}+f'^2)-\pi F_\pi^2M_\pi^2r^2(\cos f-1)\right.$$
$$+\frac{\pi r^2}{f_s^2}(\dot f^{\,2}-f'^2)(3\dot f^{\,2}+f'^2)+\frac{1}{8\pi}(\dot\phi^2+\phi'^2)$$
$$\left.-\frac{M^2}{4\pi}(\cos\phi-1)+\frac{e^2}{32\pi^3r^2}(f-\phi)^2\right]\ , \tag{19a}$$

$$\mathcal{P}(r,t)=-\left[\pi F_\pi^2r^2\dot f f'\left[1+\frac{4}{F_\pi^2f_s^2}(\dot f^{\,2}-f'^2)\right]+\frac{1}{4\pi}\dot\phi\phi'\right]\ , \tag{19b}$$

obey the local conservation equation $\partial\mathcal{E}/\partial t+\partial\mathcal{P}/\partial r=0$.

Simple analytic solutions exist for the field equations (18) in certain limits. Assuming that the Coulomb term is switched off, as is appropriate for large r, then f and ϕ decouple and ϕ obeys the familiar (1+1)-dimensional sine-Gordon equation while f obeys (for $f_s^{-2}=0$) a (3+1)-dimensional radial sine-Gordon equation. The familiar kink and antikink soliton solutions, $\phi(r,t)=4\arctan(e^{\pm A})$, $A=M(r-\beta t)/\sqrt{1-\beta^2}$, describe electron and positron particle states moving with velocity β. By contrast, the equation for f lacks Lorentz covariance and fails to have stable stationary solitons. However, if one were to insert back the Coulomb term, but still ignore the coupling to ϕ, and also set $f_s^{-2}=0$, a static solution appears $f(r)=2\pi e^{-e/(4\pi^2F_\pi r)}$ with finite energy $E=\int_0^\infty dr\,\mathcal{E}(r)=(\pi/2)eF_\pi$. As suggested by Callan and Witten,[15] this solution could describe a monopole-proton deeply bound state of unconventional type, whose binding originates in the induced Coulomb interaction rather than the ordinary $-\boldsymbol{\mu}_N\cdot\mathbf{B}$ magnetic interaction.

The generalization of the soliton approach to the case of three quark flavors and to effective Lagrangians incorporating the vector mesons is straightforward. The discussion of these cases is relegated to Appendix B since no numerical application was attempted for them in the present work.

III. INITIAL CONDITIONS

A field-theory description of the reaction $M+P\to M+e^+$ pions is made possible with the effective action (15) already at a classical tree-level approximation. The formulation suggested by Callan and Witten[15] consists of an initial-value problem for a baryonic chiral field $f(r,t)$, corresponding to a proton-Skyrmion incident on the monopole at impact parameter b and velocity β, which excites the leptonic field $\phi(r,t)$ to an emerging positron. The identification of a borderline between this time history and the alternative one ending in the proton reemission should then permit the determination of a critical impact parameter $b_{\rm crit}$ and lead eventually to a geometrical estimate of the cross section $\sigma_{\text{cat}} \simeq \pi b_{\text{crit}}^2$.

We shall discuss in this section the construction of the initial data for the radial kink field $f(r,t)$, defined by $U_K(r,t) = e^{i\tau_3 f(r,t)}$, in terms of the Skyrmion shape function, defined by $U_{\text{Sk}}(\mathbf{x}) = e^{i\tau\cdot\hat{\mathbf{x}}F(r)}$. The classical field configuration of a Skyrmion whose center, initially at $(\mathbf{b}, z_0)$, moves with uniform velocity β along the linear trajectory described in Cartesian coordinates by $\mathbf{x}_0(t) = (\mathbf{b}, z_0(t)) \equiv (\mathbf{b}, z_0 + \beta t)$, is obtained from the familiar static solution by means of a translation and a Lorentz boost. One finds

$$U_{\text{Sk}}(\mathbf{x},t) = e^{i\tau\cdot(\mathbf{R}/R)F(R)} = \cos F(R) + i\tau\cdot\frac{\mathbf{R}}{R}\sin F(R) , \tag{20}$$

where $F(R)$ is subject to the boundary conditions $F(0) = \pi$, $F(\infty) = 0$, and

$$\mathbf{R} = (\mathbf{x}_\perp - \mathbf{b}, \gamma(z - z_0 - \beta t)), \quad \gamma = (1-\beta^2)^{-1/2}, \quad R = |\mathbf{R}| .$$

The correspondence between the configuration of a neutral, spherically symmetric radial kink and the charged, spatially deformed, displaced Skyrmion appears intuitively as a sort of $l=0$ partial s-wave projection. It is important, however, not to confuse our solitonic (dual classical field theory and particle) description with a quantum-mechanical description. In particular, the finite Skyrmion size invalidates the familiar association of an l partial wave with impact parameters in the range l/k, $(l+1)/k$. Thinking of the underlying quark picture, there are the participating quarks which must propagate in s waves in order to interact with the monopole, via such subprocesses as

$$u_{1L} + u_{2L} + M \to d_{3R}^c + e_R^+ + M ,$$
$$u_{1L} + M \to \tfrac{1}{2}(u_{1R} + u_{2L}^c + d_{3R}^c + e_R^+) + M, \ldots ,$$

but also the spectator quarks which may propagate in all partial waves. As usual in the classical field theory description, the comparison of $U_K(r,t)$ with $U_{\text{Sk}}(\mathbf{x},t)$ is made by considering continuous deformations which interpolate between the two configurations. These are viewed as a possible virtual evolution in time of the system. The corresponding motion may or may not be classically allowed. In the event it meets an energy barrier, one needs to consider a time-dependent deformation and envisage a quantum treatment for an estimate of the tunneling probability. The continuous interpolations between U_K and U_{Sk} are described as paths in the space of the field configurations $U(x) \in \text{SU}(2)$ and are defined in terms of the one-dimensional mapping $\lambda \in [0,1] \to U^\lambda(\mathbf{x},t) \in \text{SU}(2)$, such that $U^{\lambda=0}(\mathbf{x},t) = U_K(r,t)$ and $U^{\lambda=1}(\mathbf{x},t) = U_{\text{Sk}}(\mathbf{x},t)$. In the presence of the Dirac string, both U_K and U_{Sk} have zero winding number. The crucial item which permits the unwrapping of the topological baryon number is the modified continuity constraint, Eq. (11), which is to be imposed on $U^\lambda(\mathbf{x},t)$ for all λ. The path is easier to construct if one starts from the radial kink and, for fixed orientation θ,φ, one slips off the closed path in group space $f(r=0,t)=0$, $f(r=\infty,t)=2\pi$ to a point. To keep away from the Dirac string singularity, Callan and Witten[15] have suggested to introduce as a prefactor to λ a selective deformation function $h(\theta,r)$,

$$h(\theta,r) = \begin{cases} 0 & (0 \le \theta \le \theta_1) , \\ h(r) & (\theta_1 \le \theta \le \theta_2) , \\ 1 & (\theta_2 \le \theta \le \pi) , \end{cases} \tag{21}$$

corresponding to no deformation at $\theta \le \theta_1$, intermediate at $\theta_1 \le \theta \le \theta_2$, and full at $\theta > \theta_2$. We have supplied an additional r dependence to $h(\theta,r)$ [$h(r)$ is assumed monotonic ~ 1] for a reason to be clarified at the end of the present section. The interpolation path of Callan and Witten[15] is

$$\begin{aligned} U^\lambda(r,\theta,\varphi,t) &= e^{i(\tau_3/2)\varphi}(\lambda h(\theta,r) + [1-\lambda h(\theta,r)]e^{if(r,t)\tau_3} + \{2\lambda h(\theta,r)[1-\lambda h(\theta,r)][1-\cos f(r,t)]\}^{1/2} i\tau_1)e^{-i(\tau_3/2)\varphi} \\ &= \cos f + \lambda h(\theta,r)(1-\cos f) + i\sin f[1-\lambda h(\theta,r)]\tau_3 \\ &\quad + i\{2\lambda h(\theta,r)[1-\lambda h(\theta,r)](1-\cos f)\}^{1/2}(\tau_1\cos\phi - \tau_2\sin\phi) . \end{aligned} \tag{22}$$

This renders possible an identification with the translated Skyrmion of Eq. (20) through the natural prescription

$$U^{\lambda=1}(r,\theta,\varphi,t) = U_{\text{Sk}}(r,\theta,\varphi,t) . \tag{23}$$

The above definition of $f(r,t)$ involves the physical parameters b and β but also introduces two other auxiliary parameters, the location z_0 at the initial time $t=0$ and the function $h(\theta,r)$, with no evident physical content. We recall that space translational invariance is lost upon reducing to the radial coordinate. To be meaningful, the physical results should be weakly sensitive to changes in these auxiliary parameters, or else allow for a plausible interpretation of any possible dependence on these parameters.

Solving Eq. (23) for $f(r,t)$ in full generality is difficult and really not necessary. A simple solution may be obtained by forming suitable averages of Eq. (23) over spatial orientations and flavor indices. Thus, an identification in terms of the two integrals

$$\begin{bmatrix} I_1(r,t) \\ I_2(r,t) \end{bmatrix} \equiv \frac{1}{8\pi} \int d\Omega_{\hat{x}} \mathrm{Tr} \left[U_{\mathrm{Sk}}(\mathbf{R},t) \begin{bmatrix} 1 \\ -i\tau_3 \end{bmatrix} \right] = \frac{1}{4\pi} \int_{-1}^{+1} d(\cos\theta) \int_0^{2\pi} d\varphi \begin{bmatrix} \cos F(R) \\ \frac{1}{R}(r\cos\theta - z_0 - \beta t)\sin F(R) \end{bmatrix} \tag{24}$$

allows for the solution

$$\cos f(r,t) = \frac{2I_1(r,t) - H(r,t)}{2 - H(r,t)} , \quad \sin f(r,t) = \frac{2I_2(r,t)}{2 - H(r,t)} , \tag{25}$$

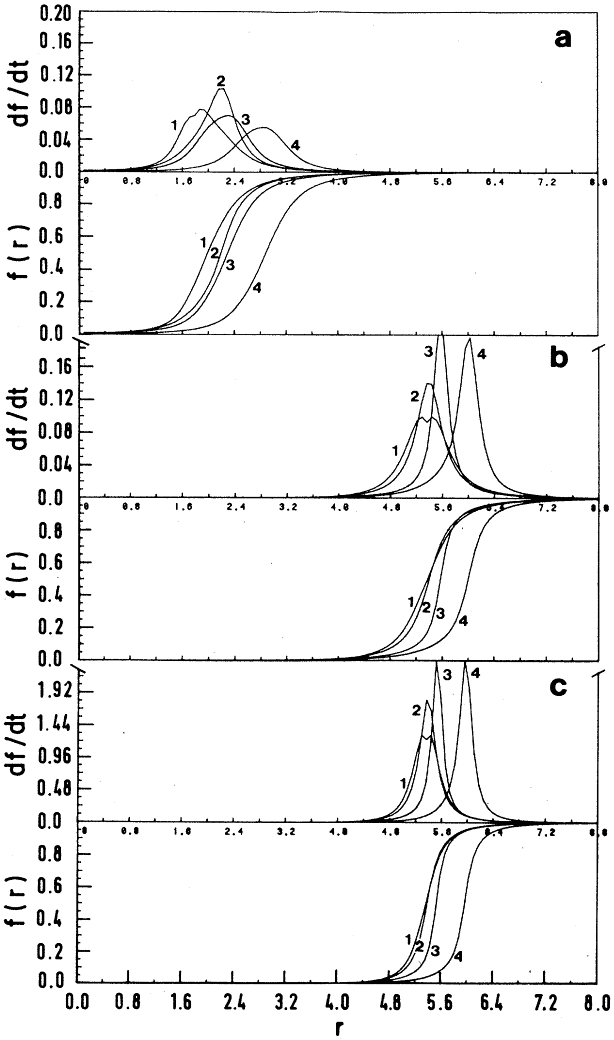

FIG. 1. Initial conditions for $f(r,t)$ and $\dot{f}(r,t) = \partial f/\partial t$ (scaled by 2π) based on a Skyrmion profile $f(r)$ given by an approximate analytic form obtained in Ref. 18: $f(r) = \pi e^{-M_\pi r}(1+M_\pi r)/[(1+1.83r)(1+9.18r^2)^{1/2}]$. Case (a) is for $|z_0| = 0.2R$, $\beta = 0.1$. Case (b) is for $|z_0| = 0.6R$, $\beta = 0.1$. Case (c) is for $|z_0| = 0.6R$, $\beta = 0.8$. For each case the curve labels 1, 2, 3, 4 refer to $b = 0.01$, 0.5, 1.0, 2.0. We use units $F_\pi = 1$.

where the sole dependence on $h(\theta,r)$ lies in the integral $H(r,t) \equiv \int_{-1}^{+1} d(\cos\theta) h(\theta,r)$. The need for an r dependence appears clearly at this point, since otherwise little freedom would be left to satisfy the unitarity constraint $\cos^2 f + \sin^2 f = 1$. On the other hand, our generalized r-dependent deformation path is perfectly legitimate, to the extent that $h(\theta,r)$ presents no singularity or zero in r, except at $r=0$. Since the situation at hand is one in which we dispose of a Skyrmion profile $F(r)$ and hence of fixed inputs for $I_1(r,t)$ and $I_2(r,t)$, the unitarity constraint completely fixes for us the auxiliary function

$$H(r,t) = \frac{I_1^2(r,t) + I_2^2(r,t) - 1}{I_1(r,t) - 1} . \tag{26}$$

The more complete treatment based on the quantized rotating Skyrmion, with $A(t) U_{\mathrm{Sk}}(\mathbf{x},t) A^\dagger(t)$ replacing $U_{\mathrm{Sk}}(\mathbf{x},t)$, is in principle straightforward. The projection on the spin and charge states of the nucleon is obtained by forming the appropriate expectation values with respect to the collective-coordinate $A(t) \in \mathrm{SU}(2)$ wave function. The particular solution given by Eqs. (24) and (25) may be found in a similar manner. The situation is simple to the extent that $A(t)$ is treated in a nonrelativistic approximation. By contrast, a fully covariant treatment of the collective motion represents a technically harder task.

We have been unable to find an appropriate form for $F(R)$ permitting an analytic calculation of $f(r,t)$, from Eq. (25). On the other hand, the numerical quadrature involved in the evaluation of I_1 and I_2, and their time derivatives, represents a relatively easy task. The use in Eq. (24) of transverse coordinates x,y for which $\mathbf{b}$ lies along the x axis results in helpful simplifications. Our numerical results are depicted in Fig. 1 for a range of values for the parameters. The computed function $H(r)$ is found to vary very smoothly with r. It starts from zero $[H(0)=0]$ at the monopole location and increases with r so as to saturate at large r to $H(\infty) \simeq 2$. We observe in Fig. 1 the expected kink shapes. An increase of b results simply in a shift of the kink center to larger r but a slight steepening in the r and t dependence also takes place. The space and time dependence of $f(r,t)$ also become stronger with an increase in b and in z_0.

IV. DISCRETIZED ACTION

In this section we shall discuss the classical proton-electron-monopole problem by considering a spatial discretization of the continuum action, Eq. (15), and solving subsequently for the time evolution numerically. Let us compactify the half-line to the segment $r \in [0,R]$ and impose on it a lattice of $N+1$ equally spaced points $r=aj$ $(j=0,1,\ldots,N)$, with fields defined at the nodes $f_j(t)=f(aj,t)$, $\phi_j(t)=\phi(aj,t)$ and spatial derivatives substituted by the forward finite differences

$$f_j'(t)=\frac{f_{j-1}(t)-f_j(t)}{(-a)} .$$

Our definition for the discrete Lagrangian is

$$\frac{L}{a}=\sum_{j=1}^{N}\left[\pi F_\pi^2(aj)^2\left\{\frac{1}{2}\left[\dot f_j^2(t)-\left[\frac{f_{j-1}(t)-f_j(t)}{a}\right]^2\right]+M_\pi^2[\cos f_j(t)-1]\right\}\right.$$
$$+\frac{\pi}{f_s^2}(aj)^2\left[\dot f_j^2(t)-\left[\frac{f_{j-1}(t)-f_j(t)}{a}\right]^2\right]^2+\frac{1}{8\pi}\left[\dot\phi_j^2(t)-\left[\frac{\phi_{j-1}(t)-\phi_j(t)}{a}\right]^2\right]$$
$$\left.+\frac{M^2}{4\pi}[\cos\phi_j(t)-1]-\frac{e^2}{32\pi^3(aj)^2}[f_j(t)-\phi_j(t)]^2\right] . \tag{27}$$

Variation with respect to f_j and ϕ_j $(j=1,2,\ldots,N)$ yields the equations of motion

$$\ddot f_j+\frac{1}{a^2}[(1+1/j)^2(f_j-f_{j+1})(1-\delta_{jN})-(f_{j-1}-f_j)(1-\delta_{j1})]$$
$$+\frac{4}{F_\pi^2 f_s^2}\left[\frac{d}{dt}(F_j\dot f_j)+\frac{1}{a^2}[(1+1/j)^2F_{j+1}(f_j-f_{j+1})(1-\delta_{jN})-F_j(f_{j-1}-f_j)(1-\delta_{j1})]\right]$$
$$+M_\pi^2\sin f_j+\frac{e^2}{16\pi^4(aj)^4F_\pi^2}(f_j-\phi_j)=0 ,$$
$$\ddot\phi_j+\frac{1}{a^2}[(\phi_j-\phi_{j+1})(1-\delta_{jN})-(\phi_{j-1}-\phi_j)(1-\delta_{j1})]+M^2\sin\phi_j-\frac{e^2}{4\pi^2(aj)^2}(f_j-\phi_j)=0 \tag{28}$$
$$\left[F_j(t)\equiv\dot f_j^2(t)-\left[\frac{f_{j-1}(t)-f_j(t)}{a}\right]^2\right]$$

which represent a discrete set of $2N$ coupled one-dimensional differential equations. The Lagrangian system (27) allows for a conserved energy given by the discretized counterpart of Eq. (19a). The discretization version that we have adopted here is known in numerical analysis as the method of lines. While this is not the most effective numerical method[19] (simultaneous discretization of space and time or Fourier series transformation are generally judged superior) it suits well our limited purpose in this work. One consequence of our special definition of discrete Lagrangian, Eq. (27), excluding the inner boundary point, is that $f_0(t)$ and $\phi_0(t)$ are auxiliary variables fixed by the relations $f_0=f_1$, $\phi_0=\phi_1$. In the alternative discretization in which fields are defined at half-nodes, $r=a(j+\frac{1}{2})$, and the point $j=0$ is then included in (27), f_0 and ϕ_0 would become dynamical variables. Let us note here that the end results are found to be insensitive to the specific choice adopted for the discretization procedure.

The system of equations (27) is to be solved as an initial-value problem by entering as inputs f_j, $\dot f_j$, ϕ_j, $\dot\phi_j$ at an initial time $t=0$. The evolution in time leads thereby to a unique solution, the only freedom residing in the conditions at the boundaries. Considering first the inner boundary $(r=0)$, the electric-charge-conservation condition translates to $\phi_0(t)\equiv\phi_1(t)=f_1(t)\equiv f_0(t)$. The extent to which this constraint continues to be satisfied automatically in the discrete model depends on the closeness to the continuum limit and on the magnitude of the Coulomb energy term. In practice, for reasonable choices of spatial mesh and step sizes, which are not computationally too prohibitive, the Coulomb energy coupling term fails generally by itself to secure the charge conservation condition. The reason is that the distinct $(3+1)$-dimensional character of the f field (reflected by the extra r^2 factor) weakens the feedback on the ϕ field induced by the Coulomb term. The alternative is to enforce the boundary condition directly on the field equations. One could, for example, consider f_1 and ϕ_1 as dependent variables and invoke the conditions $\ddot f_1-\ddot\phi_1=0$ to determine these variables, which leads to $f_1=\phi_1=(4f_2-\phi_2)/3$. However, this procedure is not satisfactory because it introduces spurious energy source effects at the origin. A simpler prescription free of this deficiency consists of imposing $f_2=f_1=\phi_2=\phi_1$ and determining the common value of these variables by replacing the pair of equations $\ddot f_2=\mathcal{P}_2(f,\phi)$, $\ddot\phi_2=\mathcal{S}_2(f,\phi)$ by the single equation $\ddot f_2=\ddot\phi_2=\frac{1}{2}(\mathcal{P}_2+\mathcal{S}_2)$.

The outer boundary $(r=R)$ poses a different sort of problem. Several possibilities are suggested here. One is the choice of free boundary conditions, letting $\phi_N(t)$ and $f_N(t)$ evolve freely in time. Another consists of imposing fixed values specific to the f- and ϕ-field vacuum sectors, e.g., $f_N=2\pi$, $\phi_N=0$. The physics of linear wave propagation[20] suggests one other choice which consists of the reflectionless boundary conditions $\dot{f}_N=f'_N=0$, $\dot{\phi}_N+\phi'_N=0$. Note, however, that we deal here with a distinct problem, involving nonlinear interactions and finite mass terms, so that these conditions prohibit wave reflections at the boundary in an approximate sense only. While the second choice of (fixed) boundary conditions conserves the energy, the first (free) choice and the third (reflectionless) choice are not energy conserving. Indeed, these cases allow energy to be radiated away at infinity and hence cause the field energy to decrease with time.

V. NUMERICAL RESULTS

We set the parameters in the effective Lagrangian to their physical values, namely, $F_\pi=174.8$ MeV, $M_\pi=140.4$ MeV (including pion loop corrections), $e^2/4\pi\simeq 1/137$, and make for the electron mass parameter the plausible choice $M=M_\pi$. We consider a spatial mesh of size $R=10$ fm, $N=120$ (step size $a=0.074$ fm). These choices define our reference set of parameters and we note immediately that the end results are insensitive to reasonable variations about these reference values. Henceforth, we quote all dimensional quantities in units of F_π.

We shall only treat the case $f_s^{-2}=0$ since the numerical stability and convergence of the solution for $f_s^{-2}\neq 0$ is not easily achieved in our discretization method based on Eq. (27).

To numerically solve the system of $2N$ equations of motion we have transformed Eq. (28) to a system of $4N$ first-order differential equations, written symbolically

$$\begin{bmatrix}\dot{\mathcal{F}}\\ \dot{\Phi}\end{bmatrix}=\begin{bmatrix}P(\mathcal{F}) & Q(\Phi)\\ R(\mathcal{F}) & S(\Phi)\end{bmatrix}\begin{bmatrix}\mathcal{F}\\ \Phi\end{bmatrix},$$
$$\mathcal{F}^T(t)=[f_1(t)\dot{f}_1(t)\cdots f_N(t)\dot{f}_N(t)]\,, \qquad (29)$$
$$\Phi^T(t)=[\phi_1(t)\dot{\phi}_1(t)\cdots\phi_N(t)\dot{\phi}_N(t)]$$

and adopted an algorithm of solution [the International Mathematics and Scientific Library (IMSL) subroutine named GEAR] based on the Adams predictor-corrector method and a diagonal approximation for the Jacobian of the rigidity matrix in Eq. (29). Time steps of 10^{-2}–10^{-3} are required to reach an accuracy of 10^{-6}. The algorithm is unconditionally stable and global stability is checked via the energy-conservation test.

In the presentation of intermediate results we shall concentrate mainly on the choice of a reflectionless outer boundary but will also offer a brief comparison with the fixed-boundary case. (The reflectionless case is implemented by imposing both $\ddot{f}_N=\ddot{\phi}_N=0$ and $\dot{f}_N+f'_N=0$, $\dot{\phi}_N+\phi'_N=0$.) The time evolution presents some characteristic features which appear clearly on the numerical results presented in Fig. 2. As the initial radial kink $f(r,t)$ approaches the origin, its shrinkage is accelerated to velocities $\simeq 1$ and it acquires on reflection at the origin a strongly enhanced amplitude. While the reflected structure moves outwards and expands it gets gradually depleted and the resulting profile of $f(r,t)$ is quite smooth and also evolves smoothly in time. The expansion seems to be halted at some distance between the origin and the outer boundary, at which the structure returns back to the origin where it is again reflected. The initial motion repeats itself, only with a substantially reduced amplitude.

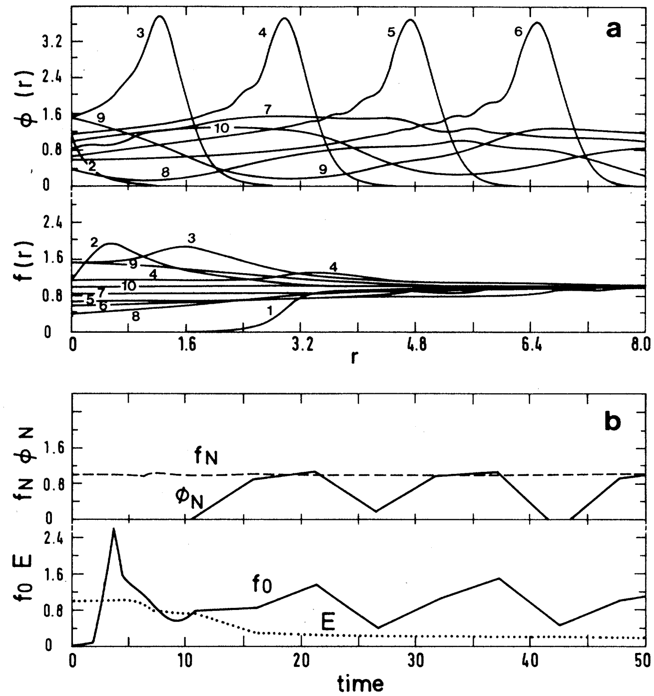

FIG. 2. Evolution in time for an initial $B=1$ chiral kink at impact $b=1.0$, velocity $\beta=0.1$, center $|z_0|=0.3R$ in the reflectionless outer boundary case. (a) shows the profiles of $f(r,t)$ and $\phi(r,t)$ (scaled by 2π) at ten times: $t_i=0$, 2.6, 4.4, 6.2, 8.0, 9.7, 15.9, 26.6, 37.2, 47.8, the index $i=1,\ldots,10$ labeling the curves. (b) shows the field values at the origin $f(0,t)=\phi(0,t)$ (scaled by 2π) and at the outer boundary $f_N(t)=f(R,t)$, $\phi_N(t)=\phi(R,T)$ and also the scaled field energy $E(t)/E(0)$.

Concerning the electron field $\phi(r,t)$, it is at first gradually excited and, at the first reflection, grows to a large amplitude structure. This structure travels outwards and seems to return back to the origin in phase with $f(r,t)$. We observe that the first return is associated with a disruption in the outer boundary value $\phi_N(t)$ which later oscillates in phase with the inner boundary value $\phi_1(t)$.

A clearer view of the time evolution is provided by the field values at the origin and at the outer boundary (Fig. 2). We note that the calculated field energy decreases with time, as expected, and that the sharp drop it suffers coincides with the arrival of waves at the outer boundary. We shall mostly concentrate our presentation of results on the time dependence at the origin since this represents the most significant characteristic of the time evolution. The dependence on the impact b is depicted in Fig. 3. The time of the first reflection increases slightly with b (from 3 at $b=0.1$ to 4.5 at $b=3$) and similarly for the re-

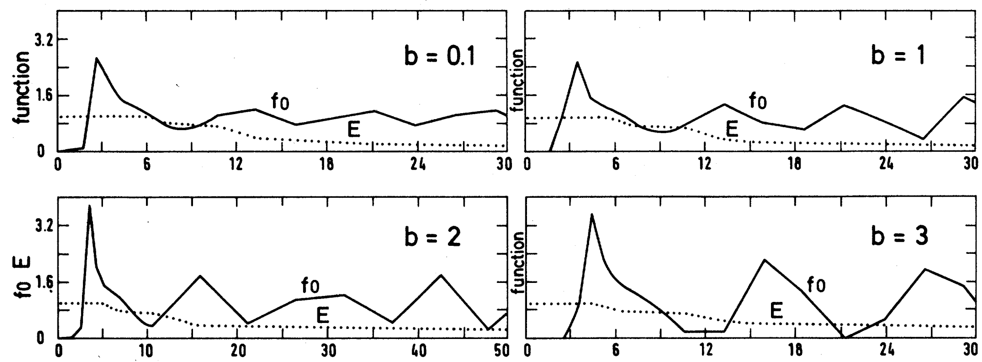

FIG. 3. Values of the chiral field (scaled by 2π) at the inner boundary and the field energy $E(t)/E(0)$ at $\beta=0.1$ for $b=0.1$, 1.0, 2.0, 3.0. This solution corresponds to the reflectionless outer boundary case with $|z_0|=0.3R$.

turn time (from 12 at $b=0.1$ to 16 at $b=3$). Larger b leads essentially to a stronger amplitude at reflection and stronger amplitudes in the time oscillations but with no modification in the period. No significant qualitative change takes place with increased values of b except that around $b\simeq 2-3$ the time oscillations acquire impressively large amplitudes. The dependence on the initial velocity also shows no substantial changes, as seen in the results given in Fig. 4.

The time evolution presents some dependence on the auxiliary parameter z_0 (initial Skyrmion center). We show in Fig. 5 results for two values of z_0 which bracket the value of z_0 adopted in the above results. The oscillations in $f_0(t)$ are smoother for an initial center closer to the origin and stronger for a remote initial center. They grow to impressively large amplitudes at $z_0/R\simeq 0.5$.

The numerical results for the fixed outer boundary condition case are presented in Fig. 6. The field energy is seen to be conserved to a good precision. The time history of $f(r,t)$ is essentially the same as in the previous case; the only change is in the presence for $\phi(r,t)$ of a reflected wave at the outer boundary. The fields at the origin possess again the same time oscillation about $f(0,t)/2\pi\simeq 1$ growing with the impact and with the initial distance from the origin and weakly dependent on the initial velocity.

Among the effects that could suppress the catalysis of baryon decay, one possible mechanism could be r-dependent mass terms growing as one approaches the monopole. We have examined the case where both the chiral and electron mass parameters are replaced as $M^2\to M^2+(\gamma/r)^\alpha$, $M_\pi^2\to M_\pi^2+(\gamma/r)^\alpha$ with $\alpha=1,2$. The singular behavior of the mass terms results in an infinite energy but the discretized action remains well defined. We find that as long as $\gamma\lesssim 10^{-1}$ the above standard picture remains valid. However, as γ increases and reaches $\gamma\simeq 1$, the time oscillations in $f(0)$ acquire very large magnitudes which average to zero. Recalling the estimate for γ found by Bennett[5] ($\gamma\lesssim 10^{-3}$) we conclude

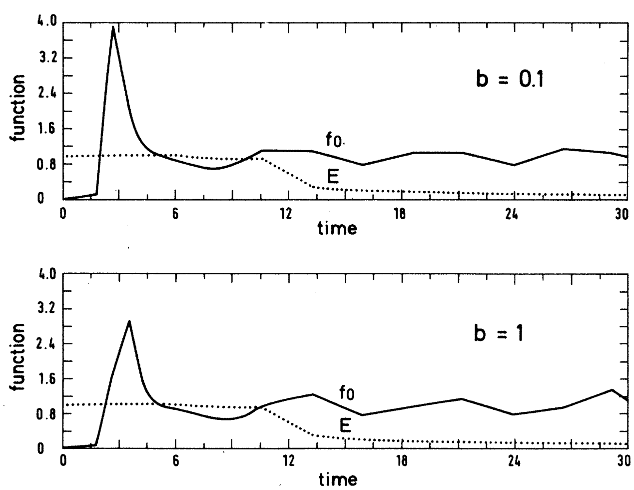

FIG. 4. Values of the chiral field (scaled by 2π) at the origin and of the field energy $E(t)$ [scaled by its initial value $E(0)$] at $\beta=0.5$ for $b=0.1$, 1.0. This solution is obtained for $|z_0|=0.3R$ in the reflectionless boundary case.

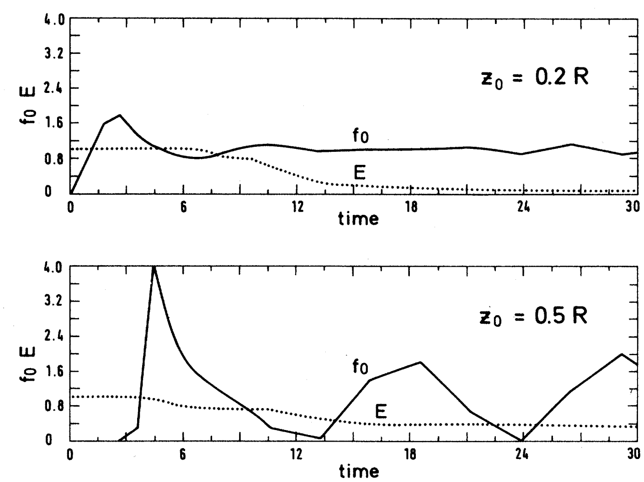

FIG. 5. Dependence of the solution in the initial center position z_0. We show the time evolution of the chiral field (scaled by 2π) at the origin and the field energy $E(t)$ [scaled by $E(0)$] for the case $\beta=0.1$, $b=0.01$ in the reflectionless boundary case. Two values of z_0 are considered: $|z_0|=0.2R$ and $0.5R$.

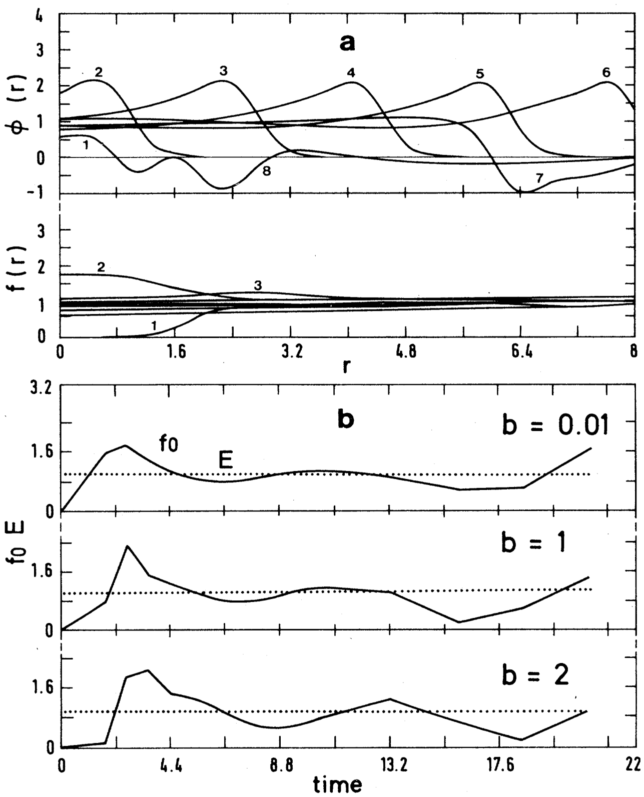

FIG. 6. Evolution in time of the $B=1$ radial kink in the free outer-boundary case $[\phi_N(t)=0,\ f_N(t)=2\pi]$. (a) shows the profile functions for $\beta=0.1$, $b=0.01$, and $|z_0|=0.2R$ at eight times: $t_i=0$, 2.66, 4.43, 6.20, 7.9, 9.7, 13.3, 18.6, the index $i=1,\ldots,8$ labeling the curves. (b) shows the time variation of the fields at the origin (scaled by 2π) and of the energy $E(t)$ [scaled by $E(0)$] for $\beta=0.1$ at three values of the impact parameter, $b=0.01$, 1, 2.

that although the mass growth at the monopole could suppress catalysis, the expected size of the mass term makes this mechanism ineffective.

The antimonopole ($g_D<0$) case is related to the monopole case by a change of sign in Eq. (6). The initial conditions for an incoming proton are simply obtained from the ones used above by the replacement $f(r,t)\to -f(r,t)+2\pi$, $\phi(r,t)=2\pi$. This case is interesting because it realizes, in the naive impulse approximation applied to the Skyrmion, the reaction $P+\overline{M}\to\overline{M}+P+P+e^-$. Nevertheless, the results obtained by numerical integration of the field equations lead here to the same transition as before $P\to e^+$ and so fail to lead, even at large velocities $\beta\simeq 0.8$, to $P\to P+P+e^-$.

VI. DISCUSSION OF THE RESULTS

The first remark to be made is that our initial expectation of observing a time evolution leading to either positron emission or proton reemission, and of observing a crossover between these two distinct time histories in terms of a critical impact parameter, is clearly frustrated. This expectation was in a sense motivated by the analogy with the case of (1+1)-dimensional systems of coupled sine-Gordon fields, which has been employed to model monopole-induced charge exchange at the quark-lepton level (cf. the paper of Dawson and Schellekens in Ref. 5). In spite of several similarities, this analogy is invalidated in the present proton case by the distinct (3+1)-dimensional character of the chiral field $f(r,t)$.

Let us first attempt to clarify the time evolution that is observed numerically. One possible interpretation of the solution found is that it describes the excitation of a solitonic mode in both f and ϕ of the breather or pulsating type. The bounded spatial motion and oscillating time evolution imply here that proton decay proceeds through the excitation of a long-lived resonance state of the proton-electron-monopole system. Another distinct interpretation, more in line with the (1+1)-dimensional case, is that proton decay takes place at the first passage of the monopole through an exchange of kink structures from f to ϕ. The large enhancement in $\phi(r,t)$ and the time oscillations in $f(r,t)$ and $\phi(r,t)$ are then associated with the interference between linear standing waves.

We shall successively furnish some semiquantitative arguments which justify each of these two interpretations. The existence of nonstationary soliton solutions to spherically symmetric nonlinear equations in 3+1 dimensions has been investigated so far mainly numerically in the literature.[21] Pulsating breatherlike solutions, exhibiting a similar sort of return effect as the one we observe, have been previously reported for the (3+1)-dimensional sine-Gordon equation[22] and also for related equations.[21] The method of analysis employed by Christiansen and Olsen[22] may be readily adapted to our problem. The starting assumption is that localized pulsating nonlinear components exists for both f and ϕ and that these are weakly coupled to the linear modes. Let us adopt for f and ϕ the familiar kink and antikink parametrizations

$$f(r,t)=4\arctan\left[\exp\left[M_\pi\frac{r-R(t)}{\sqrt{1-\dot{R}^2(t)}}\right]\right],$$
$$\phi(r,t)=4\arctan\left[\exp\left[-\frac{M[r-\rho(t)]}{\sqrt{1-\dot{\rho}^2(t)}}\right]\right],\qquad(30)$$

with moving centers $R(t)$ and $\rho(t)$. The main justification for using (30) is that numerical results more or less suggest such a behavior. To analyze the time evolution we substitute (30) in the equations of motion, Eqs. (18), and set successively $r=R(t)$ and $r=\rho(t)$. This results in considerable simplifications and we obtain

$$2\ddot{R}\frac{1+\dot{R}^2}{(1-\dot{R}^2)^{3/2}}+4\frac{1}{R(1-\dot{R}^2)^{1/2}}-\frac{e^2}{16\pi^4 M_\pi}\frac{\pi-\phi(R)}{R^4}=0\ ,\qquad(31a)$$

$$2\ddot{\rho}\frac{1+\dot{\rho}^2}{(1-\dot{\rho}^2)^{3/2}}-\frac{e^2}{4\pi^2 M}\frac{f(\rho)-\pi}{\rho^2}=0\ .\qquad(31b)$$

To progress further we need some sort of truncation of the coupling terms and invoke again the numerically observed fact that the Coulomb energy term is relevant mainly in inducing the boundary condition at the origin. For the proton field case, the coupling term is small $[O(1/R^4)]$ for R sufficiently away from the origin. Dropping the coupling term in Eq. (31a) reduces this to the case treated in Ref. 22. There, it is found that the equation admits the first integral $(1-\dot{R}^2)/R^2e^{+\dot{R}^2/2}=\text{const}$, which implies then a maximum radius

$$R_{\max}/R_0=e^{-u^2/4}(1-u^2)^{-1/2}\approx\left[1+\frac{u^2}{4}\right],$$

where $R(0)=R_0$ and $\dot{R}(0)=u$ are the initial position and velocity. For small $\dot{R}(t)$ an analytic solution exists,

$$R(t)\simeq R_{\max}\sin\left[\left[\frac{2}{R_{\max}}\right]^{1/2}(t-t_0)\right],$$

which shows that the return time [defined by $R(t_{\text{ret}})=R_{\max}$] is $t_{\text{ret}}\simeq R_0u$. The above estimates of $R_{\max}$ and t_{ret} are consistent with the numerical results.

For the lepton field, Eq. (31b), the presence of the coupling term is necessary. Let us simplify its form somewhat by assuming a constant coefficient

$$\alpha\equiv\frac{e^2[\pi-f(\rho(t),t)]}{4\pi^2M},$$

which is again justified by the numerically observed smooth spatial dependence of $f(r,t)$. In this approximation, Eq. (31b) admits the first integral

$$2[(1-\dot{\rho}^2)^{1/2}+2(1-\dot{\rho}^2)^{-1/2}]-\alpha/\rho=\text{const}.$$

The maximal value of ρ (obtained from $\dot{\rho}_{\max}=0$) is found to be

$$\rho_{\max}=\rho_0\left[1+\frac{2\rho_0}{\alpha}[3-(1-u^2)^{1/2}-2(1-u^2)^{-1/2}]\right]^{-1},$$

where $\rho(0)=\rho_0$, $\dot{\rho}(0)=u$. The necessary condition for a return effect, $\rho_{\max}>0$, is seen to be satisfied provided $\alpha<0$, which sign is indeed consistent with the numerical observations, since $f(r,t)\simeq2\pi$.

Next, we return to the alternative interpretation of the time oscillations as being due to the propagation of asymptotic linear waves. This interpretation does not exclude the possible presence of a nonlinear mode but only assumes that it is rapidly damped in time. Thus, ignoring the nonlinear mode, the solution after the first reflection would look like

$$f(r,t)=2\pi+\int_0^\infty dk\,\tilde{f}(k)\frac{\sin kr}{kr}\cos\omega_kt,$$
$$\phi(r,t)=\phi_{\bar{K}}(r,t)+\int_0^\infty dk\,\tilde{\phi}(k)\cos(kr-\omega'_kt), \tag{32}$$

where $\phi_{\bar{K}}$ is the antikink traveling solution and the second terms on the right-hand side of Eq. (32) give the Fourier integral expansions for the asymptotic linear waves ($\omega_k=(k^2+M_\pi^2)^{1/2}$, $\omega'_k=(k^2+M^2)^{1/2}$). The numerically observed smooth spatial dependence of $f(r,t)$ and $\phi(r,t)$ suggests that these integrals are dominated by the long-wavelength waves, $k\simeq0$. If this is the case, then

$$f(0,t)\simeq2\pi+C\cos(M_\pi t),\quad \phi(0,t)\sim\phi_{\bar{K}}(r,t)+C'\cos(Mt),$$

i.e., a time oscillating behavior of period $\tau=2\pi/M_\pi\simeq8$, which is not too far from the observed period $\sim5-10$. The property of undamped $k\simeq0$ waves is specific to the (1+1)-dimensional case. (In higher space dimensions, the extra factors of k in the Fourier-integral measure suppress the infrared $k=0$ modes.) It is related to the property of plane waves to give finite $f(0,t)$ and $\phi(0,t)$, and hence carry finite B and L charges. It is interesting to note that similar time oscillations are found to take place in Abelian Higgs model in 1+1 dimensions with respect to the nonconservation of the topological Chern-Simons winding number[23] (sphaleron decay).

To decide between the above two interpretations one needs some analytic, even if approximate, knowledge of the periodic bounded solutions to the field equations, Eq. (18). One could apply for this purpose the scheme of Dashen, Hasslacher, and Neveu[24] and write

$$f(r,t)=\epsilon^2f_0(r)+\sum_{n=0}^{\infty}\{\epsilon^{2n+1}f_{2n+1}(r)\sin[(2n+1)\omega_0t]+\epsilon^{2n+2}f_{2n+2}(r)\cos[(2n+1)\omega_0t]\}, \tag{33}$$

with a similar expansion for $\phi(r,t)$. These expressions consist of simultaneous expansions in Fourier harmonics of a fundamental frequency, parametrized as $\omega_0=2\pi/\tau_0=M_\pi/(1+\epsilon^2)^{1/2}$, and in powers of the parameter

$$\epsilon\equiv\left[\left[\frac{M_\pi\tau_0}{2\pi}\right]^2-1\right]^{1/2}.$$

However, the general physical situation of our present problem does not warrant the nontrivial effort of solving for the $f_n(r)$, $\phi_n(r)$ and subsequently quantizing the periodic motion.

Having clarified the mathematical understanding of the results we proceed now to their physical interpretation. Naively, the observed behavior of a systematic rise of $f(0,t)/2\pi$ and $\phi(0,t)/2\pi$ to unity and their oscillation with time there, independently of b and β, is naturally interpreted as meaning an unbounded critical impact parameter and, hence, an infinite, velocity-independent proton-decay catalysis cross section. Things are as if the baryon-number-violating process is allowed classically or, by reasoning by analogy with potential model scattering, is opposed by a long-range barrier. Practically, this result implies that the classical approximation cannot supply by itself a definite prediction of the reaction scattering amplitude and one needs to consider the quantum level. The usual semiclassical methods[25] (time delay, . . .) are not directly applicable here, since the solution is not known in analytic form and also because we deal with an inelastic process. Perhaps an application of the Hartree coherent-state method, as discussed in Ref. 26, could yield useful results.

Aside from the possible quantum suppression effects, one should recognize that our classical treatment of the Skyrmion-monopole scattering has imposed a natural cutoff, namely, $b \leq R_N$, where R_N is the Skyrmion radius. This corresponds to the condition that the monopole traverses the Skyrmion and would lead to the geometrical cross section $\sigma_{\text{cat}} \sim \pi R_N^2$. On the other hand, the restriction to the $j_{\min}=0$ wave for the proton-monopole system implies that the associated critical impact parameter b_{crit} must lie somewhere between 0 and $1/k$. Setting $b_{\text{crit}}=1/k$ yields the familiar s-wave unitarity bound $\sigma_{\text{cat}} \simeq \pi/k^2 = \pi/M_N^2\beta^2 \simeq 1.4\ \text{mb}/\beta^2$.

Our numerical results contain some useful information on the time scale t_{decay} of the baryon-number-violating processes. Let us first consider the oscillations at the origin. At first sight one is tempted to interpret these as baryon-number oscillations taking place as a result of the mixing at the monopole of $B=0$ and $B=1$ states. However, such an analogy with the familiar case of neutrino or neutral-kaon systems oscillations is appropriate to a quantum-mechanical two-state interference and hence is clearly misplaced in our classical problem. Instead, one should associate the oscillations with the excitation of a bounded state of finite energy in the nonlinear mode interpretation, or with excitation of radiation in the alternative linear mode interpretation. For our geometrical picture to continue to hold, in either case, it is important that the situation be distinct from that of an isolated long-lived resonance. A simple classical estimate of the resonance width Γ may then be obtained by defining Γ as the rate for the system to radiate one energy quantum, $\Gamma=\dot{\mathcal{E}}(t)/\omega$, where ω is the oscillation frequency and $\dot{\mathcal{E}}=d\mathcal{E}/dt$ is the rate of decrease of the energy. We note that, although the amplitudes of the oscillations increase rapidly with b and z_0, the period τ stays roughly constant. The numerical results for the reflectionless boundary case indicate that τ varies from 6 to 10 as b increases from 0.1 to 2. The observed slow rate of decrease of the field energy may be roughly fitted by $[d(E(t)/E(0))/dt] \simeq 0.03$. On the other hand, for the fixed-boundary case, $\tau \simeq 10$. The corresponding estimate for the rate of energy decrease may be obtained by integrating the energy density out to some intermediate distance and gives a roughly comparable result.

To convert from the field energy $E(t)$ to the physical energy $\mathcal{E}(t)$, it seems reasonable to assume proportionality, $E(t)/E(0)=\mathcal{E}(t)/\mathcal{E}(0)$, and to identify $\mathcal{E}(0)$ with the incident center-of-mass energy. Thus, by setting approximately $\mathcal{E}(0)=M_N$, we obtain $\dot{\mathcal{E}}(t)=0.03M_N \simeq 0.14$ and hence

$$t_{\text{decay}} = \frac{1}{\Gamma} = \frac{2\pi}{\tau\dot{\mathcal{E}}} = \frac{2\pi}{0.14\tau} \sim 5-8\ .$$

For an alternative estimate of the transition time, one could take the interval between $t=0$ and the first return time at which $f(0,t)/2\pi$ settles at unity. The results furnish $t_{\text{decay}} \simeq 5-10$, independent of b and β. This estimate matches well with the above one in spite of the fact that the two reasonings cover obviously different physics.

Having obtained the transition time, we may now deduce a transition probability $P(b,\beta)$ by comparing t_{decay} with the time taken by the monopole to traverse the Skyrmion, t_{trav}. Let us set $P(b,\beta)=t_{\text{trav}}/t_{\text{decay}}$ and consider the straight-line trajectory case, for which $t_{\text{trav}}=2(R_N^2-b^2)^{1/2}/\beta$. It seems reasonable to enter $P(b,\beta)$ as an extra factor in the geometrical cross-section formula, hence we write

$$\sigma_{\text{cat}} = \frac{2\pi}{\beta t_{\text{decay}}} \int_0^{R_N^2} db^2 (R_N^2-b^2)^{1/2} = \frac{\pi R_N^2}{\beta} \left[\frac{4R_N}{t_{\text{decay}}} \right] . \tag{34}$$

The present argument has not greatly modified the magnitude of the cross section but it has brought in the welcome $1/\beta$ threshold dependence. Adopting the values $t_{\text{decay}} \simeq 10$, $R_N=0.886$ ($=1$ fm) we find from Eq. (34) $\sigma_{\text{cat}} \simeq 4\ \text{mb}/\beta$. The unitarity bound is exceeded by this estimate at $\beta \gtrsim 0.35$.

VII. CONCLUSIONS

Understanding the role of quark confinement is essential in order to reach an uncontroversial order-of-magnitude estimate of the monopole-induced proton decay cross section. The mechanisms which are known to operate at the quark level involve several unconventional aspects (boundary condition at the monopole, multibody interactions, subprocesses with a variety of initial- and final-state configurations, modified quark parameters, . . .) which are not so easily transferred to the hadronic level. In particular, the quark-model description involves the implicit assumption that the range of the interactions survives up to the hadronic scale.

The current-algebra description adopts a less intuitive but more global approach which is perhaps more satisfactory. The development leading to a $(1+1)$-dimensional radially reduced field theory in the hadronic level runs parallel to the quark level. However, an immediate concern in the Callan-Witten approach is to justify how the configuration of an extended, deformed Skyrmion is projected on the spherically symmetric radial kink configuration. The arguments we have presented are convincing but somewhat intuitive. On the other hand, Craigie, Nahm, and Rubakov[14] have demonstrated that the radially reduced QCD dynamics in the monopole sector exhibits confinement and possesses massless, neutral $q\bar{q}$ states which should then dominate the low-energy dynamics of hadrons in the presence of a monopole. The corresponding mesonic excitations are represented by a field $\pi^0(r,t)$ equivalent to $f(r,t)$, whose nonlinear self-interactions allow for solitonic modes associated with baryons. This observation is important because it completes the logical connection between chromodynamics and chiral dynamics in $3+1$ and $1+1$ dimensions and motivates a quantum interpretation of our radial effective chiral action.

We have seen that the extension from two to three flavors of quarks introduces an additional chiral field (associated with the η meson) and is straightforward. We have also considered the effective Lagrangian with vector mesons and found that the radial reduction preserves the vector-meson-dominance properties and that the standard model is recovered in the limit of large vector-meson mass.

The Callan-Witten approach establishes a remarkable correspondence between (3+1)- and (1+1)-dimensional field theories which operates already at a classical level. Its applicability at the level considered in this work is linked to the possible geometrical character of the catalysis cross section σ_{cat}. Our main contribution has been in an implementation of this approach, restricted essentially to the classical approximation. Nevertheless, we have encountered on our way some potentially important quantal effects.

One important initial step has been in obtaining the initial conditions for the radial kink field. This construction exploits the important observation that a Skyrmion pierced by the Dirac string is deformable to a zero winding-number configuration. We have singled out a particular solution by assuming a sort of sudden approximation, since our identification of the Skyrmion and kink disregards energy conservation. If this assumption is correct, then, in the light of the subsequent results, it is unlikely that a more thoughtful solution would alter our conclusions. On the other hand, quantal effects could arise at this stage, in case the deformation encounters an energy barrier. This circumstance would not invalidate the classical picture but only introduce a suppression factor accounting for the tunneling process. To proceed practically in calculating the probability factor, one could consider the same path as defined by Eq. (22), choose for $f(r,t)$ our simple solution, and generalize to a time-dependent $\lambda(t)$. Substitution in the initial action should then determine the dynamics of $\lambda(t)$.

Our principal effort in this paper has been in the resolution of the field classical equations. We found unexpectedly that charge exchange $P \leftrightarrow e^+$ takes place for impact parameters much larger than the nucleon radius, independently of the initial velocity. This frustrates the hope of establishing at a classical level a suppression of some sort of baryon decay catalysis. An interesting feature of the results is in the appearance of time oscillations in the baryon number soon after the transition. These oscillations are specific to the hadronic level. Their origin is not very clear and so their physical interpretation remains uncertain. They could originate in the excitation of either linear radiation or a nonlinear periodic mode. In the former case, σ_{cat} retains its geometrical character and the finite decay time is responsible for a probability factor in the impact-parameter integral resulting in the rough estimate $\sigma_{\text{cat}} \simeq 1$ mb/β. On the other hand, the second interpretation could invalidate the geometrical character of σ_{cat} if the reaction is dominated by a single isolated resonance. Then, the information on the initial conditions is lost and so a Breit-Wigner representation is more appropriate. Exploring the solution out to large time regimes is numerically prohibitive. This would not even be sufficient, since one needs information on the possible resonances masses, widths, and also on their density. To decide on this issue, one should obtain an approximate analytic solution and apply to it the semiclassical quantization procedure. This program involves complex mathematical analysis and tools which are not very familiar at present.

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APPENDIX A

We summarize some elementary formulas for the differential calculus in curved spherical coordinates in the formalism of general covariance for curved spaces. Tensors of rank k ($k=1,2,\ldots$), A_α, $F_{\alpha\beta}, \ldots$, are given an intrinsic content by associating to them k-forms, e.g.,

$$A = A_{\underline{\alpha}} dx^{\underline{\alpha}} = A_\alpha dx^\alpha, \quad F = \tfrac{1}{2} F_{\alpha\beta} dx^\alpha dx^\beta \tag{A1}$$

(implicit wedge product) where the indices $\alpha = t, r, \theta, \varphi$ label the curved (generally covariant) coordinates and the underlined indices $\underline{\alpha} = \underline{t}, \underline{r}, \underline{\theta}, \underline{\varphi}$ label the tangent-space (Lorentz-covariant) coordinates. The (moving frame) vielbein and inverse vielbein tensors $e^{\underline{\alpha}}_{\alpha}$, $e^{\alpha}_{\underline{\alpha}}$, which one defines as

$$e^{\underline{\alpha}}_{\alpha} = \frac{\partial x^{\underline{\alpha}}}{\partial x^\alpha}, \quad e^{\alpha}_{\underline{\alpha}} = \frac{\partial x^\alpha}{\partial x^{\underline{\alpha}}} \quad (e^{\alpha}_{\underline{\alpha}} e^{\underline{\beta}}_{\alpha} = \delta^{\underline{\beta}}_{\underline{\alpha}},\ e^{\underline{\alpha}}_{\alpha} e^{\beta}_{\underline{\alpha}} = \delta^{\beta}_{\alpha}) , \tag{A2}$$

are used to relate these two coordinate systems:

$$A_\alpha = e^{\underline{\alpha}}_{\alpha} A_{\underline{\alpha}}, \quad A^\alpha = e^{\alpha}_{\underline{\alpha}} A^{\underline{\alpha}} . \tag{A3}$$

The metric is defined by $dx^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ as $g_{\alpha\beta} = e^{\underline{\alpha}}_{\alpha} e^{\underline{\beta}}_{\beta} \bar{g}_{\underline{\alpha}\underline{\beta}}$, with $\bar{g}_{\underline{\alpha}\underline{\beta}}$=const, and the antisymmetric tensor as $\epsilon^{\alpha\beta\gamma\delta} = -(1/|e|)\epsilon_{\alpha\beta\gamma\delta}$, where $e = |\det g|^{1/2}$.

The transformation from the Cartesian to the curvilinear coordinates is achieved via the standard covariant derivative replacement, $\partial_\alpha \to D_\alpha$, where $D_\alpha f = \partial_\alpha f$, $\partial_\alpha A_\beta = (\partial_\alpha A_\beta - \Gamma^\gamma_{\alpha\beta} A_\gamma), \ldots$ and

$$\Gamma_{\alpha\beta;\gamma} \equiv \frac{1}{2} \left[\frac{\partial g_{\alpha\gamma}}{\partial x^\beta} + \frac{\partial g_{\beta\gamma}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\gamma} \right] .$$

The specialization to the spherical-coordinate case gives

$$\begin{aligned}
&dx^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\theta^2 , \\
&\bar{g}_{\underline{\alpha}\underline{\beta}} = (+1, -1, -1, -1) , \\
&\epsilon^{tr\theta\varphi} = -\frac{1}{r^2 \sin\theta} \epsilon_{tr\theta\varphi} = -\frac{1}{r^2 \sin\theta} , \\
&A = A_t dt - (A^r dr + A^\theta d\theta + A^\varphi d\varphi) \\
&\quad = A_{\underline{t}} dt - (A^{\underline{r}} dr + A^{\underline{\theta}} r \, d\theta + A^{\underline{\varphi}} r \sin\theta \, d\varphi) , \\
&\partial^\varphi = e^{\varphi}_{\underline{\alpha}} \partial^{\underline{\alpha}} = \frac{1}{r \sin\theta} \partial^{\underline{\varphi}} \quad \left[\partial^{\underline{\varphi}} = -\partial_{\underline{\varphi}} = -\frac{\partial}{\partial \varphi} \right] , \\
&B^r = \epsilon^{r\theta\varphi} (\partial_\theta A_\varphi - \partial_\varphi A_\theta) \\
&\quad = -\frac{1}{r^2 \sin\theta} \left[\frac{\partial}{\partial\theta} (r \sin A_{\underline{\varphi}}) - \frac{\partial}{\partial \underline{\varphi}} (r A_{\underline{\theta}}) \right] .
\end{aligned} \tag{A4}$$

APPENDIX B

We shall discuss in this appendix two generalizations of the chiral-soliton approach, starting with the three-flavor case. Since the electric charge is related to the $SU(3)_V$ generators as $Q=\frac{1}{2}(\lambda_3+\lambda_8/\sqrt{3})$, it follows that the chiral-field configuration describing the monopole-proton system in the $j_{\min}=0$ wave may involve two classical components associated with neutral (π^0 and η) meson excitations in the λ_3 and λ_8 directions. We shall employ the classical ansatz $U(x)=e^{if(r,t)\lambda_3+ig(r,t)\lambda_8}$ where $f(r,t)$ and $(1/\sqrt{3})g(r,t)$ are defined modulo 2π. This satisfies the continuity equation (11) by virtue of its commuting with Q. We mention parenthetically that in the presence of a θ-vacuum term the θ dependence induced by the axial $U_A(1)$ anomaly may be introduced at the chiral Lagrangian level by means of the replacement $U(x)\to e^{-i\theta/2}U(x)$, $U(x)\in SU(3)$. This constraint on the phase of the chiral field permits consistent transformation rules under $U_A(1)$. The normalization of the phase angle θ (not to be confused with θ_{QCD}) has been set so as to yield by substitution into the gauged anomalous action, Eq. (21), the expected axial anomaly term: namely,

$$-\frac{e^2\theta}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}=\frac{-e^2\theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B}\ .$$

[The θ dependence can be removed by a $U_A(1)$ transformation in the massless quark limit.]

The derivation followed in the text for the $SU(2)_F$ case applies unchanged, and the effective action which emerges nearly coincides with (15). The result reads

$$\Gamma_{\rm eff}(f,g,\phi,\phi_a)=\int\int dr\,dt\left[4\pi r^2\left[\frac{F_\pi^2}{8}[(\partial_\mu f)^2+(\partial_\mu g)^2]+\frac{F_\pi^2M_\pi^2}{4}\left\{\cos f\cos\left[\frac{g}{\sqrt{3}}\right]-1+\frac{m_s}{m_u+m_d}\left[\cos\left[\frac{2g}{\sqrt{3}}\right]-1\right]\right\}+\frac{1}{4f_s^2}[(\partial_\mu f)^2+(\partial_\mu g)^2]^2\right]+2\pi r^2(\Box\phi_a)^2+\frac{1}{8\pi}(\partial_\mu\phi)^2+\frac{M^2}{4\pi}(\cos\phi-1)+\frac{e}{2\pi}\partial^\mu\phi_a\partial_\mu\left[f+\frac{g}{\sqrt{3}}-\phi+\theta\right]\right]\ . \tag{B1}$$

The θ dependence imposes the constraint $(f+g/\sqrt{3}-\phi)\to-\theta$ as a necessary condition for a finite action. Since the electric current is given by

$$J_\mu^Q(r,t)=-\frac{e}{2\pi}\epsilon_{\mu\nu}\partial^\nu\left[f+\frac{g}{\sqrt{3}}-\phi\right]\ , \tag{B2}$$

this means that the monopole sector has an induced electric charge $-e\theta/2\pi$, which is just the familiar Witten dyonic charge.[27] The charge-conservation boundary condition at the origin is now replaced by $f(0)+g(0)/\sqrt{3}-\phi(0)=0$.

The construction of initial data for the fields f and g involves similar steps as in the two-flavor case (Sec. III). One needs to consider the interpolating field path

$$U^\lambda(\mathbf{x},t)=e^{iQ\varphi}\{\lambda h(\theta,r)+[1-\lambda h(\theta,r)]u+\delta\}e^{-iQ\varphi}\ , \tag{B3}$$

where $u=e^{if\lambda_3+ig\lambda_8}$, and to determine the 3×3 matrix $\delta=\delta(\theta,r)$ which satisfies the unitarity constraint

$$\begin{aligned}0&=(U^{\lambda\dagger}U^\lambda-1)\\&=2\lambda h(\lambda h-1)+\delta\delta^\dagger+\lambda h(1-\lambda h)(u+u^\dagger)\\&\quad+(1-\lambda h)(u^\dagger\delta+\delta^\dagger u)\ .\end{aligned}$$

To solve for δ one could consider the decomposition on U(3) generators $\delta=\delta_0+\delta_1\lambda_1+\delta_2\lambda_2+\delta_8\lambda_8$ and obtain thereby a closed system of equations for the constant coefficients δ_0, δ_1, δ_2, and δ_8. However, one faces then the somewhat intractable problem of solving for a set of coupled second-degree equations.

We next turn to the important generalization concerning the effective Lagrangians incorporating vector mesons. As is well known, these Lagrangians exist in two main versions: the linear or vector-meson-dominance version, in which vector mesons are gauge bosons of the gauged chiral symmetry, and the nonlinear realization case, in which vector mesons arise as gauge bosons of a hidden local symmetry. We shall only discuss here the former linear-basis realization, restricted to the two-flavor case and without the axial-vector mesons. This has been discussed in Refs. 28 and 29. The extended $U(1)_{em}$ gauged effective mesonic action which satisfies the current algebra and exact vector-meson dominance reads

$$\Gamma(U,\mathbf{V}_\mu,\omega_\mu,a_\mu)=\int d^4x\left\{\frac{F_\pi^2}{8}\left[(\partial_\mu\phi)^2+\left[\frac{\sin\phi}{\phi}\right]^2\frac{1}{1-\alpha\sin^2\phi}(\boldsymbol{\phi}\times\mathcal{D}_\mu\boldsymbol{\phi})^2\right]-\tfrac{1}{4}\mathbf{V}_{\mu\nu}^2-\tfrac{1}{4}\omega_{\mu\nu}^2+\tfrac{1}{2}(m_V^2\mathbf{V}_\mu^2+m_\omega^2\omega_\mu^2)\right.$$
$$-\frac{N_c g}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\left[g\boldsymbol{\phi}\cdot\mathbf{V}_{\alpha\beta}+\frac{1}{\phi^2}\left[-1+\frac{\sin2\phi}{2\phi(1-\alpha\sin^2\phi)^2}\right]\boldsymbol{\phi}\cdot(\mathcal{D}_\alpha\boldsymbol{\phi}\times\mathcal{D}_\beta\boldsymbol{\phi})\right]$$
$$\left.-\tfrac{1}{4}F_{\mu\nu}^2-\frac{e}{g}a^\mu\left[-m_V^2V_\mu^0+\frac{m_\omega^2}{3}\omega_\mu\right]\right\}. \tag{B4}$$

Here V_μ^a, ω_μ, and a_μ are the ρ-meson, ω-meson, and photon fields,

$$U=e^{i\tau\cdot\phi},\quad \phi=|\boldsymbol{\phi}|,\quad \mathcal{D}_\alpha\boldsymbol{\phi}=(\partial_\alpha\boldsymbol{\phi}-g\boldsymbol{\phi}\times\mathbf{V}_\alpha),\quad \mathbf{V}_{\alpha\beta}=\partial_\alpha\mathbf{V}_\beta-\partial_\beta\mathbf{V}_\alpha+g(\mathbf{V}_\alpha\times\mathbf{V}_\beta)\ ,$$

and conventional notation is used for the mass and coupling-constant parameters, m_V^2, m_ω^2, g, and $\alpha\equiv(F_\pi g/2m_V)^2=\frac{1}{2}$. For a truly gauge-invariant formulation of vector-meson dominance, the last contact terms in Eq. (B4) should be replaced according to $(m_V^2/g)V_\mu^0a^\mu\to J_\mu^Va^\mu+(1/2g)V_{\mu\nu}^0F^{\mu\nu}$, where J_μ^V is the vector-meson source current as defined by the field equation $\partial_\mu\mathbf{V}^{\mu\nu}+m_V^2\mathbf{V}^\nu=g\mathbf{J}^{V\nu}$. We note that the limits $m_V^2\to\infty$ and $m_\omega^2\to\infty$ render the vector mesons nondynamical and hence bring one back to the standard Skyrme model case.

The continuity condition at the Dirac string, corresponding to Eq. (11), is equivalent to that imposed on the em field, namely,

$$g(\varphi)\left[\frac{\tau^a}{2i}V_\mu^a(r,\,\theta=0,\,\varphi,t)+\frac{1}{e}\partial_\mu\right]g(\varphi)^\dagger=\frac{\tau^a}{2i}V_\mu^a(r,\,\theta=0,\,\varphi=0,\,t)\ , \tag{B5}$$

i.e, a ϕ-independent left-hand side. A similar equation holds for $(1/2i)\omega_\mu(x)$. Thus the $j_{\min}=0$ field configuration may be chosen as

$$\begin{aligned}&V_\mu^a(r,\theta,\varphi,t)=\delta_{a0}[\rho_t(r,t),\rho_r(r,t),\rho_\theta(r,\theta,\varphi,t),\rho_\varphi(r,\theta,\varphi,t)]\ ,\\&\omega_\mu(r,\theta,\varphi,t)=[\omega_t(r,t),\omega_r(r,t),\omega_\theta(r,\theta,\varphi,t),\omega_\varphi(r,\theta,\varphi,t)]\ .\end{aligned} \tag{B6}$$

Let us make here the technically simplifying assumption that the θ and φ components of the vector-meson field assume the form appropriate to the limits $m_V^2\to\infty$, $m_\omega^2\to\infty$. This implies that $\rho_\theta\equiv0$, $\omega_\theta\equiv0$ and prescribes the proportionality relations $\omega_\varphi=(e/3g)a_\varphi=-\frac{1}{3}\rho_\varphi$, so that we may then deal with notation appropriate to a $(1+1)$-dimensional space-time. The reduced chiral action reads

$$\Gamma_{\text{eff}}(f,\rho_\mu,\omega_\mu,a_\mu)=\int\int dr\,dt\left[\frac{\pi}{2}F_\pi^2r^2(\partial_\mu f)^2+\pi F_\pi^2M_\pi^2r^2(\cos f-1)\right.$$
$$+4\pi r^2\left[-\tfrac{1}{2}\omega_{\mu\nu}^2-\tfrac{1}{2}\rho_{\mu\nu}^2+\tfrac{1}{2}m_V^2\rho_\mu^2+\tfrac{1}{2}m_\omega^2\omega_\mu^2+\frac{e}{g}a_\mu(m_V^2\rho^\mu-\tfrac{1}{3}m_\omega^2\omega^\mu)-\tfrac{1}{2}a_{\mu\nu}^2\right]$$
$$\left.-\frac{(N_c/3)g}{4\pi}\epsilon^{\mu\nu}\partial_\mu(\rho_\nu-3\omega_\nu)f\right]\ , \tag{B7}$$

where

$$-\tfrac{1}{2}\omega_{\mu\nu}^2\equiv-\tfrac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu}=\tfrac{1}{2}(\dot\omega_r-\omega_t')^2,\quad \rho_\mu a^\mu=\rho_ta_t-\rho_ra_r,\quad -\tfrac{1}{2}a_{\mu\nu}^2=\tfrac{1}{2}(\dot a_r-a_t')^2,\quad \text{etc.},$$

and we have subtracted away the static monopole magnetic energy. Based on the relationship of the vector-meson fields to the em field, we shall further specialize to the simplified parametrization $\rho_\mu=(e/g)\epsilon_{\mu\nu}\partial^\nu\rho$, $\omega_\mu=-(e/3g)\epsilon_{\mu\nu}\partial^\nu\omega$. This leads us to the total effective action

$$\Gamma_{\text{eff}}(f,\rho,\omega,\phi,\phi_a)=\int\int dr\,dt\left\{\frac{\pi}{2}F_\pi^2r^2(\partial_\mu f)^2+\pi F_\pi^2M_\pi^2r^2(\cos f-1)\right.$$
$$+4\pi r^2\left[\frac{1}{2}\left[\frac{e}{3g}\right]^2[(\Box\omega)^2-m_\omega^2(\partial_\mu\omega)^2+2m_\omega^2\partial_\mu\omega\partial^\mu\phi_a]\right.$$
$$\left.+\frac{1}{2}\left[\frac{e}{g}\right]^2[(\Box\rho)^2-m_V^2(\partial_\mu\rho)^2+2m_V^2\partial_\mu\rho\partial^\mu\phi_a]+\tfrac{1}{2}(\Box\phi_a)^2\right]$$
$$\left.+\frac{1}{8\pi}(\partial_\mu\phi)^2+\frac{M^2}{4\pi}(\cos\phi-1)+\frac{e}{2\pi}\phi\Box\phi_a-\frac{N_c}{3}\frac{e}{4\pi}f(\Box\rho+\Box\omega)\right\}\ , \tag{B8}$$

where we have incorporated at this stage the electron field action. The limits $m_V \to \infty$, $m_\omega \to \infty$ permit an exact elimination of the vector-meson fields via the field equations, yielding $\rho_\mu = -(e/g)a_\mu = -3\omega_\mu$ or $\rho = \phi_a = +\omega$, reducing thereby (B8) to the standard Skyrme-model result, Eq. (15) (with $f_s^{-2}=0$). The direct elimination of ρ_μ and ω_μ from Eqs. (B7) or (B8) introduces a spurious photon mass and kinetic energy terms:

$$-\frac{1}{2}\left[\left[\frac{m_V e}{g}\right]^2 + \left[\frac{m_\omega e}{3g}\right]^2\right] a_\mu a^\mu - \frac{1}{2}\left[1+\frac{e^2}{g^2}+\left[\frac{e}{3g}\right]^2\right] a_{\mu\nu}a^{\mu\nu} .$$

These terms exactly cancel away in the extended formulation of vector-meson dominance employing the gauge-invariant contact coupling term.

The baryonic and electric currents are prescribed directly in terms of the vector-meson fields:

$$\frac{J^B_\mu(r,t)}{4\pi r^2} = 2\frac{m_\omega^2}{3g}\omega_\mu(r,t) = -\frac{2em_\omega^2}{(3g)^2}\epsilon_{\mu\nu}\partial^\nu\omega ,$$
$$\frac{J^Q_\mu(x)}{e4\pi r^2} = -\frac{m_V^2}{g}\rho_\mu(r,t) + \frac{m_\omega^2}{3g}\omega_\mu(r,t) - \bar{\chi}(r,t)\bar{\gamma}_\mu\chi(r,t)$$
$$= \epsilon_{\mu\nu}\partial^\nu\left[-\frac{1}{g^2}\left[m_V^2\rho + \frac{m_\omega^2}{9}\omega\right] + \frac{1}{\sqrt{\pi}}\phi\right] . \qquad \text{(B9)}$$

By invoking the equation of motion for the ρ field,

$$4\pi\{\partial^\nu[r^2(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu)] + m_V^2 r^2\rho_\mu\} - \frac{N_c g}{12\pi}\epsilon^{\mu\nu}\partial_\nu f = 0 , \qquad \text{(B10)}$$

with a similar equation for the ω field, and taking the integral over space, one finds that the charges B and Q are again fixed in terms of the field $f(r,t)$ through the same relations, Eq. (16), as in the standard Skyrme-model case.

The initial data for the radial vector-meson fields $\rho_\mu(r,t)$, $\omega_\mu(r,t)$ are found by identification with a displaced, moving Skyrmion configuration based on the familiar rest-frame ansatz,

$$V^{am}(\mathbf{R}) = -\epsilon_{\text{mak}}\frac{R^k}{R^2}[\hat{\phi}_2(R)]_{\text{Sk}}, \quad \omega_t(R) = [\omega(R)]_{\text{Sk}} , \qquad \text{(B11)}$$

all the other components vanishing. We consider again an average over the spatial orientations of the Skyrmion. Allowing for the action of the Lorentz-boost transformation on the vector-field t and z components and the translated center (z_0), one finds for the radial ρ-meson field $\rho_\mu(r,t)=0$ $(\mu=t,r)$ and, for the radial ω meson field,

$$[\omega^t(r,t), \omega^r(r,t)] = \gamma\int\frac{d\Omega_{\hat{x}}}{4\pi}[\omega(R)]_{\text{Sk}}[1,\beta\cos\theta] , \qquad \text{(B12)}$$

where $\mathbf{R}=(\mathbf{b},\gamma(z-\beta t-z_0))$, $R=|\mathbf{R}|$, $\gamma=(1-\beta^2)^{-1/2}$. The radial pseudoscalar kink field $f(r)$ is the same as in the standard case.

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