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Radiative quark and lepton masses through soft supersymmetry breaking

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A supersymmetric extension of the standard strong and electroweak gauge model has the property that light quarks and leptons acquire radiative masses as the result of the breaking of a chiral symmetry by soft supersymmetry-breaking terms. This radiative mechanism is then extended to include all known quarks and leptons in the context of a supersymmetric SU(3)×SU(2)×U(1)×U(1)′ gauge model based on E_6 particle content as the possible low-energy limit of a superstring theory.

The u and d quarks have masses of just a few MeV, and the electron mass is only 0.5 MeV. These are all very small compared to the masses of the other known quarks (s,c,b) and leptons (μ,τ), apart from the neutrinos (ν_e,ν_μ,ν_τ) which are all consistent with being massless. Whereas the mass of any fermion in the standard SU(2)×U(1) gauge model[1] is a free parameter, the fact that m_u, m_d, and m_e are many orders of magnitude smaller than the electroweak symmetry-breaking scale of about 10^2 GeV may well be indicative of a radiative mechanism at work for these masses. The following is an example of how this can be done within the context of a supersymmetric extension of the standard model. A more comprehensive program in which all known quarks and leptons acquire radiative masses will also be discussed from the perspective of an E_6 superstring model.

Consider a minimal supersymmetric extension[2] of the standard model with two Higgs-boson doublets H_1 and H_2 of opposite weak hypercharge, i.e., $-\frac{1}{2}$, $+\frac{1}{2}$, respectively. The d, s, and b quarks as well as the e, μ, and τ leptons will have masses proportional to the vacuum expectation value $\langle H_1^0\rangle$, whereas the u, c, and t quarks will have masses proportional to $\langle H_2^0\rangle$, through their various Yukawa couplings. The neutrinos are a special case, because their right-handed components are inert under SU(2)×U(1) and do not have to be included in the standard model. In their absence and assuming lepton-number conservation, the neutrinos will remain massless. On the other hand, in the presence of right-handed components, they will obtain masses through $\langle H_2^0\rangle$. To prevent u, d, and e from picking up tree-level masses, all one needs to do at this stage is to impose a discrete odd-even symmetry (Z_2) on the Lagrangian such that all fields are even under Z_2 except u_R, d_R, and e_R (together with their supersymmetric scalar partners $\tilde{u}_R$, $\tilde{d}_R$, and $\tilde{e}_R$) which are odd. Hence terms such as $\bar{u}_R u_L H_2^0$, $\bar{d}_R d_L H_1^0$, and $\bar{e}_R e_L H_1^0$ are forbidden, and a chiral symmetry exists to keep u, d, and e massless. Now since $H_{1,2}$ are even under Z_2, this discrete symmetry is maintained even in the presence of the spontaneous breaking of the gauge symmetry. Consequently, radiative masses are not possible unless there are soft gauge-invariant terms in the Lagrangian which are odd under Z_2. In particular, the terms $\tilde{u}_R^*\tilde{u}_L H_2^0$, $\tilde{d}_R^*\tilde{d}_L H_1^0$, and $\tilde{e}_R^*\tilde{e}_L H_1^0$ will be needed. Happily, these are precisely some of the soft supersymmetry-breaking terms[3] assumed to be present in a supersymmetric extension of the standard model. Taken together with the mass terms $\tilde{u}_R^*\tilde{u}_R$, $\tilde{u}_L^*\tilde{u}_L$, etc., which also break supersymmetry but are even under Z_2, it means that the scalar quarks $\tilde{u}_{L,R}$ will mix to form two mass eigenstates

$$\tilde{u}_1=\tilde{u}_L\cos\theta+\tilde{u}_R\sin\theta , \tag{1}$$

$$\tilde{u}_2=-\tilde{u}_L\sin\theta+\tilde{u}_R\cos\theta , \tag{2}$$

and similarly for $\tilde{d}_{L,R}$ and $\tilde{e}_{L,R}$. The resulting mass-generating radiative mechanism is depicted in Fig. 1, where $\tilde{g}$ and $\tilde{B}$ are the supersymmetric fermion partners of the gluon and the U(1) gauge boson, respectively. Because of $\tilde{u}_{L,R}$ mixing, the individual divergences coming from $\tilde{u}_1$ and $\tilde{u}_2$ exchange do indeed cancel in the sum and a finite mass is obtained. Note that if u had a nonzero bare mass to begin with, then the one-loop diagram from gluon exchange would be infinite and proportional to this mass.

Let $m_{1,2}$ be the masses of $\tilde{u}_{1,2}$ and M_3 be the mass of $\tilde{g}$ then their contribution to m_u is given by

$$m_u(\tilde{g})=\frac{\alpha_s\sin2\theta}{\pi}M_3\left[\frac{m_1^2\ln(m_1^2/M_3^2)}{m_1^2-M_3^2}-\frac{m_2^2\ln(m_2^2/M_3^2)}{m_2^2-M_3^2}\right] , \tag{3}$$

where $\alpha_s=g_s^2/4\pi$ is the characteristic interaction strength of quantum chromodynamics. Using $\alpha_s=0.136$ and assuming that M_3 is of the order 10^2 GeV, $m_u(\tilde{g})$ may well be a few MeV if the unknown factor in Eq. (3)

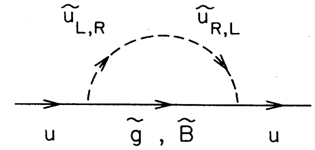

FIG. 1. Radiative mechanism for the mass of the u quark in a supersymmetric extension of the standard model.

due to $\tilde{u}_{L,R}$ mixing is of the order 10^{-3}. The contribution of $\tilde{B}$ is a little more involved because $\tilde{B}$ is definitely not a mass eigenstate. The well-known 4×4 mass matrix spanning $\tilde{B}$, $\tilde{W}_3$, $\tilde{h}_1$, and $\tilde{h}_2$ is given by

$$\underline{M}=\begin{bmatrix} M_1 & 0 & -M_Z s_w c_\beta & M_Z s_w s_\beta \\ 0 & M_2 & M_Z c_w c_\beta & -M_Z s_w s_\beta \\ -M_Z s_w c_\beta & M_Z c_w c_\beta & 0 & -\mu_0 \\ M_Z s_w s_\beta & -M_Z c_w s_\beta & -\mu_0 & 0 \end{bmatrix}, \tag{4}$$

where $\tilde{W}_3$ is the neutral member of the SU(2) gauge fermions and $\tilde{h}_{1,2}$ are the supersymmetric fermion partners of $H^0_{1,2}$. Also, $s_w=\sin\theta_w$, $c_w=\cos\theta_w$, $s_\beta=\sin\beta$, $c_\beta=\cos\beta$, with θ_w the well-known weak mixing angle and $\tan\beta=\langle H_2^0\rangle/\langle H_1^0\rangle$, M_Z is the mass of the Z boson, μ_0 is the supersymmetric Higgs-boson mass, and $M_{1,2}$ are soft supersymmetry-breaking Majorana mass terms which preserve the gauge symmetries U(1) and SU(2), respectively. Let

$$\tilde{B}=N_1\chi_1+N_2\chi_2+N_3\chi_3+N_4\chi_4\ , \tag{5}$$

where χ_i are the mass eigenstates of $\underline{M}$ with eigenvalues μ_i. Then

$$m_u(\tilde{B})=\frac{\alpha}{4\pi}\frac{\sin2\theta}{\cos^2\theta_w}|Y_L Y_R|\sum_i|N_i|^2\mu_i\left[\frac{m_1^2\ln(m_1^2/\mu_i^2)}{m_1^2-\mu_i^2}-\frac{m_2^2\ln(m_2^2/\mu_i^2)}{m_2^2-\mu_i^2}\right], \tag{6}$$

where $\alpha=e^2/4\pi$ and $Y_{L,R}$ are the weak hypercharges of the u quark, i.e., $\frac{1}{6}$ and $\frac{2}{3}$, respectively. Note that if $\mu_0=M_1=M_2=0$ in Eq. (4), then the mass eigenvalues of $\underline{M}$ are just 0, 0, and $\pm M_Z$, in which case $m_u(\tilde{B})$ is zero in Eq. (6). Hence M_1 may be taken to be a rough estimate of the effective Majorana mass contributing to Eq. (6) and $m_u(\tilde{B})/m_u(\tilde{g})$ becomes of the order of $\alpha M_1/36\alpha_s\cos^2\theta_w M_3\approx2\times10^{-3}M_1/M_3$, which should be negligible. Note that in supergravity models,[4] M_1/M_3 is given by $5\alpha/3\alpha_s\cos^2\theta_w\approx0.12$. Similar statements hold for the mass of the d quark, which will be mostly given by the analog of Eq. (3). In the case of the electron mass, $\tilde{g}$ does not contribute, so it is entirely given by the analog of Eq. (6), i.e.,

$$m_e=\frac{\alpha}{8\pi}\frac{\sin2\theta_e}{\cos^2\theta_w}M_{\text{eff}}\ , \tag{7}$$

which yields 0.5 MeV if $M_{\text{eff}}\sin2\theta_e$ is about 1.25 GeV. As for neutrinos, the analog of Eq. (7) will give them nonzero Dirac masses. If these turn out to be of the order m_e, then a large Majorana mass for each right-handed component will be needed to generate a small mass for the observed left-handed component.[5]

Consider now a supersymmetric $SU(3)\times SU(2)\times U(1)\times U(1)'$ gauge model populated by particles belonging to the **27** representation of E_6. This is a candidate model[6] for the low-energy limit of a superstring theory. In Table I the various components of the **27** are listed against their $SU(3)\times SU(2)\times U(1)\times U(1)'$ contents. The known particles are of course Q, u^c, d^c, L, and e^c, where the superscript c denotes charge conjugation and all states are taken to be left handed. Note that d^c, L, and N^c are indistinguishable from h^c, E, and S, respectively, through their gauge interactions. However, they do have distinguishable Yukawa interactions up to a simultaneous exchange of the two sets of fields.[7] There are 11 possible terms in the superpotential. In all such models proposed to date,[8] three of these terms, i.e., $Qu^c\bar{E}$, Qd^cE, and Le^cE, are always assumed to be present because they provide masses for the known quarks and leptons as the neutral-scalar components of $\bar{E}$ and E pick up vacuum expectation value. In the following, they will be assumed to be absent through the application of a single Z_2 discrete symmetry as shown in Table I. Consequently, there are only five allowed terms in the superpotential:

$$W=\lambda_1 SE\bar{E}+\lambda_2 Shh^c+\lambda_3 QLh^c+\lambda_4 u^ce^ch+\lambda_5 d^cN^ch\ . \tag{8}$$

Note that the terms QQh and $u^cd^ch^c$ are forbidden at the same time, thus avoiding rapid proton decay. The masses of the exotic color singlets E and $\bar{E}$, as well as those of the exotic color triplets h and h^c, are proportional to the vacuum expectation value of S, but all known quarks and

TABLE I. Transformation properties of the various components of the **27** representation under $SU(3)\times SU(2)\times U(1)\times U(1)'\times Z_2$.

	SU(3)	SU(2)	U(1)	U(1)′	Z_2
Q	3	2	$\frac{1}{6}$	$\frac{1}{3}$	+
u^c	3*	1	$-\frac{2}{3}$	$\frac{1}{3}$	−
d^c	3*	1	$\frac{1}{3}$	$-\frac{1}{6}$	−
L	1	2	$-\frac{1}{2}$	$-\frac{1}{6}$	−
e^c	1	1	1	$\frac{1}{3}$	+
E	1	2	$-\frac{1}{2}$	$-\frac{1}{6}$	+
$\bar{E}$	1	2	$\frac{1}{2}$	$-\frac{2}{3}$	+
S	1	1	0	$\frac{5}{6}$	+
N^c	1	1	0	$\frac{5}{6}$	+
h	3	1	$-\frac{1}{3}$	$-\frac{2}{3}$	−
h^c	3*	1	$\frac{1}{3}$	$-\frac{1}{6}$	−

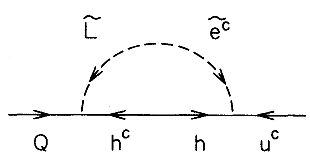

FIG. 2. Radiative mechanism for the mass of the u quark in the supersymmetric $SU(3)\times SU(2)\times U(1)\times U(1)'\times Z_2$ gauge model based on E_6 particle content.

leptons remain massless at the tree level. It is also assumed here that N^c does not pick up a vacuum expectation value.[9]

To obtain radiative masses for the quarks and leptons, soft breaking of Z_2 is again required. In particular, the soft supersymmetry-breaking terms $\tilde{Q}\tilde{u}^c\tilde{\bar{E}}$, $\tilde{Q}\tilde{d}^c\tilde{E}$, $\tilde{L}\tilde{e}^c\tilde{E}$, and $\tilde{L}\tilde{N}^c\tilde{\bar{E}}$ are assumed to be present. Hence the radiative mechanism already discussed in this paper is again at work. In addition, there will be contributions from h exchange as depicted in Fig. 2. The resulting mass for the u quark is given by

$$m_u(h)=\frac{\lambda_3\lambda_4\sin 2\theta_e}{32\pi^2}\times m_h\left[\frac{m_1^2\ln(m_1^2/m_h^2)}{m_1^2-m_h^2}-\frac{m_2^2\ln(m_2^2/m_h^2)}{m_2^2-m_h^2}\right], \quad (9)$$

where $m_{1,2}$ now refer to the scalar-electron mass eigenvalues. Since the Yukawa couplings $\lambda_{3,4,5}$ are not necessarily small and m_h is related to the $U(1)'$-breaking scale, this mechanism is also capable of generating large masses. Because each radiative mass is proportional to the product of four quantities (two couplings, one mass, and one mixing angle), it is easy to get a large enhancement even if no single quantity changes its value by very much. Thus all known quarks and leptons may very well owe their masses to the mixing among their supersymmetric scalar partners (u from $\tilde{e}$, d from $\tilde{\nu}$, e from $\tilde{u}$, ν from $\tilde{d}$) and the existence of the $U(1)'$ scale of symmetry breaking. By doing away with quark and lepton masses at the tree level, the embarrassing issue of small Yukawa couplings in E_6 superstring models is now avoided. However, small ratios within a generation, such as $m_\nu/m_d\sim\sin 2\theta_d/\sin 2\theta_\nu$, which come from soft supersymmetry-breaking terms are yet to be understood. Additional discrete symmetries beyond the single Z_2 discussed so far may perhaps shed some light.

It may be argued that it is not very natural to have a discrete symmetry of the superpotential which is not shared by the full theory. On the other hand, the full theory is certainly not supersymmetric; hence, the assumed soft breaking of the discrete symmetry is not much different in spirit to the assumed soft breaking of the supersymmetry, and in this model, they actually come from the same terms. However, the renormalized kinetic terms will have finite off-diagonal entries; hence, there will be induced corrections to the mass matrix beyond those which are computed here, but they amount only to a redefinition of the scalar quark and lepton mass matrices which are arbitrary to begin with. Fine-tuning of parameters is of course still necessary to obtain the very small quark and lepton masses, but the degree of fine-tuning is much less because there is already a strong suppression factor [$\lambda_3\lambda_4/32\pi^2$ in Eq. (9)] from the inherent radiative mechanism.

In conclusion, radiative quark and lepton masses are easily obtained in models with soft supersymmetry breaking. In a simple extension of the standard model, the u and d quarks pick up masses through gluino exchange,[10] while the electron does so through the exchange of the $U(1)$ gauge fermion. For larger radiative masses, a higher mass scale of symmetry breaking is called for. This can be accommodated within the context of a supersymmetric $SU(3)\times SU(2)\times U(1)\times U(1)'$ gauge model based on E_6 particle content as the possible low-energy limit of a superstring theory. A single simple Z_2 discrete symmetry is used to limit the superpotential of this model to five terms as given by Eq. (8). Radiative quark and lepton masses are then obtained with the breaking of Z_2 by soft supersymmetry-breaking terms. They are expressible in terms of a set of heavy masses belonging to color-triplet fermions h with baryon number $B=\frac{1}{3}$ and lepton number $L=1$, and the mixing among scalar leptons and among scalar quarks. To verify these ideas, supersymmetric particles will have to be discovered and that is of course what one can hope to achieve with the next round of high-energy accelerators.

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