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Double-cascade scheme for QCD jets in e^+e^- annihilation

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We propose a new scheme for a QCD cascade model in the lightlike axial gauge. In this new scheme the valence antiquark can evolve as well as the valence quark, while in the conventional scheme with the lightlike axial gauge only the valence quark evolves. Since the proposed scheme is formulated in terms of well-defined variables, we can apply this scheme to the next-to-leading-logarithmic model in e^+e^- annihilation. We show that new and conventional schemes give us the same results for energy distributions of the quark and for emission probabilities for gluons.

I. INTRODUCTION

In previous papers[1,2] we developed the QCD cascade model that includes the full next-to-leading-logarithmic (NLL) correction in e^+e^- annihilation. Since in this model we use the lightlike axial gauge where the gauge vector is parallel to the momentum of the valence antiquark, it does not emit any collinear gluons. Because of asymmetric treatments for quarks and antiquarks, there were slight differences between energy or transverse-momentum distributions for the valence quark and antiquark. In order to correct this unfavorable characteristic of the model we propose a new scheme where the valence antiquarks can evolve as well as the valence quark. We call the new scheme a double-cascade scheme while the old one is called a single-cascade scheme. In this double-cascade scheme it is possible to treat cascades of quarks and antiquarks symmetrically.

In e^+e^- annihilation the consistent treatment of quark and antiquark jets has been a difficult problem. In the lightlike axial gauge an asymmetric treatment is required[3] though it is consistent kinematically. On the other hand, in the timelike axial gauge kinematical constraints for energy fraction depend strongly on detailed description of models so that consistency for two jets is not clear.[4] Another model was presented by Marchesini and Webber,[5] who adopted angular variables instead of the virtual mass. Because of this variable it is possible to separate two jets in kinematically consistent manner. The angular variable, however, introduces another problem; since it is not manifestly Lorentz invariant, the total energy of the system cannot be determined until all cascades finish completely. This fact is not favorable from a practical point of view. In addition, it is difficult to extend the model with the angular variable to the NLL approximation because virtual masses must be given definitely in this approximation. To keep the virtual masses throughout the cascade we adopted the lightlike axial gauge in the construction of the NLL model.[1,2]

In the single-cascade scheme the reason for no cascade of the valence antiquark is simple. In the cross section for three jets $q\bar{q}G$ only two variables are left after integrating the cross section over angles around the beam axis. The remaining variables are energy fractions for the quark and antiquark in the center-of-mass system. In the cascade model we have two variables, the virtual mass and the light-cone variable, at one branching, so that if they are given in the first branching, the energy of the antiquark is determined as well as that of the quark. Therefore it is impossible for both quark and antiquark to evolve independently.

If the gauge vector is taken to be parallel to the momentum of the valence antiquark, energy distributions of the valence quark are correctly given by the single-cascade scheme. An essential point of the double-cascade scheme is that the quark distributions will be obtained correctly even if one restricts the kinematical region of the variables in the cascade. By this restriction we can make the quark and the antiquark evolve independently. In other words the models in the single- and double-cascade schemes give the same distributions for the valence quark. This equivalence will be discussed theoretically and be shown by Monte Carlo data.

Since the double-cascade scheme is formulated in terms of the well-defined variables, it is easy to include the NLL corrections in this scheme. In this point our scheme should be distinguished from the other methods that have been presented.

In the next section we will present discussions which lead to the double-cascade scheme and a detailed description for it. Also we will show that the single-cascade and double-cascade schemes give us the same results for probabilities that gluons are emitted from the valence quark and antiquark. This equivalence will be also discussed in Appendix A for moments of the inclusive cross section for valence quark. In Sec. III we will discuss the application of the double-cascade scheme to the NLL model. Here we would like to point out an improvement on a treatment of hard scattering in the NLL model. Also some results of our model will be given. The last section is devoted to a discussion and conclusion.

II. DOUBLE-CASCADE SCHEME

We make several discussions for the double-cascade scheme in e^+e^- annihilation. The first observation is that the most singular part of the probability for gluon emission from the quark in the order of α_s is symmetric on the fraction of the virtual mass squared and the light-cone variable. It is

$$\frac{4\alpha_s}{3\pi}\frac{dt}{t}\frac{dx}{x} . \quad (1)$$

Here t is a fraction of virtual mass squared, K^2/Q^2, where Q^2 is the total energy squared and x is a fraction of the light-cone variable for the gluon.[1] In the lightlike axial gauge there is a simple constraint between t and x. If Q_0^2 is a minimum virtual mass squared of partons and small enough compared with Q^2, the constraint is

$$xt > \epsilon = \frac{Q_0^2}{Q^2}, \quad x<1 \text{ and } t<1 .$$

Also we use the transverse momentum squared as the argument of α_s so that

$$\alpha_s = \alpha_s(xtQ^2)$$

for small x and t. Therefore we can see that the most singular part (1) is unchanged if t and x are interchanged.

A next observation is that for the three jets ($q\bar{q}G$) energy fractions $x_q, x_{\bar{q}}$ for quark and antiquark in the center-of-mass system are

$$x_{\bar{q}} = 1-t, \quad x_q = 1-x+tx \sim 1-x .$$

For small x and t, $x\,(t)$ corresponds to the energy fraction of the quark (antiquark). Therefore in the single-cascade scheme, t and x play roles of the fractions of the light cone and the virtual mass squared for the antiquark, respectively. From this argument it is clearly understood that in the single-cascade model the energy distribution for the valence antiquark can show correct behavior as the energy fraction goes to one.

To understand the roles of t and x in the lightlike axial gauge it is useful to calculate a one-loop diagram (Fig. 1) in this gauge. In Fig. 1 the gauge vector n equals the momentum of the antiquark $p_{\bar{q}}$. This diagram does not contain any singular contribution with t, but does contain contributions of dx/x, where $t = 2p_q\cdot k/Q^2$ and

$$x = \frac{n\cdot k}{n\cdot Q} = \frac{2p_{\bar{q}}\cdot k}{2p_{\bar{q}}\cdot Q} = \frac{(p_{\bar{q}}+k)^2}{2p_{\bar{q}}\cdot Q} .$$

Although it is usually said that dx/x is infrared singularity, this is not true in this case. Because the infrared singularity implies that both x and t become zero as the gluon momentum k becomes zero, but the singularity of Fig. 1 occurs for finite t. Therefore one should consider this singularity as the collinear one.

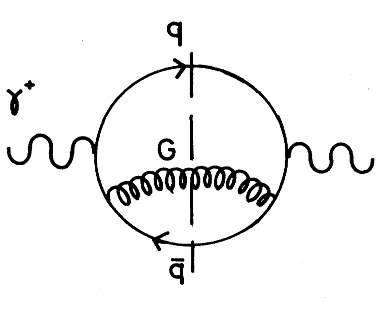

FIG. 1. A Feynman diagram for $q\bar{q}G$. The gauge vector is parallel to the momentum of the antiquark. Momenta of quark, antiquark, and gluon are p_q, $p_{\bar{q}}$, and k, respectively.

In the single-cascade scheme one sums all order of $[\alpha_s \ln(1/t)]^n$ terms using the renormalization-group equation. More precisely the contribution is of the form

$$\left\{\frac{4}{3}\frac{\alpha_s}{\pi}\ln\left[\frac{1}{t}\right]\left[\ln\left[\frac{1}{x}\right]+\text{finite } x\right]\right\}^n .$$

One should note that even in the single-cascade scheme we have a precise energy distribution for the antiquark as $x_{\bar{q}} \to 1$. The energy of the valence antiquark is determined by a virtual mass squared in the first branching of the valence quark; that is, $x_{\bar{q}} = 1-t$. Then it is easy to calculate the branching probability Π_B for small t:

$$\frac{1}{\sigma}\frac{d\sigma}{dx_q} = \Pi_B(t=1-x_q) ,$$

$$\propto t^{-1+c\{\ln[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)]\}} .$$

Here $c = \frac{4}{3}\beta_0$ with $\beta_0 = 11-2N_F/3$. N_F is the number of flavors. This behavior is obtained only if the transverse momentum squared is used as the argument of α_s. Furthermore if one wants to sum collinear singularities for the valence antiquark, one must include all order of $[\alpha_s \ln(1/x)(\text{finite } t)]^n$ terms.

Summarizing the above arguments, if one uses $\alpha_s dt\,dx/tx$ in the full kinematical region, a single cascade for a quark or antiquark is enough to give the correct energy distribution for both the quark and antiquark at least in $x_q(x_{\bar{q}}) \to 1$. We would like to divide this behavior into contributions by the quark and antiquark cascades. For this purpose we present a trivial identity: that is,

$$(1) = \frac{4\alpha_s}{3\pi}\frac{dt}{t}\frac{dx}{x}\theta(x-t) + \frac{4\alpha_s}{3\pi}\frac{dt}{t}\frac{dx}{x}\theta(t-x) . \quad (2)$$

The first term with $\theta(x-t)$ is used for the cascade of the valence quark while the second one with $\theta(t-x)$ is for the cascade of the valence antiquark. This separation is well understood if one considers the Dalitz plane with the energy of the quark and the antiquark for the $q\bar{q}G$ event (Fig. 2). For $E_{\bar{q}} > E_q$ the event is generated by the quark cascade, while for $E_q > E_{\bar{q}}$ the event is generated by the antiquark cascade. Therefore this treatment is theoretically consistent for $q\bar{q}G$, i.e., for events with one branching.

We apply the constraint the constraint $\theta(x-t)$ in any branching of the quark. By using this constraint the antiquark also evolves independently. In the double-cascade scheme the branching probability for the quark and the antiquark cascades is

$$\frac{2\alpha_s(tx(1-x)Q^2)}{3\pi}\frac{dt}{t}dx\frac{1+(1-x)^2}{x}\theta(x-t)\ , \tag{3}$$

in the leading-logarithmic (LL) approximation. Also we show the argument of α_s explicitly. The difference between probabilities in single-cascade and double-cascade schemes is of order α_s^2 so that this can be included in a consistent way in the NLL approximation using the scheme dependence. This point will be discussed in Appendix B.

Next, we will show that the single-cascade and the double-cascade schemes give us the same probability P_n for emission of n gluons from the quark and the antiquark in the LL approximation. For simplicity α_s is fixed. In the single-cascade scheme,

$$\begin{aligned}P_n&=\left[\frac{4\alpha_s}{3\pi}\right]^n\int^1 dt_1\frac{1}{t_1}\int^1 dx_1\theta(x_1t_1-\epsilon)\cdots\int^{t_{n-1}}dt_n\frac{1}{t_n}\int^1 dx_n\frac{1}{x_n}\theta(x_nt_n-\epsilon)\\&=\left[\frac{4\alpha_s}{3\pi}\right]^n\frac{1}{n!}\left[\int^1 dt\frac{1}{t}\int^1 dx\frac{1}{x}\theta(xt-\epsilon)\right]^n .\end{aligned}\tag{4}$$

Here the Altarelli-Parisi function

$$P(x)=\frac{1+(1-x)^2}{x}$$

is approximated by $2/x$. Then we use the trivial identity (2):

$$\begin{aligned}P_n&=\left[\frac{4\alpha_s}{3\pi}\right]^n\sum_{m=0}^{n}\frac{1}{m!(n-m)!}\left[\int^1 dt\frac{1}{t}\int^1 dx\frac{1}{x}\theta(xt-\epsilon)\theta(x-t)\right]^m\left[\int^1 dt\frac{1}{t}\int^1 dx\frac{1}{x}\theta(xt-\epsilon)\theta(t-x)\right]^{(n-m)}\\&=\sum_{m=0}^{n}\left[\frac{4\alpha_s}{3\pi}\right]^m\int^1 dt_1\frac{1}{t_1}\int^1 dx_1\frac{1}{x_1}\theta(x_1t_1-\epsilon)\theta(x_1-t_1)\cdots\\&\qquad\times\int^{t_{m-1}}dt_m\frac{1}{t_m}\int^1 dx_m\frac{1}{x_m}\theta(x_mt_m-\epsilon)\theta(x_m-t_m)\\&\quad\times\left[\frac{4\alpha_s}{3\pi}\right]^{n-m}\int^1 dx_1\frac{1}{x_1}\int^1 dt_1\frac{1}{t_1}\theta(x_1t_1-\epsilon)\theta(t_1-x_1)\cdots\\&\qquad\times\int^{x_{n-m-1}}dx_{n-m}\frac{1}{x_{n-m}}\int^1 dt_{n-m}\frac{1}{t_{n-m}}\theta(x_{n-m}t_{n-m}-\epsilon)\theta(t_{n-m}-x_{n-m})\ .\end{aligned}\tag{5}$$

The first factor is realized in the quark cascade while the latter is in the antiquark cascade. As a result the probability for n-gluon emission in the single-cascade scheme is equal to the sum of the products of probabilities, $P_m^{(q)}$ and $P_{n-m}^{(\bar q)}$ for quark jet and antiquark jet, where the total number of emitted gluons is n:

$$P_n=\sum_{m=0}^{n}P_m^{(q)}P_{n-m}^{(\bar q)}\ . \tag{6}$$

This equation shows that the correct Sudakov form factors in the double-cascade scheme can be obtained. The Sudakov form factor is the nonemission probability for the one jet. In the single-cascade scheme it should be the product of form factors for the double-cascade scheme. In the former approach it is

$$S_{\text{single}}=\exp\left[-\frac{2}{3\pi}\int\frac{dt}{t}\int dx\,\alpha_s(tx(1-x)Q^2)\frac{1+(1-x)^2}{x}\theta(xt-\epsilon)\right], \tag{7}$$

while the form factor in the double-cascade scheme is given by

$$S_{\text{double}}=\exp\left[-\frac{2}{3\pi}\int\frac{dt}{t}\int dx\,\alpha_s(tx(1-x)Q^2)\frac{1+(1-x)^2}{x}\theta(xt-\epsilon)\theta(x-t)\right]. \tag{8}$$

It is easily shown that the leading term of S_{single} is equal to that of $[S_{\text{double}}]^2$. In Appendix A we will show that moments of the inclusive cross section for quark are the same in the single- and double-cascade schemes.

III. NLL MODEL

When we apply the double-cascade scheme to the NLL model we can solve a problem on the infrared singularity for the hard cross section. First we will explain this problem. In the NLL model[1] we use the fixed-order cross section of α_s

order for generation of clean three jets. This cross section is

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma^{(1)}}{dx\,dt}=\frac{2\alpha_s}{3\pi}\frac{1}{t}\left[\frac{1+(1-x)^2}{x}+2t\frac{x(1-x)-1}{x}+t^2\frac{1+x^2}{x}\right], \tag{9}$$

where $t=p_T^2/x(1-x)Q^2$. For a definition of clean three jets, p_T^2 is larger than $x(1-x)\delta Q^2$, i.e., $t>\delta$. However this cross section has the infrared singularity as x goes to zero. Then total cross sections for the clean three jets is

$$\int dt\,dx\frac{1}{\sigma^{(0)}}\frac{d\sigma^{(1)}}{dx\,dt}=\frac{2\alpha_s}{3\pi}\left\{\left[2\ln\left[\frac{1}{\delta}\right]-2(1-\delta)+\tfrac{1}{2}(1-\delta^2)\right]\ln\left[\frac{Q^2}{Q_0^2}\right]-\ln^2\left[\frac{1}{\delta}\right]-\frac{3}{2}\ln\left[\frac{1}{\delta}\right]+3+O(\delta)\right\}. \tag{10}$$

This shows that we could not choose a small Q_0^2 at a very high energy. This difficulty forced us to tune parameter δ for various Q^2 in Ref. 1.

When we construct the NLL model in the double-cascade scheme, the above problem does not exist. In the double-cascade scheme the singularity for $x\to 0$ is included in the cascade of the valence antiquark. Therefore x should be larger than δ. As a result the total cross section for clean three jets is given by

$$\int dt\,dx\frac{1}{\sigma^{(0)}}\frac{d\sigma^{(1)}}{dx\,dt}=\frac{2\alpha_s}{3\pi}\left[2\ln^2\left[\frac{1}{\delta}\right]-3\ln\left[\frac{1}{\delta}\right]-\frac{\pi^2}{3}+\frac{5}{2}+O(\delta)\right]. \tag{11}$$

Since the cross section is finite for any Q_0^2 and Q^2, we do not need to worry about the tuning.

In Fig. 3 we present results of our new Monte Carlo model including the NLL correction, based on the double-cascade scheme for e^+e^- annihilation. One can find almost perfectly symmetric distributions for quarks and antiquarks. It is interesting to make a comparison between energy distributions of the valence quark in the single-cascade and the double-cascade schemes. Figure 4 shows almost the same results for them, which shows the equivalence between two schemes for quark distributions.

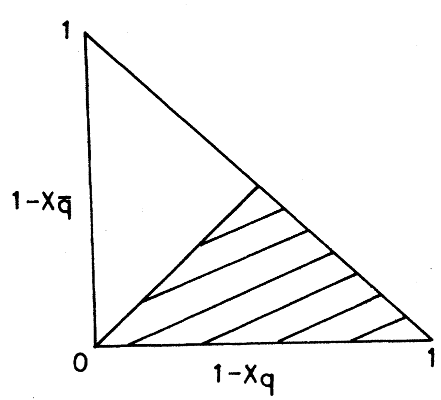

FIG. 2. The Dalitz plane for $q\bar{q}G$ events. Events in hatched area are generated by the branching of the quark, while events in another area are done by the branching of the antiquark. Here we assume that three partons are massless. $x_q=2E_q/\sqrt{Q^2}$, $x_{\bar{q}}=2E_{\bar{q}}/\sqrt{Q^2}$. E_q ($E_{\bar{q}}$) is the energy of the quark (antiquark) in the c.m. frame.

IV. CONCLUSION

In this paper we proposed the double-cascade scheme for QCD jets in the lightlike gauge, then applied it to the NLL model in e^+e^- annihilation. In this scheme the valence antiquark evolves as well as the valence quark.

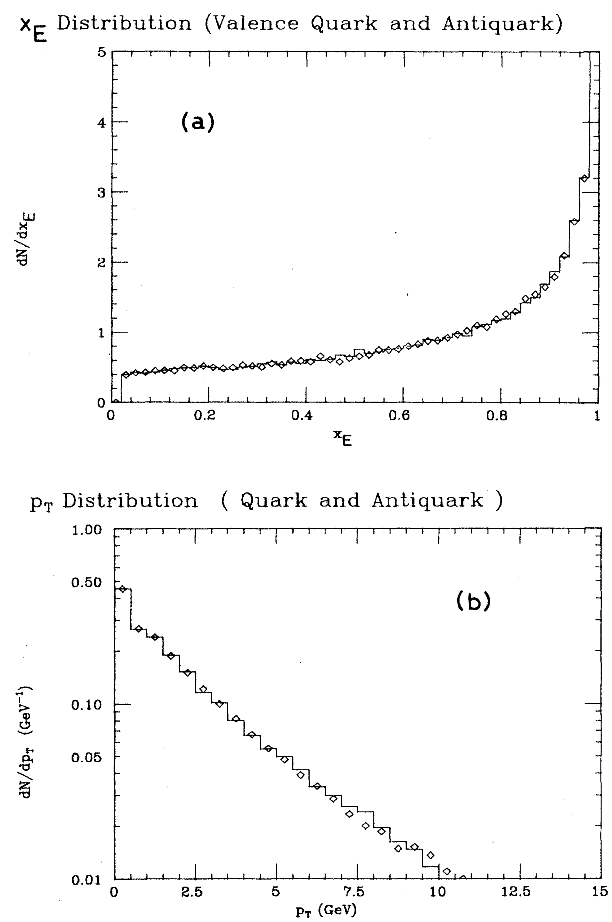

FIG. 3. The quark (solid line) and the antiquark (diamond) distributions. Results of the Monte Carlo simulation are obtained by the following parameters: $Q^2=(100\text{ GeV})^2$, $\Lambda_{\overline{\rm MS}}=0.2$ GeV ($\overline{\rm MS}$ denotes the modified minimal-subtraction scheme), $Q_0^2=1$ GeV^2, $N_F=4$, and $\delta=0.05$. (a) Energy distributions. Here $x_E=2E/\sqrt{Q^2}$. Here E is the energy of the valence quark or antiquark in the c.m. frame. (b) Distributions of transverse momentum. It is defined with respect to the thrust axis.

This scheme enables us to separate the quark and antiquark jets and to treat them symmetrically. We show the equivalence between the new scheme and the old one for the moment of the inclusive cross section and the probability of gluon emission from the valence quark and the valence antiquark. By the double-cascade scheme in the NLL model we solved the infrared problem in the hard cross section, which required us to tune a parameter in a previous work.[1]

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APPENDIX A

In this appendix we will show equivalence between the single-cascade and double-cascade schemes for the moment of the inclusive cross section for the valence quark. Here we confine ourselves to the leading behavior and neglect K^2 dependence of α_s. In the single-cascade scheme the moment is an exponential function of $I(N,Q^2)$:

$$M(N,Q^2)=\exp\left[\frac{4\alpha_s}{3\pi}I(N,Q^2)\right], \tag{A1}$$

$$I(N,Q^2)=\int\frac{dt}{t}\int\frac{dx}{x}[(1-x)^{N-1}-1]\theta(xt-\epsilon), \tag{A2}$$

where $\epsilon=Q_0^2/Q^2$. We would like to study its correspondence in the double-cascade scheme. First we will present two functions $I_1(N,Q^2)$ and $I_2(N,Q^2)$:

$$I_1(N,Q^2)=\int\frac{dt}{t}\int\frac{dx}{x}[(1-x)^{N-1}-1]\theta(xt-\epsilon)\theta(x-t), \tag{A3}$$

$$I_2(N,Q^2)=\int\frac{dt}{t}\int\frac{dx}{x}[(1-x)^{N-1}-1]\theta(xt-\epsilon)\theta(t-x). \tag{A4}$$

Of course,

$$I(N,Q^2)=I_1(N,Q^2)+I_2(N,Q^2). \tag{A5}$$

Next we will present explicit expressions of these functions, (A2), (A3), and (A4). For N not large,

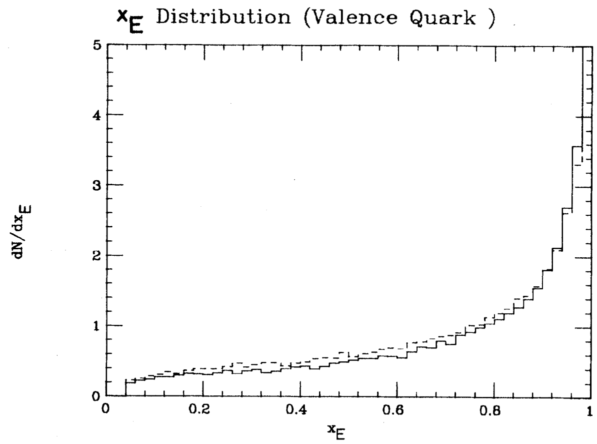

FIG. 4. Energy distributions for the valence quark in the single-cascade and double-cascade schemes. In the former scheme data are the same as that of Fig. 12 in Ref. 1, which is denoted by a dashed line. Data for the double-cascade scheme is shown by a line. Monte Carlo data are obtained by the following parameters: $Q^2=(100\text{ GeV})^2$, $\Lambda_{\overline{\rm MS}}=0.2$ GeV, $Q_0^2=4$ GeV^2, and $N_F=4$. $\delta=0.05$ for the double-cascade scheme, while $\delta=0.25$ for the single-cascade scheme. Also $x_E=2E/\sqrt{Q^2}$.

$$I(N,Q^2)\sim I_1(N,Q^2)=-\ln\left[\frac{1}{\epsilon}\right]\left[\sum_{K=1}^{N-1}\frac{1}{K}\right]+O(\epsilon) \tag{A6}$$

and

$$I_2(N,Q^2)=O(\epsilon). \tag{A7}$$

For large N,

$$I(N,Q^2)\sim-\ln N\ln\left[\frac{1}{\epsilon\sqrt{N}}\right]. \tag{A8}$$

On the other hand there are two cases for (A3) and (A4). In the first case ($N<1/\sqrt{\epsilon}$),

$$I_1(N,Q^2)\sim-\ln N\ln\left[\frac{1}{\epsilon N}\right], \tag{A9}$$

$$I_2(N,Q^2)\sim-\tfrac{1}{2}\ln^2 N. \tag{A10}$$

In this case I_2 has no Q^2 dependence, so that it does not contribute to the moment. In the second case ($N>1/\sqrt{\epsilon}$),

$$I_1(N,Q^2)\sim-\tfrac{1}{4}\ln^2\left[\frac{1}{\epsilon}\right], \tag{A11}$$

$$I_2(N,Q^2)\sim-\ln N\ln\left[\frac{1}{\epsilon\sqrt{N}}\right]+\tfrac{1}{4}\ln^2\left[\frac{1}{\epsilon}\right]. \tag{A12}$$

If in the double-cascade scheme the moment for the quark is given only by $I_1(N,Q^2)$, we could not obtain a correct one for large N. For this case one has to include effects due to the virtual mass squared K_q^2 of the antiquark that first decayed. From simple kinematical consideration the energy fraction x for the quark has changed into $x(1-K_q^2/Q^2)$. Then the moment $M_q(N,Q^2)$ is the product of the following parts:

$$M_q(N,Q^2)=M_1(N,Q^2)M_2(N,Q^2) , \tag{A13}$$

$$M_1(N,Q^2)=\exp\left[\frac{4\alpha_s}{3\pi}I_1(N,Q^2)\right] , \tag{A14}$$

$$M_2(N,Q^2)=\int_0^1 dt(1-t)^{N-1}\Pi_B(t) , \tag{A15}$$

where $M_2(N,Q^2)$ is calculated from the branching probability $\Pi_B(t)$:

$$\Pi_B(t)=\frac{d\Pi_{NB}(t)}{dt} ,$$
$$\Pi_{NB}(t)=\exp\left[-\frac{4\alpha_s}{3\pi}\int_t^1\frac{dt'}{t'}\int\frac{dx'}{x'}\theta(x't'-\epsilon)\theta(x'-t')\right] . \tag{A16}$$

This moment should correspond to $\exp[I_2(N,Q^2)]$. Actually, if one notices that $(1-t)^N$ can be approximated by $\theta(1/N-t)$ for large N, it is easy to check this correspondence.

As a result we show the equivalence between the single-cascade and the double-cascade schemes for the moment of valence quarks.

APPENDIX B

In this appendix we will give a difference between P functions in single-cascade and double-cascade schemes. First we present it in the LL order.

In the single-cascade scheme the cross section for three jets is given by

$$J_{\text{single}}=C\int_0^1\frac{dt}{t}\int_0^1\frac{dx}{x}[1+(1-x)^2]\theta(xt-\epsilon) , \tag{B1}$$

where $C=2\alpha_s/3\pi$. While in the double-cascade scheme we use

$$J_{\text{double}}=C\int_0^1\frac{dt}{t}\int_0^1\frac{dx}{x}[1+(1-x)^2]\theta(xt-\epsilon)\theta(x-t)+C\int_0^1\frac{dx}{x}\int_0^1\frac{dt}{t}[1+(1-t)^2]\theta(xt-\epsilon)\theta(t-x) . \tag{B2}$$

Then

$$\Delta J=J_{\text{single}}-J_{\text{double}}=-C\int dx\left[(2-x)\ln\left[\frac{1}{x}\right]-\frac{3}{2x}+2-\frac{x}{2}\right] . \tag{B3}$$

This is the NLL correction.

In the NLL model[1] using the double-cascade scheme we use a fixed-order cross section (9) for x and t that is larger than δ; that is,

$$J_3=C\int_\delta^1 dt\int_\delta^1 dx\frac{1}{t}\left[\frac{1+(1-x)^2}{x}+2t\frac{x(1-x)-1}{x}+t^2\frac{1+x^2}{x}\right] . \tag{B4}$$

On the other hand the cross section of the first cascade is

$$J'_{\text{double}}=C\int_0^\delta\frac{dt}{t}\int_0^1\frac{dx}{x}[1+(1-x)^2]\theta(xt-\epsilon)\theta(x-t)+C\int_0^\delta\frac{dx}{x}\int_0^1\frac{dt}{t}[1+(1-t)^2]\theta(xt-\epsilon)\theta(t-x) . \tag{B5}$$

Then the cross section for three jets is given by the sum of J_3 and J'_{double}. Therefore the difference between J_{single} in (B1) and this sum is presented here:

$$\Delta J'=J_{\text{single}}-(J_3+J'_{\text{double}}) . \tag{B6}$$

As a result we change ΔC in (A21) of Ref. 1 by

$$\Delta C_{\text{double}}=-\frac{\beta_0}{2}\left[C_2^{(1)}(x)+\frac{d\Delta\tilde{J}'}{dx}+3C_L^{(1)}\right] , \tag{B7}$$

where

$$\frac{d\Delta\tilde{J}'}{dx}=C_F\left[\frac{3}{2x}-2+\frac{3x}{2}+2\delta\left[1-x-\frac{1}{x}\right]+\frac{\delta^2}{2}\left[\frac{1}{x}+x\right]\right]\theta(x-\delta)+\left[(-2+x)\ln\frac{1}{x}+\frac{3}{2x}-2+\frac{x}{2}\right]\theta(\delta-x) . \tag{B8}$$

[1]K. Kato and T. Munehisa, Phys. Rev. D **36**, 61 (1987).
[2]K. Kato and T. Munehisa, Mod. Phys. Lett. **A1**, 345 (1986); T. Munehisa, in *Hadrons, Quarks and Gluons,* proceedings of XXII Rencontre de Moriond, Les Arcs, France, 1987, edited by J. Tran Thanh Van (Editions Frontiéres, Gif-sur-Yvette, 1987), p. 277; K. Kato and T. Munehisa, in *Perspectives of Particle Physics,* edited by R. Sasaki *et al.* (World Scientific, Singapore, 1988).
[3]R. Odorico, Nucl. Phys. **B172**, 157 (1980); P. Mazzanti and R. Odorico, Phys. Lett. **95B**, 133 (1980).
[4]G. C. Fox and S. Wolfram, Nucl. Phys. **B168**, 285 (1980); R. D. Field and S. Wolfram, *ibid.* **B213**, 65 (1983); T. D. Gottschalk, *ibid.* **B214**, 201 (1983); **B227**, 413 (1983); **B239**, 325 (1984); **B239**, 349 (1984); K. Kajantie and E. Pietarinen, Phys. Lett. **93B**, 269 (1980).
[5]G. Marchesini and B. R. Webber, Nucl. Phys. **B238**, 1 (1984); B. R. Webber, *ibid.* **B238**, 492 (1984).
[6]K. Kato and T. Munehisa, Yamanashi, Z. Phys. C **40**, 85 (1988).