Are 't Hooft indices constrained in preon models with complementarity?

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(Received 10 August 1988)

We present a counterexample to the conjecture that the 't Hooft indices for composite models satisfying complementarity are bounded in magnitude by 1. The model is based on the metacolor group $SU(9)_{MC}$ with two preons in the representation **36** and two preons in the representation $\overline{\mathbf{126}}$. We obtain the 't Hooft index 12 for this model.

In a previous paper[1] we made a conjecture that in a preon model in accord with complementarity[2] the 't Hooft indices which correspond to the solution of the 't Hooft anomaly-matching conditions[3] do not exceed 1 in magnitude:

$$|l_i| \leq 1 \ . \tag{1}$$

This conjecture is based on a physical point of view that if the indices exceed 1, it is hard to interpret physically the implied degeneracies in preon models with complementarity. The conjecture was further elaborated in Ref. 4, which showed that the only known example that seemed to violate the conjecture does indeed satisfy the conjecture. However, a rigorous mathematical proof of the conjecture was lacking. In this paper we shall discuss possible sources of degeneracies which might occur in preon models based on complementarity and then give a counterexample to our conjecture. The model is based on the metacolor gauge symmetry $SU(9)_{MC}$ and the global color-flavor symmetry $SU(2)_1 \times SU(2)_2 \times U(1)$.

We first give a general discussion of how and where degeneracies of fermion representations might occur in preon models satisfying complementarity. The degeneracies, if they exist, will imply that the 't Hooft indices corresponding to those degenerate representations exceed 1 in magnitude, thus violating the conjecture. Let us denote the metacolor group by G_{MC} and the global color-flavor group by G_{CF}. In general, there are three possible sources of degeneracies.

Case 1. Branching in G_{MC} or G_{CF}. When the metacolor group G_{MC} gets broken into G'_{MC} by tumbling,[5] two preons with representations $P_1 = (r_1, R_1)$ and $P_2 = (r_2, R_2)$ under $G_{MC} \times G_{CF}$ might result in degeneracies of representations under $G'_{MC} \times G_{CF}$. Namely, if $R_1 = R_2 = R$ and if r_1 and r_2 include the same representation under G'_{MC}, then there will be two fermions with exactly the same representation under $G'_{MC} \times G_{CF}$. For example, let $G_{MC} = SU(N)$ and $G'_{MC} = SU(N-1)$, and suppose that there exist $P_1 = (\square; R)$ and $P_2 = (\begin{smallmatrix}\square\\\square\end{smallmatrix}; R)$ in the model. Then by the following branching rules of $SU(N)$ into $SU(N-1)$,

$$\begin{aligned} &\square \to 1 + \square \ , \\ &\begin{smallmatrix}\square\\\square\end{smallmatrix} \to \begin{smallmatrix}\square\\\square\end{smallmatrix} + \square \ , \end{aligned} \tag{2}$$

we find that there exist two fermions with representation $(\square; R)$ under $G'_{MC} \times G_{CF}$. If these fermions remain massless after tumbling stops, we will violate the conjecture. The same argument follows for the case of the breaking of the global chiral symmetry G_{CF}.

In case 1 degeneracies come from two different fermions (P_1 and P_2 above). In cases 2 and 3 below we discuss possibilities of degeneracies resulting from one fermion.

Case 2. Breaking into a diagonal subgroup. It commonly happens in preon models with complementarity that two same groups (often one from G_{MC} and the other from G_{CF}) break into the diagonal subgroup (as part of G'_{CF}). When this happens, the representations in the diagonal subgroup might have degeneracies. For example, suppose $G_{MC} \times G_{CF} = SU(3) \times SU(3)$ breaks into the diagonal subgroup $G'_{MC} = SU(3)$. If a fermion $P = (15; 8)$ under $G_{MC} \times G_{CF}$ exists, it results in two **15** representations under G'_{CF}. This is a possible source of degeneracies.

Case 3. Intrinsic degeneracies in branching. This is the most naive case. In the breaking of a group G (G_{MC} or G_{CF}) into its subgroup G', a certain representation might produce degeneracies in branching. For instance, when $G = SO(10)$ and $G' = SU(5) \times U(1)$, **560** of G branches into two copies of $\mathbf{10}(-1)$ under G' (and other representations). Also, for $G = E_6$ and $G' = F_4$, **650** of G includes two **26** of G' in the branching.

We have so far considered various possibilities for violation of the conjecture, Eq. (1). The question is then whether any of these three cases actually occurs in specific models. Here we give an example in which cases 1 and 2 are realized. The model is based on the metacolor group $G_{MC} = SU(9)_{MC}$ with two preons in the **36** representation of $SU(9)_{MC}$ and two preons in the $\overline{\mathbf{126}}$ representation of $SU(9)_{MC}$. The global color-flavor-symmetry group is thus $SU(2)_1 \times SU(2)_2 \times U(1)_F$, and the preons have the representations

$$P_1 = (36; 2, 1, 5), \quad P_2 = (\overline{126}; 1, 2, -1) \tag{3}$$

under $SU(9)_{MC} \times SU(2)_1 \times SU(2)_2 \times U(1)_F$.

In the Higgs phase this model goes through seven steps of tumbling, and we end up with 12 massless fermions with the representation

$$(1;9) \tag{4}$$

under $G_{MC}\times G_{CF}=SU(4)_{MC}\times U(1)_E$. Hence, this presents a counterexample to the conjecture, Eq. (1). The seven steps of tumbling are summarized in Fig. 1.

We shall now give the details in the Higgs phase. For the first tumbling the condensate in the most attractive channel (MAC) is given by combining two preons P_2 in (3) into the symmetric **9**, i.e.,

$$\overline{\mathbf{126}}\times\overline{\mathbf{126}}\to\mathbf{9}_S \tag{5}$$

$SU(9)_{MC}\times SU(2)_1\times SU(2)_2\times U(1)_F$

Tumbling 1 ⇓

$SU(8)_{MC}\times SU(2)_1\times U(1)_1\times U(1)_2$

Tumbling 2 ⇓

$SU(8)_{MC}\times SU(2)_1\times U(1)_{12}$

Tumbling 3 ⇓

$SU(7)_{MC}\times U(1)_3\times U(1)_4$

Tumbling 4 ⇓

$SU(6)_{MC}\times U(1)_3\times U(1)_4{}'$

Tumbling 5 ⇓

$SU(5)_{MC}\times U(1)_3\times U(1)_4{}''$

Tumbling 6 ⇓

$SU(4)_{MC}\times U(1)_3\times U(1)_4{}'''$

Tumbling 7 ⇓

$SU(4)_{MC}\times U(1)_E$

FIG. 1. The seven steps of tumbling in the $SU(9)_{MC}$ model.

under $SU(9)_{MC}$. By the meta-Pauli principle, this MAC condensate has the representation

$$\Phi_1=(9_S;1,3_S,-2) \tag{6}$$

under $SU(9)_{MC}\times SU(2)_1\times SU(2)_2\times U(1)_F$. This breaks the symmetry down to

$$SU(8)_{MC}\times SU(2)_1\times U(1)_1\times U(1)_2\ , \tag{7}$$

where $U(1)_1$ and $U(1)_2$ are linear combinations of three U(1)'s: $U(1)_F$, $U(1)_{S2}$ coming from the breaking of $SU(2)_2$, and $U(1)_{MC8}$ coming from the breaking of $SU(9)_{MC}$ into $SU(8)_{MC}\times U(1)_{MC8}$. For the U(1) quantum numbers, we can set, for instance,

$$\begin{aligned} Q(U(1)_1)&=Q(U(1)_F)-6Q(U(1)_{S2})+Q(U(1)_{MC8})\ ,\\ Q(U(1)_2)&=Q(U(1)_F)-2Q(U(1)_{S2})+\tfrac{1}{2}Q(U(1)_{MC8})\ . \end{aligned} \tag{8}$$

The remaining massless fermions are then

$$(8;2,12,\tfrac{17}{2})+(28;2,3,4)+(\overline{56};1,-9,-\tfrac{9}{2})+(\overline{56};1,-3,-\tfrac{5}{2})+(70;1,6,2) \tag{9}$$

under (7).

For the second tumbling the MAC condensate is given by

$$\mathbf{70}\times\mathbf{70}\to\mathbf{1}_S \tag{10}$$

under $SU(8)_{MC}$. This condensate has the representation

$$\Phi_2=(1_S;1,12,4) \tag{11}$$

under (7) and breaks the symmetry down to

$$SU(8)_{MC}\times SU(2)_1\times U(1)_{12}\ , \tag{12}$$

where $U(1)_{12}$ is a linear combination of $U(1)_1$ and $U(1)_2$. One can choose, for example,

$$Q(U(1)_{12})=\tfrac{2}{3}Q(U(1)_1)-2Q(U(1)_2)\ . \tag{13}$$

The remaining massless fermions are then

$$(8;2,-9)+(28;2,-6)+2(\overline{56};1,3) \tag{14}$$

under (12).

For the third tumbling the MAC condensate is given by

$$\mathbf{28}\times\overline{\mathbf{56}}\to\overline{\mathbf{8}} \tag{15}$$

under $SU(8)_{MC}$. This condensate has the representation

$$\Phi_3=(\overline{8};2,-3) \tag{16}$$

under (12) and breaks the symmetry down to

$$SU(7)_{MC}\times U(1)_3\times U(1)_4\ , \tag{17}$$

where $U(1)_3$ and $U(1)_4$ are linear combinations of three U(1)'s: $U(1)_{12}$, $U(1)_{S1}$ coming from the breaking of $SU(2)_1$, and $U(1)_{MC7}$ coming from the breaking of $SU(8)_{MC}$ into $SU(7)_{MC}\times U(1)_{MC7}$. We set

$$Q(\mathrm{U}(1)_3)=Q(\mathrm{U}(1)_{12})-8Q(\mathrm{U}(1)_{S1})-Q(\mathrm{U}(1)_{\mathrm{MC}7}) ,$$
$$Q(\mathrm{U}(1)_4)=Q(\mathrm{U}(1)_{12})+20Q(\mathrm{U}(1)_{S1})+Q(\mathrm{U}(1)_{\mathrm{MC}7}) . \quad (18)$$

The massless fermions are then, under (17),

$$(1;-20,8)+(1;-12,-12)+(7;-16,10)+(7;-12,0)+(7;-8,-10)+(7;-4,-20)+(21;0,-18)+(\overline{21};8,-2)+2(\overline{35};0,6) . \quad (19)$$

For the fourth tumbling the MAC condensate is given by

$$21\times\overline{35}\rightarrow\overline{7} \quad (20)$$

under $\mathrm{SU(7)_{MC}}$. This condensate has the representation

$$\Phi_4=(\overline{7};0,-12) \quad (21)$$

under (17) and breaks the symmetry down to

$$\mathrm{SU}(6)_{\mathrm{MC}}\times\mathrm{U}(1)_3\times\mathrm{U}(1)'_4 , \quad (22)$$

where $\mathrm{U}(1)'_4$ is a linear combination of $\mathrm{U}(1)_4$ and $\mathrm{U}(1)_{\mathrm{MC}6}$ coming from the breaking of $\mathrm{SU}(7)_{\mathrm{MC}}$ into $\mathrm{SU}(6)_{\mathrm{MC}}\times\mathrm{U}(1)_{\mathrm{MC}6}$. We choose

$$Q(\mathrm{U}(1)'_4)=Q(\mathrm{U}(1)_4)-2Q(\mathrm{U}(1)_{\mathrm{MC}6}) . \quad (23)$$

The remaining massless fermions are then, under (22),

$$(1;-20,8)+(1;-16,-2)+2(1;-12,-12)+(1;-8,-22)+(1;-4,-32)+(6;-16,12)+(6;-12,2)+(6;-4,-18)+(6;0,-28)+(\overline{15};8,-6)+(\overline{15};0,14) . \quad (24)$$

Note that we have a degeneracy of metacolor-singlet fermion $(1;-12,-12)$, which will remain massless all the way to the last tumbling.

For the fifth tumbling the MAC condensate is given by

$$6\times\overline{15}\rightarrow\overline{6} \quad (25)$$

under $\mathrm{SU(6)_{MC}}$. This condensate has the representation

$$\Phi_5=(\overline{6};0,-14) \quad (26)$$

under (22) and breaks the symmetry down to

$$\mathrm{SU}(5)_{\mathrm{MC}}\times\mathrm{U}(1)_3\times\mathrm{U}(1)''_4 , \quad (27)$$

where $\mathrm{U}(1)''_4$ is a linear combination of $\mathrm{U}(1)'_4$ and $\mathrm{U}(1)_{\mathrm{MC}5}$ coming from the breaking of $\mathrm{SU}(6)_{\mathrm{MC}}$ into $\mathrm{SU}(5)_{\mathrm{MC}}\times\mathrm{U}(1)_{\mathrm{MC}5}$. We set

$$Q(\mathrm{U}(1)''_4)=Q(\mathrm{U}(1)'_4)-\tfrac{14}{5}Q(\mathrm{U}(1)_{\mathrm{MC}5}) . \quad (28)$$

The remaining massless fermions are then, under (27),

$$(1;-20,8)+2(1;-16,-2)+3(1;-12,-12)+(1;-8,-22)+2(1;-4,-32)+(1;0,-42)+(5;-16,\tfrac{74}{5})+(5;-12,\tfrac{24}{5})+(5;-4,-\tfrac{76}{5})+(\overline{5};8,\tfrac{26}{5})+(\overline{10};8,-\tfrac{58}{5})+(\overline{10};0,\tfrac{42}{5}) . \quad (29)$$

For the sixth tumbling the MAC condensate is given by

$$\overline{10}\times\overline{10}\rightarrow 5_S \quad (30)$$

under $\mathrm{SU(5)_{MC}}$. This condensate has the representation

$$\Phi_6=(5_S;0,\tfrac{84}{5}) \quad (31)$$

under (27) and breaks the symmetry down to

$$\mathrm{SU}(4)_{\mathrm{MC}}\times\mathrm{U}(1)_3\times\mathrm{U}(1)'''_4 , \quad (32)$$

where $\mathrm{U}(1)'''_1$ is a linear combination of $\mathrm{U}(1)''_4$ and $\mathrm{U}(1)_{\mathrm{MC}4}$ coming from the breaking of $\mathrm{SU}(5)_{\mathrm{MC}}$ into $\mathrm{SU}(4)_{\mathrm{MC}}\times\mathrm{U}(1)_{\mathrm{MC}4}$. We choose

$$Q(\mathrm{U}(1)'''_4)=Q(\mathrm{U}(1)''_4)-\tfrac{21}{5}Q(\mathrm{U}(1)_{\mathrm{MC}4}) . \quad (33)$$

The remaining massless fermions are then, under (32),

$$(1;-20,8)+3(1;-16,-2)+4(1;-12,-12)+3(1;-4,-32)+(1;0,-42)+(4;-16,19)+(4;-12,9)+(4;-4,-11)+2(\overline{4};8,1)+(\overline{4};0,21)+(6;8,-20) . \quad (34)$$

For the seventh and the last tumbling the MAC condensate is given by

$$\mathbf{6}\times\mathbf{6}\rightarrow\mathbf{1}_S \tag{35}$$

under $SU(4)_{MC}$. This condensate has the representation

$$\Phi_7=(1_S;16,-40) \tag{36}$$

under (32) and breaks the symmetry down to

$$SU(4)_{MC}\times U(1)_E\ , \tag{37}$$

where $U(1)_E$ is a linear combination of $U(1)_3$ and $U(1)_4'''$. We set

$$Q(U(1)_E)=-\tfrac{15}{28}Q(U(1)_3)-\tfrac{3}{14}Q(U(1)_4''')\ . \tag{38}$$

The remaining massless fermions are then, under (37),

$$12(1;9)\ . \tag{39}$$

Since we have only metacolor singlets as massless fermion, tumbling stops here. Note that we ended up with the 12-fold degeneracy.

Finally, let us consider the model in the confining phase. Following the Higgs phase, we assume that $SU(2)_1\times SU(2)_2$ in the color-flavor symmetry is completely broken so that we should apply 't Hooft anomaly matching[3] to the preon model based on $SU(9)_{MC}\times U(1)_F$. The branching of the fermions in (3) gives four preons:

$$\begin{aligned} &P_1'=(36;5),\quad P_1''=(36;5)\ ,\\ &P_2'=(\overline{126};-1),\quad P_2''=(\overline{126};-1) \end{aligned} \tag{40}$$

under $SU(9)_{MC}\times U(1)_F$. Let l be the 't Hooft index for the composite fermion (1;9) under $SU(9)_{MC}\times U(1)_F$. Then the 't Hooft anomaly-matching equations are[3,6]

$$\begin{aligned} &U(1)_F:\ \ 108=9l+\cdots\ ,\\ &(U(1)_F)^3:\ \ 8748=729l+\cdots\ , \end{aligned} \tag{41}$$

where the ellipsis indicates contributions from other composite fermions. Thus, we have the solution

$$l=12\quad \text{with all other } l_i=0\ . \tag{42}$$

This solution corresponds to the same massless fermions as in the Higgs phase [see (39)]. That is, complementarity holds in this model. Equation (42) gives an explicit counterexample to our conjecture, Eq. (1).

In this paper we considered possible sources of degeneracies in preon models which are in accord with complementarity and presented a counterexample to the conjecture about the constraint on the 't Hooft indices, thus disproving it. However, mathematical understanding of the whole subject is still missing.

[1] J.-M. Gérard, Y. Okamoto, and R. E. Marshak, Phys. Lett. **169B**, 386 (1986); the constraint of Eq. (1) had already been imposed by I. Bars [Nucl. Phys. **B207**, 77 (1982)] on a different basis.

[2] S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. **B173**, 208 (1980).

[3] G. 't Hooft, in *Recent Development in Gauge Theories*, proceedings of the NATO Advanced Study Institute, Cargèse, 1979, edited by G. 't Hooft *et al.* (Plenum, New York, 1980).

[4] Y. Okamoto and R. E. Marshak, Phys. Rev. D **34**, 3923 (1986).

[5] S. Raby, S. Dimopoulos, and L. Susskind, Nucl. Phys. **B169**, 373 (1980); M. Peskin (unpublished).

[6] L. Alvarez-Gaumé and E. Witten, Nucl. Phys. **B234**, 269 (1983).