Renormalization of the flavor-changing neutral currents

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The flavor-changing amplitudes $\gamma q_i q_j$ and Zq_iq_j are computed for off-shell particles and without approximations. The calculations are done in the 't Hooft–Feynman gauge, and the on-shell renormalization scheme is used. The Ward-Slavnov-Taylor identities are derived and used to check the results. Putting the external quarks on shell, we found that the corresponding flavor-changing S-matrix elements are not renormalized, i.e., the sum of all counterterms vanishes. Using the renormalization scheme proposed by Sirlin the meaning of this cancellation is clarified.

I. INTRODUCTION

In the Glashow-Weinberg-Salam (GWS) model[1] of the electroweak interactions, there is no tree-level coupling of the neutral gauge bosons γ and Z to quarks of different flavor. Such a flavor-changing neutral current (FCNC) arises at the one-loop level, as a result of quark mixing in the charged weak current. The same happens with the gluon-quark vertex in QCD, for which the results can be easily obtained from the analogous photon vertex, including the appropriate coupling constant and the generators of $SU(3)_{\text{color}}$. Since the mixing occurs through a unitary matrix, there is a suppression [Glashow-Iliopoulos-Maiani (GIM) mechanism[2]], roughly of the order $\Delta m^2/M_W^2$, where Δm^2 stands for the largest difference of squared masses of the quarks inside the loop. Hence, processes whose amplitudes originate from such a suppressed FCNC will be very rare and a good test of the GWS model.

In the early days of the GWS model, constraints for the mass of the charmed quark,[3,4] and for the mixing angle[5] in the Kobayashi-Maskawa (KM) matrix, were derived from the rare decays of K mesons, and from K^0-$\bar{K}^0$ mixing. Nowadays, with the advent of B-meson physics, a similar procedure is possible for the mass, and the mixing angles, of the top quark. Radiative B-meson decays, from a $b \to s\gamma$ transition, can be used[6] to put an upper limit on the mass of the top quark, and it has been suggested that a strong QCD enhancement,[7] in the $B \to K^*\gamma$ decay, will put this achievement within reach of the forthcoming experiments. Analogous analysis[8–10] of the rare decays of K, Σ^+, and Λ are more affected by long-distance effects, and thus they are less reliable. Exploring FCNC's is also possible in e^+e^- physics, where rare Z decays can give a top-mass-dependent rate (e.g., $Z \to b\bar{s}$), or even a top production mechanism below the $t\bar{t}$ threshold (e.g., $Z \to t\bar{c}$) (Ref. 11). Also, FCNC's are of some importance to CP violation. An older result[12] predicts a strong contribution, from "penguin diagrams," to the CP-violating parameter ϵ'/ϵ, and, more recently, it has been suggested[13] to look for a CP-violating asymmetry in the flavor-changing (FC) Z decays mentioned above. Still, it is the fact that all these processes are so strongly suppressed that makes the search worthwhile, in the sense that any abnormally high rate would definitely signal some new physics beyond the GWS model or, at least, a heavy fourth generation.

The flavor-changing vertex Zq_iq_j, where i and j are two different flavor indices, has been computed, in Refs. 8 and 9, for on-shell external quarks, with the approximation of neglecting external masses and momenta, and, in Refs. 3 and 4, with the further approximation of keeping only the leading terms in $(m/M_W)^2$, where m is the mass of the heaviest internal quark. For off-shell external quarks, exact calculations have been performed for the $\gamma q_i q_j$ vertex by Deshpande, Eilam, and Nazerimonfared,[14,15] and for the gq_iq_j vertex by Chia.[16] However, an exact calculation, at one-loop order, for the FC Zq_iq_j vertex, for both on- and off-shell external quarks, is still missing, and this is our aim. We also repeat and confirm Deshpande and Eliam results[14] for the $\gamma q_i q_j$ vertex. This is done in Secs. II A and II B, where the results are given in such a way that they are easily available to future users. The Green's functions involved are of one-loop order, and so some attention must be paid to their renormalization, i.e., a renormalization scheme has to be chosen, the counterterms calculated, and their contributions added to the one-loop diagrams. We adopt the on-shell renormalization scheme and derive the counterterms according to the procedure outlined by Ross and Taylor,[17] and later developed by Sakakibara.[18] The Ward-Slavnov-Taylor (WST) identities for the vertices, which were also missing in the literature, are derived in Sec. III. They are used as a check on our results, and also as an alternative and equivalent way to derive the counterterms.[14,15]

In Sec. II C we concentrate on the on-shell result. When calculating the complete vertex, with the external quarks on shell, it is seen that the ultraviolet divergences cancel away, so that the sum of the one-loop diagrams is finite. In principle, one still has to go through the renormalization procedure, and consider the contribution from the counterterms, although now we know that this must be a finite correction. Remarkably, the contribution from the counterterms adds up to zero. This is a result which has been quoted several times,[3,19] but it has never been properly justified. We show that it follows directly from the structure of the counterterms, and does not depend on their values, as fixed by the renormalization conditions. Moreover, it stems from a relation between the counterterms, that can be obtained from the WST identities, i.e., that is due to the symmetries of the theory.

In fact, the cancellation of the on-shell counterterms is a consequence of the absence of the tree-level couplings in the Lagrangian. This implication is not explicit, in the sense that it is not a trivial result, in the renormalization prescription which has been adopted. This is so, because of the introduction of field renormalization constants, so that the divergences of all one-particle-irreducible (1PI) Green's functions are absorbed. In the more "natural" prescription, proposed by Sirlin,[20] where only the parameters are renormalized, and only the physical quantities (S-matrix elements) are guaranteed to be finite, the result follows immediately. In order to further illustrate this observation, we conclude with a few comments on a similar situation, that occurs with the counterterms for the $Z\to H\gamma$ decay.[21]

The calculations are performed, using the notation and the metric of Ref. 22. We have chosen to work in the 't Hooft–Feynman gauge. Throughout the paper, for up-type quarks in the vertex, the matrix element V_{ji} represents $U_{jl}U_{li}^{\dagger}$ and, for down-type quarks, it represents $U_{jl}^{\dagger}U_{li}$, where U is the KM matrix. Summation over the internal quark index l is assumed. So, for the FC vertices and due to the unitarity of the matrix U, terms where V_{ji} multiplies factors which are independent of the internal quark mass, are zero and will be dropped. Dimensional regularization is used to deal with divergent diagrams, and we use the notation

$$\zeta=2/\epsilon-\gamma+\ln 4\pi-\ln(M_W/\mu)^2\ , \qquad (1)$$

where $\epsilon=4-d$, and d stands for the dimension of momentum space, γ is the Euler constant and μ is some arbitrary mass.

II. FLAVOR-CHANGING $\gamma q_i q_j$ AND Zq_iq_j VERTICES

At one-loop order, in the 't Hooft–Feynman gauge and using the on-shell renormalization scheme, the renormalized FC $\gamma q_i q_j$ and Zq_iq_j vertices, $i\Xi^{\mu}_{ji\,\mathrm{ren}}$, can be written in the form

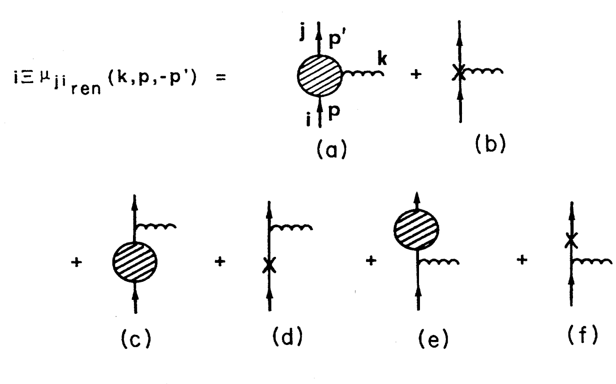

FIG. 1. The one-loop FC effective vertex.

$$\begin{aligned}\Xi^{\mu}_{ji\,\mathrm{ren}}=C[(&A_1\not p p^{\mu}+A_2\not p k^{\mu}+A_3\not k p^{\mu}+A_4\not k k^{\mu}+A_5\not k\gamma^{\mu}\not p\\&+A_6\gamma^{\mu}+A_7k^2\gamma^{\mu}+A_8p\cdot k\gamma^{\mu}+A_9p^2\gamma^{\mu}\\&+A_{10}\gamma^{\mu}\not p+A_{11}\gamma^{\mu}\not k+A_{12}p^{\mu}+A_{13}k^{\mu})\gamma_L\\&+(A_r\to B_r)\gamma_R]\ ,\end{aligned} \qquad (2)$$

with

$$C=\left\{\begin{matrix} e \\ -g/\cos\theta_W\end{matrix}\right\}\frac{g^2}{16\pi^2}V_{ji}\ , \qquad (3)$$

where the upper line corresponds to the $\gamma q_i q_j$ vertex and the lower one to the Zq_iq_j vertex. The incoming momenta corresponding to the neutral gauge boson γ or Z and the quarks q_i and q_j are denoted by k, p, and $-p'=-(k+p)$, respectively. The coefficients A_r and B_r ($r=1,\ldots,13$) are scalar functions of the momenta of the external particles and they depend on the masses and quantum numbers of the external and internal quarks. They are listed in Table I, where contributions from the proper vertex, $i\Lambda^{\mu}_{ji\,\mathrm{ren}}$ [diagrams 1(a) and 1(b), in Fig. 1], and from the corrections to the external legs, $i\Omega^{\mu}_{ji\,\mathrm{ren}}$ [diagrams 1(c), 1(d), 1(e), and 1(f)], as well as from unrenormalized diagrams and counterterms, are kept separate. The final result for the renormalized vertex, $i\Xi^{\mu}_{ji\,\mathrm{ren}}$, is obtained adding all contributions, as shown in Fig. 1. External particles are kept off shell, and no approximations are made.

A. Flavor-changing quark self-energy

At one-loop order, the unrenormalized FC quark self-energy $-i\Sigma_{ji}$ shown in Fig. 2 receives contributions from loops with charged gauge bosons [diagram 2(a)] and from loops with charged unphysical scalars $\phi^{\pm}$ [diagram 2(b)]. If q denotes the momentum of the external lines, m_l the mass of the internal quark, and m_i and m_j the masses of the external quarks, the result is

$$\Sigma_{ji}=(g^2/16\pi^2)V_{ji}[D\not q\gamma_L+E\not q\gamma_R+F(m_i\gamma_R+m_j\gamma_L)]\ , \qquad (4)$$

where D, E, and F are the following functions of q^2:

$$\begin{aligned}D=&[1+(m_l/M_W)^2/2]\int_0^1 dx\,(1-x)\ln\delta\\&-\zeta(m_l/M_W)^2/4\ ,\end{aligned} \qquad (5a)$$

$$E=\tfrac{1}{2}(m_im_j/M_W^2)\int_0^1 dx(1-x)\ln\delta\ , \qquad (5b)$$

$$F=\tfrac{1}{2}(m_l/M_W)^2\left[\zeta-\int_0^1 dx\,\ln\delta\right]\ , \qquad (5c)$$

with

$$\delta=1-x+(m_l/M_W)^2x-(q^2/M_W^2)x(1-x)\ . \qquad (6)$$

The divergent terms in D and F, proportional to the squared mass of the internal quark, are due to the coupling of the charged unphysical scalar, in diagram 2(b). Those stemming from diagram 2(a), which only contributes to the function D, cancel upon summation over the internal quark flavor.

TABLE I. Coefficients A_r and B_r ($r=1,\ldots,13$) for the renormalized flavor-changing vertices. For A_1–A_{13}, the entries in columns $i\Lambda^{\mu}_{ji}$ [Fig. 3(a)], $i\Lambda^{\mu}_{ji}$ [Fig. 3(b)], and $i\Lambda^{\mu}_{ji}$ [Fig. 3(c)] are the integrands $A_r(x_1,x_2)$ in $\int_0^1 dx_1 \int_0^{x_1} dx_2\, A_r(x_1,x_2)$. For A_1–A_{13}, the entries in column $i\Lambda^{\mu}_{ij}$ [Fig. 3(a)] must be multiplied by m_l^2/M_W^2. For B_1–B_{13}, the entries in columns $i\Lambda^{\mu}_{ji}$ [Fig. 3(a)] and $i\Lambda^{\mu}_{ji}$ [Fig. 3(c)] are the integrands $B_r(x_1,x_2)$ in $\int_0^1 dx_1 \int_0^{x_1} dx_2 B_r(x_1,x_2)$.

	$i\Lambda^{\mu}_{ji}$ [Fig. 3(a)]	$i\Lambda^{\mu}_{ji}$ [Fig. 3(b)]	$i\Lambda^{\mu}_{ji}$ [Fig. 3(c)]	$i\lambda^{\mu}_{ji}$ [Fig. 1(b)]
A_1		$-\begin{Bmatrix} Q_W \\ Q_W\cos^2\theta_W \end{Bmatrix} \frac{1}{\Delta M_W^2} 2x_1(1-x_1)$ $-\begin{Bmatrix} Q' \\ T_3'-Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2} 2(1-x_1)^2$	$-\begin{Bmatrix} Q_W \\ Q_W\frac{1}{2}\cos2\theta_W \end{Bmatrix} \frac{1}{\Delta M_W^2} x_1(1-x_1)$ $-\begin{Bmatrix} Q' \\ -Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2} (1-x_1)^2$	
A_2		$\begin{Bmatrix} Q_W \\ Q_W\cos^2\theta_W \end{Bmatrix} \frac{1}{\Delta M_W^2} [x_2-2x_1(x_2-x_1)]$ $-\begin{Bmatrix} Q' \\ T_3'-Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2} 2(1-x_1)$ $\times(1+x_2-x_1)$	$-\begin{Bmatrix} Q_W \\ Q_W\frac{1}{2}\cos2\theta_W \end{Bmatrix} \frac{1}{\Delta M_W^2} x_1(\frac{1}{2}+x_2-x_1)$ $-\begin{Bmatrix} Q' \\ -Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2} (1-x_1)(x_2-x_1)$	
A_3		A_2	$\left[\begin{Bmatrix} Q_W \\ Q_W\frac{1}{2}\cos2\theta_W \end{Bmatrix} \frac{1}{\Delta M_W^2}\right.$ $\left.-\begin{Bmatrix} Q' \\ -Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2}\right] (1-x_1)(x_2-x_1)$	
A_4		$\left[\begin{Bmatrix} Q_W \\ Q_W\cos^2\theta_W \end{Bmatrix} \frac{1}{\Delta M_W^2} [1+2(x_2-x_1)]\right.$ $-\begin{Bmatrix} Q' \\ T_3'-Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2}$ $\left.\times 2(1+x_2-x_1)\right] (x_2-x_1)$	$\left[\begin{Bmatrix} Q_W \\ Q_W\frac{1}{2}\cos2\theta_W \end{Bmatrix} \frac{1}{\Delta M_W^2} (\frac{1}{2}+x_2-x_1)\right.$ $\left.-\begin{Bmatrix} Q' \\ -Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2} (1+x_2-x_1)\right]$ $\times(x_2-x_1)$	
A_5		$-\begin{Bmatrix} Q_W \\ Q_W\cos^2\theta_W \end{Bmatrix} \frac{1}{\Delta M_W^2} \frac{3}{2} x_1$ $+\begin{Bmatrix} Q' \\ T_3'-Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2} (1-x_1)$	$-\begin{Bmatrix} Q' \\ -Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2} \frac{1}{2} (1-x_1)$	
A_6	$-\begin{Bmatrix} Q_W \\ -Q_W\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta M_W^2} m_l^2$	$\begin{Bmatrix} Q_W \\ Q_W\cos^2\theta_W \end{Bmatrix} 3\ln\Delta$ $+\begin{Bmatrix} Q' \\ T_3'-Q'\sin^2\theta_W \end{Bmatrix} \ln\Delta'$ $-\begin{Bmatrix} Q' \\ -Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2} m_l^2$	$\begin{Bmatrix} Q_W \\ Q_W\frac{1}{2}\cos2\theta_W \end{Bmatrix} \frac{1}{2}(\ln\Delta-\zeta)$ $+\begin{Bmatrix} Q' \\ -Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{2}(\ln\Delta'+1-\zeta)$ $-\begin{Bmatrix} Q' \\ T_3'-Q'\sin^2\theta_W \end{Bmatrix} \frac{1}{\Delta' M_W^2} \frac{1}{2} m_l^2$	$-\begin{Bmatrix} Q \\ T_3-Q\sin^2\theta_W \end{Bmatrix} a$

TABLE I. (*Continued*).

	$i\Lambda^{\mu}_{ji}$ [Fig. 3(a)]	$i\Lambda^{\mu}_{ji}$ [Fig. 3(b)]	$i\Lambda^{\mu}_{ji}$ [Fig. 3(c)]	$i\lambda^{\mu}_{ji}$ [Fig. 1(b)]
A_7		$\left[\begin{bmatrix} Q_W \\ Q_W\cos^2\theta_W \end{bmatrix}\frac{1}{\Delta M_W^2}(\tfrac{1}{2}+x_2-x_1) + \begin{bmatrix} Q' \\ T'_3-Q'\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta' M_W^2} \times(1+x_2-x_1)\right](x_2-x_1)$	$\begin{bmatrix} Q' \\ -Q'\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta' M_W^2}\frac{1}{2}(x_2-x_1)(1+x_2-x_1)$	
A_8		$\begin{bmatrix} Q_W \\ Q_W\cos^2\theta_W \end{bmatrix}\frac{1}{\Delta M_W^2} \times[x_2-x_1-2x_1(1+x_2-x_1)] + \begin{bmatrix} Q' \\ T'_3-Q'\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta' M_W^2} \times 2(1-x_1)(1+x_2-x_1)$	$\begin{bmatrix} Q' \\ -Q'\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta' M_W^2}(1-x_1)(x_2-x_1)$	
A_9		$-\begin{bmatrix} Q_W \\ Q_W\cos^2\theta_W \end{bmatrix}\frac{1}{\Delta M_W^2}x_1(1-x_1) + \begin{bmatrix} Q' \\ T'_3-Q'\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta' M_W^2}(1-x_1)^2$	$\begin{bmatrix} Q' \\ -Q'\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta' M_W^2}\frac{1}{2}(1-x_1)^2$	
A_{10}	$-\begin{bmatrix} Q_W \\ -Q_W\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta M_W^2} \times\tfrac{1}{2}m_j x_1$		$-\begin{bmatrix} 0 \\ T'_3 \end{bmatrix}\frac{1}{2}m_j\frac{1}{\Delta' M_W^2}(1-x_1)$	
A_{11}	$\begin{bmatrix} Q_W \\ -Q_W\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta M_W^2}\frac{1}{2}m_j \times(x_2-x_1)$		$\left[\begin{bmatrix} Q' \\ -Q'\sin^2\theta_W \end{bmatrix}(x_2-x_1) - \begin{bmatrix} Q' \\ T'_3-Q'\sin^2\theta_W \end{bmatrix}(1+x_2-x_1)\right]\tfrac{1}{2}m_j\frac{1}{\Delta' M_W^2}$	
A_{12}	$\begin{bmatrix} Q_W \\ -Q_W\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta M_W^2}m_j x_1$		$\left[\begin{bmatrix} Q_W \\ Q_W\tfrac{1}{2}\cos2\theta_W \end{bmatrix}\frac{1}{\Delta M_W^2} + \begin{bmatrix} Q' \\ T'_3-Q'\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta' M_W^2}\right](1-x_1)m_j$	
A_{13}	$-\begin{bmatrix} Q_W \\ -Q_W\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta M_W^2}m_j \times(x_2-x_1)$		$\begin{bmatrix} Q_W \\ Q_W\tfrac{1}{2}\cos2\theta_W \end{bmatrix}\frac{1}{\Delta M_W^2}(\tfrac{1}{2}+x_2-x_1)m_j + \begin{bmatrix} Q' \\ T'_3-Q'\sin^2\theta_W \end{bmatrix}\frac{1}{\Delta' M_W^2}(1+x_2-x_1)m_j$	

TABLE I. (*Continued*).

	$i\Lambda^{\mu}_{ji}$ [Fig. 3(a)]	$i\Lambda^{\mu}_{ji}$ [Fig. 3(b)]	$i\Lambda^{\mu}_{ji}$ [Fig. 3(c)]	$i\lambda^{\mu}_{ji}$ [Fig. 1(b)]
B_1			$-\left[\left[\begin{matrix}Q_W\\ Q_W\frac{1}{2}\cos2\theta_W\end{matrix}\right]\frac{1}{\Delta M_W^2}x_1+\left[\begin{matrix}Q'\\ T'_3-Q'\sin^2\theta_W\end{matrix}\right]\frac{1}{\Delta' M_W^2}(1-x_1)\right]\times\frac{m_im_j}{M_W^2}(1-x_1)$	
B_2			$-\left[\left[\begin{matrix}Q_W\\ Q_W\frac{1}{2}\cos2\theta_W\end{matrix}\right]\frac{1}{\Delta M_W^2}x_1(\frac{1}{2}+x_2-x_1)+\left[\begin{matrix}Q'\\ T'_3-Q'\sin^2\theta_W\end{matrix}\right]\frac{1}{\Delta' M_W^2}\times(1-x_1)(x_2-x_1)\right]\frac{m_im_j}{M_W^2}$	
B_3			$\left[\left[\begin{matrix}Q_W\\ Q_W\frac{1}{2}\cos2\theta_W\end{matrix}\right]\frac{1}{\Delta M_W^2}-\left[\begin{matrix}Q'\\ T'_3-Q'\sin^2\theta_W\end{matrix}\right]\frac{1}{\Delta' M_W^2}\right]\times\frac{m_im_j}{M_W^2}(1-x_1)(x_2-x_1)$	
B_4			$\left[\left[\begin{matrix}Q_W\\ Q_W\frac{1}{2}\cos2\theta_W\end{matrix}\right]\frac{1}{\Delta M_W^2}(\frac{1}{2}+x_2-x_1)-\left[\begin{matrix}Q'\\ T'_3-Q'\sin^2\theta_W\end{matrix}\right]\frac{1}{\Delta' M_W^2}\times(1+x_2-x_1)\right]\frac{m_im_j}{M_W^2}(x_2-x_1)$	
B_5			$-\left[\begin{matrix}Q'\\ T'_3-Q'\sin^2\theta_W\end{matrix}\right]\frac{1}{\Delta' M_W^2}\frac{1}{2}\frac{m_im_j}{M_W^2}(1-x_1)$	
B_6			$\left[\left[\begin{matrix}Q_W\\ Q_W\frac{1}{2}\cos2\theta_W\end{matrix}\right]\ln\Delta+\left[\begin{matrix}Q'\\ T'_3-Q'\sin^2\theta_W\end{matrix}\right]\ln\Delta'-\left[\begin{matrix}Q'\\ -Q'\sin^2\theta_W\end{matrix}\right]\frac{1}{\Delta' M_W^2}m_l^2\right]\frac{1}{2}\frac{m_im_j}{M_W^2}$	$-\left[\begin{matrix}Q\\ -Q\sin^2\theta_W\end{matrix}\right]b$
B_7			$\left[\begin{matrix}Q'\\ T'_3-Q'\sin^2\theta_W\end{matrix}\right]\frac{1}{\Delta' M_W^2}\frac{1}{2}\frac{m_im_j}{M_W^2}\times(x_2-x_1)(1+x_2-x_1)$	

TABLE I. (*Continued*).

	$i\Lambda^{\mu}_{ji}$ [Fig. 3(a)]	$i\Lambda^{\mu}_{ji}$ [Fig. 3(b)]	$i\Lambda^{\mu}_{ji}$ [Fig. 3(c)]	$i\lambda^{\mu}_{ji}$ [Fig. 1(b)]
B_8			$\begin{bmatrix} Q' \\ T'_3 - Q'\sin^2\theta_W \end{bmatrix} \frac{1}{\Delta' M_W^2} \frac{m_i m_j}{M_W^2} (1-x_1)(x_2-x_1)$	
B_9			$\begin{bmatrix} Q' \\ T'_3 - Q'\sin^2\theta_W \end{bmatrix} \frac{1}{\Delta' M_W^2} \frac{1}{2} \frac{m_i m_j}{M_W^2} (1-x_1)^2$	
B_{10}	$\begin{bmatrix} Q_W \\ -Q_W\sin^2\theta_W \end{bmatrix} \frac{1}{2} \times \frac{1}{\Delta M_W^2} m_i x_1$		$\begin{bmatrix} 0 \\ T'_3 \end{bmatrix} \frac{1}{\Delta' M_W^2} \frac{1}{2} \frac{m_l^2}{M_W^2} m_i (1-x_1)$	
B_{11}	$-\begin{bmatrix} Q_W \\ -Q_W\sin^2\theta_W \end{bmatrix} \frac{1}{2} \times \frac{1}{\Delta M_W^2} m_i (x_2-x_1)$		$\begin{bmatrix} 0 \\ T'_3 \end{bmatrix} \frac{1}{\Delta' M_W^2} \frac{1}{2} \frac{m_l^2}{M_W^2} m_i (x_2-x_1) - \begin{bmatrix} Q' \\ -Q'\sin^2\theta_W \end{bmatrix} \frac{1}{\Delta' M_W^2} \frac{1}{2} \frac{m_l^2}{M_W^2} m_i$	
B_{12}			$\left[\begin{bmatrix} Q_W \\ Q_W \frac{1}{2}\cos 2\theta_W \end{bmatrix} \frac{1}{\Delta M_W^2} + \begin{bmatrix} Q' \\ -Q'\sin^2\theta_W \end{bmatrix} \frac{1}{\Delta' M_W^2} \right] \frac{m_l^2}{M_W^2} m_i (1-x_1)$	
B_{13}			$\left[\begin{bmatrix} Q_W \\ Q_W \frac{1}{2}\cos 2\theta_W \end{bmatrix} \frac{1}{\Delta M_W^2} (\tfrac{1}{2}+x_2-x_1) + \begin{bmatrix} Q' \\ -Q'\sin^2\theta_W \end{bmatrix} \frac{1}{\Delta' M_W^2} (1+x_2-x_1) \right] \frac{m_l^2}{M_W^2} m_i$	

	$i\Omega^{\mu}_{ji}$ [Fig. 1(c)]	$i\omega^{\mu}_{ji}$ [Fig. 1(d)]	$i\Omega^{\mu}_{ji}$ [Fig. 1(e)]	$i\omega^{\mu}_{ji}$ [Fig. 1(f)]
A_1				
A_2				
A_3				
A_4				
A_5				
A_6	$-\frac{1}{p^2-m_j^2} \begin{bmatrix} Q \\ T_3 - Q\sin^2\theta_W \end{bmatrix} \times m_j^2 F(p^2)$	$\frac{1}{p^2-m_j^2} \begin{bmatrix} Q \\ T_3 - Q\sin^2\theta_W \end{bmatrix} m_j c$	$-\frac{1}{(p+k)^2-m_i^2} \begin{bmatrix} Q \\ T_3 - Q\sin^2\theta_W \end{bmatrix} m_i^2 F((p+k)^2)$	$\frac{1}{(p+k)^2-m_i^2} \times \begin{bmatrix} Q \\ T_3 - Q\sin^2\theta_W \end{bmatrix} m_i d$
A_7			$-\frac{1}{(p+k)^2-m_i^2} \begin{bmatrix} Q \\ T_3 - Q\sin^2\theta_W \end{bmatrix} D((p+k)^2)$	$\frac{1}{(p+k)^2-m_i^2} \begin{bmatrix} Q \\ T_3 - Q\sin^2\theta_W \end{bmatrix} a$

TABLE I. (*Continued*).

	$i\Omega^{\mu}_{ji}$ [Fig. 1(c)]	$i\omega^{\mu}_{ji}$ [Fig. 1(d)]	$i\Omega^{\mu}_{ji}$ [Fig. 1(e)]	$i\omega^{\mu}_{ji}$ [Fig. 1(f)]
A_8			$2A_7$	$2A_7$
A_9	$-\frac{1}{p^2-m_j^2}\left[T_3-Q\sin^2\theta_W\right] \times D(p^2)$	$\frac{1}{p^2-m_j^2}\left[T_3-Q\sin^2\theta_W\right]a$	A_7	A_7
A_{10}	$-\frac{1}{p^2-m_j^2}\left[-Q\sin^2\theta_W\right] \times m_j[D(p^2)+F(p^2)]$	$\frac{1}{p^2-m_j^2}\left[-Q\sin^2\theta_W\right] \times (m_j a+c)$	$\frac{1}{(p+k)^2-m_i^2}\left[T_3-Q\sin^2\theta_W\right] \times [m_j F((p+k)^2)+m_i E((p+k)^2)]$	$-\frac{1}{(p+k)^2-m_i^2}\left[T_3-Q\sin^2\theta_W\right] \times (m_i b+c)$
A_{11}			A_{10}	A_{10}
A_{12}			$-2A_{10}$	$-2A_{10}$
A_{13}			$-2A_{10}$	$-2A_{10}$
B_1				
B_2				
B_3				
B_4				
B_5				
B_6	$-\frac{1}{p^2-m_j^2}\left[-Q\sin^2\theta_W\right] \times m_j m_i F(p^2)$	$\frac{1}{p^2-m_j^2}\left[-Q\sin^2\theta_W\right]m_j d$	$-\frac{1}{(p+k)^2-m_i^2}\left[-Q\sin^2\theta_W\right] \times m_i m_j F((p+k)^2)$	$\frac{1}{(p+k)^2-m_i^2}\left[-Q\sin^2\theta_W\right]m_i c$
B_7			$-\frac{1}{(p+k)^2-m_i^2}\left[-Q\sin^2\theta_W\right]E((p+k)^2)$	$\frac{1}{(p+k)^2-m_i^2}\left[-Q\sin^2\theta_W\right]b$
B_8			$2B_7$	$2B_7$
B_9	$-\frac{1}{p^2-m_j^2}\left[-Q\sin^2\theta_W\right] \times E(p^2)$	$\frac{1}{p^2-m_j^2}\left[-Q\sin^2\theta_W\right]b$	B_7	B_7
B_{10}	$-\frac{1}{p^2-m_j^2}\left[T_3-Q\sin^2\theta_W\right] \times [m_j E(p^2)+m_i F(p^2)]$	$\frac{1}{p^2-m_j^2}\left[T_3-Q\sin^2\theta_W\right] \times (m_j b+d)$	$\frac{1}{(p+k)^2-m_i^2}\left[-Q\sin^2\theta_W\right] \times m_i[F((p+k)^2)+D((p+k)^2)]$	$-\frac{1}{(p+k)^2-m_i^2}\left[-Q\sin^2\theta_W\right] \times (m_i a+d)$
B_{11}			B_{10}	B_{10}
B_{12}			$-2B_{10}$	$-2B_{10}$
B_{13}			$-2B_{10}$	$-2B_{10}$

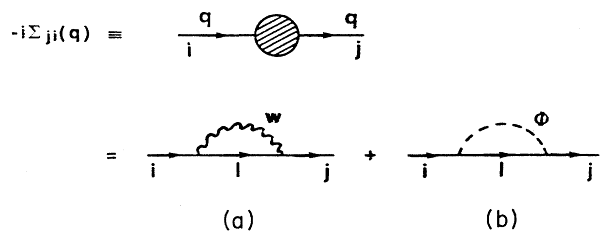

FIG. 2. The one-loop FC quark self-energy.

The structure of the counterterm $i\sigma_{ji}$ for the FC quark self-energy can be obtained with the prescription developed by Sakakibara.[18] In the free Lagrangian, the bare left-handed ($L_k^0=\gamma_L\Phi_k^0$) and right-handed (R_k^0) quark fields are scaled by a matrix in flavor space, which deviates from the identity at order g^2: i.e.,

$$L_k^0=\sqrt{Z_{Lki}}L_i=(\delta_{ki}+\tfrac{1}{2}\delta Z_{Lki})L_i\ , \tag{7a}$$

$$R_k^0=\sqrt{Z_{Rki}}R_i=(\delta_{ki}+\tfrac{1}{2}\delta Z_{Rki})R_i\ , \tag{7b}$$

where L_i and R_i are the renormalized fields. In terms of the renormalized quantities, the Lagrangian acquires an extra term $\overline{\Phi}_j\sigma_{ji}\Phi_i$ given by

$$\sigma_{ji}=a_{ji}\not{q}\gamma_L+b_{ji}\not{q}\gamma_R+c_{ji}\gamma_L+d_{ji}\gamma_R\ , \tag{8}$$

with

$$a_{ji}=\tfrac{1}{2}(\delta Z_L+\delta Z_L^\dagger)_{ji}\ , \tag{9a}$$

$$b_{ji}=\tfrac{1}{2}(\delta Z_R+\delta Z_R^\dagger)_{ji}\ , \tag{9b}$$

$$c_{ji}=-\tfrac{1}{2}(m_j\delta Z_L+m_i\delta Z_R^\dagger)_{ji}\ , \tag{9c}$$

$$d_{ji}=-\tfrac{1}{2}(m_i\delta Z_L^\dagger+m_j\delta Z_R)_{ji}\ . \tag{9d}$$

Notice that σ_{ji} obeys the relation

$$\sigma_{ji}=\gamma^0\sigma^\dagger{}_{ji}\gamma^{0\dagger}\ , \tag{10}$$

imposed by the Hermiticity of the Lagrangian.

Since we are only interested in the FC counterterm ($i\neq j$), we have omitted the mass renormalization constant $\delta m_k=m_k^0-m_k$. Still, we have to determine all the constants in Eqs. (9), which, for three generations, amount to 24 real numbers. Notice that Eq. (8) is the most general structure we could have built for the counterterm σ_{ji}, i.e., in this case, the prescription given by Eqs. (7) has not given us any additional information. This will not be so when we derive the counterterm for the proper vertex.

The renormalized FC quark self-energy $-i\Sigma_{ji\,\mathrm{ren}}$ is obtained adding the counterterm to the unrenormalized self-energy: i.e.,

$$-i\Sigma_{ji\,\mathrm{ren}}=-i\Sigma_{ji}+i\sigma_{ji}\ . \tag{11}$$

The values of the constants a_{ji}, b_{ji}, c_{ji}, and d_{ji} of Eq. (8) are now determined by the renormalization conditions imposed upon $-i\Sigma_{ji\,\mathrm{ren}}$. In the on-shell renormalization these conditions are[23]

$$\bar{u}_j(\not{q})\Sigma_{ji\,\mathrm{ren}}(\not{q})|_{\not{q}=m_j}=0\ , \tag{12a}$$

where no summation over j is implied and where u_k is a solution of the Dirac equation with mass m_k. Notice that Eq. (12a) is only imposed upon the nonabsorptive parts of the self-energy, i.e., only the real parts of D, E, and F will appear in the renormalization condition. From Eq. (12a), and using the property (10), it is easy to obtain

$$\Sigma_{ji\,\mathrm{ren}}(\not{q})u_i(\not{q})|_{\not{q}=m_i}=0\ . \tag{12b}$$

Separating the terms in γ_L from those in γ_R, in Eqs. (12a) and (12b), we obtain the necessary 24 conditions, which give

$$a_{ji}=C'a,\quad b_{ji}=C'b,\quad c_{ji}=C'c,\ \text{and}\ d_{ji}=C'd\ , \tag{13}$$

with

$$a=\{m_j^2D(m_j^2)-m_i^2D(m_i^2)+m_im_j[E(m_j^2)-E(m_i^2)]+(m_i^2+m_j^2)[F(m_j^2)-F(m_i^2)]\}/(m_j^2-m_i^2)\ , \tag{14a}$$

$$b=\{m_j^2E(m_j^2)-m_i^2E(m_i^2)+m_im_j[D(m_j^2)-D(m_i^2)]+2m_im_j[F(m_j^2)-F(m_i^2)]\}/(m_j^2-m_i^2)\ , \tag{14b}$$

$$c=m_j\{m_i^2[D(m_i^2)-D(m_j^2)]+m_im_j[E(m_i^2)-E(m_j^2)]-2m_i^2F(m_j^2)+(m_i^2+m_j^2)F(m_i^2)\}/(m_j^2-m_i^2)\ , \tag{14c}$$

$$d=m_i\{m_j^2[D(m_i^2)-D(m_j^2)]+m_im_j[E(m_i^2)-E(m_j^2)]+2m_j^2F(m_i^2)-(m_i^2+m_j^2)F(m_j^2)\}/(m_j^2-m_i^2)\ , \tag{14d}$$

and

$$C'=\frac{g^2}{16\pi^2}V_{ji}\ . \tag{15}$$

Using these results in Eq. (11) it is easy to see that the renormalized self-energy is independent of ζ, and thus finite.

B. Flavor-changing effective vertex

Once we have derived the renormalized off-diagonal self-energy, it is straightforward to obtain the contribution to the effective vertex from the corrections to the external legs $i\Omega^\mu_{ji\,\mathrm{ren}}$ shown in Fig. 1. We write

$$i\Omega^\mu_{ji\,\mathrm{ren}}=i\Omega^\mu_{ji}+i\omega^\mu_{ji}\ , \tag{16}$$

where the first and the second terms on the right-hand side of Eq. (16) correspond to the unrenormalized diagrams and the counterterms, respectively. Then, from Figs. 1(c) and 1(d) we have

$$\Omega^\mu_{ji\,\mathrm{ren}}=X\frac{\not{p}+m_j}{p^2-m_j^2}[-\Sigma_{ji}(\not{p})+\sigma_{ji}(\not{p})]\ , \tag{17a}$$

while Figs. 1(e) and 1(f) give

$$\Omega^{\mu}_{ji\,\mathrm{ren}}=[-\Sigma_{ji}(\not{p}')+\sigma_{ji}(\not{p}')]\frac{\not{p}'+m_i}{p'^2-m_i^2}X\ , \qquad (17\mathrm{b})$$

with

$$X=\begin{bmatrix} e \\ -g/\cos\theta_W \end{bmatrix}\left[\begin{bmatrix} Q \\ T_3-Q\sin^2\theta_W \end{bmatrix}\gamma^{\mu}\gamma_L + \begin{bmatrix} Q \\ -Q\sin^2\theta_W \end{bmatrix}\gamma^{\mu}\gamma_R\right], \qquad (18)$$

where Q denotes the charge, in units of $e>0$, and T_3 is the third component of the weak isospin of the quarks in the external legs. Replacing the values of Σ_{ji} and σ_{ji}, in Eqs. (17), these can be written in the general form of Eq. (2). The corresponding A's and B's are in Table I.

In Fig. 3, we show the one-loop diagrams that contribute to the proper vertex $i\Lambda^{\mu}_{ji}$. The diagrams where the neutral gauge boson couples to the internal quark are infinite: the divergent terms from the one with the W boson inside the loop cancel upon summation over the internal quark flavor, whereas those from the diagram with the charged unphysical scalar $\phi^{\pm}$ are proportional to m_l^2 and do not cancel. Again, the final result can be written in the form given by Eq. (2), and the coefficients for graphs 3(a), 3(b), and 3(c) are in Table I. The divergent term is of the form $\gamma^{\mu}\gamma_L$, and it will appear in the coefficient A_6. We denote by Q_W the W charge, which is $+1$ or -1 for up or down quarks, respectively, in the external legs, and by $Q'=Q-Q_W$ and $T_3'=-Q_W/2$ the quantum numbers of the internal quarks. The denominators Δ and Δ' are

$$\begin{aligned}\Delta=&\,x_1+(m_l/M_W)^2(1-x_1)\\&+(k^2/M_W^2)(x_2-x_1)(1+x_2-x_1)\\&-(p^2/M_W^2)x_1(1-x_1)\\&+(p\cdot k/M_W^2)2(1-x_1)(x_2-x_1)\ ,\end{aligned} \qquad (19\mathrm{a})$$

$$\begin{aligned}\Delta'=&\,1-x_1+(m_l/M_W)^2x_1\\&+(k^2/M_W^2)(x_2-x_1)(1+x_2-x_1)\\&-(p^2/M_W^2)x_1(1-x_1)\\&+(p\cdot k/M_W^2)2(1-x_1)(x_2-x_1)\ .\end{aligned} \qquad (19\mathrm{b})$$

The renormalized proper vertex $i\Lambda^{\mu}_{ji\,\mathrm{ren}}$ is obtained adding the contribution from the counterterm $i\lambda^{\mu}_{ji}$. The latter is generated following the same prescription as for the self-energy counterterm, i.e., in the γ- or Z-quark interaction terms in the Lagrangian, we perform a scaling of the bare fields and of the coupling constants. For the FC counterterm, at one-loop order, there are no contributions from the γ or Z field renormalization, nor from the renormalization of the couplings, and the result is

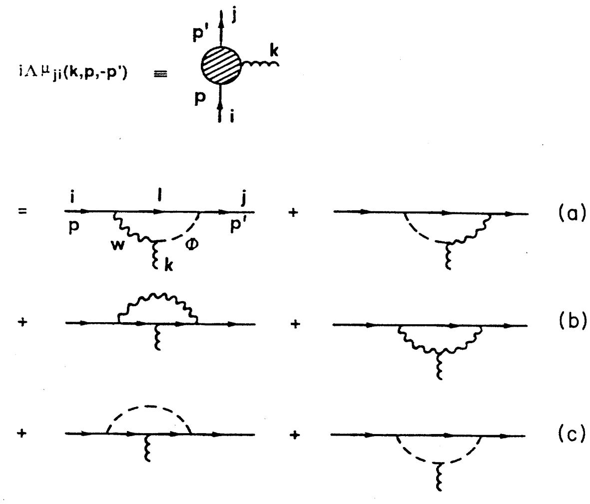

FIG. 3. The one-loop FC proper vertex.

$$i\lambda^{\mu}_{ji}=-i\begin{bmatrix} e \\ -g/\cos\theta_W \end{bmatrix}\left[\begin{bmatrix} Q \\ T_3-Q\sin^2\theta_W \end{bmatrix}a_{ji}\gamma^{\mu}\gamma_L + \begin{bmatrix} Q \\ -Q\sin^2\theta_W \end{bmatrix}\times b_{ji}\gamma^{\mu}\gamma_R\right], \qquad (20)$$

where a_{ji} and b_{ji} are the same as in the self-energy counterterm and are given in Eqs. (9a) and (9b). Notice that the prescription adopted is now determinant in defining the structure of $i\lambda^{\mu}_{ji}$, which only includes terms in $\gamma^{\mu}\gamma_L$ and in $\gamma^{\mu}\gamma_R$, among all the terms in the general form [cf. Eq. (2)]. Furthermore, their values are fixed, in Eqs. (14a) and (14b), without requiring any additional renormalization condition. The results are listed in Table I, and one can see that the divergent term in A_6 is absorbed and a finite correction remains.

The relation between the counterterm for the proper vertex and the counterterm for the self-energy can be obtained explicitly, using the WST identities which are derived in Sec. III [cf. Eqs. (33) and (37)]. Since these identities are valid, order by order, both for the renormalized and for the unrenormalized Green's functions, they lead to a relationship for the counterterms, which is

$$k_{\nu}\lambda^{\nu}_{ji}(k,p,-p')=-eQ[\sigma_{ji}(\not{p}')-\sigma_{ji}(\not{p})] \qquad (21\mathrm{a})$$

for the $\gamma q_i q_j$ vertex, and

$$\begin{aligned}k_{\nu}\lambda^{\nu}_{ji}&(k,p,-p')\\&=iM_Z\lambda_{ji}(k,p,-p')\\&\quad-(g/\cos\theta_W)[\sigma_{ji}(\not{p}')(Q\sin^2\theta_W-T_3\gamma_L)\\&\qquad-(Q\sin^2\theta_W-T_3\gamma_R)\sigma_{ji}(\not{p})]\end{aligned} \qquad (21\mathrm{b})$$

for the Zq_iq_j vertex. In the previous equation, $i\lambda_{ji}$ is the counterterm for the $\phi_Z q_i q_j$ FC proper vertex $i\Lambda_{ji}$, where ϕ_Z is the neutral unphysical scalar.

Notice that the terms proportional to T_1 and T_1', in both Eqs. (33) and (37), are finite and have no counterterms. Hence, they do not contribute to either Eqs. (21a) or (21b). These terms in the WST identities are gauge dependent; for instance, they do not arise in a nonlinear R_ξ gauge,[15] nor in the unitary gauge.[24] The fact that they have no influence in deriving Eqs. (21), signals the gauge independence of the structure of the proper vertex counterterm. Using, on the right-hand side of Eqs. (21), the most general structure for the self-energy counterterm $i\sigma_{ji}$ [cf. Eq. (8)], it is simple to obtain the result given by Eq. (20). At the same time one derives the following expression for the $\phi_Z q_i q_j$ proper vertex counterterm:

$$i\lambda_{ji}=(gT_3/M_W)(d_{ji}\gamma_R-c_{ji}\gamma_L)\ , \tag{22}$$

which is in agreement with the result one could have obtained with the scaling prescription, applied to the appropriate term in the Lagrangian.

It is interesting to point out that, the WST identities can be used to determine directly the value of the proper vertex counterterm, in Eq. (20), without computing the self-energy. In fact, the renormalization conditions, Eqs. (12a) and (12b), applied to Eqs. (33) and (37), give

$$k_\nu \Lambda^\nu_{ji\,\mathrm{ren}}|_{\mathrm{on\ shell}}=0 \tag{23a}$$

and

$$k_\nu \Lambda^\nu_{ji\,\mathrm{ren}}|_{\mathrm{on\ shell}}=iM_Z\Lambda_{ji\,\mathrm{ren}}|_{\mathrm{on\ shell}}\ , \tag{23b}$$

for the photon and Z vertices, respectively, and from these equations one can obtain the value of the counterterms $i\lambda^\nu_{ji}$ and $i\lambda_{ji}$.

C. On-shell effective vertex

For on-shell quarks, the general form of $i\Xi^\mu_{ji\,\mathrm{ren}}$ reduces to a sum of terms proportional to $k^\mu\gamma_{L,R}$, $p^\mu\gamma_{L,R}$, and $\gamma^\mu\gamma_{L,R}$. Alternatively, using Gordon's identities, we write

$$\Xi^\mu_{ji\,\mathrm{ren}}|_{\mathrm{on\ shell}}=C[(\alpha_1k^\mu+\alpha_2\gamma^\mu+\alpha_3 i\sigma^{\mu\nu}k_\nu)\gamma_L + (\alpha_i\to\beta_i)\gamma_R]\ , \tag{24}$$

with

$$\alpha_1=m_i(B_2-B_4)+m_jA_4+A_{11}+A_{13}-\tfrac{1}{2}m_jA_3 -\tfrac{1}{2}A_{12}-\tfrac{1}{2}m_i(B_1-B_3-2B_5)\ , \tag{25a}$$

$$\alpha_2=\tfrac{1}{2}m_im_jB_1+\tfrac{1}{2}(m_j^2-m_i^2)A_3+\tfrac{1}{2}m_jA_{12}+\tfrac{1}{2}m_i^2A_1 +\tfrac{1}{2}m_iB_{12}+m_i^2A_9+m_iB_{10}+A_6+k^2A_7 +\tfrac{1}{2}(m_j^2-m_i^2-k^2)A_8\ , \tag{25b}$$

$$\alpha_3=[-m_i(B_1-B_3-2B_5)-m_jA_3-2A_{11}-A_{12}]/2\ , \tag{25c}$$

and similar equations for β_i ($i=1,2,3$), but interchanging the A's and B's. These can be read from Table I, where the momenta should be set equal to their on-shell values (for the sake of generality, and since this will not affect our subsequent discussion, k^2 has not been fixed). Since we are working in the on-shell renormalization scheme, the contributions from the renormalized corrections to the external legs, $i\Omega^\mu_{ji\,\mathrm{ren}}$, are zero, as can be seen from the renormalization conditions, given in Eqs. (12a) and (12b). Thus, only diagrams (a) and (b) in Fig. 1, i.e., the renormalized proper vertex, contribute to the on-shell effective vertex.

However, let us examine in more detail all the diagrams in Fig. 1, considering on-shell external quarks. Using Eq. (24), with the appropriate entries from Table I, it is easy to see that the contribution of the unrenormalized diagrams is finite, and thus, the same must happen to the contribution from the counterterms. Moreover, this contribution, given by graphs $(b)+(d)+(f)$, adds up to zero, which means that the effective vertex, for on-shell quarks, or the corresponding S-matrix element, is not renormalized, i.e., it can be obtained simply by summing up the one-loop diagrams, (a), (c), and (e), in Fig. 1. In the context of the on-shell renormalization scheme, this is a rather surprising result and, despite the fact that it has been stated several times,[3,19] we have not yet seen a full discussion of its origin. This is the purpose of this section.

For off-shell quarks, the counterterms for the corrections to the external legs, shown in graphs 1(d) and 1(f), write

$$\omega^\mu_{ji}=X\frac{\not p+m_j}{p^2-m_j^2}\sigma_{ji}(\not p)+\sigma_{ji}(\not p')\frac{\not p'+m_i}{p'^2-m_i^2}X\ . \tag{26}$$

If we replace σ_{ji} by its expression given in Eq. (8) and add the contribution of the counterterm for the proper vertex, shown in graph 1(b) and given by Eq. (20), we obtain an expression in terms of the constants a_{ji}, b_{ji}, c_{ji}, and d_{ji}. For on-shell quarks, this is easily seen to be zero, independently of the explicit values of those constants, which are fixed by the renormalization conditions. Hence, the cancellation of the counterterms follows directly from their structure, irrespective of the renormalization scheme which has been chosen, and can be traced back to the relation given in Eqs. (21), which was derived from the WST identities. This means that such a result arises from the symmetry of the Lagrangian, and, in this sense, it is a fundamental result in the renormalization of the theory.

In the renormalization scheme that we are using, additional counterterms are generated by renormalizing the fields, with the sole purpose of absorbing the divergences in all 1PI Green's functions. Such is the origin of the $i\lambda^\nu_{ji}$ counterterm that renormalizes the FC proper vertex, and of the counterterm $i\sigma_{ji}$ for the FC self-energy. Field renormalization is required for a formal proof of the renormalizability, based on the analysis of the generating functional of the 1PI Green's functions (e.g., Ref. 22). However, it can be ignored, if we are simply interested in the calculation of S-matrix elements. In fact, the Green's functions may remain divergent, provided that the S-matrix elements, which are the physically meaningful predictions of a field theory, are finite. Such a renormalization program has been proposed by Sirlin.[20] There, the counterterms are generated only from the renormalization of the parameters, which is necessary for the consistency of the theory, and there is no renormalization of the fields. So, if the diagonalization of the mass matrices, both in the quark sector and in the neutral-gauge-boson sector, is achieved rotating the fields through bare mixing angles, off-diagonal counterterms do not arise,[20] and the counterterm Lagrangian has the same structure as the tree-level one. In particular, counterterms for both the FC quark self-energies and the FC vertices do not exist, so that the result

$$\Xi^{\mu}_{ji\ \mathrm{ren|on\ shell}}=(\Lambda^{\mu}_{ji}+\Omega^{\mu}_{ji})_{\mathrm{on\ shell}}\ , \tag{27}$$

which has been derived previously, now follows trivially.

Notice that, in this scheme, the corrections to the external legs, must be introduced, when calculating the S-matrix element. They follow[25] from the Lehmann-Symanzik-Zimmermann (LSZ) reduction (e.g., Ref. 22), and are avoided in the standard on-shell picture because they are canceled by the counterterms from the field renormalization.

It is interesting to remark that an analogous result has been found by one of us[21] when calculating the $Z\to H\gamma$ decay. Again it was checked that, on shell, the counterterm for the proper vertex cancels with the counterterms for the corrections to the external legs. Proceeding, for the γZ mixing, in a similar way as for the quark mixing, it is trivial to justify[25] the cancellation using Sirlin's scheme.

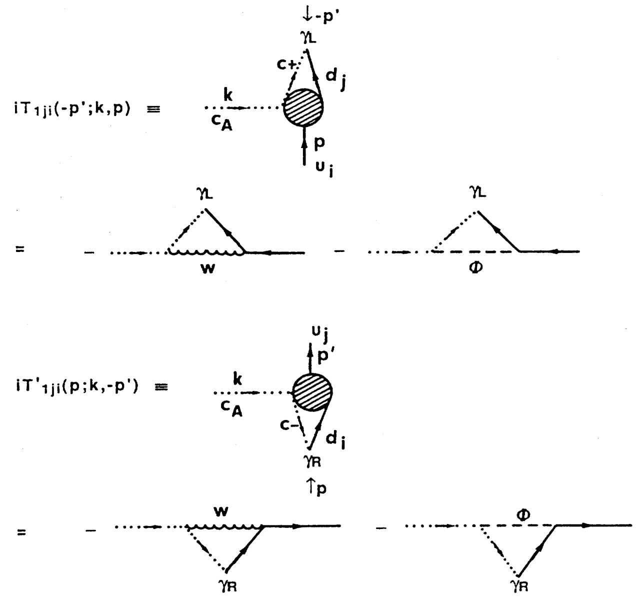

FIG. 4. The Green's functions T_1 and T_1' up to one-loop order.

III. WARD-SLAVNOV-TAYLOR IDENTITIES

The WST identities for the $\gamma q_i q_j$ and Zq_iq_j proper vertices, $i\Lambda^{\nu}_{ji}(k,p,-p')$, are derived from the invariance of the appropriate Green's functions, under a Becchi-Rouet-Stora (BRS) transformation, or, equivalently, under the action of the Slavnov-Taylor (ST) operator s. For completeness and to fix our notation, we write in the Appendix the ST transformation of the neutral gauge fields A_μ and Z_μ of the corresponding Faddeev-Popov (FP) ghost fields c_A and c_Z and of the up- (u-) and down- (d-) quark fields. All Green's functions will be written as functions of incoming momenta and, for clearness, we derive the WST identities for up quarks in the vertices.

Let us start with the identity for the $\gamma q_i q_j$ vertex. From

$$s\langle 0|T\bar{c}_A(x)u_j(y)\bar{u}_i(z)|0\rangle=0\ , \tag{28}$$

and using the ST transformation of the fields, we obtain, in momentum space, the relation

$$k^{\mu}G_{\mu\nu}(k)S_{jk}(p')i\Lambda^{\nu}_{kl}S_{li}(p)=-(g/\sqrt{2})\Delta_c(k)[U_{jn}iT_{1nl}S_{li}(p)-S_{jl}(p')iT'_{1ln}U^{\dagger}_{ni}]+eQ_u\Delta_c(k)[iT_{2jl}S_{li}(p)-S_{jl}(p')iT'_{2li}]$$
$$+(g/\cos\theta_W)\Delta_c(k)[iT_{3jl}S_{li}(p)-S_{jl}(p')iT'_{3li}]\ , \tag{29}$$

where Δ_c and $G_{\mu\nu}$ are the propagators for c_A and for the photon, respectively, and S_{jk} is the two-point Green's function for up quarks. The composite operators iT_1 and iT_1' are shown in Fig. 4, up to one-loop order, whereas to the same order $iT_{2ji}=iT'_{2ji}=-\delta_{ji}$ and $T_{3ji}=T'_{3ji}=0$. Noting that[26]

$$k^{\nu}G_{\mu\nu}(k)=-k_{\mu}\Delta_c(k)+ie\Delta_c(k)(F_{\mu}-F'_{\mu})\ , \tag{30}$$

where the composite operators F and F' are of one-loop order, and, replacing Eq. (30) into Eq. (29), we obtain the WST identity

$$ik_{\nu}\Lambda^{\nu}_{ji}=-e(F_{\nu}-F'_{\nu})\Lambda^{\nu}_{ji}+i[S_{kj}(p')]^{-1}[(g/\sqrt{2})U_{kn}T_{1ni}-eQ_uT_{2ki}-(g/\cos\theta_W)T_{3ki}]$$
$$-i[(g/\sqrt{2})T'_{1jn}U^{\dagger}_{nl}-eQ_uT'_{2jl}-(g/\cos\theta_W)T'_{3jl}][S_{il}(p)]^{-1}\ . \tag{31}$$

At the tree level this equation gives the trivial result $p'=p+k$ and, writing the one-loop vertex as

$$i\Lambda^{1\nu}_{ji}=i\Lambda^{\nu}_{ji}+ieQ_u\gamma^{\nu}\delta_{ji}\ , \tag{32}$$

one obtains, at one-loop order, for the FC vertex,

$$ik_{\nu}\Lambda^{1\nu}_{ji}=ieQ_u[\Sigma_{ji}(\not p')-\Sigma_{ji}(\not p)]$$
$$-i(g/\sqrt{2})[(\not p'-m_j)U_{jn}iT_{1ni}$$
$$-iT'_{1jn}U^{\dagger}_{ni}(\not p-m_i)]\ , \tag{33}$$

where.

$$\Sigma_{ji}(\not p)=-i[S_{ij}(\not p)]^{-1}+(\not p-m_i)\delta_{ji} \quad (34)$$

is the one-loop quark self-energy, and T_1 and T'_1 are to be replaced by their one-loop results shown in Fig. 4. If one notices that there are no one-loop counterterms for these composite operators, Eq. (21a) follows immediately from the expression above.

The procedure for down-type quarks is analogous and the final result can be obtained replacing U by its conjugate $U^\dagger$ (and vice versa) and c by the negatively charged conjugate $c^\dagger$ (and vice versa).

The WST identity for the Zq_iq_j vertex is derived along similar steps as for the photon vertex. The result is somehow less simple, due to the longitudinal polarization of the Z boson, which, in the HF gauge, is embodied in the neutral unphysical scalar ϕ_Z. We start with the BRS invariance of the Green's function $\langle 0|T\bar{c}_z(x)u_j(y)\bar{u}_i(z)|0\rangle$, and obtain, in momentum space, an equation similar to Eq. (29): namely,

$$\begin{aligned}k^\mu G_{\mu\nu}(k)S_{jk}(p')i\Lambda^\nu_{kl}S_{li}(p)=&-(g/\sqrt{2})\Delta_c(k)[U_{jn}iT_{1nl}S_{li}(p)-S_{jl}(p')iT'_{1ln}U^\dagger_{ni}]\\&+eQ_u\Delta_c(k)[iT_{2jl}S_{li}(p)-S_{jl}(p')iT'_{2li}]\\&+(g/\cos\theta_W)\Delta_c(k)[iT_{3jl}S_{li}(p)-S_{jl}(p')iT'_{3li}]+M_ZG(k)S_{jk}(p')\Lambda_{kl}S_{li}(p)\ . \quad (35)\end{aligned}$$

One should remember that now Δ_c and $G_{\mu\nu}$ are the propagators for c_Z and the Z boson, respectively, G is the ϕ_Z propagator, T_{1ji} and T'_{1ji} are composite operators similar to the ones defined before, but with the incoming c_A replaced by c_Z, also, to one-loop order, $T_{2ji}=T'_{2ji}=0$, $iT_{3ji}=-\delta_{ji}(Q\sin^2\theta_W-T_3\gamma_L)$, and $iT'_{3ji}=-\delta_{ji}(Q\sin^2\theta_W-T_3\gamma_R)$, and finally $i\Lambda_{ji}$ is the $\phi_Z u_j u_i$ proper vertex. Again, we derive[26] an auxiliary WST identity: starting with the BRS invariance of the Green's function $\langle 0|TZ_\mu(x)\bar{c}_z(y)|0\rangle$, we obtain

$$k^\nu G_{\mu\nu}(k)=-k_\mu\Delta_c(k)-ig\cos\theta_W\Delta_c(k)(F_\mu-F'_\mu)-iM_ZE_\mu(k)\ , \quad (36)$$

where the composite operators, F_μ and F'_μ are similar to the ones in Eq. (30), and again of one-loop order, and E_μ is the $Z\phi_Z$ mixing two-point Green's function. Replacing this result in Eq. (35), we obtain the WST identity. At one-loop order, for the FC vertex, it is

$$\begin{aligned}ik_\nu\Lambda^{1\nu}_{ji}=&\,i(g/\cos\theta_W)[\Sigma_{ji}(\not p')(Q\sin^2\theta_W-T_3\gamma_L)-(Q\sin^2\theta_W-T_3\gamma_R)\Sigma_{ji}(\not p)]\\&-i(g/\sqrt{2})[(\not p'-m_j)U_{jn}iT_{1ni}-iT'_{1jn}U^\dagger_{ni}(\not p-m_i)]-M_Z\Lambda^1_{ji}\ , \quad (37)\end{aligned}$$

where $\Lambda^{1\nu}_{ji}$ is the one-loop proper Z vertex

$$i\Lambda^{1\nu}_{ji}=i\Lambda^\nu_{ji}+i(g/\cos\theta_W)\gamma^\nu(Q\sin^2\theta_W-T_3\gamma_L)\delta_{ji}\ , \quad (38)$$

and, similarly,

$$i\Lambda^1_{ji}=i\Lambda_{ji}+g(m_i/M_W)T_3\gamma_5\delta_{ji} \quad (39)$$

is the one-loop proper ϕ_Z vertex. Again, T_1 and T'_1 are not renormalized; thus, Eq. (21b) follows easily.

Adopting the on-shell renormalization scheme, it is clear that, for on-shell quarks, the first two terms on the right-hand side of Eq. (37) vanish. However the last term, proportional to M_Z is not zero and therefore one obtains Eq. (23b). Similarly, Eq. (23a) follows from Eq. (33).

IV. CONCLUSION

We have computed the amplitude for the FC Zq_iq_j and γq_iq_j effective vertices, for off-shell external particles, in the HF gauge. The calculations were done to one-loop order, with no approximations, and the renormalization was achieved, using the on-shell scheme to generate and determine the counterterms. Notice that such FC counterterms do indeed exist, in this scheme, both for the proper vertex, and for the self-energy, although there are no corresponding terms in the tree-level Lagrangian; they stem from the renormalization of the quark fields. The results can be read from Table I, using Eq. (2) which has the same form as the analogous equation for the photon vertex, in Ref. 14. Hence, it is straightforward to check this result against our own: every contribution to the effective photon vertex in Ref. 14 was confirmed [notice the misprints in Eqs. (23) and (35a) and in the entry B_6 of Table I of Ref. 14] either analytically or numerically (this was the case for the proper vertex counterterm). This partially checks our result for the Z vertex, since the calculation was done simultaneously for both vertices, whenever this was possible, keeping track of the different couplings for the γ and the Z. Moreover, setting the external quarks on shell and performing the appropriate approximations, the previous results for the Z vertex[3,8,9] were recovered. A check has also been made of the exact WST identities [Eqs. (33) and (37)], which we have derived; in particular, it seemed necessary to test the gauge-dependent terms that arise in the off-shell identity. This was done numerically, using the results that we had obtained for the vertices, and the WST identity was proved to be correct.

Putting the external particles on shell, it was found that the counterterms cancel, i.e., that the S-matrix element is not renormalized. This result can be derived using the WST identity which relates the counterterms for the proper vertex and for the self-energy, which means that it originates from some feature of the tree-level Lagrangian. However, no explanation can be provided within the context of the on-shell renormalization scheme, where the relevant objects are the 1PI Green's functions.

Using the renormalization scheme proposed by Sirlin,[20] the result follows trivially. In this scheme, only the parameters are renormalized, so that the redundant counterterms, which came from field renormalization, are eliminated. In this way, it is immediate to see that those S matrix elements which do not have a tree-level contribution will not be renormalized, whereas in the usual on-shell scheme, this behavior only appears after the calculation, and with no obvious connection to the absence of the tree-level coupling.

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APPENDIX

The Slavnov operator has the following action on the fields:

$$sA_\mu = \partial_\mu c_A + ie(c^\dagger W_\mu - cW_\mu^\dagger) , \tag{A1}$$

$$sZ_\mu = \partial_\mu c_Z - ig\cos\theta_W(c^\dagger W_\mu - cW_\mu^\dagger) , \tag{A2}$$

$$s\bar{c}_A = \partial_\mu A^\mu , \tag{A3}$$

$$s\bar{c}_Z = \partial_\mu Z^\mu - M_Z\phi_Z , \tag{A4}$$

$$su = -i(g/\sqrt{2})Uc\gamma_L d + ieQ_u c_A u + i(g/\cos\theta_W)c_Z(Q_u\sin^2\theta_W - T_{3u}\gamma_L)u , \tag{A5}$$

$$s\bar{u} = -i(g/\sqrt{2})\bar{d}\gamma_R c^\dagger U^\dagger + ieQ_u\bar{u}c_A + i(g/\cos\theta_W)\bar{u}(Q_u\sin^2\theta_W - T_{3u}\gamma_R)c_Z , \tag{A6}$$

$$sd = -i(g/\sqrt{2})U^\dagger c^\dagger\gamma_L u + ieQ_d c_A d + i(g/\cos\theta_W)c_Z(Q_d\sin^2\theta_W - T_{3d}\gamma_L)d , \tag{A7}$$

$$s\bar{d} = -i(g/\sqrt{2})\bar{u}\gamma_R cU + ieQ_d\bar{d}c_A + i(g/\cos\theta_W)\bar{d}(Q_d\sin^2\theta_W - T_{3d}\gamma_R)c_Z . \tag{A8}$$

W_μ is the field that annihilates a W^+.

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