Energy-momentum tensor in theories with scalar fields and two coupling constants. III. A model with two scalars

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(Received 25 October 1988)

In this work, we continue our investigation of the improvement term needed for the energy-momentum tensor in field theories with scalar fields and two coupling constants. Here, we shall discuss a model having two scalar fields. As in earlier works, we shall consider improvement terms which are (i) a finite function of bare quantities, (ii) a finite function of renormalized quantities. We show that, unlike the case of $\lambda\phi^4$ theory with a single scalar field, in the present case neither form of improvement leads to a finite energy-momentum tensor.

I. INTRODUCTION

Energy-momentum tensors and their finiteness have received a good deal of attention on account of their relevance in physics.[1-12] As is well known, finiteness of the energy-momentum tensor in theories with scalar fields is a nontrivial question on account of a need for an improvement term and has been studied in great detail by various authors.[2-4,6-9] These investigations have been restricted to theories with scalar field(s) and a single coupling constant[6-8]

In Refs. 10–12 (henceforth referred to as I, II, III, respectively) we investigated the cases containing scalar field(s) and two coupling constants, viz., scalar electrodynamics, non-Abelian theories with scalar fields, and Yukawa theory. In this work we complete this investigation by considering a scalar field model with two fields and two coupling constants.

We have motivated this study in Ref. 11. The improvement term needed is generally of the *form*

$$\tilde{g}(\partial_\mu\partial_\nu-\partial^2 g_{\mu\nu})\phi^2 \;.$$

As in earlier works, we shall consider two forms[11] for improvement terms: (i) one in which $\tilde{g}$ is a finite function of bare quantities (i.e., finite at $\epsilon=0$), (ii) one in which $\tilde{g}$ is a finite function of renormalized quantities. Unlike a $\lambda\phi^4$ theory (with one scalar field) but like the cases dealt with in I–III, neither form leads to a finite energy-momentum tensor. The physical significance of this result is explained in II. As the discussion in this case is very similar to that in I–III, we shall present our results with brevity.

Our conclusion is that in all (renormalizable) theories with scalar field(s) and two coupling constants neither kind of improvement term leads to a finite energy-momentum tensor, and that a new renormalization is needed to make $\theta_{\mu\nu}$ finite. This is unlike theories without scalar fields and also unlike $\lambda\phi^4$ theory.

II. PRELIMINARY

We consider a theory of two interacting scalar fields ϕ_1 and ϕ_2. We would like to restrict ourselves to the case in which there are only two independent couplings. Hence we shall assume that the couplings of quartic interactions ϕ_1^4 and ϕ_2^4 are the same. We shall also assume that their masses are equal: this simplifies the treatment without altering the final conclusion. We consider the Lagrange density

$$\mathcal{L}=\tfrac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1-\tfrac{1}{2}m_0^2\phi_1^2-\frac{\lambda_0}{4!}\phi_1^4 +\tfrac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2-\tfrac{1}{2}m_0^2\phi_2^2-\frac{\lambda_0}{4!}\phi_2^4-\tfrac{1}{4}\kappa_0\phi_1^2\phi_2^2 \;. \quad (2.1)$$

We shall use dimensional regularization and the minimal-subtraction scheme throughout.

We define the renormalization transformations:

$$\phi_i=Z^{1/2}\phi_i^R,\quad i=1,2,\quad m_0^2=Z_m m^2 \;,$$
$$\lambda_0=\mu^\epsilon[\lambda Z_\lambda+\delta\lambda(\kappa_0)],\quad \kappa_0=\mu^\epsilon\kappa Z_\kappa \;. \quad (2.2)$$

Here λ_0 is not multiplicatively renormalizable and $\delta\lambda(\kappa_0)$ starts as κ_0^2.

The canonical energy-momentum tensor is

$$\theta^c_{\mu\nu}=-g_{\mu\nu}\mathcal{L}+\partial_\mu\phi_1\partial_\nu\phi_1+\partial_\mu\phi_2\partial_\nu\phi_2 \;. \quad (2.3)$$

We define an improved energy-momentum tensor

$$\theta^{\rm imp}_{\mu\nu}=\theta^c_{\mu\nu}+\left[\frac{n-2}{4(1-n)}+\frac{\tilde{g}}{1-n}\right](\partial_\mu\partial_\nu-\partial^2 g_{\mu\nu})\times(\phi_1^2+\phi_2^2) \;, \quad (2.4)$$

where the second term on the right-hand side arises when one considers the conformally invariant action in curved space-time and obtains $\theta_{\mu\nu}$ from it via

$$\theta_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}\bigg|_{g^{\mu\nu}=\eta^{\mu\nu}} \;,$$

and $\tilde{g}$ is an additional improvement to be determined. The improvement term is symmetric under $\phi_1\leftrightarrow\phi_2$ because so is the Lagrange density. A term of the kind $(\partial_\mu\partial_\nu-\partial^2 g_{\mu\nu})(\phi_1\phi_2)$ is not an allowed improvement because $\mathcal{L}$ is invariant under $\phi_1\to-\phi_1$ (and also under $\phi_2\to-\phi_2$ separately).

A simple calculation shows that

$$\theta_\mu^{\text{imp}\,\mu}=(n-4)\left[-\frac{\lambda_0\phi_1^4}{4!}-\frac{\lambda_0\phi_2^4}{4!}-\tfrac{1}{4}\kappa_0\phi_1^2\phi_2^2\right]+m_0^2(\phi_1^2+\phi_2^2)+\frac{n-2}{2}\left[\frac{\delta S}{\delta\phi_1}\phi_1+\frac{\delta S}{\delta\phi_2}\phi_2\right]+\bar g\partial^2(\phi_1^2+\phi_2^2)\ . \tag{2.5}$$

To obtain $\langle\theta_\mu^{\text{imp}\,\mu}\rangle$ one needs to consider renormalization of operators appearing in the above equation. To this end we shall consider the following set:

$$O_1=-\left[\frac{\lambda\phi_1^4}{4!}+\frac{\lambda\phi_2^4}{4!}+\tfrac{1}{4}\kappa_0\phi_1^2\phi_2^2\right],$$
$$O_2=m_0^2(\phi_1^2+\phi_2^2),\quad O_3=\phi_1\frac{\delta S}{\delta\phi_1},\quad O_4=\phi_2\frac{\delta S}{\delta\phi_2}, \tag{2.6}$$
$$O_5=-\tfrac{1}{4}\kappa_0\phi_1^2\phi_2^2,\quad O_6=\partial^2(\phi_1^2+\phi_2^2)\ .$$

The above set is closed under renormalization. A slight generalization of the argument in Ref. 10 shows that O_2 is a finite operator so that

$$O_2^{\text{UR}}=O_2^R\ . \tag{2.7}$$

Equations of motion further imply that[10] O_3 and O_4 are also finite operators:

$$O_3^{\text{UR}}=O_3^R,\quad O_4^{\text{UR}}=O_4^R\ . \tag{2.8}$$

O_6 is a multiplicatively renormalizable operator[10] with

$$O_6^{\text{UR}}=Z_m^{-1}O_6^R\ , \tag{2.9}$$

where Z_m^{-1} is defined below.

We note the following:

$$O_1=\lambda_0\frac{\partial S}{\partial\lambda_0}+\kappa_0\frac{\partial S}{\partial\kappa_0}\ ,$$
$$O_2=-2m_0^2\frac{\partial S}{\partial m_0^2}\ , \tag{2.10}$$
$$O_5=\kappa_0\frac{\partial S}{\partial\kappa_0}\ ,$$

and thus define renormalized operators[10] in terms of the generating functional for connected renormalized Green's function $Z^R[J_i^R,\lambda,\kappa]$ as

$$O_1^R=\lambda\frac{\partial Z^R}{\partial\lambda}+\kappa\frac{\partial Z^R}{\partial\kappa}\ ,$$
$$O_2^R=-2m^2\frac{\partial Z^R}{\partial m^2}\ ,$$
$$O_3=-J_1^R\frac{\delta Z^R}{\delta J_1^R}\ , \tag{2.11}$$
$$O_4=-J_2^R\frac{\delta Z^R}{\delta J_2^R}\ ,$$
$$O_5=\kappa\frac{\partial Z^R}{\partial\kappa}\ .$$

[The definition of O_1^R is valid only up to $O(\kappa^0)$. In this definition, we have equated $\langle\lambda_0(\partial S/\partial\lambda_0)\rangle^R=\lambda\partial Z^R/\partial\lambda$. But this cannot be true in $O(\kappa_0^2)$ because the right-hand side vanishes at $\lambda=0$ whereas the left-hand side does not vanish at $\lambda=0$; as then $\lambda_0=\mu^\epsilon\delta\lambda(\kappa_0)\neq0$ in $O(\kappa_0^2)$. However, this definition is valid in $O(\kappa_0)$ because $\delta\lambda_0$ vanishes to this order. Our treatment needs only $O(\kappa)$ quantities and hence the above definition of O_1^R is correct for our purpose.]

We introduce the following renormalization-group definitions and indicate the nontrivial leading term whenever they are needed in the future:

$$\gamma(\lambda,\kappa,\epsilon)=\mu\frac{\partial}{\partial\mu}\ln Z\bigg|_{\lambda_0,\kappa_0,\epsilon}=\gamma(\lambda,\kappa)\ ,$$
$$\gamma_m(\lambda,\kappa,\epsilon)=-\tfrac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_m=\gamma_m(\lambda,\kappa)=\gamma_{m(1)}\lambda+\gamma_{m(1)}\kappa+\cdots\ , \tag{2.12}$$
$$\beta^\lambda(\lambda,\kappa,\epsilon)=\mu\frac{\partial\lambda}{\partial\mu}=\beta^\lambda(\lambda,\kappa)-\lambda\epsilon=\lambda^2\frac{\partial}{\partial\lambda}Z_\lambda^{(1)}+\kappa\lambda\frac{\partial Z_\lambda^{(1)}}{\partial\kappa}+\kappa\frac{\partial}{\partial\kappa}\delta\lambda^{(1)}-\delta\lambda^{(1)}=-\lambda\epsilon+\beta_2\lambda^2+\cdots\ ,$$
$$\beta^\kappa(\lambda,\kappa,\epsilon)=\mu\frac{\partial\kappa}{\partial\mu}=\beta^\kappa(\lambda,\kappa)-\kappa\epsilon=\kappa^2\frac{\partial Z_\kappa^{(1)}}{\partial\kappa}+\lambda\kappa\frac{\partial Z_\kappa^{(1)}}{\partial\lambda}=-\kappa\epsilon+\frac{2}{16\pi^2}\lambda\kappa+\cdots\ .$$

For the six operators of Eq. (2.6), the renormalization matrix is defined by

$$O_i^{\text{UR}}=Z_{ij}O_j^R\ . \tag{2.13}$$

It can be shown that

$$Z_{ik}^{-1}\mu\frac{\partial}{\partial\mu}Z_{kj}=\gamma_{ij}=\text{finite at }\epsilon=0\ . \tag{2.14}$$

Equations (2.6)–(2.9) imply the following structure for Z_{ij}:

$$Z=\begin{bmatrix}Z_{11}&Z_{12}&Z_{13}&Z_{14}&Z_{15}&Z_{16}\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\Z_{51}&Z_{52}&Z_{53}&Z_{54}&Z_{55}&Z_{56}\\0&0&0&0&0&Z_m^{-1}\end{bmatrix}. \tag{2.15}$$

III. EXPRESSIONS FOR Z_{1j}

From Eqs. (2.5)–(2.9) the expression for the improved trace can be written as

$$\langle \theta_\mu^{\text{imp}\,\mu} \rangle = -\epsilon \langle O_1 \rangle^{\text{UR}} + \langle O_2 \rangle^R + \frac{n-2}{2} \langle O_3 + O_4 \rangle^R + \tilde{g} Z_m^{-1} \langle O_G \rangle^R$$
$$= -\epsilon \langle O_1 \rangle^{\text{UR}} + \tilde{g} Z_m^{-1} \langle O_G \rangle^R + \text{finite}$$
$$= -\epsilon \sum_{j=1}^{5} Z_{1j} \langle O_j \rangle^R + (-\epsilon Z_{16} + \tilde{g} Z_m^{-1}) \langle O_6 \rangle^R + \text{finite} . \quad (3.1)$$

In the future discussion, we shall need Z_{1j} ($j=1,2,\ldots,5$). These are obtained by noting the identity

$$\int d^n x \, O_1 = -S + \tfrac{1}{2} \int d^n x (O_3 + O_4) . \quad (3.2)$$

An expression for $\langle S \rangle$ can be obtained along the lines of Ref. 10. Here we shall give the final result:

$$\left\langle \int d^n x \, O_1 \right\rangle^{\text{UR}} = -\langle S \rangle + \tfrac{1}{2} \left\langle \int d^n x \, O_3 + \int d^n x \, O_4 \right\rangle$$
$$= \left[1 - \frac{\beta^\lambda}{\lambda \epsilon} \right] \left\langle \int d^n x \, O_1^R \right\rangle^R + \frac{\gamma_m}{\epsilon} \left\langle \int d^n x \, O_2 \right\rangle + \frac{\gamma}{\epsilon} \left\langle \int d^n x \, O_3 + \int d^n x \, O_4 \right\rangle^R + \left[\frac{\beta^\lambda}{\lambda \epsilon} - \frac{\beta^\kappa}{\kappa \epsilon} \right] \left\langle \int d^n x \, O_5 \right\rangle^R .$$

This yields

$$Z_{11} = 1 - \frac{\beta^\lambda}{\lambda \epsilon}, \quad Z_{12} = \frac{\gamma_m}{\epsilon} ,$$
$$Z_{13} = \frac{\gamma}{\epsilon} = Z_{14}, \quad Z_{15} = \frac{\beta^\lambda}{\lambda \epsilon} - \frac{\beta^\kappa}{\kappa \epsilon} . \quad (3.3)$$

As Z_{1j} ($j=1,\ldots,5$) have only simple poles in ϵ, it follows from Eq. (3.1) that at zero momentum (where O_6 does not contribute), $\langle \theta_\mu^{\text{imp}\,\mu} \rangle$ is finite.

IV. IMPROVEMENT-TERM DEPENDENCE OF THE FORM $\tilde{g}(\epsilon, \kappa_0 \mu^{-\epsilon}, \lambda_0 \mu^{-\epsilon})$

In this section, we shall consider an improvement coefficient $\tilde{g}(\epsilon, \kappa_0 \mu^{-\epsilon}, \lambda_0 \mu^{-\epsilon})$ which is a finite function of bare couplings κ_0 and λ_0, at $\epsilon=0$. Using the renormalization-group equation satisfied by Z_{16} we shall show that such an improvement term cannot yield a finite energy-momentum tensor $\theta_{\mu\nu}^{\text{imp}}$ for any choice of $\tilde{g}(\epsilon, \kappa_0 \mu^{-\epsilon}, \lambda_0 \mu^{-\epsilon})$ even to first order in κ_0.

We rewrite Eq. (3.1) (noting that Z_{1j} are finite for $j=1,\ldots,5$) as

$$\langle \theta_\mu^{\text{imp}\,\mu} \rangle = -\epsilon X \langle O_6 \rangle^R + \text{finite} , \quad (4.1)$$

where

$$X = Z_{16} - \frac{\tilde{g}}{\epsilon} Z_m^{-1} = Z_{16} + g Z_m^{-1} . \quad (4.2)$$

Here

$$g \equiv -\frac{\tilde{g}(\epsilon, \kappa_0 \mu^{-\epsilon}, \lambda_0 \mu^{-\epsilon})}{\epsilon}$$

can have $1/\epsilon$ terms when expanded in a series in powers of $(\kappa_0 \mu^{-\epsilon})$ and $(\lambda_0 \mu^{-\epsilon})$.

Z_{16} satisfies the differential equation (obtained along the lines of Appendix C of I)

$$(-\lambda\epsilon + \beta^\lambda) \frac{\partial Z_{16}}{\partial \lambda} + (-\kappa\epsilon + \beta^\kappa) \frac{\partial Z_{16}}{\partial \kappa} - 2\gamma_m Z_{16} = Z_{11}\gamma_{16} + Z_{15}\gamma_{56} . \quad (4.3)$$

Using the Eqs. (4.2) and (4.3), one can obtain an equation satisfied by X:

$$(-\lambda\epsilon + \beta^\lambda) \frac{\partial X}{\partial \lambda} + (-\kappa\epsilon + \beta^\kappa) \frac{\partial X}{\partial \kappa} - 2\gamma_m X - Z_{11}\gamma_{16} - Z_{15}\gamma_{56}$$
$$= \left[\mu \frac{\partial}{\partial \mu} g(\epsilon, \kappa_0 \mu^{-\epsilon}, \lambda_0 \mu^{-\epsilon}) \right] Z_m^{-1}$$
$$= -\epsilon \sum_{n=0}^{\infty} (\kappa_0 \mu^{-\epsilon})^n \left[n g_n(\epsilon, \lambda_0 \mu^{-\epsilon}) + \frac{\partial g_n(\epsilon, \lambda_0 \mu^{-\epsilon})}{\partial (\lambda_0 \mu^{-\epsilon})} (\lambda_0 \mu^{-\epsilon}) \right] Z_m^{-1} , \quad (4.4)$$

where we have expanded g in powers of $(\kappa_0 \mu^{-\epsilon})$:

$$g(\epsilon, \kappa_0 \mu^{-\epsilon}, \lambda_0 \mu^{-\epsilon}) = \sum_{n=0}^{\infty} (\kappa_0 \mu^{-\epsilon})^n g_n(\epsilon, \lambda_0 \mu^{-\epsilon}) . \quad (4.5)$$

Now, if it is possible to choose $\tilde{g}$ such that X has no worse than simple poles in ϵ, the left-hand side of Eq. (4.4) would have no worse than simple poles and hence

$$\epsilon^2 \sum_{n=0}^{\infty} (\kappa_0 \mu^{-\epsilon})^n \left[n g_n(\epsilon, \lambda_0 \mu^{-\epsilon}) + \frac{\partial g_n(\epsilon, \lambda_0 \mu^{-\epsilon})}{\partial(\lambda_0 \mu^{-\epsilon})} \lambda_0 \mu^{-\epsilon} \right] Z_m^{-1} = \text{finite} . \quad (4.6)$$

We have analyzed a similar equation in Sec. V of I and shown there that Eq. (4.6) in particular implies that

$$g_0(\epsilon, \lambda_0 \mu^{-\epsilon}) \equiv g_0(\epsilon), \quad g_1(\epsilon, \lambda_0 \mu^{-\epsilon}) = 0 \quad (4.7)$$

implying that the improvement coefficient obtained in $O(\kappa^0)$, viz., $g_0(\epsilon)$, should be sufficient to $O(\kappa)$ to make $\theta_{\mu\nu}^{\text{imp}}$ finite to $O(\kappa\lambda^n)$. But this contradicts the result in the Appendix that in $O(\kappa\lambda^3)$, $X = Z_{16} + g_0(\epsilon) Z_m^{-1}$ does have double poles as verified explicitly. Hence such an improvement term does not yield a finite $\theta_{\mu\nu}^{\text{imp}}$ even to $O(\kappa\lambda^n)$.

V. IMPROVEMENT-TERM DEPENDENCE OF THE FORM $\tilde{g}(\epsilon,\kappa,\lambda)$

In this section we shall consider an improvement coefficient which is a finite function (at $\epsilon=0$) of the renormalized coupling constants and λ. As in the previous section,

$$\langle \theta_\mu^{\text{imp}\,\mu} \rangle = -\epsilon X \langle O_6 \rangle^R + \text{finite} , \quad (5.1)$$

where

$$X = Z_{16} - \frac{\tilde{g}(\epsilon,\kappa,\lambda)}{\epsilon} Z_m^{-1} \equiv Z_{16} + g(\epsilon,\kappa,\lambda) Z_m^{-1} . \quad (5.2)$$

Using Eq. (4.3) we obtain a differential equation satisfied by X of Eq. (5.2), viz.,

$$(-\lambda\epsilon + \beta^\lambda) \frac{\partial X}{\partial \lambda} + (-\kappa\epsilon + \beta^\kappa) \frac{\partial X}{\partial \kappa} - 2\gamma_m X - Z_{11}\gamma_{16} - Z_{15}\gamma_{56} = \left[(-\lambda\epsilon + \beta^\lambda) \frac{\partial g}{\partial \lambda} + (-\kappa\epsilon + \beta^\kappa) \frac{\partial g}{\partial \kappa} \right] Z_m^{-1} . \quad (5.3)$$

As in the previous section, if it is possible to choose $\tilde{g}(\epsilon,\kappa,\lambda)$ such that X has no worse than simple poles, then this implies

$$\epsilon \left[(-\lambda\epsilon + \beta^\lambda) \frac{\partial g}{\partial \lambda} + (-\kappa\epsilon + \beta^k) \frac{\partial g}{\partial \kappa} \right] Z_m^{-1} = \text{finite at } \epsilon = 0 . \quad (5.4)$$

We expand

$$g(\epsilon,\kappa,\lambda) = \sum_{n=0}^{\infty} \kappa^n g_n(\epsilon,\lambda) \quad (5.5)$$

and consider Eq. (5.4) in various powers of κ. Following the same procedure as in Sec. VI of II (β^λ and β^κ have identical leading terms as β^λ and β^e of II) we obtain

$$g_0(\epsilon,\lambda) = g_0(\epsilon), \quad g_1(\epsilon,\lambda) = 0 \quad (5.6)$$

implying that the improvement coefficient obtained in $O(\kappa^0)$, viz., $g_0(\epsilon)$ should be sufficient to $O(\kappa)$. This is of course wrong as explained at the end of the last section. Hence this kind of an improvement function does not yield a finite $\theta_{\mu\nu}^{\text{imp}}$ even to $O(\kappa\lambda^n)$.

APPENDIX

In this appendix we shall explicitly show that $X = Z_{16} + g_0(\epsilon) Z_m^{-1}$ does have double poles in ϵ in $O(\kappa\lambda^3)$. This we shall deduce from a calculation of simple pole divergence in $O(\kappa\lambda^2)$ in Z_{16}, use of renormalization-group equation for Z_{16}, viz., Eq. (4.3) and the knowledge of $O(\epsilon^2)$ terms in $g_0(\epsilon)$.

In $O(\kappa\lambda^3)$, the double-pole terms in X come from those in Z_{16} and the $O(\kappa\lambda^3/\epsilon^4)$ terms in Z_m^{-1} multiplied by $O(\epsilon^2)$ term in $g_0(\epsilon)$.

A direct calculation shows that the order-$\kappa\lambda^2$ terms in Z_{16} are given by

$$Z_{16} = -\frac{1}{18(16\pi^2)\epsilon} \kappa\lambda^2 + \text{other terms} . \quad (A1)$$

Using the RG equation satisfied by Z_{16} [Eq. (4.3)] one can relate the $O(\kappa\lambda^3/\epsilon^2)$ terms in Z_{16} to those of $O(\kappa\lambda^2/\epsilon)$. The result is

$$Z_{16} = -\frac{1}{18(16\pi^2)^2\epsilon^2} \kappa\lambda^3 + \text{other terms} . \quad (A2)$$

We further use

$$g_0(\epsilon) = g_{02}\epsilon^2 + O(\epsilon^3) = -\tfrac{1}{480}\epsilon^2 + O(\epsilon^3) \quad (A3)$$

and

$$Z_m^{-1} = -\frac{1}{16\pi^2\epsilon} \lambda + \text{other terms} \quad (A4)$$

and use the renormalization-group equation for Z_m^{-1} to obtain

$$Z_m^{-1} = -\frac{1}{(16\pi^2)^4}\lambda^3\kappa + \text{other terms} . \tag{A5}$$

Using the results of Eqs. (A2)–(A5) we obtain

$$X = Z_{16} + g_0(\epsilon) Z_m^{-1}$$
$$= \frac{1}{(16\pi^2)^4}\left(-\tfrac{1}{18}+\tfrac{1}{480}\right)\kappa\lambda^3 + \text{other terms} .$$

Thus X does have double poles in ϵ in order $\kappa\lambda^3$, a result used in Secs. IV and V.

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