T-odd and CP-odd aplanarities in e^+e^- colliders

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We consider possible tests of CP violation or new strong-interacting sectors at future colliders, for instance e^+e^-. These tests minimize the flavor identification needed. In the case of the standard model no sizable observable is found, even if the Higgs sector becomes strongly interacting. The mean values of the simplest T-odd operators involving electron and positron polarizations and initial or final momenta are forbidden by the chiral-conserving interactions. More complicated observables involving correlations between final momenta and electron or positron polarizations can in principle have nonvanishing expectation values. However, the effect is proportional to a very small phase shift between left-handed and right-handed amplitudes. On the contrary, in theoretical schemes beyond the standard model, with significant chirality-flip couplings to fermions, either new sources of CP violation or new strong-interacting forces could be tested below the threshold of production of new particles.

I. INTRODUCTION

In this paper we consider whether the search for aplanarities in e^+e^- colliders with polarized beams can give indications either on CP violation or on the existence of a strongly interacting Higgs sector or any new source of strong interactions. Momenta and polarizations are reversed by time reversal. Thus a cross product $O=(\mathbf{v}_1\times\mathbf{v}_2)\cdot\mathbf{v}_3$, where $\mathbf{v}_i$ are momenta or polarizations, is a T-odd observable. Consider a reaction $e^+e^-\to X$, where X is any final state, and call p_1, p_2, and p the momenta of any ingoing or outgoing particle. Possible T-odd observables could be, for instance,

$$O_1=\mathbf{S}^{e^-}\cdot(\mathbf{p}_1\times\mathbf{p}_2)\ , \tag{1}$$

$$O_2=(\mathbf{S}^{e^-}\times S^{e^+})\cdot\mathbf{p}\ . \tag{2}$$

The nonvanishing of certain mean values $\langle O_1\rangle_\sigma$, $\langle O_2\rangle_\sigma$ would be signals of T-odd effects, either final-state strong interactions or CP violation.

Since operators (1) and (2) will involve initial and final variables as well, *by mean value we will mean the following.* Let $O(\{v_i\},\{v_f\})$ be an operator dependent on initial and final variables. *We will take the average of O, that we will denote $\langle O\rangle_\sigma$, by summing over some final states and some initial states with the corresponding differential cross* section as the weight, as is made precise below by Eqs. (12)–(14). Of course, as we want to isolate T-odd effects due to the dynamics, we will add to each final state its symmetrical state under T, and similarly for initial states. This is a procedure to define T-odd aplanarities on which we will establish theorems within the standard model and beyond it.

Sufficient conditions of T-odd effects would also be the asymmetries

$$A_1=\frac{N(\mathbf{S}^{e^-}\cdot(\mathbf{p}_1\times\mathbf{p}_2)>0)-N(\mathbf{S}^{e^-}\cdot(\mathbf{p}_1\times\mathbf{p}_2)<0)}{N(\mathbf{S}^{e^-}\cdot(\mathbf{p}_1\times\mathbf{p}_2)>0)+N(\mathbf{S}^{e^-}\cdot(\mathbf{p}_1\times\mathbf{p}_2)<0)} \tag{3}$$

or

$$A_2=\frac{N((\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot\mathbf{p}_1>0)-N((\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot\mathbf{p}_1<0)}{N((\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot\mathbf{p}_1>0)+N((\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot\mathbf{p}_1<0)}\ . \tag{4}$$

$\mathbf{S}^{e^-}$ ($\mathbf{S}^{e^+}$) denotes the spin of the electron (positron). It is assumed that the cross section is inclusive; i.e., there is a sum over the dynamical variables which has not been made explicit, or at least there is a sum to eliminate any angular correlation between variables, although this hypothesis will be relaxed later on.

Note that both T-odd aplanarities $\langle O_1\rangle_\sigma$, $\langle O_2\rangle_\sigma$ and A_1, A_2 are different, although superficially A_1, A_2 look like averages of O_1,O_2. However, they are not, since the events in the numerators of (3) and (4), count for $+1$ or -1 according to the sign of O_1,O_2.

Let us consider the standard model. The e^- (e^+) are coupled only to vector bosons, the couplings of Higgs particles being negligible, as they are proportional to the masses. Chirality conservation implies that, to a very good approximation, the amplitude for any e^+e^- annihilation process can be expressed as

$$A^{(f)}=\bar{e}^+(A_L^{(f)}\gamma_{\mu L}+A_R^{(f)}\gamma_{\mu R})e^-\epsilon^{(f)\mu}\ , \tag{5}$$

where A_L, A_R, and ϵ are functions of the considered final state, labeled by the superindex f, and e^-,e^+ are Dirac spinors. It is always possible to take ϵ^μ real, including all the phases in the couplings A_L, A_R. To simplify the notation, we will often delete the index (f) that characterizes a particular final state or a particular family of final states. Diagrams such as those of Figs. 1 and 2 give amplitudes of type (5). On the contrary, the QED radiative corrections (Fig. 3) lead, among other terms, to terms of the anomalous magnetic moment form $\bar{e}^+ i\sigma_{\mu\nu}q^\nu e^- \epsilon^{(f)\mu}$ not considered in (5). We will still neglect these kinds of term as they are suppressed by a factor α.

Our conclusion will be that the mean values $\langle O_1 \rangle_\sigma$, $\langle O_2 \rangle_\sigma$, or such aplanarities as A_1 and A_2, vanish for amplitudes of type (5). We think that the reason for this result is that light Dirac fermions are close to Weyl fermions, for which spin is not really an independent degree of freedom, being aligned or antialigned to the momentum. However, some T-odd effects are allowed if additional correlations exist between the polarizations and momenta involved. This could be useful if, for instance, the Higgs sector of the standard model becomes strongly interacting. Our results can obviously be generalized to any couple of ingoing or outgoing fermions verifying Eq. (5). To explore such possible generalizations we will keep a notation that distinguishes between m_{e^+}, m_{e^-}, and we will sometimes relax the equality $m_{e^+} = m_{e^-}$. We will consider the case where heavy top quarks are produced.

We will now recall some known results concerning time reversal, CP symmetry, and final-state interactions, and discuss which combinations of mean values of T-odd operators and which aplanarities are signal of either CP violation or new strong interactions. Then we will prove that there are T-odd operators of a structure more complicated than (1) and (2) that can have nonvanishing expectation values. We will first present a proof using a polarized cross-section formalism, followed by a proof using Dirac algebra, which will clarify the physical meaning of our results.

II. T-ODD AMPLITUDES, CP VIOLATION, AND FINAL-STATE INTERACTION

Consider a transition matrix T, expressed in terms of its Hermitian and anti-Hermitian components:

$$T = \tfrac{1}{2}(T + T^\dagger) + \tfrac{1}{2}(T - T^\dagger) .$$

The effect of time reversal τ gives, if one assumes T invariant under τ,

$$\langle b|T|a\rangle = \eta[\langle \tau a|\tfrac{1}{2}(T+T^\dagger)|\tau b\rangle + \langle \tau a|\tfrac{1}{2}(T-T^\dagger)|\tau b\rangle] ,$$

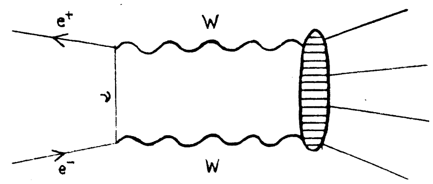

FIG. 2. e^+e^- annihilation diagram via two W and ν exchange.

where η is a phase. By Hermiticity,

$$\langle b|T|a\rangle = \eta[\langle \tau b|\tfrac{1}{2}(T+T^\dagger)|\tau a\rangle^* - \langle \tau b|\tfrac{1}{2}(T-T^\dagger)|\tau a\rangle^*] .$$

In the lowest-order Born approximation, $T \sim H$, and only the Hermitian part, the first term on the right-hand side, contributes. This means that, if τ is an exact symmetry, we have equal probability of finding negative or positive eigenvalues of the observable $O = (\mathbf{v}_1 \times \mathbf{v}_2)\cdot\mathbf{v}_3$, where $\mathbf{v}_i$ are momenta or polarizations. The reason is that O being T odd, for each positive eigenvalue $\lambda > 0$ there will be a negative one λ_τ, where the subindex τ means that it corresponds to the τ-reversed eigenstate. $\lambda > 0$ and $\lambda_\tau < 0$ occur with equal probability because only the Hermitian part of the transition matrix T can contribute in the Born approximation, and the expectation value of O vanishes. We conclude that to *first order in H* a nonvanishing expectation value of a T-odd operator O is a signal of τ *noninvariance* of the Hamiltonian, that is from the CPT theorem, *a signal of CP violation.*[1]

But higher-order terms in H induce an anti-Hermitian contribution to T, often referred to as unitarity corrections because it is the absorptive part of the amplitude:

$$(T - T^\dagger)_{if} = i \sum_n T_{in} T^\dagger_{nf} \delta^4(p - p_n) .$$

Suppose that the mean value of a T-odd observable O (may be more general than O_1 and O_2) is measured in a collider. How do we discriminate if it is due to CP violation or strong interactions? As we will see, this distinction can be made if we know how to identify at least one flavor. First it should be noted than an aplanarity always appears through terms of the type $i\epsilon^{\mu\nu\rho\sigma} v_{1\mu} v_{2\nu} v_{3\rho} v_{4\sigma}$. The i has to be combined with some imaginary interference between two terms in the amplitude, in order to get a real contribution to the cross section. To clarify the notations we will parametrize the latter by $i\sin(\delta_{CP} + \delta_{st})$, where δ_{CP} is any possible CP-violating phase, and δ_{st} is

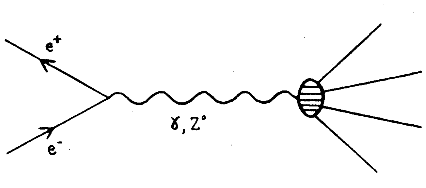

FIG. 1. e^+e^- annihilation diagram via γ and Z^0.

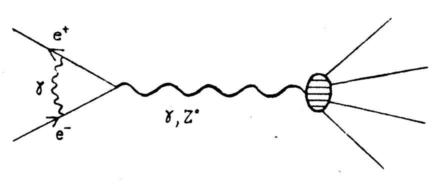

FIG. 3. e^+e^- annihilation diagram with radiative QED corrections.

any phase due to unitarity corrections in a strong-interacting process such as a final-state interaction.[2]

For any transition $\langle b|T|a\rangle$, we consider the CP-reversed amplitude $\langle \bar{b}|T|\bar{a}\rangle$ where $|\bar{b}\rangle = CP|b\rangle$, $|\bar{a}\rangle = CP|a\rangle$. Let us consider a T-odd operator O such as (1) or (2) and its matrix element between an initial state $|i\rangle$ and a particular final state $|f\rangle$, $M=\langle f|S|i\rangle\langle i|O|i\rangle$, and let us consider the CP-transformed matrix element $\overline{M}=\langle\bar{f}|S|\bar{i}\rangle\langle\bar{i}|\overline{O}|\bar{i}\rangle$ (we are considering as an example the particular case in which O acts only on initial observables; S is the S matrix) where all the momenta and polarizations are replaced by the CP-transformed ones of the corresponding antiparticle in the CP-reversed process. Since $\langle\bar{b}|T|\bar{a}\rangle = \langle b|(CP)^{-1}T(CP)|a\rangle$, it follows that, for CP-conserving transitions, $(CP)^{-1}T(CP)=T$ and hence it is proved that, for CP-conserving processes, $M=\overline{M}$. It follows that the phase $\delta_{\rm st}$ is the same in $\langle b|T|a\rangle$ and $\langle\bar{b}|T|\bar{a}\rangle$ while obviously, if there is CP violation, the phase δ_{CP} is reversed. It is thus obvious that a nonvanishing value for $M-\overline{M}$ *is a signal of a CP-violating phase* while a nonvanishing $M+\overline{M}$ *is a signal of unitarity corrections.*

In the case where the initial state $|a\rangle$ is e^+e^-, $|a\rangle$ may be, depending on e^- and e^+ polarizations, a CP eigenstate. Then $\overline{M}$ can be looked for in the same reaction than M, but one needs to identify at least one final flavor, as we will see below.

III. T-ODD OPERATORS WITH VANISHING EXPECTATION VALUES

Let us consider the inclusive process $e^+e^-\to f+\cdots$ where f is some observed final state. We will here consider the T-odd observables of the types $O_1(1)$ and $O_2(2)$, where the momenta p_1, p_2 can be p_{e^+}, p_{e^-}, or any final momenta p_f. We will prove that the mean values of these T-odd operators vanish. We will first use for our proof a formalism suitable for polarized cross sections and, to clarify the physical meaning of our results, we will give an independent proof using Dirac algebra.

A. Helicity formalism

We quantize the spin along the initial electron momentum. Let us consider the amplitude for the process $e^+e^-\to f+X$,

$$(2\pi)^4\delta_4(p_{e^+}+p_{e^-}-p_f-p_X)A^{(f)}_{\mu_{e^-}\mu_{e^+}}(p_{e^-},p_{e^+},f+X) = \langle p_{e^+}p_{e^-}\mu_{e^+}\mu_{e^-}|S|f+X\rangle\ , \quad (6)$$

and let us consider the auxiliary quantity

$$\sigma^{(f)}_{\mu_{e^-}\mu_{e^+};\mu'_{e^-}\mu'_{e^+}} = \sum A^{(f)}_{\mu_{e^-}\mu_{e^+}} A^{(f)*}_{\mu'_{e^-}\mu'_{e^+}}\ , \quad (7)$$

where the sum extends over the unobserved final states X and the superindex f indicates the observed final state. The trace of this matrix, summed over f, is proportional to the total cross section. The left (right) amplitudes (6) involve a spin $-\frac{1}{2}$ $(+\frac{1}{2})$ electron and a spin $-\frac{1}{2}$ $(+\frac{1}{2})$ positron, since the left-handed coupling couples to a right-handed positron that has spin $-\frac{1}{2}$ along $\mathbf{p}_{e^-}$. The only nonzero amplitudes are then A_{--} and A_{++} and the only nonzero matrix element of the matrix σ (7) are

$$\begin{bmatrix} \sigma_{++,++} & 0 & 0 & \sigma_{++,--} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sigma_{--,++} & 0 & 0 & \sigma_{--,--} \end{bmatrix} . \quad (8)$$

It is convenient to define the vectors:

$$\mathbf{M}_1 = \sum \sigma^{(f)}_{\mu_{e^-}\mu_{e^+};\mu'_{e^-}\mu'_{e^+}} \mathbf{S}_{\mu'_{e^-}\mu_{e^-}}\ , \quad (9)$$

$$\mathbf{M}_2 = \sum \sigma^{(f)}_{\mu_{e^-}\mu_{e^+};\mu'_{e^-}\mu'_{e^+}} (\mathbf{S}_{\mu'_{e^-}\mu_{e^-}} \times \mathbf{S}_{\mu'_{e^+}\mu_{e^+}})\ , \quad (10)$$

where the sum extends over the repeated spin indices and

$$\mathbf{S}_{\mu'\mu} = \left\langle \mu' \left| \frac{\sigma}{2} \right| \mu \right\rangle . \quad (11)$$

The mean values of operators (1) and (2) will be given by

$$M_1 = \langle \mathbf{S}^{e^-}\cdot(\mathbf{p}_1\times\mathbf{p}_2)\rangle_\sigma = \frac{1}{N_1}\langle \mathbf{M}_1\cdot(\mathbf{p}_1\times\mathbf{p}_2)\rangle_\sigma\ , \quad (12)$$

$$M_2 = \langle (\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot\mathbf{p}\rangle_\sigma = \frac{1}{N_2}\langle \mathbf{M}_2\cdot\mathbf{p}\rangle_\sigma\ , \quad (13)$$

where N_i are convenient normalization factors and the expectation value in the last expression is taken over all the variables besides the spin, already summed up in (9) and (10). Of course, if $\mathbf{p}$ in (12) is $\mathbf{p}_{e^+}$ or $\mathbf{p}_{e^-}$, we understand (7) as being summed over f; $\mathbf{p}_1$ and $\mathbf{p}_2$ in (11) can be $\mathbf{p}_{e^+}$, $\mathbf{p}_{e^-}$, or $\mathbf{p}_f$, for instance. These are the special kinds of mean values that we have outlined in the Introduction.

Note that we are considering as possible observable $(\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot\mathbf{p}_{e^-}$. Although all variables are initial, the average defined above, *when a particular final state is observed,* is dependent on the dynamics. This is the reason why we make explicit the superindex f in the cross section in (9) and (10).

(1) We now consider the observables of the type $O_1(1)$. From (9) and (8) we obtain that *only* M_1^z can be nonvanishing. M_1^z is proportional to $\langle \mathbf{S}^{e^-}\cdot\mathbf{p}_{e^-}\rangle$ since we choose Oz along the $\mathbf{p}_{e^-}$. But $\mathbf{S}^{e^-}\cdot\mathbf{p}_{e^-}$ is actually a T-even, CP-even oservable. Thus, no genuine T-odd aplanarity can be nonzero, either coming from strong interactions or from CP violation: both $M_1+\overline{M}_1$ (signal of final-state interactions) and $M_1-\overline{M}_1$ (signal of CP violation) vanish.

(2) Let us now consider the observables of the type $O_2(2)$.

(i) We will examine first the case where $\mathbf{p}=\mathbf{p}_{e^-}$. Then the observable $(\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot\mathbf{p}_{e^-}$ is both T odd and CP odd since the e^+e^- initial state transforms under CP by simply exchanging the spin

$$CP|e^-,p,s;e^+,p',s'\rangle = e^{i\psi}|e^-,p,s';e^+,p',s\rangle$$

(ψ is an arbitrary phase related to the definition of C) and the cross product $\mathbf{S}^{e^-}\times\mathbf{S}^{e^+}$ is antisymmetric in the spin indices. Thus, a nonvanishing M_2 is a signal of CP violation only.

(ii) We consider now the expectation value of an operator of the type O_2, $\langle (\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot\mathbf{v}\rangle_\sigma$, where $\mathbf{v}$ is some final particle momentum $\mathbf{p}_f$. Denoting by $\overline{\mathbf{v}}$ the momentum of the antiparticle $\bar{f}$, $\langle (\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot(\mathbf{v}+\overline{\mathbf{v}})\rangle_\sigma$ is CP even and signals unitarity corrections, while $\langle (\mathbf{S}^{e^-}\times\mathbf{S}^{e^+})\cdot(\mathbf{v}-\overline{\mathbf{v}})\rangle_\sigma$ is CP odd and signals only CP violation. The observation of the latter needs flavor identification. The mean value $\overline{M}_2$ of this type of operator is given in terms of the vector $\mathbf{M}_2$ (10) by (13). Quantizing as usual the spin along the electron momentum, the components of M_2 read

$$\sum_{\substack{\mu_{e^-},\mu_{e^+}\\ \mu'_{e^-},\mu'_{e^+}}} \sigma^{(f)}_{\mu_{e^-}\mu_{e^+};\mu'_{e^-}\mu'_{e^+}} \times (S^x_{\mu'_{e^-}\mu_{e^-}} S^y_{\mu'_{e^+}\mu_{e^+}} - S^y_{\mu'_{e^-}\mu_{e^-}} S^x_{\mu'_{e^+}\mu_{e^+}}) \tag{14}$$

and the cyclic permutations $x\to y\to z$. Using the explicit form (8), it is straightforward to show that the vector $\mathbf{M}_2$ is zero, and thus the mean value M_2 vanishes for any $\mathbf{v}$.

An important remark is in order here. We have just proven that $\langle S^x_{e^-}S^y_{e^+}-S^y_{e^-}S^x_{e^+}\rangle_\sigma=0$, but it is not true that $\langle S^x_{e^-}S^y_{e^+}\rangle_\sigma=0$. The latter observable is T odd, but it is not rotational invariant around the Oz axis. Thus such an observable may be nonzero only if we have singled out some direction perpendicular to the z axis or, in other words, if p_f depends on the initial polarizations. An example of this phenomenon will be discussed below. On the contrary, whenever p_f does not know of the initial polarizations or momenta implied in the observable, the previous theorem holds, i.e., $\mathbf{M}_2=0$. Note that $\langle S^x_{e^-}S^y_{e^+}\rangle_\sigma$ is transformed by CP into $\langle S^y_{e^-}S^x_{e^+}\rangle_\sigma$. The combinations

$$\langle (S^x_{e^-}S^y_{e^+}+S^y_{e^-}S^x_{e^+})(\mathbf{v}\pm\overline{\mathbf{v}})_T\rangle_\sigma \tag{15}$$

correspond to CP-odd and CP-even observables ($\mathbf{v}_T$ means perpendicular to Oz). Note in particular that if no final-state identification is made the combination $\mathbf{v}+\overline{\mathbf{v}}$ is implicitly considered. It follows then that

$$\langle (S^x_{e^-}S^y_{e^+}+S^y_{e^-}S^x_{e^+})(\mathbf{v}+\overline{\mathbf{v}})_T\rangle_\sigma \tag{16}$$

is a pure CP-odd signal. As stated, no final flavor identification is needed, at the prize of tagging a transverse axis (x or y) through some correlation between polarizations and final momenta. This is a particular case of our O_2' observables to be defined below.

B. Dirac algebra

Starting from (5), the cross section is proportional to the following trace of Dirac matrices:

$$\sigma\propto \mathrm{Tr}\left[(A_L\not\epsilon_L+A_R\not\epsilon_R)(\not p_{e^-}+m_{e^-})\frac{1+\gamma_5\not S_{e^-}}{2} \times (A_L^*\not\epsilon_L^*+A_R^*\not\epsilon_R^*)(\not p_{e^+}-m_{e^+})\frac{1+\gamma_5\not S_{e^+}}{2}\right]. \tag{17}$$

The ϵ_μ are vectors depending on the final-state momenta and polarizations [to simplify the notation we delete from now on the index (f) from the general expression (5)] according to the considered diagrams. For example, in the process $e^+e^-\to H^+H^-$ where H^+, H^- are two spin-zero particles, one has $\epsilon\propto \mathbf{p}_{H^+}-\mathbf{p}_{H^-}$.

We consider the $m_e=0$ limit, and the two kinds of observables, (1) and (2).

(1) Observables of the type O_1. In the $m_e=0$ limit, no expectation value M_1 can appear since it would come from the term in (17) which contains $\gamma_5\not S_{e^-}$ but not $\gamma_5\not S_{e^+}$. This term is a trace over an odd number of Dirac matrices, that vanishes.

(2) Observables of the type O_2. Dealing now with observables of the form (2), we are left with the trace

$$\sigma^{(2)}\propto -\tfrac{1}{4}\mathrm{Tr}[(A_L\not\epsilon_L+A_R\not\epsilon_R)\not p_{e^-}\not S_{e^-} \times (A_L^*\not\epsilon_L^*+A_R^*\not\epsilon_R^*)\not p_{e^+}\not S_{e^+}], \tag{18}$$

where $\sigma^{(2)}$ is the part in (17) which depends on S_{e^-} and S_{e^+}. From $S_{e^-}\cdot p_{e^-}=S_{e^+}\cdot p_{e^+}=0$ (since S_{e^-}, S_{e^+} are polarization vectors) we can anticommute momenta and spin to get alternatively the form $\mathrm{Tr}(\cdots \not p_{e^-}\not\epsilon^*\not p_{e^+}\cdots)$ or $\mathrm{Tr}(\cdots\not p_{e^+}\not\epsilon\not p_{e^-}\cdots)$, whence we can conclude that $\epsilon_{L,R}$ and $\epsilon^*_{L,R}$ have to be orthogonal to the (p_{e^-},p_{e^+}) hyperplane. Indeed, assuming $\epsilon=ap_{e^-}+bp_{e^+}$, we get $\not p_{e^-}\not\epsilon\not p_{e^+}=ap^2_{e^-}\not p_{e^+}+bp^2_{e^+}\not p_{e^-}=0$, since we neglect m_e. The vectors S_{e^-} and S_{e^+} are also orthogonal to the (p_{e^-},p_{e^+}) hyperplane, since, in the massless case, the longitudinal polarization becomes proportional to the momentum

$$S^L_{e^-}=\pm\left[\frac{1}{m_{e^-}}p_{e^-}\pm O(m_e)p_{e^+}\right], \tag{19}$$

and therefore $p_{e^-}S^L_{e^-}=O(m_e)$. Straightforward manipulations from (17) lead us to the result that the longitudinal polarization has a nonvanishing contribution:

$$\sigma^{(m)}\propto\sum \mathrm{Tr}(A_L\not\epsilon_L+A_R\not\epsilon_R)m_e \times\frac{\gamma_5\not S^L_{e^-}}{2}(A_R^*\not\epsilon_L^*+A_R^*\not\epsilon_R^*)m_{e^+}\frac{\gamma_5\not S^L_{e^+}}{2} \tag{20}$$

since the $1/m_e$ in (19) cancels the m_e factor here. But these contributions lead to no T-odd effect. They simply lead to the expected selection of left (right) helicities when coupled to left (right) currents. Finally, we obtain

$$\sigma^{(2)} \propto \mathrm{Tr}[(1-\boldsymbol{\sigma}\cdot\mathbf{p}_{e^-})(\mathbf{S}^{e^-}\cdot\boldsymbol{\sigma})(\boldsymbol{\epsilon}\cdot\boldsymbol{\sigma})(1+\boldsymbol{\sigma}\cdot\mathbf{p}_{e^+})(\mathbf{S}^{e^+}\cdot\boldsymbol{\sigma})(\boldsymbol{\epsilon}^*\cdot\boldsymbol{\sigma})]A_L A_R^* + \mathrm{Tr}[(1-\boldsymbol{\sigma}\cdot\mathbf{p}_{e^-})(\mathbf{S}^{e^-}\cdot\boldsymbol{\sigma})(\boldsymbol{\epsilon}^*\cdot\boldsymbol{\sigma})(1+\boldsymbol{\sigma}\cdot\mathbf{p}_{e^+})(\mathbf{S}^{e^+}\cdot\boldsymbol{\sigma})(\boldsymbol{\epsilon}\cdot\boldsymbol{\sigma})]A_L^* A_R \ . \tag{21}$$

We see that the part *in the cross section which depends on the polarizations is only the cross product between left and right couplings.* In any process where only left (or only right) couplings are involved, the polarized electron and positron act exactly as if they were unpolarized. If we sum over all possible values of ϵ in order to wash out any correlation with $\mathbf{S}_{e^-}$, $\mathbf{S}_{e^+}$, we recover the result of Sec. III: $\mathbf{S}_{e^+}$, $\mathbf{S}_{e^-}$ appear to be parallel, and no cross product $\mathbf{S}^{e^-}\times\mathbf{S}^{e^+}$ can have a nonzero average.

IV. *T*-ODD OPERATORS WITH NONVANISHING EXPECTATION VALUES

A. Example: Correlation between initial polarizations and a final state

Let us assume that we do not sum over all final states, i.e., over all the possible values of ϵ_μ that characterize possible final states in (5). We will assume, furthermore, that there is a correlation between the particular final state considered and the e^- or e^+ polarizations, i.e., a correlation between $\boldsymbol{\epsilon}$ and $\mathbf{S}^{e^-}$ or $\mathbf{S}^{e^+}$. Then, *a nonzero average for a T-odd observable is possible.* One example could be the mean value of the operator

$$O_2' = [(\mathbf{S}_{e^-}\times\mathbf{S}_{e^+})\cdot\hat{\mathbf{p}}_e](\boldsymbol{\epsilon}\cdot\mathbf{S}_{e^+})^2 \ . \tag{22}$$

To obtain the mean value of this operator it is enough to multiply the right-hand side (RHS) of (21) by $(\boldsymbol{\epsilon}\cdot\mathbf{S}_{e^+})^2$. If we now assume ϵ to be real so that all phases are included in A_L, A_R in (5), we obtain

$$\langle [(\mathbf{S}_{e^-}\times\mathbf{S}_{e^+})\cdot\hat{\mathbf{p}}_e](\boldsymbol{\epsilon}\cdot\mathbf{S}_{e^+})^2\rangle_\sigma \propto \mathrm{Im}(A_L A_R^*) \ . \tag{23}$$

The *mean value of such an observable does not vanish in the $m_e=0$ limit* and moreover it can only be nonzero if there is *a relative phase between left and right amplitudes.* To select a genuine *CP*-violating effect or a *T*-odd effect due to strong interactions we will have to consider, respectively, the combinations

$$\langle [(\mathbf{S}_{e^-}\times\mathbf{S}_{e^+})\cdot\hat{\mathbf{p}}_e][(\boldsymbol{\epsilon}\cdot\mathbf{S}_{e^-})^2\pm(\boldsymbol{\epsilon}\cdot\mathbf{S}_{e^+})^2]\rangle_\sigma \ , \tag{24}$$

the upper (lower) signs corresponding to *CP*-odd (-even) operators. For example, if we consider the production of two (pseudo)scalar final particles H^+H^-, then $\boldsymbol{\epsilon}\propto(\mathbf{p}_{H^+}-\mathbf{p}_{H^-})$. One may think of Higgs particles whose direct coupling to electrons is negligible and thus does not spoil our starting hypothesis (5). An asymmetry of the *CP*-even type would be, for example,

$$A_2' = \frac{A(|p_{H_x^+}|>|p_{H_y^+}|)-A(|p_{H_x^+}|<|p_{H_y^+}|)}{A(|p_{H_x^+}|>|p_{H_y^+}|)+A(|p_{H_x^+}|<|p_{H_y^+}|)} \ , \tag{25}$$

where A is given by the expression

$$A \propto N(S_{e^-}^x=+1, S_{e^+}^y=+1) + N(S_{e^-}^x=-1, S_{e^+}^y=-1) - N(S_{e^-}^x=-1, S_{e^+}^y=+1) - N(S_{e^-}^x=+1, S_{e^+}^y=-1) - \{S_{e^+}\leftrightarrow S_{e^-}\} \ . \tag{26}$$

A nonvanishing value of (25) would be a sufficient condition of the existence of *T*-odd *CP*-even effects due to a strongly interacting sector. Our conclusion is then that such correlated aplanarities as A_2' need not vanish in the $m_e=0$ limit. However, in the framework of the standard model, any estimate that we have made leads to very small predictions for such aplanarities, since the phase shift between right and left amplitudes (5) turns, in practice, to be very small. This is so even if we consider a strongly interacting sector, where final-state interactions between *W*, *Z*, and Higgs particles do not discriminate among bosons which have been produced through a right (or left) coupling to e^+e^-. On the contrary, any new strong-interacting sector which could appear in the colliders at high energies could be detected by this kind of observables *before reaching* the production threshold of the particles responsible for the new force. We have in mind, for instance, models where, instead of the coupling in (5), the fermions are coupled through forces which change chirality as, for instance, $\bar{e}e\phi$ where ϕ is a scalar.

This holds also for the O_1 and O_2 type of observables and asymmetries A_1, A_2 discussed before as signals of unitarity corrections. To quantify these effects, one needs the discussion of precise models, which we do not intend here.

B. An example of *T*-odd asymmetry in supersymmetry

An example of asymmetry of the type considered in this section is studied by Kizukuri[3], that considers in a supersymmetric model a decay: $Z'\to W_h^+W_e^-\to e^+\tilde{\nu}+e^-\tilde{\nu}^c$ where W_h, W_l are *W*-inos, i.e., fermions coupled to the *Z* boson via a Lagrangian: $L^{\text{int}}=eZ_\mu\overline{W}_h(G_L\gamma_{\mu L}+G_R\gamma_{\mu R})W_l+$ H.c. The decay amplitude is at the tree level of the form (5) except that the fermions are in the final state instead of the initial state. The author predicts an aplanarity of the type $\boldsymbol{\epsilon}\cdot(\mathbf{S}_h\times\mathbf{S}_l)$ where ϵ is the *Z* polarization. Looking at formula (13) in Ref. 3, it appears that this aplanarity is proportional to scalar products $(\mathbf{p}_h\cdot\mathbf{S}_l)$, etc. (labeled p_x, p_y in Ref. 3), which means that it is of the type A_2' (25). The author takes advantage of the fact that G_L and G_R may have a relative phase. This is an example of our main conclusion: T-odd observables (whether they are related to CP violation or to unitarity corrections) may exist with polarized fermions interacting through chiral-invariant interactions if one considers situations in which some final momentum coupled to the current is correlated to the fermion polarizations. Furthermore, some relative phase between right and left amplitudes is needed.

C. General T-odd terms: Nonvanishing masses

We start from Eq. (17), but making explicit now the sum over possible final states that could be observed:

$$\sigma \propto \sum_{f,f'} \mathrm{Tr}\left[(A_L^{(f)}\not\epsilon_L^{(f)} + A_R^{(f)}\not\epsilon_R^{(f)})(\not p_{e^-} + m_{e^-})\frac{1+\gamma_5 \not S_{e^-}}{2} (A_L^{(f')*}\not\epsilon_L^{(f')*} + A_R^{(f')*}\not\epsilon_R^{(f')*})(\not p_{e^+} - m_{e^+})\frac{1+\gamma_5 \not S_{e^+}}{2} \right] . \tag{27}$$

It is always possible to take ϵ_μ real in (5), putting the phases in A_L, A_R. We now make the remark that any T-odd term in the sum (17) must be proportional to $\mathrm{Im}(A_L A_R^*)$ since any antisymmetric vector product comes from $\mathrm{Tr}(\gamma_5 \not a \not b \not c \not d) = 4i\epsilon_{\alpha\beta\gamma\delta} a^\alpha b^\beta c^\gamma d^\delta$ and the i on the RHS has to be canceled by another factor i to give a real contribution to the cross section. It is now easy to check that all terms proportional to $\mathrm{Im}(A_L^{(f)} A_R^{(f')*})$ in (27) are odd under the exchange of $(p_{e^-}, S_{e^-}) \leftrightarrow (p_{e^+}, S_{e^+})$. Indeed, after some manipulations we get, from (27),

$$\begin{aligned} \sigma((p_{e^-},S_{e^-}),(p_{e^+},S_{e^+})) - \sigma((p_{e^+},S_{e^+}),(p_{e^-},S_{e^-})) = \sum_{f,f'} & \mathrm{Tr}\left[(A_L^{(f)*}\not\epsilon_L^{(f)} + A_R^{(f)*}\not\epsilon_R^{(f)})(\not p_{e^+} - m_{e^+})\frac{1+\gamma_5 \not S_{e^+}}{2} \right. \\ & \left. \times (A_L^{(f')}\not\epsilon_L^{(f')} + A_R^{(f')}\not\epsilon_R^{(f')})(\not p_{e^-} + m_{e^-})\frac{1+\gamma_5 \not S_{e^-}}{2} \right] \\ & - \mathrm{Tr}\left[(A_L^{(f)}\not\epsilon_L^{(f)} + A_R^{(f)}\not\epsilon_R^{(f)})(\not p_{e^+} + m_{e^+})\frac{1+\gamma_5 \not S_{e^+}}{2} \right. \\ & \left. \times (A_L^{(f')*}\not\epsilon_L^{(f')} + A_R^{(f')*}\not\epsilon_R^{(f')})(\not p_{e^-} - m_{e^-})\frac{1+\gamma_5 \not S_{e^-}}{2} \right] , \end{aligned} \tag{28}$$

where we have permuted the matrices under the trace and exchanged $f \leftrightarrow f'$ in the first trace. From (28) it is clear that only terms of the form $\mathrm{Im}(A_{L(R)}^{(f)} A_{L(R)}^{(f)*})$ are odd for the exchange $(p_{e^-}, S_{e^-}) \leftrightarrow (p_{e^+}, S_{e^+})$ while the part in σ which is even for the latter exchange will include only real parts of interferences in which we are not interested here. We thus come to the important conclusion, confirmed by explicit computations, that no term proportional to $i\epsilon_{\alpha\beta\gamma\delta} p_{e^-}^\alpha p_{e^+}^\beta S_{e^-}^\gamma S_{e^+}^\delta$ can appear, leading to no aplanarity when the vectors ϵ are averaged on, i.e., to no mean value of the type M_2 (13) nor aplanarity of the type A_2 (4). We are left with only two types of T-odd terms in σ. First, terms involving six vectors, analogous to

$$(\epsilon_{\alpha\beta\gamma\delta} p_{e^-}^\alpha p_{e^+}^\beta S_{e^-}^\gamma S_{e^+}^\delta)(\epsilon^{(f')} \cdot S_{e^+}) \, \mathrm{Im}(A_L^{(f)} A_R^{(f')*} - A_R^{(f)} A_L^{(f')*}) - (e^- \leftrightarrow e^+) \tag{29}$$

or, similarly,

$$(\epsilon_{\alpha\beta\gamma\delta} p_{e^-}^\alpha S_{e^-}^\beta S_{e^+}^\gamma \epsilon^{(f)\delta})(\epsilon^{(f')} \cdot p_{e^+}) \, \mathrm{Im}(A_L^{(f)} A_R^{(f')*} - A_R^{(f)} A_L^{(f')*}) - (e^- \leftrightarrow e^+) . \tag{30}$$

These lead to aplanarities of the type A_2' (25).

Second, since we do not anymore assume vanishing masses, we have now new terms involving only four-vectors:

$$\epsilon_{\alpha\beta\gamma\delta} p_{e^-}^\alpha p_{e^+}^\beta \epsilon^{(f)\gamma} \epsilon^{(f')\delta} \, \mathrm{Im}(A_L^{(f)} A_L^{(f')*} - A_R^{(f)} A_R^{(f')*}) , \tag{31}$$

$$\epsilon_{\alpha\beta\gamma\delta} S_{e^-}^\alpha S_{e^+}^\beta \epsilon^{(f)\gamma} \epsilon^{(f')\delta} m_{e^-} m_{e^+} \, \mathrm{Im}(A_L^{(f)} A_L^{(f')*} - A_R^{(f)} A_R^{(f')*}) . \tag{32}$$

These terms do not depend anymore on a nonvanishing phase between left and right amplitudes unlike the case in Eqs. (29) and (30), but on a nonvanishing phase between left (right) amplitudes corresponding to vectors $\epsilon_\mu^{(f)}$ vs $\epsilon_\nu^{(f')}$. This means that their existence depends on the interference between different processes in the final-state interaction. We will leave aside the study of this equation in this paper.

When we speak of non-negligible masses we have first in mind the t quarks. These will furthermore introduce in the scheme non-negligible coupling with the Higgs particles which are outside our hypothesis (5), which in turn lead to some contributions to the aplanarities.

V. CONCLUSION

To summarize, we have studied possible tests of T-odd effects in e^+e^- colliders, either due to genuine CP violation or to final-state strong interactions. We have shown that the simplest T-odd observables of the form $(\mathbf{v}_1 \times \mathbf{v}_2)\cdot\mathbf{v}_3$, where $\mathbf{v}_i$ are e^+ or e^- polarizations and initial or final momenta have vanishing expectation values in the standard model. This fact follows in spite of the CP violation of the standard model and even if the Higgs sector is strongly interacting, and it is essentially due to the chirality-conserving character of the gauge interactions; i.e., it holds exactly in the $m_e=0$ limit. More complicated observables correlating final momenta and initial lepton polarizations are on the contrary allowed, but they are in practice very small in the standard model, since they are proportional to a small phase shift Im $(A_L A_R^*)$ between the left-handed and right-handed amplitudes. We give examples of this type of effect in supersymmetry, and in the production of Higgs-particle pairs. More generally, we have studied in detail the general from of the T-odd observables that can allow to detect new CP-violation sources or unitarity correction effects due to nonstandard strong-interacting forces. Finally we have briefly discussed the case of heavy quarks in the standard model.

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