Symmetry breaking in three-generation Calabi-Yau manifolds

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The spontaneous breaking of $[SU(3)]^3$ symmetry to $SU(3)\times SU(2)\times U(1)$ due to nonrenormalizable interactions in the three-generation Calabi-Yau superstring model is considered. It is seen that these models lead naturally to intermediate scales $M_I \sim 10^{15}$ GeV with simultaneous formation of N and ν^c vacuum expectation values (VEV's). The lowest-lying extrema automatically preserve matter parity and hence protect the model against too rapid proton decay. Models with matter parity also protect against electroweak Higgs VEV's from forming at M_I. The intermediate mass scale produces some particles with mass $O(M_I)$. It also implies the existence of others with mass ~ 1 TeV, which are possibly accessible to low-energy experiments.

I. INTRODUCTION

The last several years has seen a large amount of analysis of the properties of the compactified ten-dimensional heterotic string.[1] While a number of difficult problems must be solved before a fundamental understanding of the consequences of this theory is obtained,[2] the phenomenological aspects of models that maintain four-dimensional $N=1$ supergravity after compactification[3,4] remain a compelling framework for unification of interactions. In this picture, the ten-dimensional space-time compactifies to $M_4\times K$, where M_4 is Minkowski space and K is a compact six-dimensional Calabi-Yau manifold. The only known Calabi-Yau manifold with three generations[5] can be defined as $CP^3\times CP^3/Z_3$, which is the manifold with coordinates x_i, y_i, $i=0,1,2,3$ obeying

$$P_1 \equiv \sum x_i^3 + a_1 x_0 x_1 x_2 + a_2 x_0 x_1 x_3 = 0 \ , \tag{1.1a}$$

$$P_2 \equiv x_0 y_0 + c_1 x_1 y_1 + c_2 x_2 y_2 + c_3 x_3 y_3 + c_4 x_2 y_3 + c_5 x_3 y_2 = 0 \ , \tag{1.1b}$$

$$P_3 \equiv \sum y_i^3 + b_1 y_0 y_1 y_2 + b_2 y_0 y_1 y_3 = 0 \ . \tag{1.1c}$$

Thus K depends on nine, *a priori* arbitrary complex parameters: a_i, b_i, c_i. The zero-mass states at the compactification scale M_c are given by the Hodge numbers of K. There are $h_{2,1}=9$ generations of **27** representation and $h_{1,1}=6$ generations of $\overline{\mathbf{27}}$ representation of the gauge group E_6. Here M_c is $O(M_P)$ where the Planck mass M_P is 2.4×10^{18} GeV. The nonsimply connected nature of $CP^3\times CP^3/Z_3$ allows for a flux breaking[6] of E_6 to the rank-6 group $[SU(3)]^3$ at M_c (Ref. 4), where

$$[SU(3)]^3 \equiv SU(3)_C \times SU(3)_L \times SU(3)_R \ . \tag{1.2}$$

After flux breaking, the remaining massless matter fields can be characterized by their $[SU(3)]^3$ content. There remain[4] nine families of leptons $L_r^l(1,3,\overline{3})$ (from the nine generations of **27**), six families of "mirror" leptons $\overline{L}_l^r(1,\overline{3},3)$ (from the six generations of $\overline{\mathbf{27}}$); seven families of quarks $Q_l^a(3,\overline{3},1)$ and four families of "mirror" quarks $\overline{Q}_a^l(\overline{3},3,1)$; and seven families of antiquarks $(Q^c)_a^r(\overline{3},1,3)$ and four families of "mirror" antiquarks $(\overline{Q}^c)_r^a(3,1,\overline{3})$. The letters $(a,l,r)=1,2,3$ label the SU(3) (color, left, right) component. The nonets of particles in L, Q, Q^c are given in terms of standard-model particles in Appendix A.

The SU(5) content of the **27** of E_6 is

$$\mathbf{27} = (M_{10} + M_{\overline{5}}) + (H_5 + H_{\overline{5}}) + (\nu^c + N) \ , \tag{1.3}$$

where $M_{10}+M_{\overline{5}}$ are the standard-model quarks and leptons, H_5 and $H_{\overline{5}}$ are the **5** and $\overline{\mathbf{5}}$ representations of Higgs particles needed in supersymmetry (SUSY) unification, and ν^c and N are SU(5) singlets. [N is also an O(10) singlet.] A phenomenologically acceptable theory requires that the $[SU(3)]^3$ symmetry at M_c break to the standard model at some lower intermediate scale M_I. This can be accomplished if the scalar components of both N and ν^c grow vacuum expectation values (VEV's). The cubic (renormalizable) contributions to the superpotential W are generally F flat [see Eq. (A5)] and so it has been suggested[4] that N and ν^c VEV growth arise from the nonrenormalizable quartic or higher contributions to W. Qualitative analyses[4] have indicated the possible validity of this scenario, though a number of difficulties have been raised.[7] It is the purpose of this paper to examine this question in detail and to see under what circumstances one might expect N and ν^c VEV's to occur at sufficiently large M_I.

A fundamental investigation of this question would require the detailed calculation of both the renormalizable and nonrenormalizable Yukawa coupling constants from the properties of the Calabi-Yau manifold. The full Calabi-Yau manifold of Eqs. (1.1), however, is quite complicated, depending on 18 real parameters. Thus, obtaining even the cubic coupling constants[8] for the general case would be quite difficult, while very symmetric special cases are probably not phenomenologically viable. In addition, there are the difficult problems of calculating the kinetic energy normalizations and the nonrenormalizable couplings. While some progress has recently been made on these problems,[9] it is not yet possible to obtain all the needed Yukawa couplings from first principles.

We will, therefore, take a less fundamental approach in this paper and investigate what the structure of the Yukawa couplings must be in order to achieve N and ν^c VEV's at $M_I \approx 10^{15}$ GeV without specifying the exact values of the Yukawa coupling constants. A central idea in obtaining a phenomenologically acceptable theory is the idea of matter parity M_2 (Refs. 4 and 10). Violation of matter parity would allow rapid proton decay. We thus restrict our discussion to Calabi-Yau manifolds that maintain matter parity. However, it is also necessary that the spontaneous breaking of $[\mathrm{SU}(3)]^3$ to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ at M_I does not also break M_2. We will see below, from explicit minimization of the effective potential, that there exist reasonable superpotentials where the M_2-preserving minimum does indeed lie *below* the M_2-violating extrema. These potentials imply *simultaneous* VEV growth of N and ν^c (with $\langle N \rangle, \langle \nu^c \rangle \approx M_I$) so that $[\mathrm{SU}(3)]^3$ breaks directly to the standard model at $M_I \approx 10^{15}$ GeV. In addition, the VEV's of the Higgs doublets, $\langle H \rangle$ and $\langle H' \rangle$, remain zero so that $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ breaking does not incorrectly occur at M_I. Thus the matter-parity-preserving Calabi-Yau manifolds can lead naturally to potentials in general accord with low-energy phenomenology.

In Sec. II we will review the definition of matter parity and summarize the constraints it produces on the superpotential and mass matrices. In Sec. III we exhibit the form of the superpotential and discuss the minimization of the effective potential leading to the spontaneous breaking at M_I. Section IV is devoted to the analysis of the size of the N and ν^c VEV's and masses. One of the unexpected features of the intermediate-scale scenario is that even though $\langle N \rangle$ is very large (i.e., $\gtrsim 10^{14}$ GeV to prevent too rapid proton decay[11]) the effective potential always leads to new exotic particles of electroweak mass which may be accessible to colliders. The phenomenology of these particles will be discussed elsewhere.[12]

II. MATTER-PARITY CONSTRAINTS

Matter parity for the three-generation superstring theory is defined by[4,10]

$$M_2 = CU_z \ , \tag{2.1}$$

where U_z is an element of $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$,

$$U_z = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & & \\ & -1 & \\ & & -1 \end{bmatrix} \otimes \begin{bmatrix} -1 & & \\ & -1 & \\ & & 1 \end{bmatrix} \tag{2.2}$$

and C is a transformation of the Calabi-Yau coordinates

TABLE I. C parity of lepton and quark generations (Ref. 4). The labeling of the generations is that of Ref. 4, $L_{1\pm} = (L_1 \pm L_2)/\sqrt{2}$, etc.

C-even states	C-odd states
$L_{1+}, L_{3+}, L_5, L_7, L_{8+}$	$L_{1-}, L_{3-}, L_6, L_{8-}$
$Q_1, Q_2, Q_3, Q_{4+}, Q_{6+}$	Q_{4-}, Q_{6-}
$Q_1^c, Q_2^c, Q_3^c, Q_{4+}^c, Q_{6+}^c$	Q_{4-}^c, Q_{6-}^c
$\bar{L}_1, \bar{L}_2$	$\bar{L}_3, \bar{L}_4, \bar{L}_5, \bar{L}_6$
$\bar{Q}_{1+}, \bar{Q}_{3+}; \bar{Q}_{1+}^c, \bar{Q}_{3+}^c$	$\bar{Q}_{1-}, \bar{Q}_{3-}; \bar{Q}_{1-}^c, \bar{Q}_{3-}^c$

$$C[(x_0,x_1,x_2,x_3) \times (y_0,y_1,y_2,y_3)] = (x_0,x_1,x_3,x_2) \times (y_0,y_1,y_3,y_2) \ . \tag{2.3}$$

The most general Calabi-Yau manifold having C as a discrete symmetry is then

$$P_1 = \sum x_i^3 + a_1(x_0x_1x_2 + x_0x_1x_3) = 0 \ , \tag{2.4a}$$

$$P_2 = x_0y_0 + c_1x_1y_1 + c_2(x_2y_2 + x_3y_3) + c_3(x_2y_3 + x_3y_2) = 0 \ , \tag{2.4b}$$

$$P_3 = \sum y_i^3 + b_1(y_0y_1y_2 + y_0y_1y_3) = 0 \ . \tag{2.4c}$$

This is still a rather general manifold, depending on five complex parameters. From Appendix A we see that q^λ, u^c, d^c and l^λ, e^c, ν^c are odd under U_z while H^λ H'_λ, D, D^c, and N are even. The properties under C have been obtained from the polynomial representation of the generations in Ref. 4. The C-even and C-odd generations of leptons and quarks are listed in Table I. In the following we will adopt the following notation for generation indices:

$$i = (n,r), \quad n = C \text{ even}, \quad r = C \text{ odd} \tag{2.5}$$

(e.g., from Table I, for the lepton nonet L_i one has $n = 1+, 3+, 5, 7, 8+$ and $r = 1-, 3-, 6, 8-$.) Combining Table I with the above stated properties of the particle states under U_z, one obtains the matter parity of each state. These are listed in Table II. (The mirror states have the same M_2 parity.)

Matter-parity invariance restricts the allowed coupling structures. The restrictions on the cubic interactions of Eq. (A5) are

$$\begin{aligned} &\lambda_{rst}^{1,2,3} = 0 = \lambda_{mnr}^{1,2,3} \ , \\ &\lambda_{rst}^4 = 0 = \lambda_{mnr}^4 = \lambda_{mrn}^4 = \lambda_{rmn}^4 \ . \end{aligned} \tag{2.6}$$

Equation (2.5) holds for the general Calabi-Yau mani-

TABLE II. M_2 (matter) parity of particle states. Generation notation is given in Eq. (2.5), and particle notation is given in Appendix A.

M_2-even states	M_2-odd states
l_r, e_r^c, ν_r^c	l_n, e_n^c, ν_n^c
q_r, u_r^c, d_r^c	q_n, u_n^c, d_n^c
D_n, D_n^c, N_n	D_r, D_r^c, N_r
H_n, H_n'	H_r, H_r'

folds Eqs. (2.4). After symmetry breaking at M_I, quark and lepton mass terms will grow in the superpotential of the form

$$W_{\rm mass}=M_{ij}^{(q)}q_i\bar{q}_j+M_{ij}^{(l)}l_i\bar{l}_j\ . \qquad (2.7)$$

If matter parity is preserved by the symmetry breaking, one sees from Table II that the only surviving terms will be

$$W_{\rm mass}=(M_{rs}^{(q)}q_r\bar{q}_s+M_{mn}^{(q)}q_m\bar{q}_n)+(M_{rs}^{(l)}l_r\bar{l}_s+M_{mn}^{(l)}l_m\bar{l}_n)\ . \qquad (2.8)$$

From Table I we see for the quarks that $r=4-,6-$ and $s=1-,3-$ so that $M_{rs}^{(q)}$ is a 2×2 matrix. Barring accidental zeros in $M_{rs}^{(q)}$, we expect all the C-odd q_r states therefore to acquire masses. However, $M_{mn}^{(q)}$ is a 5×2 matrix so that only two combinations of q_m become massive while three linear combinations of q_m remain massless. These latter are the three light generations of low-energy physics. Similarly all four C-odd leptons l_r become heavy, while two C-even combinations become massive and three C-even combinations of l_m remain massless. All the mirror quarks and leptons will grow masses.

Thus, if matter parity is preserved under gauge symmetry the three light generations we see at low energies must come from the C-even states of the Calabi-Yau manifold and all other quarks and leptons are massive. How massive these exotic particles are, i.e., how large the matrices $M^{(q)}$ and $M^{(l)}$ are, depends on the dynamical details of the symmetry breaking at M_I, which is discussed in Sec. III. We will see that, remarkably, even though $M_I\sim10^{15}$ GeV, the intermediate-scale dynamics generally implies the existence of new exotic light states (i.e., $\lesssim1$ TeV) which may lead to phenomenological signals at colliders.

III. N AND ν^c VEV GROWTH

The origin of supersymmetry breaking in superstring theory remains unclear. For the purposes of this paper we assume that some mechanism exists at M_c leading to the growth of soft breaking terms there (e.g., gaugino masses). These terms then feed into the physical sector allowing the running mass of one or more multiplets to turn negative at a scale[13] $\mu=M_I$. The nonrenormalizable terms in the superpotential then allow for the possibility of gauge symmetry breaking via N and ν^c VEV growth.[4] If matter parity is preserved in this symmetry breaking, Table II implies that only N_n and ν_r^c can grow VEV's. One could then make a linear transformation in generation space so that the nonvanishing N VEV lies in one C-even generation, and the nonvanishing ν^c VEV lies in one C-odd generation, and similarly for the mirror generations.

We consider, therefore, a model containing four generations: one C-odd generation and one C-even generation for both particles and mirror particles. The effective potential then has the form

$$V=V_m+V_F+V_D\ . \qquad (3.1)$$

The running mass term V_m arising from supersymmetry breaking contains the terms $-m_i^2L_iL_i^\dagger-\bar{m}_i^2\bar{L}_i\bar{L}_i^\dagger$. Retaining only those (neutral) fields that can grow VEV's one has

$$V_m=-\sum m_i^2(x_i+y_i+z_i+\omega_i)-\sum\bar{m}_i^2(\bar{x}_i+\bar{y}_i+\bar{z}_i+\bar{\omega}_i)\ . \qquad (3.2)$$

Here $i=1,2$ in Eq. (3.2) where now $i=1$ is the C-even state and $i=2$ is the C-odd state and

$$\begin{aligned}&x_i=N_iN_i^\dagger,\quad y_i=\nu_i^c\nu_i^{c\dagger},\quad z_i=H_i^2H_i^{2\dagger},\quad \omega_i=H_{2i}'H_{2i}'^\dagger\ ,\\ &\bar{x}_i=\bar{N}_i\bar{N}_i^\dagger,\quad \bar{y}_i=\bar{\nu}_i^c\bar{\nu}_i^{c\dagger},\quad \bar{z}_i=\bar{H}_i^2\bar{H}_i^{2\dagger},\quad \bar{\omega}_i=\bar{H}_{2i}'\bar{H}_{2i}'^\dagger\ .\end{aligned} \qquad (3.3)$$

$-m_i^2$ and $-\bar{m}_i^2$ are the running masses (which we assume have turned negative) and are ~1 TeV. Note that $[\mathrm{SU}(3)]^3$ invariance requires there be a common mass in each multiplet.

The F term is derived from the nonrenormalizable terms of the superpotential of form $(27\times\overline{27})^n/(M_c)^{2n-3}=(L_r^l\bar{L}_r^l)^n/M_c^{2n-3}$. For the neutral fields one has

$$W_{nr}=\sum\frac{\lambda_i^2}{(M_c)^{2n-3}}(f_i)^n\ , \qquad (3.4a)$$

$$f_i\equiv N_i\bar{N}_i+\nu_i^c\bar{\nu}_i^c+H_i^2\bar{H}_i^2+H_{2i}'\bar{H}_{2i}'\ , \qquad (3.4b)$$

where λ_i is the effective coupling constant of the heavy fields to the light fields. One has for V_F then

$$V_F=\frac{n^2}{M_c^{4n-6}}\sum\lambda_i^4(f_if_i^\dagger)^{n-1}(x_i+\bar{x}_i+y_i+\bar{y}_i+z_i+\bar{z}_i+w_i+\bar{w}_i)\ . \qquad (3.5)$$

The D terms are constructed from the $\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R$ gauge transformation properties of the fields of Appendix A. One finds

$$\begin{aligned}V_D=&\frac{g_L^2}{24}\left[\sum[x_i-\bar{x}_i+y_i-\bar{y}_i-\tfrac{1}{2}(z_i-\bar{z}_i+w_i-\bar{w}_i)]\right]^2+\frac{g_L^2}{32}\left[\sum(-z_i+\bar{z}_i+w_i-\bar{w}_i)\right]^2\\ &+\frac{g_L^2}{8}\left|\sum(H_{2i}'^\dagger\nu_i^c-\bar{\nu}_i^{c\dagger}\bar{H}_{2i}')\right|^2+\frac{g_R^2}{24}\left[\sum[x_i-\bar{x}_i-\tfrac{1}{2}(y_i-\bar{y}_i+z_i-\bar{z}_i+w_i-\bar{w}_i)]\right]^2\\ &+\frac{g_R^2}{32}\left[\sum(y_i-\bar{y}_i-z_i+\bar{z}_i+w_i-\bar{w}_i)\right]^2+\frac{g_R^2}{8}\left|\sum(N_i^\dagger\nu_i^c-\bar{\nu}_i^{c\dagger}\bar{N}_i)\right|^2\ .\end{aligned} \qquad (3.6)$$

Here $g_{L,R}$ are the $SU(3)_{L,R}$ gauge coupling constants. We note that unless m_i^2 and $\overline{m}_i^2$ are accidentally equal the D term will make significant contributions to the extrema equations (and to the mass matrices). There is no reason to expect the cubic couplings [Eq. (A5)] of the Calabi-Yau manifold of Eqs. (2.4) to produce equal running masses for the **27** and $\overline{\mathbf{27}}$ multiplets (the Yukawa coupling constants are different). The usual neglect of D terms[4] thus does not appear justified.

It is straightforward now to write down the extrema equations for Eqs. (3.1), (3.2), (3.5), and (3.6). We consider here only the case when all VEV's are real, and discuss in detail $n=2$ of Eq. (3.4). The extrema equations allow one to divide the class of solutions into two cases: those with vanishing Higgs VEV's (i.e., $z_i=\overline{z}_i=w_i=\overline{w}_i=0$) and those where the Higgs VEV's are nonzero. We consider the former case first. The potential then contains a number of extrema, some of which both preserve or break matter parity (including the case with no symmetry breaking at all where all the VEV's vanish). However, the lowest-lying extrema is the one where *matter parity is conserved.*[14] We find

$$N_1^2=\frac{\Sigma_1 M_c}{\sqrt{24}\lambda_1^2}+\frac{1}{g_R^2}\frac{(g_L^2+g_R^2)\Delta_1^2+(\frac{1}{2}g_R^2-g_L^2)\Delta_2^2}{g_L^2+\frac{1}{4}g_R^2}+O\left[\frac{\Delta_1^4}{\Sigma_1 M_c}\right], \tag{3.7a}$$

$$(\nu_2^c)^2=\frac{\Sigma_2 M_c}{\sqrt{24}\lambda_2^2}+\frac{1}{g_R^2}\frac{(\frac{1}{2}g_R^2-g_L^2)\Delta_1^2+(g_L^2+g_R^2)\Delta_2^2}{g_L^2+\frac{1}{4}g_R^2}+O\left[\frac{\Delta_2^4}{\Sigma_2 M_c}\right], \tag{3.7b}$$

$$N_2=0=\nu_1^c\ , \tag{3.7c}$$

$$\overline{N}_1^2=\frac{\Sigma_1 M_c}{\sqrt{24}\lambda_1^2}-\frac{1}{g_R^2}\frac{(g_L^2+g_R^2)\Delta_1^2+(\frac{1}{2}g_R^2-g_L^2)\Delta_2^2}{g_L^2+\frac{1}{4}g_R^2}+O\left[\frac{\Delta_1^4}{\Sigma_1 M_c}\right], \tag{3.7d}$$

$$(\overline{\nu}_2^c)^2=\frac{\Sigma_2 M_c}{\sqrt{24}\lambda_2^2}-\frac{1}{g_R^2}\frac{(\frac{1}{2}g_R^2-g_L^2)\Delta_1^2+(g_L^2+g_R^2)\Delta_2^2}{g_L^2+\frac{1}{4}g_R^2}+O\left[\frac{\Delta_2^4}{\Sigma_2 M_c}\right], \tag{3.7e}$$

$$\overline{N}_2=0=\overline{\nu}_1^c\ , \tag{3.7f}$$

where

$$\Sigma_i^2\equiv m_i^2+\overline{m}_i^2,\quad \Delta_i^2\equiv m_i^2-\overline{m}_i^2\ . \tag{3.8}$$

The potential at this extrema is

$$V_{\text{extrema}}=-\frac{2}{3}\left[\frac{(\Sigma_1)^3 M_c}{\sqrt{24}\lambda_1^2}+\frac{(\Sigma_2)^3 M_c}{\sqrt{24}\lambda_2^2}\right]-\frac{1}{2g_R^2}\frac{(g_L^2+g_R^2)(\Delta_1^4+\Delta_2^4)-(2g_L^2-g_R^2)\Delta_1^2\Delta_2^2}{g_L^2+\frac{1}{4}g_R^2}+O\left[\frac{\Delta^4\Sigma}{M_c}\right]. \tag{3.9}$$

We note that solutions exist provided

$$(\Sigma_1)^2>0,\quad (\Sigma_2)^2>0\ ; \tag{3.10}$$

i.e., it is not necessary that *both* running masses $-m_i^2$ and $-\overline{m}_i^2$ be negative, but only that Eq. (3.10) hold. Since $M_c\sim M_P=2.4\times10^{+18}$ GeV, Eq. (3.9) lies much deeper than the symmetric state $V=0$. We note also that the solution of Eq. (3.7) requires that *both* N_1 and ν_2^c grow VEV's. Thus $[SU(3)]^3$ breaks completely to the standard model $SU(3)_C\times SU(2)_L\times U(1)_Y$ in *one* step at the intermediate-mass scale M_I.

Returning now to the case where the Higgs VEV's H^2 and H_2' are nonzero, the extrema equations are considerably more complicated. There are, in fact, solutions where the VEV's $(H^2)^2$ and $(H_2')^2$ are large, i.e., $O(\Sigma M_c)$ (which would be phenomenologically disastrous). The case where the Higgs VEV growth preserves matter parity is discussed in Appendix B, where it is shown that those extrema lie $O(\Sigma^3 M_c)$ *higher* than Eq. (3.9). Thus, the matter-parity-preserving solution Eqs. (3.7) is the lowest-lying extrema for this class of solutions and has the physically valid property of N and ν^c VEV's but no H^2 and H_2' VEV's. A more general treatment of the Higgs VEV's will be given elsewhere.

IV. VEV AND MASS SIZES

One may analyze the extrema equations for a superpotential of Eq. (3.4) for arbitrary n. The leading term for the N_1 and ν_2^c VEV's is given by

$$N_1^2\simeq\left[\frac{(\Sigma_1)^2 M_c^{4n-6}}{2n^2(2n-1)\lambda_1^4}\right]^{1/(2n-2)},\qquad (\nu_2^c)^2\simeq\left[\frac{(\Sigma_2)^2 M_c^{4n-6}}{2n^2(2n-1)\lambda_2^4}\right]^{1/(2n-2)}. \tag{4.1}$$

Proton decay data require that[11] $\langle N_1\rangle \gtrsim 10^{14}$ GeV. Table III lists the values of $\langle N_1\rangle$ for various n. We see

TABLE III. Value of intermediate-scale VEV $\langle N_1\rangle$ for $M_c=2.4\times10^{18}$ GeV and supersymmetry-breaking mass $\Sigma_1=1$ TeV for the $(\mathbf{27}\times\overline{\mathbf{27}})^n$ superpotential.

	$\langle N_1\rangle$ (GeV)	
n	$\lambda_1=1$	$\lambda_1=10^{-2}$
2	2.2×10^{10}	2.2×10^{14}
3	2.0×10^{14}	2.0×10^{15}
4	4.2×10^{15}	1.9×10^{16}
5	2.0×10^{16}	6.2×10^{16}

that $n=2$ is consistent with proton decay data, provided the coupling constant λ_1 of the heavy fields to the light fields is somewhat suppressed. Should a discrete symmetry of the Calabi-Yau manifold make the $n=2$ term vanish,[9] satisfactory values of $\langle N_1 \rangle$ are obtained for a wide range of n and λ_1 for $n>2$.

While the nonzero VEV's are very large, one of the remarkable features of symmetry breaking with intermediate-mass scales is that the masses of the fields are not all large. This occurs because M_c can cancel out of some of the mass eigenvalues, leaving only supersymmetry-breaking size masses in the eigenvalues. To calculate the masses of N_1 and ν_2^c, it is convenient to decompose the fields into their Hermitian and skew-Hermitian parts:

$$N_1 = N_1^{(1)} + iN_1^{(2)}, \quad \nu_2^c = \nu_2^{c(1)} + i\nu_2^{c(2)} ,$$
$$\overline{N}_1 = \overline{N}_1^{(1)} + i\overline{N}_1^{(2)}, \quad \overline{\nu}_2^c = \overline{\nu}_2^{c(1)} + i\overline{\nu}_2^{c(2)} , \tag{4.2}$$

where we chose phases so that $\langle N_1^{(2)} \rangle = 0 = \langle \nu_2^{c(2)} \rangle$. The mass matrices for these fields can be directly calculated by differentiating the potential of Eq. (3.1) [using Eqs. (3.2), (3.5), and (3.6)]. One may easily verify that the imaginary parts remain massless at this stage. [Some of these components are Goldstone bosons,[12] and the rest presumably grow mass at a lower scale from renormalization-group (RG) corrections and electroweak breaking.]

The calculation of the masses for the Hermitian parts is quite lengthy[12] and we consider here only the situation when $\langle \nu_2^c \rangle \ll \langle N_1 \rangle$ [which, e.g., by Eq. (4.1) could arise if $\lambda_2 \gg \lambda_1$], and examine the case for N_1. In this limit, the mass matrix is 2×2 coupling $N_1^{(1)}$ and $\overline{N}_1^{(1)}$. The two eigenvalues $m_\pm^2$ are

$$m_+^2 \simeq \tfrac{2}{3}(g_L^2 + g_R^2)\langle N_1 \rangle^2 - 8\frac{n^2(n-1)\lambda_1^4}{M_c^{4n-6}}\langle N_1 \rangle^{4n-4} , \tag{4.3a}$$

$$m_-^2 \simeq \frac{8n^2(n-1)\lambda_1^4}{M_c^{4n-6}}\langle N_1 \rangle^{4n-4} . \tag{4.3b}$$

Inserting in Eq. (4.1) for $(N_1)^{4n-4}$, we see M_c precisely cancels in these terms yielding

$$m_+^2 \simeq \tfrac{2}{3}(g_L^2 + g_R^2)\langle N_1 \rangle^2 - \frac{4(n-1)}{2n-1}\Sigma_1^2 , \tag{4.4a}$$

$$m_-^2 \simeq 4(n-1)\Sigma_1^2 . \tag{4.4b}$$

Thus m_+ is heavy, scaled by $\langle N_1 \rangle$, and hence of size M_I. However m_- is light, scaled by the supersymmetry-breaking mass (i.e., $\approx$electroweak mass). Note also that the coupling constant λ_1 also cancels out of the result for m_-. The importance of the D term to prevent m_+^2 from becoming tachyonic is evident above.

V. CONCLUSIONS

One of the more difficult problems of superstring models based on Calabi-Yau compactification is to see how one can achieve VEV growth of both N and ν^c at the intermediate scale $M_I \sim 10^{15}$ GeV so that $[\mathrm{SU}(3)]^3$ breaks to the standard model.[7] We have seen here that the original scenario[4] of having this arise from the combination of low-energy supersymmetry breaking and the nonrenormalizable interactions can achieve this. Indeed, the detailed calculations with models given above imply that N and ν^c must *simultaneously* break at M_I.

Matter parity plays a crucial role in superstring models[4,10] as it is necessary to prevent the overt type of rapid proton decay.[15] It is thus pleasing to see in the models considered that the deepest-lying extrema automatically preserve matter parity. In addition, extrema where the $\mathrm{SU}(2)\times\mathrm{U}(1)$-breaking Higgs bosons would grow VEV's at M_I lie significantly higher than the $\mathrm{SU}(2)\times\mathrm{U}(1)$-conserving solution. Thus, the models automatically give rise to the phenomenologically needed results: $[\mathrm{SU}(3)]^3$ breaks to $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ at $M_I\sim 10^{15}$ GeV with matter parity conserved and $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ breaking not occurring until a lower-mass scale.

One of the most important features of superstring theory is its prediction of towers of particles with Planck masses. It is the interactions of these particles which presumably makes the theory finite. Direct detection of these particles is of course impossible. However, they effect the low-energy domain in that it is their interaction with the light particles that produces the nonrenormalizable interactions. Thus the appearance of the intermediate scale is a direct consequence of the Planck-mass particles. Most significant is the fact that even though M_I is scaled by a power of the Planck mass and hence is very large, the intermediate-scale information produces particles of mass ~ 1 TeV. One may view these low-mass particles as an unexpected low-energy signature of the Planck-mass domain. The phenomenology of such particles will be considered in a subsequent paper.

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APPENDIX A

As discussed in Sec. I, the **27** representation of E_6 can be broken into three $[\mathrm{SU}(3)]^3$ nonets of particles $L_r^l(1,3,\overline{3})$, $Q_l^a(3,\overline{3},1)$, and $(Q^c)_a^r(\overline{3},1,3)$. We write $l=(\lambda,3)$, $r=(\rho,3)$ where $(\lambda,\rho)=1,2$ run over the $(\mathrm{SU}(2)_L,\mathrm{SU}(2)_R)$ indices [and $a=1,2,3$ is the $\mathrm{SU}(3)_C$ index]. The lepton multiplet has the components

$$L_\rho^\lambda = \begin{bmatrix} H^\lambda \\ \epsilon^{\lambda\mu}H'_\mu \end{bmatrix}, \quad L^3\rho = \begin{bmatrix} e^c \\ \nu^c \end{bmatrix} ,$$
$$L_3^\lambda = l^\lambda, \quad L_3^3 = N . \tag{A1}$$

The doublets in Eq. (A1) are $\mathrm{SU}(2)_R$ doublets. The quantity $l^\lambda \equiv (\nu, e)$ is the usual lepton $\mathrm{SU}(2)_L$ doublet, H^λ and H'_μ are the usual $\mathrm{SU}(2)_L$ Higgs doublets, and $\epsilon \equiv i\tau_2$ in $\mathrm{SU}(2)_L$ space. The quark and antiquark multiplets are

$$Q_\lambda^a = \epsilon_{\lambda\mu}q^{a\mu}, \quad Q_3^a = -D^a \tag{A2}$$

and

$$(Q^c)^\rho_a = \begin{bmatrix} u^c_a \\ -d^c_a \end{bmatrix}, \quad (Q^c)^3_a = -D^c_a \ , \qquad \text{(A3)}$$

where $q^{a\mu} \equiv (u^a, d^a)$ is the quark $SU(2)_L$ doublet. D^a and H^λ form the H_5 of SU(5), while D^c_a and H'_λ form the $H_{\bar{5}}$. All fields are left-handed chiral multiplets.

The most general $[SU(3)]^3$-invariant cubic superpotential depends on four Yukawa coupling constants $\lambda^1, \lambda^2, \lambda^3, \lambda^4$:

$$W_3 = \lambda^1 \det Q^c + \lambda^2 \det Q^c + \lambda^3 \det L - \lambda^4 \mathrm{Tr} Q L Q^c \qquad \text{(A4)}$$

and from Eqs. (A1)–(A3) one has

$$\begin{aligned} W_3 = {} & \lambda^1_{ijk} \epsilon_{aa'a''} d^a_i u^{a'}_j D^{a''}_k + \lambda^2_{ijk} \epsilon^{aa'a''} u^c_{ai} d^c_{a'j} D^c_{a''k} \\ & + \lambda^3_{ijk} (- H^\lambda_i H'_{\lambda j} N_k - H^\lambda_i \nu^c_j l_{\lambda k} + H'_{\lambda i} e^c_j l^\gamma_k) \\ & - \lambda^4_{ijk} (D^a_i N_j D^c_{ak} - D^a_i e^c_j u^c_{ak} + D^a_i \nu^c_j d^c_{ak} \\ & \qquad + q^{a\lambda}_i l_{\lambda j} D^c_{ak} - q^a_{\lambda i} H^\lambda_j u^c_{ak} - q^{a\lambda}_i H'_{\lambda j} d^c_{ak}) \ , \end{aligned} \qquad \text{(A5)}$$

where $q_\lambda \equiv \epsilon_{\lambda\mu} q^\mu$, etc., and the i,j,k indices run over generations. ($i=1,\dots,9$ for leptons and $i=1,\dots,7$ for quarks.) A similar expression holds for the cubic contribution from the mirror generations. (Here $i=1,\dots,6$ for leptons and $i=1,\dots,4$ for quarks.) The Yukawa coupling constants λ^1_{ijk}, etc., are in principle determinable from the Calabi-Yau manifold specified in Eqs. (1.1), though in practice they have only been determined for symmetric cases and only up to normalization factors.[8]

APPENDIX B

We treat here the calculation of the extrema when the Higgs fields H^2_i and H'_{2i} are allowed to grow VEV's. We assume matter parity is preserved so that only Higgs field in the C-even state ($i=1$) can grow VEV's, and will consider the case $n=2$. It is convenient to introduce $x_i \pm \bar{x}_i$, $y_i \pm \bar{y}_i$, etc. [in the notation of Eq. (3.3)]. Minimizing Eqs. (3.1), (3.2), (3.5), and (3.6) with respect to $x_1+\bar{x}_1$, $z_1+\bar{z}_1$, $w_1+\bar{w}_1$ (which enters only in the mass and F terms) yields

$$\frac{\bar{x}_1}{x_1} = \frac{\bar{z}_1}{z_1} = \frac{\bar{w}_1}{w_1} \equiv \gamma_1^2 \ . \qquad \text{(B1)}$$

This shows that the H^2 and H'_2 VEV's become as large as the N_1 VEV. Minimizing with respect to the other variables yields the solutions

$$\begin{aligned} x_1 &= \frac{\Sigma_1 M_c}{6\lambda_1^2} \frac{1}{(\gamma_1^4+4\gamma_1^2+1)^{1/2}} + \frac{1}{3} \frac{g_R^2 - 2g_L^2}{g_L^2+g_R^2} \frac{\gamma_2^2-1}{\gamma_1^2-1} \frac{\Sigma_2 M_c}{2\lambda_2^2} \frac{1}{(\gamma_2^4+4\gamma_2^2+1)^{1/2}} \ , \\ z_1 &= \frac{\Sigma_1 M_c}{6\lambda_1^2} \frac{1}{(\gamma_1^4+4\gamma_1^2+1)^{1/2}} + \frac{1}{3} \frac{\gamma_2^2-1}{\gamma_1^2-1} \frac{\Sigma_2 M_c}{2\lambda_2^2} \frac{1}{(\gamma_2^4+4\gamma_2^2+1)^{1/2}} \ , \\ w_1 &= \frac{\Sigma_1 M_c}{6\lambda_1^2} \frac{1}{(\gamma_1^4+4\gamma_1^2+1)^{1/2}} + \frac{1}{3} \frac{g_L^2 - 2g_R^2}{g_L^2+g_R^2} \frac{\gamma_2^2-1}{\gamma_1^2-1} \frac{\Sigma_2 M_c}{2\lambda_2^2} \frac{1}{(\gamma_2^4+2\gamma_2^2+1)^{1/2}} \ , \\ y_2 &= \frac{\Sigma_2 M_c}{2\lambda_2^2} \frac{1}{(\gamma_2^4+4\gamma_2^2+1)^{1/2}} \quad \bar{y}_2 = \gamma_2^2 y_2 \ , \end{aligned} \qquad \text{(B2)}$$

where

$$\gamma_1^2 = \frac{2\Delta_1^2/\Sigma_1^2 + (1+3\Delta_1^4/\Sigma_1^4)^{1/2}}{1-\Delta_1^2/\Sigma_1^2} \ , \qquad \text{(B3a)}$$

$$\gamma_2^2 = 1 - 4\sqrt{6} \frac{g_L^2+g_R^2}{g_R^2 g_L^2} \frac{\lambda_2^2 \Delta_2^2}{\Sigma_2 M_c} + O\left[\frac{\Delta^2}{M_c^2} \right] . \qquad \text{(B3b)}$$

Since $\gamma_1^2 > 0$, there are acceptable solutions, only if $\Delta_1^2 > 0$, $\Delta_1^2 < \Sigma_1^2$ or $\Delta_1^2 < 0$, $-\Delta_1^2 < \Sigma_1^2$. Inserting Eqs. (B2) and (B3) into V gives

$$\begin{aligned} V_{\text{extrema}} = {} & -\frac{1}{8} \frac{\Sigma_1^3 M_c}{\lambda_1^2} \frac{(\gamma_1^4+6\gamma_1^2+1)(\gamma_1^2+1)}{(\gamma_1^4+4\gamma_1^2+1)^{3/2}} \\ & + \frac{1}{8} \frac{\Delta_1^2 \Sigma_1 M_c}{\lambda_1^2} \frac{\gamma_1^2-1}{(\gamma_1^4+4\gamma_1^2+1)^{1/2}} \\ & + (\Sigma_1, \lambda_1, \gamma_1 \to \Sigma_2, \lambda_2 \gamma_2) \ . \end{aligned} \qquad \text{(B4)}$$

One may verify that Eq. (B4) lies a distance $O(\Delta_1^2 \Sigma_1 M_c; \Sigma_1^3 M_c)$ higher than Eq. (3.9) and hence lies significantly *above* the physically acceptable extrema with vanishing H^2, H'_2 VEV's.

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