B-$\bar{B}$ mixing and two-scalar-doublet model predicting a Fritzsch structure

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We investigate the Yukawa interactions of a two-scalar-doublet scheme which yields Fritzsch zeros in the mass matrices. Our motivation is to study the plausibility of obtaining the observed large B_d-$\bar{B}_d$ mixing without an exceptionally large t-quark mass. The difficulty with models of this type is the existence of flavor-changing neutral currents. An interesting feature of the present model is that all the relevant Yukawa couplings are determined in terms of quark masses. We show that a low-mass charged-scalar exchange cannot account for the B_d-$\bar{B}_d$ mixing in this model without yielding much too large a value of the CP-violation parameter ϵ. A possible exotic explanation of the large B_d-$\bar{B}_d$ mixing in terms of flavor-changing neutral-scalar exchange exists due to the peculiar neutral-scalar couplings arising in the model.

I. INTRODUCTION

The relatively large amount of B_d-$\bar{B}_d$ mixing measured by the ARGUS Collaboration[1] and later confirmed by the CLEO Collaboration[2] has stimulated a great deal of interest in the question of quark masses and mixing angles. In the standard model (which includes *one* Higgs doublet) the mixing parameter is calculated from the well-known box diagram with W-boson exchanges. Since the dominant term is proportional (other factors being more or less "known") to the square of the top-quark mass m_t, the implication[3] is that m_t is on the large side: $m_t \gtrsim 50$ GeV with $m_t \sim 100$ GeV corresponding to "central" values of the parameters. Such a conclusion is, of course, unpleasant for the possibility of observing the t quark in the laboratory but another relevant question is whether it disagrees with any theories or models for quark masses and mixings. The most well-known model of this type is that of Fritzsch.[4] Before the ARGUS Collaboration result it was considered to give a fairly promising account of the situation. Actually in order to make a fair comparison of this model with experiment it is necessary to take account of the full allowed range of input parameter variation. This was carried out before the ARGUS result and it was seen[5] that a value for m_t roughly around 90 GeV is the highest that can be achieved in the Fritzsch model. After the ARGUS result several authors[6] pointed out that because of the upper limit on the top-quark mass the Fritzsch model is in possible danger, though not quite ruled out.

Since the Fritzsch model has some attractive features, a number of attempts to "save" it were made. For example, the possibility of a (fourth-generation) t'-dominated B_d-$\bar{B}_d$ transition amplitude was investigated[7] with a Fritzsch structure and it was found that (independent of parameter choices) the model would not work because it gave too large a value for the "CP impurity" ϵ. Another small extension of the standard model is to include *two* scalar doublets. Then there is another contribution to the B_d-$\bar{B}_d$ amplitude coming from box diagrams with charged-scalar-boson exchange.[8] For suitably small mass (~ 25 GeV) of the charged scalar boson these diagrams can easily dominate.[9] Hence the Fritzsch structure might possibly be maintained, if desired, even if B_d-$\bar{B}_d$ mixing is larger than its central value. In this paper we shall discuss a speculative model with two scalar doublets as well as a Fritzsch structure to examine this question.

To be specific let us take seriously the standard model with two scalar doublets present. Then we might also include provision for an (invisible) axion[10] to solve the "Θ problem." Furthermore we must worry about flavor-changing processes mediated by neutral-scalar exchange. The standard solution[11] for the latter problem is to have one scalar doublet supply masses to the up quarks and the other to the down quarks. However, this solution provides no "natural" explanation for an assumed Fritzsch form of the mass matrix. In an investigation designed to study the Fritzsch structure including the effects of scalar exchange it is clearly desirable to consider models in which the Fritzsch structure is automatic. This is not difficult to achieve. Fritzsch[12] showed that the characteristic zeros in the mass matrices could be obtained by introducing several scalars and axial-type global symmetries. Davidson *et al.*[13–15] provided a more compact solution by utilizing the U(1) Peccei-Quinn (PQ) symmetry as the axial-type symmetry and by pointing out that, in the two-scalar case, the requirement of different U(1) numbers for different generations uniquely leads to the Fritzsch zeros. We shall investigate the Yukawa structure for this model to see what it has to say about the problem of large B_d-$\bar{B}_d$ mixing. Actually, as we shall see and as was already implied by Ref. 15, the most natural interpretation of the model with a true invisible axion present is that of an effective low-energy single-Higgs-boson theory in which all other scalar particles are pushed to an extremely large mass. Here, however, we shall not insist on the axion constraints from cosmology and just regard the model as a convenient toy one for encompassing an automatic Fritzsch structure with only two scalar doublets.

The difficulty of the present model is that because it is technically natural (modulo a small extra assumption to be discussed later) there are flavor-changing neutral-scalar couplings.[16] It will be seen that the characteristic factor for the coupling of a particular neutral scalar to fermions of masses m_1 and m_2 is[17] $\sqrt{m_1 m_2}$. (This generalizes the characteristic factor m for the flavor-diagonal coupling.) The observed K_L-K_S mass difference then requires a particular neutral-scalar particle to be heavier than about 5 TeV. A crucial question is what consequence this has for the charged-scalar mass. If the scale 5 TeV is considered to be a characteristic one of some new physics, then the charged scalar should be similarly heavy with consequent small contribution to B_d-$\bar{B}_d$ mixing. The model itself would be consistent with the minimal standard model at low energies. It *is* possible to have a low-mass charged scalar particle but this would require a special choice of nonperturbatively large parameters in the scalar sector. Furthermore, as will be discussed later, explaining a large B_d-$\bar{B}_d$ mixing through charged-scalar exchange would lead to an unacceptably large *CP*-violation parameter ϵ. Another possibility is to allow the charged-scalar particle to be heavy and try to put the responsibility for the large B_d-$\bar{B}_d$ mixing on the flavor-changing neutral-scalar exchange directly. This does not affect the predicted *CP* violation in the *K*-meson decays but does, if one assumes a large scale for all scalars other than the normal one, give a too large K_L-K_S mass difference. There is the amusing possibility, however, that if one is willing to accept an unnaturally low-mass extra neutral scalar, the B_d-$\bar{B}_d$ mixing could be explained without violating other experimental bounds. This might be interpreted as a hint that if m_t is measured to be low a more general theory might choose to explain the large B_d-$\bar{B}_d$ mixing by neutral-scalar exchanges at the tree level rather than by charged scalars contributing to a loop diagram.

In Sec. II the potential and masses for the two-scalar-doublet model are discussed based on the invisible-axion formalism. As mentioned, however, we shall just use the axion symmetries to get restrictions on the two-scalar-doublet members. Thus we can only interpret the model as a toy one for learning what consequences the requirement of natural Fritzsch zeros might have. The Yukawa interactions of the doublet scalar mesons are calculated in Sec. III. Even though the characteristic Fritzsch structure is symmetric the fact that its eigenvalues alternate in sign introduces an interesting asymmetry in the flavor-changing coupling matrix. This asymmetry turns out to play a physical role in that it leads to the exchange of different neutral scalars dominating the weak mass differences in the *K* and *B* system. In Sec. IV we estimate the most relevant flavor-changing effects of neutral-scalar exchange in order to get information on the scalar masses. Section V contains the calculation of the charged-scalar contributions to the box diagram in the *K* and *B* systems. Finally a brief summary is provided in Sec. VI.

II. THE SCALAR PARTICLES

We shall work in the framework of a conventional invisible-axion scheme. The appropriate scalar content of the model is

$$\phi_1=\begin{bmatrix}\phi_1^+\\ \phi_1^0\end{bmatrix},\quad \phi_2=\begin{bmatrix}\phi_2^+\\ \phi_2^0\end{bmatrix},\quad \sigma\ , \tag{2.1}$$

where ϕ_1 and ϕ_2 are two SU(2) doublets with weak hypercharge one and the complex singlet σ is introduced in a standard way to make the axion invisible. Without any loss of generality the U(1) PQ transformation properties of these scalar fields may be taken as

$$\phi_1\to e^{i\tau}\phi_1,\quad \phi_2\to e^{-i\tau}\phi_2,\quad \sigma\to e^{2i\tau}\sigma\ , \tag{2.2}$$

corresponding to quantum numbers 1, -1, and 2 for ϕ_1, ϕ_2, and σ. The relevant terms in the pure meson Lagrangian are

$$\begin{aligned}\mathcal{L}_{\text{meson}}=&-\tfrac{1}{2}(D_\mu\phi_1)^\dagger(D_\mu\phi_1)-\tfrac{1}{2}(D_\mu\phi_2)^\dagger(D_\mu\phi_2)\\ &-\partial_\mu\sigma^*\partial_\mu\sigma-V(\phi_1,\phi_2,\sigma)\ .\end{aligned} \tag{2.3}$$

Here the potential[18] is

$$\begin{aligned}V=&c_1\phi_1^\dagger\phi_1+c_2\phi_2^\dagger\phi_2+c_3(\phi_1^\dagger\phi_1)^2+c_4(\phi_2^\dagger\phi_2)^2\\ &+c_5(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)+c_6(\phi_2^\dagger\phi_1)(\phi_1^\dagger\phi_2)\\ &+d_1\sigma^*\sigma+d_2(\sigma^*\sigma)^2+d_3\phi_1^\dagger\phi_2\sigma+d_3^*\phi_2^\dagger\phi_1\sigma^*\\ &+d_4\sigma^*\sigma\phi_1^\dagger\phi_1+d_5\sigma^*\sigma\phi_2^\dagger\phi_2\ .\end{aligned} \tag{2.4}$$

Notice that the symmetry (2.2) has eliminated some terms which might otherwise have been present in (2.4). Except for d_3, all the coefficients c_i and d_i are real. We expect the scale of those c_i which are not dimensionless to be very roughly of the order of the usual weak scale 250 GeV; e.g., $c_1\sim(250\text{ GeV})^2$. On the other hand, d_i might normally be related to a scale Λ appropriate to the invisible axion; e.g., $d_1=O(\Lambda^2)$. From a variety of astrophysical and other considerations on axion interactions it is expected that[19] $\Lambda>10^4-10^6$ TeV. For the axion of our model that has flavor-changing couplings, the scale must be even larger, as will be discussed in Sec. IV. How these different scales could arise naturally is nothing more than the familiar hierarchy problem if one embeds the model in a grand unified theory. We will take the position that this problem can be solved in some more comprehensive model and in particular we take Λd_3 as a free parameter. This allows one to interpret some of our results in a more general framework.

The scalar vacuum values may be consistently taken to be

$$\langle \phi_1^0 \rangle = v_1, \quad \langle \phi_2^0 \rangle = v_2, \quad \langle \sigma \rangle = \Lambda e^{ib} , \tag{2.5}$$

where, because of (2.2) and weak hypercharge symmetry, v_1 and v_2 can be chosen real and positive. We define

$$(v_1)^2 + (v_2)^2 \equiv v^2 = \frac{1}{\sqrt{2}G_F} \simeq (250 \text{ GeV})^2 . \tag{2.6}$$

Furthermore we have

$$\cos(w+b) = -1 \tag{2.7}$$

with $d_3 \simeq |d_3| e^{iw}$. In order to discuss the mass spectrum it is convenient to define for the two-scalar-doublet fields the combinations ϕ_G and ϕ_P by

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \frac{1}{v} \begin{bmatrix} v_1 & -v_2 \\ v_2 & v_1 \end{bmatrix} \begin{bmatrix} \phi_G \\ \phi_P \end{bmatrix} . \tag{2.8}$$

For the neutral-scalar fields we define real and imaginary pieces by

$$\phi^0 = \phi_R^0 + i\phi_I^0 . \tag{2.9}$$

Now by explicit computation from (2.4) we may find the mass spectrum. The three zero-mass fields which are absorbed by the $W^\pm$ and Z bosons in the unitary gauge are $\phi_G^\pm$ and ϕ_{GI}^0. The physical charged scalars are the fields $H^\pm = (1/\sqrt{2})\phi_P^\pm$ with squared mass

$$M_H^2 = 2v^2 \left[-c_6 + \frac{\Lambda |d_3|}{v_1 v_2} \right] . \tag{2.10}$$

We shall not discuss the σ field interactions here. The invisible-axion field is essentially $\Lambda \arg\sigma$ (for large Λ) with a minute admixture of ϕ_{PI}^0 given by $(v_1 v_2 / v\Lambda)\phi_{\mathrm{PI}}^0$ while $|\sigma|$ is a very heavy singlet, whose mass may actually be taken much heavier than the scale Λ. Of more interest are the three remaining neutral combinations of the doublet fields. ϕ_{PI}^0 does not mix with ϕ_{PR}^0 or ϕ_{GR}^0 (in the absence of σ it would be the original axion) and has the squared mass

$$M^2(\phi_{\mathrm{PI}}^0) = \frac{2v^2 \Lambda |d_3|}{v_1 v_2} . \tag{2.11}$$

Finally, the two fields ϕ_{GR}^0 and ϕ_{PR}^0 mix with each other; the mass-squared matrix in $\phi_{\mathrm{GR}}^0, \phi_{\mathrm{PR}}^0$ space being

$$\begin{aligned} M^2 &= \begin{bmatrix} A & B \\ B & C+D \end{bmatrix} , \\ A &= 8v^2 \left[c_3 \left[\frac{v_1}{v} \right]^4 + c_4 \left[\frac{v_2}{v} \right]^4 + (c_5 + c_6) \left[\frac{v_1 v_2}{v^2} \right]^2 \right] , \\ B &= 4 v_1 v_2 \left\{ -2c_3 \left[\frac{v_1}{v} \right]^2 + 2c_4 \left[\frac{v_2}{v} \right]^2 \right. \\ &\qquad \left. + (c_5 + c_6) \left[\left[\frac{v_1}{v} \right]^2 - \left[\frac{v_2}{v} \right]^2 \right] \right\} , \\ C &= 8 \frac{v_1^2 v_2^2}{v^2} (c_3 + c_4 - c_5 - c_6) , \\ D &= M^2(\phi_{\mathrm{PI}}^0) . \end{aligned} \tag{2.12}$$

Notice that $|d_3|$ is the only one of the d_i's to appear in this formula. Thus we might have neglected the σ field from the beginning and just included in the potential a $\phi_1^\dagger \phi_2$ type coupling with an arbitrary strength of order Λd_3 which may come from some unspecified U(1)-breaking interaction. In this sense Λd_3 may be considered *a priori* arbitrary.

We first notice from (2.10)–(2.12) that if $\Lambda |d_3| \gg v^2$, the four fields $H^\pm$, ϕ_{PI}^0, and ϕ_{PR}^0 are very massive and approximately degenerate with each other. In this case the field ϕ_{GR}^0 does not acquire an extremely large mass and plays the role of the usual Higgs field. This is the most conservative and probably the most reasonable interpretation of the model. However, from a strictly phenomenological standpoint one might be interested in asking if it is possible to get a low $H^\pm$ mass while keeping ϕ_{PI}^0 very heavy. Comparison of (2.10) and (2.11) shows that this is possible provided[20] c_6 is positive and much greater than unity (which, of course, mangles any attempt at a perturbative treatment of the scalar sector). In this situation there are sufficient free parameters in V so that ϕ_{GR}^0 and ϕ_{PR}^0 mix with an arbitrary angle to yield two eigenstates of arbitrary mass.

III. THE YUKAWA SECTOR

It is an interesting exercise to work out completely the Yukawa sector for this two-scalar-doublet model which automatically yields the Fritzsch zeros. We assign axial-type quantum numbers under the U(1) transformation (2.2) to quarks q_i of the first, second, and third generations as follows:

$$\begin{aligned} &q_{1L}: \tfrac{5}{2}, \quad q_{2L}: -\tfrac{3}{2}, \quad q_{3L}: \tfrac{1}{2} , \\ &q_{1R}: -\tfrac{5}{2}, \quad q_{2R}: +\tfrac{3}{2}, \quad q_{3R}: -\tfrac{1}{2} . \end{aligned} \tag{3.1}$$

This assignment guarantees the Fritzsch zeros in the up- and down-quark mass matrices which arise from the trilinear Yukawa terms. It may be verified[14] that this is actually the unique choice which leads to a nontrivial intergenerational mixing while insisting on a different quantum number for each generation. For ease of computation we shall make the assumption that each quark coupling matrix is Hermitian. We do not feel that this is a serious lack of naturalness because it does *not* suppress flavor-changing neutral-scalar-exchange amplitudes. The same results can also be achieved by assuming the quark coupling matrices to be symmetric. The latter choice can be motivated by embedding the model[15] in SO(10). Here, however, we wish to stay in the SU(2)×U(1) framework so that (whatever disadvantages may exist) the calculation can be carried out as completely as possible with just two scalar doublets.

The Yukawa part of the Lagrangian may then be written as

$$\mathcal{L}_Y = -\bar{\Psi}_L^0 \phi_1 P_d \begin{bmatrix} 0 & g_1 & 0 \\ g_1 & 0 & 0 \\ 0 & 0 & \tilde{g}_1 \end{bmatrix} P_d^\dagger d_R^0 - \bar{\Psi}_L^0 \phi_2 P_d \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & g_2 \\ 0 & g_2 & 0 \end{bmatrix} P_d^\dagger d_R^0 - \bar{\Psi}_L^0 (i\sigma_2 \phi_1^*) P_u \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & g_1' \\ 0 & g_1' & 0 \end{bmatrix} P_u^\dagger u_R^0 - \bar{\Psi}_L^0 (i\sigma_2 \phi_2^*) P_u \begin{bmatrix} 0 & g_2' & 0 \\ g_2' & 0 & 0 \\ 0 & 0 & \tilde{g}_2' \end{bmatrix} P_u^\dagger u_R^0 + \text{H.c.} , \quad (3.2)$$

where $\bar{\Psi}_L^0 = (\bar{u}_L^0 \bar{d}_L^0)$ and the superscript 0 on the fermion fields indicates that they are not yet mass eigenstates. P_u and P_d are each diagonal matrices of phases while all the g coefficients are real. The lepton terms (assuming zero-mass neutrinos) are exactly analogous to the first two terms of (3.2).

We remind the reader that the real symmetric matrix

$$M = \begin{bmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{bmatrix} \approx \begin{bmatrix} 0 & \sqrt{m_1 m_2} & 0 \\ \sqrt{m_1 m_2} & 0 & \sqrt{m_2 m_3} \\ 0 & \sqrt{m_2 m_3} & m_3 \end{bmatrix} \quad (3.3)$$

of Fritzsch type is diagonalized[21] by

$$R^T M R = \hat{M} X , \quad (3.4)$$

wherein

$$\hat{M} = \text{diag}(m_1, m_2, m_3) ,$$
$$X = \text{diag}(1, -1, 1) , \quad (3.5)$$
$$R \approx \begin{bmatrix} 1 & -\sqrt{m_1/m_2} & \frac{m_2}{m_3}\sqrt{m_1/m_3} \\ \sqrt{m_1/m_2} & 1 & \sqrt{m_2/m_3} \\ -\sqrt{m_1/m_3} & -\sqrt{m_2/m_3} & 1 \end{bmatrix} .$$

We identify the Kobayashi-Maskawa (KM) matrix U as

$$U = R_u^T P_u^\dagger P_d R_d \quad (3.6)$$

and by introducing the mass eigenstate fermion fields (no superscript 0) find the trilinear Yukawa terms (including leptons):

$$\begin{aligned} \mathcal{L}_y^{(3)} \approx {} & \bar{u}_L \left[-\frac{1}{v} \left[\bar{\phi}_G^0 + \frac{v_1}{v_2} \bar{\phi}_P^0 \right] \hat{M}_u + \frac{v}{v_1 v_2} \bar{\phi}_P^0 K_u \right] u_R + \bar{d}_L \left[-\frac{1}{v} \left[\phi_G^0 - \frac{v_2}{v_1} \phi_P^0 \right] \hat{M}_d - \frac{v}{v_1 v_2} \phi_P^0 K_d \right] d_R \\ & + \bar{e}_L \left[-\frac{1}{v} \left[\phi_G^0 - \frac{v_2}{v_1} \phi_P^0 \right] \hat{M}_e - \frac{v}{v_1 v_2} \phi_P^0 K_e \right] e_R + \bar{d}_L \left[\frac{1}{v} U^\dagger \hat{M}_u \left[\phi_G^- + \frac{v_1}{v_2} \phi_P^- \right] - \frac{v}{v_1 v_2} U^\dagger K_u \phi_P^- \right] u_R \\ & + \bar{u}_L \left[-\frac{1}{v} U \hat{M}_d \left[\phi_G^+ - \frac{v_2}{v_1} \phi_P^+ \right] - \frac{v}{v_1 v_2} U K_d \phi_P^+ \right] d_R + \bar{v}_L \left[-\frac{1}{v} \hat{M}_e \left[\phi_G^+ - \frac{v_2}{v_1} \phi_P^+ \right] - \frac{v}{v_1 v_2} \phi_P^+ K_e \right] e_R + \text{H.c.} , \end{aligned} \quad (3.7)$$

where the matrix K is[22]

$$K = R^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & B & 0 \end{bmatrix} R X \approx \begin{bmatrix} -2m_1 & 2\sqrt{m_1 m_2} & \sqrt{m_1 m_3} \\ -2\sqrt{m_1 m_2} & 2m_2 & \sqrt{m_2 m_3} \\ \sqrt{m_1 m_3} & -\sqrt{m_2 m_3} & 2m_2 \end{bmatrix} . \quad (3.8)$$

Notice that the presence of X prevents K from being a symmetric matrix.

The expansion[23] of (3.7) evidently contains many terms. As an example, let us focus on flavor-changing down-type quark bilinears coupling to neutral scalar fields. These will come from the term in (3.7) proportional to $\phi_P^0 \bar{d}_L K_d d_R$ and are explicitly

$$\begin{aligned} \mathcal{L}_d \simeq \frac{-v}{v_1 v_2} [& \sqrt{m_d m_b}\, \bar{d}(\phi_{\text{PR}}^0 - i\gamma_5 \phi_{\text{PI}}^0) b \\ & + 2\sqrt{m_d m_s}\, \bar{d}(i\phi_{\text{PI}}^0 - \gamma_5 \phi_{\text{PR}}^0) s \\ & + \sqrt{m_s m_b}\, \bar{s}(i\phi_{\text{PI}}^0 - \gamma_5 \phi_{\text{PR}}^0) b] + \text{H.c.} , \end{aligned} \quad (3.9)$$

wherein we have introduced the usual (e.g., $s = s_L + s_R$) Dirac field for each quark. It is interesting that the flavor-changing neutral-scalar coupling to quarks of masses m_i and m_j is always proportional to $\sqrt{m_i m_j}$ in this model. We saw that ϕ_{PI}^0 is always a mass-eigenstate field while ϕ_{PR}^0 is a mass-eigenstate field only for large scale factor Λ. If Λ is allowed to be low ϕ_{PR}^0 and ϕ_{GR}^0 mix with some angle ψ to give mass eigenstates ϕ_A^0 and ϕ_B^0. Then we should replace ϕ_{PR}^0 by $\phi_B^0 \cos\psi + \phi_A^0 \sin\psi$.

IV. PHENOMENOLOGY OF THE NEUTRAL SCALARS

It is well known that flavor-changing neutral currents are severely suppressed in nature and provide strong constraints on a nontrivial scalar sector. In models of the kind we are discussing where the neutral-scalar couplings are not diagonalized in the physical basis, one is forced to suppress unwanted processes with large scalar masses. In fact, bounds on the axion scale Λ can be set considering[24] the decays $\mu \to e +$axion and $K^+ \to \pi^+ +$axion. In our model these decays would proceed through the minute mixing of the axion with the scalar $\phi_{\rm PI}$. Using the bound[25] $B(\mu \to e + \text{axion}) < 2.6\times 10^{-6}$ we estimate $\Lambda \gtrsim 3\times 10^6$ TeV while the bound[26] $B(K^+ \to \pi^+ + \text{axion}) < 3.8\times 10^{-8}$ yields a somewhat stronger limit of $\Lambda \gtrsim 3\times 10^7$ TeV. However, our purpose is to study the consequences of a more general scheme for flavor violation at a possibly lower scale. As discussed above, this may be done by simply including a symmetry-breaking mass mixing term in the potential with an arbitrary scale and without introducing the σ particle.

In this section we will consider measurements on $K_S \to \mu^+\mu^-$ as well as the K_L-K_S mass difference to further restrict[27] our model and then examine predictions for B_d-$\bar{B}_d$ mixing. We will see that a large B_d-$\bar{B}_d$ mixing through a neutral-scalar exchange is possible but only if restrictions from a perturbative scalar potential can be circumvented in some way.

The relevant effective Lagrangian for $K \to \mu^+\mu^-$ can be written

$$\mathcal{L}_{\rm eff} = G_H(\bar{d}\gamma_5 s - \bar{s}\gamma_5 d)\bar{\mu}\mu \ , \tag{4.1}$$

where G_H can be computed from Eqs. (3.7)–(3.9):

$$G_H \simeq \frac{2\sqrt{m_d m_s}\, m_\mu}{M^2(\phi_{\rm PR})}\left[\frac{v}{v_1 v_2}\right]^2\left[\left[\frac{v_2}{v}\right]^2 - 2\right] . \tag{4.2}$$

$M^{-2}(\phi_{\rm PR})$ is defined as the (2,2) element of the inverse of the mass matrix in (2.12). Note that to this order of calculation, the effective Lagrangian (4.1) contributes only to K_S decay and not to the more experimentally constrained K_L decay. For the decay width we find

$$\Gamma(K_S \to \mu^+\mu^-) = G_H^2 |\langle 0|\bar{d}\gamma_5 s|\bar{K}^0\rangle|^2 \times\left[\frac{M_k}{4\pi}\left[1-\frac{4m_\mu^2}{M_k^2}\right]^{3/2}\right] . \tag{4.3}$$

The matrix element can be estimated using standard current-algebra techniques:

$$\langle 0|\bar{d}\gamma_5 s|\bar{K}^0\rangle \simeq \left\langle 0\left|\frac{\partial_\mu \bar{d}\gamma^\mu\gamma_5 s}{m_d+m_s}\right|\bar{K}^0\right\rangle = \frac{-iF_k M_k^2}{m_d+m_s} . \tag{4.4}$$

Using the experimental limit[28] $\Gamma(K_S \to \mu^+\mu^-) < 2\times 10^{-21}$ GeV and a minimal scalar coupling, corresponding to the case $v_1 = v_2$, we find the mass scale for the Higgs scalar to be constrained by

$$M(\phi_{\rm PR}) \gtrsim 30 \text{ GeV} . \tag{4.5}$$

The point here is that the Yukawa couplings are so small in this model for the first and second generations that rare decay processes such as $K_S \to \mu\mu$, $\mu \to eee$, etc., are sufficiently suppressed with scalar masses around the ordinary electroweak scale. The most sensitive constraint on the neutral flavor-changing scalars will come from $\bar{K}^0$-K^0 mixing which depends on an amplitude directly. The effective Lagrangian for $\bar{K}^0$-K^0 mixing in this model is

$$\mathcal{L}_{\rm eff} \simeq \tfrac{1}{2}G_P(\bar{d}\gamma_5 s)^2 + \tfrac{1}{2}G_S(\bar{d}s)^2 \tag{4.6}$$

with G_P and G_S determined from (3.9):

$$G_P \simeq \frac{4m_d m_s}{M^2(\phi_{\rm PR})}\left[\frac{v}{v_1 v_2}\right]^2 ,\qquad G_S \simeq \frac{-4m_d m_s}{M^2(\phi_{\rm PI})}\left[\frac{v}{v_1 v_2}\right]^2 . \tag{4.7}$$

Note that when evaluating the matrix element of $H_{\rm eff}$ using the vacuum-insertion approximation, the scalar term proportional to G_S will only contribute after a Fierz rearrangement and will be suppressed by roughly $\frac{1}{12}$ compared to the pseudoscalar term. In fact, inserting the vacuum in all possible ways gives

$$\langle K^0|(\bar{d}\gamma_5 s)^2|\bar{K}^0\rangle \simeq \frac{-11}{6}\left[\frac{M_k^2 F_k}{m_d+m_s}\right]^2 + \tfrac{1}{6}M_k^2 F_k^2 ,\qquad \langle K^0|(\bar{d}s)^2|\bar{K}^0\rangle \simeq \frac{1}{6}\left[\frac{M_k^2 F_k}{m_d+m_s}\right]^2 - \tfrac{1}{6}M_k^2 F_k^2 . \tag{4.8}$$

Neglecting $O(m_d/m_s)$ and $O((m_s/M_k)^2)$ corrections, one then finds, for the K_L-K_S mass splitting from neutral-scalar exchange,

$$|\Delta M_k| \simeq \frac{2}{3}\left[\frac{m_d}{m_s}\right]\left[\frac{v}{v_1 v_2}\right]^2 \times M_k^3 F_k^2\left[\frac{11}{M^2(\phi_{\rm PR})} + \frac{1}{M^2(\phi_{\rm PI})}\right] . \tag{4.9}$$

Experimentally[28] we know $|\Delta M_k| \simeq 3.5\times 10^{-15}$ GeV which implies that the neutral-scalar masses be larger than

$$M(\phi_{\rm PR}) \gtrsim 5 \text{ TeV},\quad M(\phi_{\rm PI}) \gtrsim 1.5 \text{ TeV} . \tag{4.10}$$

It is interesting to compare these scales with the mass expressions given in Sec. II. Equation (2.12) implies that $M^2(\phi_{\rm PR})$ and $M^2(\phi_{\rm PI})$ should be split by terms of order $\delta M^2 \lesssim 4\pi\,(250\text{ GeV})^2 = 0.8\text{ TeV}^2$ where we have used the perturbative limit that the quartic couplings appearing in (2.12) be less than 4π. Thus one would expect that if $M(\phi_{\rm PR}) \gtrsim 5$ TeV then $M(\phi_{\rm PI})$ should also satisfy $M(\phi_{\rm PI}) \gtrsim 5$ TeV.

In the case of B_d mixing the situation is slightly different. First the dominant pseudoscalar term in the effective Lagrangian comes from the exchange of $\phi_{\rm PI}$ rather than $\phi_{\rm PR}$ as can be seen from Eq. (3.9). Second, m_b is approximately equal to M_B so we cannot drop axial-vector terms compared to pseudoscalar terms when Fierz rearranging. The result for the mass splitting of the $\bar{B}\,^0_d$-B^0_d mesons is

$$|\Delta M_{B_d}| \simeq \frac{1}{6}\left[\frac{m_d}{m_b}\right]\left[\frac{v}{v_1 v_2}\right]^2 M_B^3 F_B^2 \left[\frac{10}{M^2(\phi_{\rm PI})}\right] . \quad (4.11)$$

Interestingly, in order to fit the recent ARGUS result,[1] $|\Delta M_B| = (3.4\pm 1.1)\times 10^{-13}$ GeV, would require $M(\phi_{\rm PI}) \simeq 1.3$ TeV for the case $v_1 = v_2$. This is roughly consistent with the experimental bound from the K_L-K_S mass difference given in (4.10). More generally one can obtain a useful ratio from (4.9) and (4.11) which is independent of (v_1/v_2):

$$\left|\frac{\Delta M_k}{\Delta M_{B_d}}\right| \simeq \frac{4m_b}{m_s}\frac{M_k^3 F_k^2}{M_B^3 F_B^2}\left[1.1\frac{M^2(\phi_{\rm PI})}{M^2(\phi_{\rm PR})} + 0.1\right]$$
$$\simeq 0.1\left[1.1\frac{M^2(\phi_{\rm PI})}{M^2(\phi_{\rm PR})} + 0.1\right] . \quad (4.12)$$

Experimentally this ratio is approximately $|\Delta M_K/\Delta M_B| \simeq 0.01$. It is clear from (4.12) that this small ratio can be maintained only if $M^2(\phi_{\rm PI}) \ll M^2(\phi_{\rm PR})$. In this simple two-doublet model, such a mass splitting would seem to require large nonperturbative quartic couplings in the scalar potential. We note in closing that there is no CP violation in (4.6) and so there is no further constraint on the neutral Higgs sector from the ϵ parameter. There is, however, an interesting prediction for B_s-$\bar{B}_s$ mixing. Referring to Eq. (3.9) one sees that mixing in the B_s-$\bar{B}_s$ system will be due dominantly to the heavier $\phi_{\rm PR}$ scalar and will thus have some suppression relative to the B_d-$\bar{B}_d$ mixing. We find

$$|\Delta M_{B_s}| \simeq \frac{m^s}{6m_b}\left[\frac{v}{v_1 v_2}\right]^2 M_{B_s}^3 F_{B_s}^2 \left[\frac{10}{M^2(\phi_{\rm PR})}\right] \quad (4.13)$$

and therefore $|\Delta M_{B_s}|$ should be approximately

$$|\Delta M_{B_s}| \simeq \frac{m_s}{m_d}\frac{M^2(\phi_{\rm PI})}{M^2(\phi_{\rm PR})}|\Delta M_{B_d}| \quad (4.14)$$

neglecting the B_d and B_s meson mass and form-factor differences. If we take $m_s/m_d \simeq 20$ and $M^2(\phi_{\rm PI})/M^2(\phi_{\rm PR}) < 10$ as suggested by (4.12) then we expect that $|\Delta M_{B_s}/\Delta M_{B_d}| \lesssim 2$.

In the ordinary standard model with only W-exchange diagrams and three generations, the ratio is predicted[29] to be $|\Delta M_{B_s}/\Delta M_{B_d}| \simeq |V_{ts}/V_{td}|^2 > 4.7$. Therefore, should B_s-$\bar{B}_s$ be measured to be smaller than the standard-model prediction it might possibly be explained along the lines of this toy model with flavor-changing neutral-scalar exchange.

V. PHENOMENOLOGY OF THE CHARGED SCALAR

Recently there has been a great deal of interest in the possible existence of a light charged scalar[30] which might contribute to the large B_d-$\bar{B}_d$ mixing.[9] Because of the mass relation (2.10) and the bound (4.10) the charged-scalar mass in this model must be of the order of $M(\phi_{\rm PI})$ or else the quartic coupling c_6 would have to be larger than the perturbative limit. In this case there would be no hope of getting a sizable contribution to B_d-$\bar{B}_d$ mixing from fourth-order box diagrams with a charged scalar given the bounds on second-order contributions from the neutral scalars. However, we can make an even stronger statement in this model if we consider the CP-violation parameter ϵ. That is, let us assume the coupling scheme predicted by the unique axial symmetry which was necessary to get a Fritzsch structure with two scalar doublets but allow the charged-scalar mass M_H to be a free parameter. We can write the dominant charged-scalar interactions from Eq. (3.7) keeping only the pieces proportional to up-quark masses:

$$\mathcal{L} \simeq \left[\frac{g}{2\sqrt{2}}\right] H_p^- \,\bar{d}U^\dagger G_u^\dagger(1+\gamma_5)u + \text{H.c.} , \quad (5.1)$$

where, for this model,

$$G_u^\dagger = \frac{1}{M_W}\left[\frac{v_1}{v_2}\hat{M}_u - \frac{v^2}{v_1 v_2}K_u\right] . \quad (5.2)$$

When calculating M_{12} from box diagrams, it is clear that top-quark exchange will dominate. Let us for the sake of this argument just consider those contributions. Then the relevant expression for M_{12} is

$$M_{12}^i = -\frac{G_F^2 M_W^2}{48\pi^2}(BMF^2)(\lambda_3^2 E_{33} + X_t\lambda_3\tilde{\lambda}_3 F_{33} + \tilde{\lambda}_3^2 I_{33}) ,$$
$$\lambda_3 = U_{31}^* U_{3i}, \quad \tilde{\lambda}_3 = (G_u U)_{31}^*(G_u U)_{3i} \quad (5.3)$$

where $i=2,3$ for K, B_d meson, respectively, B, M, F^2 are the relevant hadronic factor, mass, and decay constant, and kinematic factors are given by[31]

$$E_{33} = X_t\left[1 + \frac{9}{1-X_t} - \frac{6}{(1-X_t)^2}\right] - 6\left[\frac{X_t}{1-X_t}\right]^3 \ln X_t ,$$
$$I_{33} = \left[X_H^2 - X_t^2 + 2X_H X_t \ln\left[\frac{X_t}{X_H}\right]\right] \Big/ (X_H - X_t)^3 , \quad (5.4)$$

$$F_{33} = (8-2X_H)X_H \ln(X_t/X_H)/(X_H-1)(X_H-X_t)^2$$
$$+ 6\ln(X_t)/(1-X_H)(1-X_t)^2$$
$$+ (8-2X_t)/(X_t-X_H)(1-X_t) ,$$
$$X_t \equiv \frac{M_t^2}{M_W^2}, \quad X_H \equiv \frac{M_H^2}{M_W^2} .$$

In the Fritzsch model the KM matrix is determined if given all the quark masses and two mixing angles, say $|U_{12}|$ and $|U_{23}|$. Our interest here is to see if we can fit a large value of B_d-$\bar{B}_d$ mixing while keeping the top-quark mass small. Given m_t and v_1/v_2 we can fit $\Delta M_B = 2|M_{12}^B|$ by adjusting M_H. But this will also determine $\mathrm{Im}M_{12}^K$ which is accurately determined experimentally from CP violation in K-meson decays,[28] $\mathrm{Im}M_{12}^K = 2\Delta M_K \,\mathrm{Re}\epsilon \simeq -1.1\times 10^{-17}$ GeV. The computations are quite complicated and the rough approximations used so far are not dependable when there are delicate cancellations. Therefore this calculation was done exactly on a computer. The results are presented in Table I and clearly indicate that the large central value of $\bar{B}_d$-B_d mixing reported by ARGUS cannot be accommodated by charged-scalar interactions in this model without producing too much CP violation for the kaon decays. For example, with a top-quark mass of $m_t = 45$ GeV, we varied v_1/v_2 from 1 to 8 and adjusted M_H to fit the central value of ΔM_B. For all fits the CP-violation parameter ϵ was 20 to 30 times too large. As you raise the top-quark mass, less of a contribution from the scalar is necessary to fit ΔM_B and so the situation improves slightly. For $m_t = 70$ GeV, the value of ϵ was 10 to 15 times too large over a range of v_1/v_2 and M_H. To fit all of the K and B meson phenomenology in this model it is still necessary to push the top-quark mass to its upper limit and to take ΔM_B in the lower end of its range. A light charged scalar does not seem to improve the overall fit.

As a separate problem, one could ask if there are any constraints on a charged-scalar interaction of the flavor-conserving type used in Ref. 9 if one assumes a Fritzsch-type KM matrix. We remark that this is unnatural in the sense that you do not have any symmetry predicting Fritzsch zeros in the mass matrices. In this case we take $K_u = 0$ in (5.2) and the expression for M_{12} from just top-quark-exchange box diagrams becomes

$$M_{12}^i \simeq \frac{-G_F^2 M_W^2}{48\pi^2}(BMF^2)_i \lambda_3^2 \left\{ E_{33} + X_t^2 \left[\left(\frac{v_1}{v_2} \right)^2 I_{33} + \frac{v_1}{v_2} F_{33} \right] \right\}. \tag{5.5}$$

It is convenient to consider the ratio

$$\left| \frac{\mathrm{Im}M_{12}^k}{\Delta M_B} \right| \simeq 0.047 \left[\frac{B_k F_k^2}{B_B F_B^2} \right] \frac{\mathrm{Im}(U_{31}^* U_{32})^2}{|U_{31}^* U_{33}|^2} , \tag{5.6}$$

which is experimentally

$$\left| \frac{\mathrm{Im}M_{12}^k}{\Delta M_B} \right| = 3.3^{+1.6}_{-0.8} \times 10^{-5} . \tag{5.7}$$

For the Fritzsch model,[7] $\mathrm{Im}(U_{31}^* U_{32})^2 \simeq |U_{31}^* U_{32}|^2 \sin 2\phi$ with $\sin 2\phi = 0.5 \pm 0.10$ for $|U_{12}| = 0.221 \pm 0.002$. If we take $|U_{32}| = 0.044 \pm 0.008$ we predict a ratio

$$\left| \frac{\mathrm{Im}M_{12}^k}{\Delta M_B} \right| \simeq \left[\frac{B_k F_k^2}{B_B F_B^2} \right] (4\pm 2)\times 10^{-5} , \tag{5.8}$$

which is of the same order of magnitude as the measured value (5.7). The reason that the Fritzsch model gives such a large value for this ratio is that it naturally has a near maximal CP phase as discussed in Ref. 32.

The prediction in (5.8) was a crude numerical approximation and did not include charm-quark-exchange contributions. Evaluating this ratio exactly including all internal quark contributions might serve to further restrict the allowed parameter space in this albeit unnatural two-Higgs-boson Fritzsch model. For example, choosing $m_t \simeq 45$ GeV, $|U_{12}| = 0.22$, and $|U_{23}| = 0.05$ one would be able to fit the central value of ΔM_B with $v_1/v_2 \simeq 2$ and $M_H \simeq 64$ GeV. But this would imply a ratio

$$\left| \frac{\mathrm{Im}M_{12}^k}{\Delta M_B} \right| \simeq \left[\frac{B_k F_k^2}{B_B F_B^2} \right] (14\times 10^{-5}) , \tag{5.9}$$

which appears to be somewhat too large. If QCD corrections for the scalar exchange follow the same general behavior as for the W-exchange diagrams, one would expect them to decrease this ratio by about a factor of 2 bringing it more in line with experiment although still slightly large. With improved experimental measurements and better knowledge of theoretical parameters this ratio could become a useful constraint on possible $\bar{B}_d$-B_d mixing via scalar exchange in this simple although unnatural model. Such a model with two flavor-conserving scalar doublets and an assumed Fritzsch structure for the quark mass matrices was considered in Ref. 33.

TABLE I. Values of charged-scalar parameters needed to fit the central value of B_d-$\bar{B}_d$ mixing and the resulting model prediction for the CP-violation parameter ϵ. The first set of values is for $m_t(m_t) = 46$ GeV, $m_s(1\ \mathrm{GeV}) = 0.175$ GeV and the second set is for $m_t(m_t) = 72$ GeV and $m_s(1\ \mathrm{GeV}) = 0.12$ GeV. In both cases we used $|U_{12}| = 0.22$, $|U_{23}| = 0.05$, and a hadronic factor $B^{1/2}F = 0.15$ GeV.

v_1/v_2	$m_s=0.175$ M_H(GeV)	$m_t=46$ $\epsilon/\epsilon_{\mathrm{expt}}$	$m_s=0.12$ M_H(GeV)	$m_t=72$ $\epsilon/\epsilon_{\mathrm{expt}}$
1	67	30	211	13.2
2	212	18.5	554	8.8
4	693	18.5	1697	8.8
8	2530	18.5	6197	8.8

VI. SUMMARY

We have explicitly worked out the Yukawa sector of a two-scalar-doublet model with an axial PQ symmetry arranged to ensure Fritzsch zeros in the fermion mass matrices. The Yukawa couplings are determined simply in terms of fermion masses and a ratio of scalar vacuum values. An interesting and nontrivial interaction structure was observed in the flavor-changing neutral currents of the model. As is well known, the Fritzsch model does not easily accommodate a large observed B_d-$\bar{B}_d$ mixing through conventional box diagrams due to its upper limit on possible top-quark masses. We have examined the possibility of whether a nontrivial scalar sector could contribute to B_d-$\bar{B}_d$ mixing in a model with Fritzsch-type symmetry and would thus allow for a small top-quark mass. In this regard it is a nice feature that the phenomenology of a two-doublet model with natural Fritzsch zeros is uniquely predicted in terms of fermion masses, one vacuum ratio and the scalar masses. We have carefully analyzed the scalar potential in this model to derive mass relations for the scalars. To make the model's axion invisible one conventionally adds a scalar singlet which breaks the PQ symmetry at a large scale Λ (or alternatively one could add explicit soft-breaking terms to the potential). The most general scalar potential with couplings in the perturbative range leads one to expect a scalar spectrum of one ordinary light neutral Higgs scalar along with two extra neutral scalars and a charged scalar whose masses should be roughly related to the large scale Λ. The mass squared differences are typically of the order of the ordinary electroweak scale squared.

Axion constraints from μ decay and K decay force the scale Λ to be greater than 10^8 TeV but the heavy scalar masses remain free parameters in the model due to an arbitrary scalar coupling. However, K^0-$\bar{K}^0$ mixing via flavor-changing neutral currents requires one of the extra neutral scalars to have a mass greater than about 5 TeV. The charged scalar and the other exotic neutrals would then have similarly large masses unless some of the couplings in the scalar potential were nonperturbatively large. In this most conservative interpretation of the model, the extra scalars would be too heavy to contribute to B_d-$\bar{B}_d$ mixing or any other present observable and one is left with essentially the minimal standard model.

It is thus extremely interesting to consider a somewhat broader interpretation of the model in which one does not strictly enforce perturbative constraints from the scalar potential. It is possible that in a more complete model more scalar multiplets may contribute to the potential or new physics may occur at some intermediate scale affecting the mass relations for the scalars but not necessarily affecting the Yukawa structure for the two doublets predicted from the axial symmetry. We have shown that even in this "toy model" with scalar masses taken as free parameters one cannot account for a large B_d-$\bar{B}_d$ mixing through a light charged-scalar exchange without producing too much CP violation in the kaon system. One is still forced into the very small region of parameter space in the Fritzsch model with a large top-quark mass.

Allowing the scalar masses to be free parameters does, however, suggest an interesting and exotic possibility for the flavor-changing neutral currents to contribute to B_d-$\bar{B}_d$ mixing. Because of a peculiar twist in the couplings of the model, different neutral scalars contribute dominantly for K^0-$\bar{K}^0$ mixing and for B_d-$\bar{B}_d$ mixing. Our rough calculations indicate that if these two scalars are allowed to have widely separated masses, one can just about explain the large B_d-$\bar{B}_d$ mixing without affecting predictions in the kaon system. If this scenario were to be realized, it would predict B_s-$\bar{B}_s$ mixing to be less than about twice that of B_d-$\bar{B}_d$ mixing. This is in contrast with the standard result from W exchange with a heavy top quark which predicts B_s-$\bar{B}_s$ mixing to be greater than about 5 times the B_d-$\bar{B}_d$ mixing and should provide a very interesting signal which can be tested in the near future.

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