Gravitational nucleation of vacuum phase transitions by compact objects

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The possibility of gravitationally condensed material objects (such as neutron stars, monopoles, etc.) acting as nucleation sites for the decay of the false vacuum is examined. The Einstein equations are solved to determine the classical motion of thin-wall bubbles of the true vacuum which form in or around spherically symmetric bodies of uniform-density perfect fluid. Certain decays of negative-energy-density false-vacuum states, which were previously considered to be forbidden, are shown to be locally possible in the neighborhood of a star. In these solutions, bubbles of the true vacuum form inside the star and then oscillate between two radii. The Euclidean action for bubbles which form at $R=3M$ around a uniform-density star is calculated and compared to the action for O(4)-symmetric decay and for decay nucleated by a black hole. The action in the presence of a star is found to be significantly smaller than the other cases. It thus appears that gravitationally condensed material objects in the appropriate mass and radius range can act as nucleation centers for bubbles of the true vacuum, greatly hastening the phase transition and making supercooling difficult or even impossible to achieve.

I. INTRODUCTION

The vacuum phase transitions associated with spontaneous symmetry breaking are central features of modern unified theories of elementary-particle interactions, from the electroweak unification in the standard model to grand unified theories (GUT's) and beyond. These phase transitions involve a downward change in the value of the vacuum energy density as the symmetries of the fundamental gauge group are spontaneously broken. It is now common to assume that the Universe, in the course of its expansion and cooling, has undergone a number of vacuum phase transitions. It is also possible that additional phase transitions will occur in the future: we may be living today in the "fool's paradise" of a long-lived metastable false-vacuum state.

The basic theory of the decay of the false vacuum was first developed by Voloshin, Kobzarev, and Okun,[1] and Coleman.[2] The role of gravitation in the decay process was first studied by Coleman and De Luccia[3] for the special cases of decay to or from zero vacuum energy density; Parke[4] then extended the analysis to arbitrary vacuum energy densities. All of these early analyses assumed O(4) symmetry for the Euclidean instanton. Since the discovery of the inflationary universe scenario,[5-8] it has been realized that the gravitational effects associated with vacuum decay might have important cosmological and astrophysical consequences. As a result, these simple O(4)-symmetric decays have now been thoroughly studied.

More recently, Lake and Wevrick,[9] Hiscock,[10] Blau, Guendelman and Guth,[11] and Berezin, Kuzmin, and Tkachev[12] have studied possible O(3)-symmetric motions of bubble walls in the context of general-relativity theory. In particular, Hiscock[10] considered the possibility that the curved spacetime of a Schwarzschild black hole could act as a nucleation center for the vacuum phase transition. A calculation of the Euclidean action for a spherically symmetric bubble forming around a black hole showed that the presence of a black hole would greatly enhance the vacuum decay rate. It thus appears to be much more difficult to achieve supercooling in the presence of appropriate mass black holes than it is in their absence.

In this paper, the study of gravitational nucleation of vacuum phase transitions is extended to consider gravitationally compact material objects as nucleation centers. The set of such objects includes neutron stars and white dwarfs, and in principle any material object which is significantly gravitationally bound [so that the spacetime differs significantly from (Euclidean) O(4) symmetry]. This could include objects such as main-sequence stars and planets, although here we will mainly be concerned with those objects which are significantly gravitationally bound. The discussion could also apply to the gravitational fields of heavy elementary particles, such as magnetic monopoles, acting as nucleation centers. A 10^{16}-GeV monopole, with an effective radius given by its Compton wavelength, has $m/R \sim 10^{-6}$, a degree of gravitational binding comparable to that of the Sun. For brevity, we will hereafter refer to all of these objects as "stars" or "stellar models," whether they are actually stars, planets, rocks, or elementary particles. We restrict our attention in this paper to studying the nucleation of spherically symmetric bubbles of true vacuum in and around uniform-density stellar models.

In general, the bulk equation of state of the matter in the star can be expected to undergo large changes when the vacuum undergoes a phase transition (e.g., massless particles may become massive). The star may then grossly change its structure, collapse to a black hole, or even evaporate, as a result of the phase transition. The change in the equation of state will depend strongly on the details of the particular particle-physics model being examined. In order to avoid being tied to a particular model, in this paper we examine the simplest possible case: we assume that the bulk equation of state is unchanged by the vacuum phase transition, i.e., that the fluid of which the star is composed is of the same uniform density, $\rho=\rho_0$, both before and after the phase transition. We also assume that the radius of the uniform-density star, R^*, is unchanged in the phase transition, so that the total fluid mass is unchanged by the phase transition. Whether there are any current particle-physics models for which our assumption is reasonable will be briefly discussed in Sec. V.

One of the interesting results of the earlier work of Coleman and De Luccia[13] and Parke[4] was their finding that an O(4)-symmetric decay of the false vacuum was not possible if the false-vacuum energy density was zero or negative and the difference in energy between the two vacua was small. Lake and Wevrick[9] and Hiscock[10] have shown that these "forbidden decays" remain forbidden in the presence of a black hole: although it is possible to nucleate a bubble of the true vacuum around a black hole for some decays which were forbidden in the O(4)-symmetric case, those bubbles always collapse into the black hole rather than expanding to infinity. In this study, we find that certain of the previously "forbidden" decays can occur by nucleating a bubble of the true vacuum inside a uniform-density star. As in the black-hole case, these bubbles cannot expand to infinity, but, unlike the black-hole case, they persist indefinitely into the future, oscillating in radius between a turning point inside the star and a turning point outside the star. It is also possible to adjust the fluid density and total mass so that such a bubble forms at the surface of the star, and is subsequently static. Thus it is possible, in the presence of a uniform-density stellar model, to form spherically symmetric, static (or radially oscillating) regions of the true vacuum, when the universal conversion of the false vacuum to the true is energetically forbidden. The solutions we find are, of course, classical, and hence may not be adequate to describe the behavior of the quantum fields near the radius at which the bubble forms; we feel, however, that the existence of these stationary points in the action will play an important role in the full quantum theory. In particular, such local condensations of the true vaccum may have important consequences in supersymmetric theories, where gravity breaks the degeneracy of the supersymmetric vacua.[13]

In the more globally interesting case where the bubble of true vacuum can expand without bound, we have calculated the Euclidean action for bubbles of true vacuum forming around uniform density stars. As in the black-hole case, we find that the decay rate is substantially greater than in the O(4)-symmetric case; the stars thus act as impurities in the false vacuum around which bubbles of the true vacuum are likely to nucleate. Stars seem to be considerably more effective than black holes at hastening the phase transition.

The paper is organized as follows. In Sec.II the extension of the interior Schwarzschild (uniform-density fluid) stellar model to the case of nonzero vacuum energy density is given, and its basic properties quickly reviewed. In Sec. III the solutions of the Einstein equations corresponding to a thin-wall bubble forming in or around a uniform-density stellar model are discussed. New solutions describing the (previously forbidden) local decay of one negative-energy-density vacuum state into another by the formation of an oscillating radius region of true vacuum are also described. In Sec. IV, the Euclidean action for a particularly simple set of stellar-nucleated bubble solutions is calculated. The bubble is assumed to form at $R=3M$ (where M is the mass of the stellar model, not including the vacuum energy contribution) in the Schwarzschild (possibly de Sitter or anti–de Sitter) exterior of the star (this was the minimum-action solution for the Schwarzschild black-hole case previously examined[10]). The Euclidean action is then calculated for a set of models, with stellar radius ranging from $R^*=3M$ downward, and for vacuum energies corresponding to decay to or from zero vacuum energy density. Finally, the possible application of these results to vacuum phase transitions in the real Universe is discussed in Sec. V.

Natural units ($G=c=\hbar=1$) are used throughout the paper. Sign conventions follow those of Misner, Thorne, and Wheeler.[14]

II. UNIFORM-DENSITY STELLAR MODELS WITH NONZERO VACUUM ENERGY DENSITY

The spherically symmetric static metric representing the interior of a star of uniform density is the well-known interior Schwarzschild metric.[15,16] Most treatments of the interior Schwarzschild metric, however, assume that the interior is joined (at the radius at which the pressure is zero) to an exterior metric which has vanishing vacuum energy density: namely, the exterior Schwarzschild metric. In the present circumstance, it is necessary to consider the case in which the uniform-density stellar model is immersed in an exterior universe which has nonvanishing vacuum energy density. In this case the appropriate exterior metric is either the Schwarzschild–de Sitter or Schwarzschild–anti–de Sitter metric, depending on whether the vacuum energy density is positive or negative, respectively. The interior and exterior metrics are joined in this case at the radius at which the pressure in the interior solution is equal to the negative of the exterior vacuum energy density (which is the exterior vacuum pressure). If the spacetime metric is put into the standard static, spherically symmetric form

$$ds^2=-e^{2\nu(r)}dt^2+\left[1-\frac{2m(r)}{r}\right]^{-1}dr^2+r^2d\Omega^2 , \tag{1}$$

where $d\Omega^2$ is the metric of a two-sphere, then the metric functions $\nu(r)$ and $m(r)$ are defined by[17]

$$m(r)=4\pi(\rho_0+\rho_v)r^3/3 , \tag{2}$$

$$e^{\nu(r)}=\frac{3[1-2m(R^*)/R^*]^{1/2}-(1-2\rho_v/\rho_0)[1-2m(r)/r]^{1/2}}{2(1+\rho_v/\rho_0)}\ , \tag{3}$$

where ρ_0 is the (uniform) fluid density, ρ_v is the vacuum energy density, and R^* is the radius of the stellar model. The pressure as a function of radius is

$$p(r)=(\rho_0+\rho_v)\frac{(1-2\rho_v/\rho_0)[1-2m(r)/r]^{1/2}-[1-2m(R^*)/R^*]^{1/2}}{3[1-2m(R^*)/R^*]^{1/2}-(1-2\rho_v/\rho_0)[1-2m(r)/r]^{1/2}}\ . \tag{4}$$

At the surface of the stellar model, $r=R^*$ and the pressure is equal to the exterior vacuum pressure $-\rho_v$ (i.e., the fluid component of the pressure is zero there). Equation (4) may be used to relate the radius of the stellar model to the central pressure p_c of the fluid sphere:

$$R^{*2}=\frac{3}{8\pi\rho}\{1-(1-2\rho_v/\rho_0)^2[(\rho+p_c)/(\rho+3p_c)]^2\}\ , \tag{5}$$

where $\rho=\rho_v+\rho_0$. Given values of ρ_0 and ρ_v, a specific stellar model is singled out by choosing a value for p_c; R^* is then found from Eq. (5). Alternatively, one could choose a value for R^*, however, although there exist stellar models for all values of $p_c>-\rho_v$, there do not exist stellar models for all values of R^*, since for some values of R^*, the pressure will diverge at a nonzero value of r inside the star (e.g., for any $R^*<9M/4$ when $\rho_v=0$). Finally, the total mass of the fluid, irrespective of the vacuum energy density, is

$$M=4\pi\rho_0 R^{*3}/3\ . \tag{6}$$

III. BUBBLE-WALL MOTIONS

In the thin-wall approximation, the motion of the bubble wall is treated using the general-relativistic equations of motion for thin shells of matter.[18] Spherically symmetric bubbles can form inside the star, at its surface, and in the exterior of the star. The motion of a spherically symmetric thin-wall bubble moving in a spherically symmetric fashion in or around a star can be determined in the usual fashion by first matching the intrinsic metrics induced on the bubble wall, and then relating the discontinuity in the extrinsic curvature tensor to the surface stress energy of the wall. The resulting form of the Einstein equation is quite similar to results previously obtained in the black-hole case:[9–12]

$$\left[1-\frac{2m_1(R)}{R}+\left[\frac{dR}{d\tau}\right]^2\right]^{1/2}-\left[1-\frac{2m_2(R)}{R}+\left[\frac{dR}{d\tau}\right]^2\right]^{1/2}=4\pi\sigma R\ , \tag{7}$$

where R is the radius of the bubble, τ is the proper time along the bubble wall, σ is the surface energy density of the bubble wall (which will be taken to be constant), and $m_1(R)$ and $m_2(R)$ are the masses inside and outside the bubble wall, respectively:

$$m_1(R)=\begin{cases}4\pi(\rho_0+\rho_1)R^3/3, & R<R^*,\\ M+4\pi\rho_1 R^3/3, & R>R^*,\end{cases} \tag{8}$$

$$m_2(R)=\begin{cases}4\pi(\rho_0+\rho_2)R^3/3, & R<R^*,\\ M+4\pi\rho_2 R^3/3, & R>R^*,\end{cases} \tag{9}$$

where ρ_1 is the true-vacuum energy density (inside the bubble) and ρ_2 is the false-vacuum energy density (outside the bubble).

The solution represented by Eqs. (7)–(9) is not completely general. First, the matching conditions on the timelike bubble-wall hypersurface do not require that the mass parameter M in Eq. (8) equal the mass parameter M in Eq. (9). Exterior solutions in which they are not equal have been studied previously in Refs. 9, 11, and 12. In this paper, we will restrict our attention to what we consider to be the simplest possible case consistent with our assumptions that ρ_0 and R^* are constant through the phase transition: namely, that the masses M in Eqs. (8) and (9) are equal in magnitude. Second, the choice of the signs of the two square-root terms on the left-hand side of Eq. (7) is somewhat arbitrary. The signs of the square roots are determined by the metric matching conditions;[18] the sign of each square root depends on whether the radial coordinate (defined in terms of the area of the invariant two-spheres) increases or decreases as one moves away from the bubble wall. The choice of signs we have made in Eq. (7) corresponds to having the area of two-spheres decrease as one moves away from the bubble wall into the interior true-vacuum (ρ_1) region and increase as one moves away from the wall into the false-vacuum exterior region (ρ_2). While this is the simplest assumption to make, it is not the only possible choice, at least in the case where at least one of ρ_1 and ρ_2 are positive. For instance, in the case where $M=0$, since the spatial topology of de Sitter space is S^3, it is possible to consider more exotic matchings such as one where, on a spacelike slice through the bubble wall at the stationary point where $dR/d\tau=0$, the radius of two-spheres initially *increases* in the true-vacuum interior, yet the three-volume of the interior is finite (this corresponds to placing the bubble wall at a polar angle $>\pi/2$ on the interior de Sitter spatial three-sphere, so that more than half of the three-sphere is joined to the exterior).

Although Eq. (7) describes the classical motion of the bubble wall for all times, only a portion of the classical solution is relevant to the problem of vacuum decay. In a first-order phase transition, bubbles of new phase appear via quantum tunneling at a turning-point radius of the classical solution. Their late time expansion is asymptotically described by the classical equation of motion. Whether the more "exotic" matchings [involving other choices of signs in Eq. (7) or differing values of M in the interior and exterior] can occur in nature is at present unresolved: while they are perfectly acceptable solutions of the Einstein equations for the timelike evolution of the bubble wall, the question of their relevance to actual vacuum decay hinges on understanding in detail the *formation* (via quantum tunneling) of such a bubble, which is intrinsically quantum in nature and probably not amenable to a treatment within classical general relativity. It should perhaps be emphasized that our choices are not known to be any more physical than the other possibilities (so long as the details of the quantum tunneling process which forms the bubble remain unknown); we have simply chosen what we feel are the simplest and most conservative assumptions: that the mass parameters M in Eqs. (8) and (9) are the same, and that the signs of the square roots are as given in Eq. (7), so that the sense of increasing radius would be the same before and after the phase transitions.

Equation (7) may now be squared twice in order to solve for $(dR/d\tau)^2$. The result is

$$\left[\frac{dR}{d\tau}\right]^2=\beta^2R^2-1+\frac{2M}{R} \tag{10}$$

outside the star and

$$\left[\frac{dR}{d\tau}\right]^2=\alpha^2R^2-1 \tag{11}$$

inside the star, where

$$\beta^2=\left[\frac{\rho_2-\rho_1-6\pi\sigma^2}{3\sigma}\right]^2+\frac{8\pi\rho_2}{3}\ , \tag{12}$$

and

$$\alpha^2=\beta^2+8\pi\rho_0/3\ . \tag{13}$$

In examining solutions of Eqs. (10) and (11), one must be cautious to check that the solutions satisfy the original Eq. (7), since additional roots are introduced by squaring. Some of these additional roots corrrespond to the alternative sign choices in front of the two square roots on the left-hand side of Eq. (7), which we will not consider further here. Others correspond to bubble-wall motions for which the proper energy density of the bubble is negative (and hence, unphysical, as will be discussed below) and still others to odd solutions in which the area of two-spheres locally increase in all (or decrease in all) spacelike directions off the bubble wall. These additional solutions will not be further studied here.

The free parameters that may be varied to describe distinct bubble solutions are ρ_0, ρ_1, ρ_2, R^*, and σ (with ρ_0 and R^* determining M). However, not all values of these parameters will yield acceptable physical solutions to Eqs. (10) and (11) [for the given assumptions in Eqs. (7)–(9)]. The necessary restrictions on these parameters are discussed in the next four paragraphs. A description of the nature of the acceptable solutions then follows.

Standard models of spontaneous symmetry breaking require that σ, the bubble-wall energy density, be positive, as it arises from the derivative terms in the Higgs field's stress energy, which have definite sign. Furthermore, since the radial coordinate of the bubble wall is restricted to non-negative values, it is clear that not all values of ρ_1 and ρ_2 will be compatible with real solutions for $dR/d\tau$ in Eq. (7). In fact, examination of Eqs. (7), (10), and (11) shows that there are no solutions with $\sigma R\geq 0$ unless

$$\rho_2\geq\rho_1+6\pi\sigma^2\ , \tag{14}$$

exactly as in the black-hole case.[10]

Equation (11) requires $\alpha^2>0$ for any bubble solution which has support inside the star, since $(dR/d\tau)^2$ must be non-negative. This constraint may be rewritten as an inequality bounding ρ_1:

$$\rho_1<\rho_2-6\pi\sigma^2-[-24\pi\sigma^2(\rho_0+\rho_2)]^{1/2} \tag{15}$$

for $\rho_0+\rho_2\leq 0$. For the case $\rho_0+\rho_2>0$, α^2 is always positive. The motion of the bubble wall outside the star is governed by Eq. (10); acceptable solutions can exist for either positive or nonpositive values of β^2, with the details of each case given below. For now it is useful to note that when $\rho_2<0$, the condition $\beta^2>0$ corresponds to

$$\rho_1<\rho_2-6\pi\sigma^2-(-24\pi\sigma^2\rho_2)^{1/2}\ . \tag{16}$$

Next, possible bubble solution parameters are limited by two physical constraints on the stellar structure. The first is the requirement that the pressure of the fluid monotonically decrease outward from the center of the star. Using the Tolman-Oppenheimer-Volkoff equation it can be shown[17] that this implies $\rho_0\geq 2\rho_v$ (assuming that $\rho_0>0$, i.e., that there is a star present). As $\rho_0\to 2\rho_v$, the fluid component of the pressure vanishes, and the "star" becomes a portion of the Einstein universe, in which pressureless dust is held in (unstable) equilibrium by the opposing forces of gravitational attraction and the repulsion caused by the nonzero vacuum energy density. A uniform density fluid sphere with $\rho_0<2\rho_v$ and non-negative fluid pressures would lack sufficient mass to hold itself together against the repulsive force associated with the vacuum energy density. Since $\rho_2>\rho_1$, taking ρ_v as ρ_2 gives the greater lower bound and so solutions in the present study must be restricted to

$$\rho_0\geq 2\rho_2\ . \tag{17}$$

Since realistic stars (particles, etc.) have a positive energy density this bound is of no consequence if $\rho_2\leq 0$. The second physical constraint is that the central pressure of the fluid must be finite. For a uniform-density fluid sphere, Eq. (4) can be used to show that this imposes a limit on M/R^* given by

$$M/R^*<2(2\rho_0-\rho_v)/9\rho_0\ . \tag{18}$$

In the case $\rho_v=0$ this reduces to the familiar $M/R^*<\frac{4}{9}$ limit of the interior Schwarzschild solution.[16] The bound in Eq. (18) is always smaller than the horizon limit for the fluid sphere. Replacing ρ_v with ρ_2 gives the least upper bound in Eq. (18); rewriting Eq. (18) as a bound on R^*, we restrict consideration to solutions which satisfy[19]

$$R^* < R_{max} = [(2\rho_0 - \rho_2)/6\pi\rho_0^2]^{1/2} \ . \tag{19}$$

Finally, consider the case where a bubble of the true vacuum forms with an initial radius less than R^*. The radius at which the bubble forms, R_0, is trivially found by setting $(dR/d\tau)^2$ equal to zero in Eq. (11):

$$R_0 = \alpha^{-1} \ . \tag{20}$$

Unless $R_0 \le R_{max}$, the solution given by Eq. (20) is obviously not physically realizable. Combining Eq. (19) for R_{max} and Eq. (20) for R_0 gives the following constraint on the energy densities:

$$\rho_1 < \rho_2 - 6\pi\sigma^2 - [6\pi\sigma^2(\rho_0 - 2\rho_2)^2/(2\rho_0 - \rho_2)]^{1/2} \ . \tag{21}$$

The value of ρ_0 is restricted by Eq. (17) such that the square-root term in Eq. (21) is always real. For a given choice of energy densities, if Eq. (21) is satisfied it is then impossible for bubbles to form in stars with radii ranging from R_0 (the bubble forms at the surface of the star) to just less than R_{max}. If Eq. (21) is violated for given values of ρ_0, ρ_1, and ρ_2, it is then impossible to nucleate a bubble inside a star since its radius would be required to exceed the limit imposed by Eq. (19); however, it should be noted that violation of Eq. (21) does not preclude the possibility of bubbles appearing outside a star with $R^* < R_{max}$.

The different classes of solutions to Eqs. (10) and (11) are now discussed, subject to the constraints of Eqs. (14)–(21). It can be shown that Eq. (15) is inconsequential since it is always satisfied if Eq. (21) is. The limit curves of the different constraints embodied in Eqs. (14)–(17) and (21) are illustrated in Fig. 1 for a typical value of ρ_0. These limit curves divide the ρ_1,ρ_2 plane into a number of regions. Each region corresponds to a different class of solutions to Eqs. (10) and (11). First the manner in which the boundary curves (and hence the regions) depend on the choice of ρ_0 is explained, followed by a discussion of the nature of the bubble solutions in each region, based on Eqs. (10) and (11). The qualitative nature of the bubble solutions in each region may be easily determined by sketching $(dR/d\tau)^2$ versus R [using Eqs. (10) and (11)] for each case.

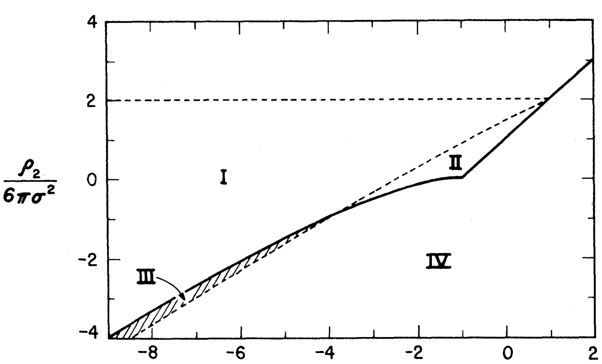

FIG. 1. Possible false-vacuum decays by the formation of thin-wall bubbles in or around uniform-density fluid spheres. The horizontal and vertical axes are the true- and false-vacuum energy densities, respectively, rendered dimensionless by dividing by $6\pi\sigma^2$. The dashed curves are functions of the value of ρ_0, the fluid density. For the case shown, $\rho_0 = 24\pi\sigma^2$. The solid curves represent decay constraints which are independent of the value of ρ_0 (these are the same as in the black-hole or O(4)-symmetric cases). In regions I and II bubbles can form and expand to complete the phase transition. In region III bubbles can form and oscillate between two radii; decay of the false vacuum is locally possible in this region, although it appeared to be forbidden in previous studies. In region IV decay of the false vacuum is still impossible.

The dashed line boundaries in Fig. 1 depend on the choice of the free parameter ρ_0 ($=24\pi\sigma^2$ in the case shown). If we imagine varying ρ_0 continuously, these dashed curves will move as will their intercepts with the solid line boundaries, hence, changing the size and shape of the labeled regions. To understand the results, note that the intercepts move away from the origin as ρ_0 increases. Thus points above region I (where ρ_0 is less than $2\rho_2$ and thus the chosen density star could not exist in the false-vacuum state) would lie in regions I and II for larger values of ρ_0. Similarly, any point shown in region I would be found in region II for sufficiently large values of ρ_0, and, conversely, any point in region II may be moved into region I by decreasing ρ_0. In general all points above the solid curves lie in regions I or II for some choice of ρ_0. Any point which can be found in region III for one value of ρ_0 can be found in region IV for a larger value of ρ_0; however, the converse is not true in this case: not all points in region IV can be moved into region III by decreasing ρ_0. Many points lie in region IV for all values of ρ_0. Last, regions II and III disappear as $\rho_0 \to 0$.

Region I. In this region it is always possible for a bubble to form in the presence of a star and subsequently expand to complete the phase transition. For $\rho_1 + 6\pi\sigma^2 \ll \rho_2$ the bubble forms inside or at the surface of the star and expands. Near the boundary to regions II and III bubbles which appear within the star will oscillate between a minimum radius less than R^* and a maximum radius greater than R^*; the radius of the star can also be chosen such that a static bubble in stable equilibrium forms at the star's surface. In this case there is a second turning point in the exterior geometry at which a bubble exterior to both the star and any oscillating or static bubble can form and then expand.

Region II. In this region bubbles cannot form inside the star, as R_0, given by Eq. (20), is greater than R_{max}. However, it is always possible in this region for a bubble to form with an initial radius greater than R^*, and then expand to complete the phase transition.

Region III. This is perhaps the most interesting region, since in the O(4)-symmetric case (no black hole or star to nucleate the phase transition) it is impossible to complete the phase transition for values of ρ_1 and ρ_2 in this region.[3,4,13] In the presence of a black hole, while it is possible to form a bubble of the true vacuum in this area,[9,10] such bubbles always subsequently collapse into the black hole and thus do not persist. In this study, however, with a star replacing the black hole, it is possible to form bubbles of the true vacuum which subsequently oscillate between turning points in the interior and exterior of the star. Static bubbles in stable equilibrium can also form at the surface of a star, for fortuitous values of R^*. However, unlike region I, in this case no expanding bubble can form outside the star to complete the phase transition. Thus, although these bubbles do not expand to convert the Universe in the large to the true vacuum, they still represent a new phenomenon: the possibility of the false vacuum decaying in a local spatially bounded region, which persists indefinitely into the future, for values of the false- and true-vacuum energy densities which previously had been thought to absolutely prohibit vacuum decay. This still holds on the boundary between this region and region I. On the boundary between regions III and IV only the unphysical solution $R_0=R^*=R_{\max}$ is possible.

These new solutions which represent previously "forbidden" decays occur only for a negative-energy-density false-vacuum state decaying into another negative-energy-density false-vacuum state. They are thus not relevant to modeling the early Universe, nor to considering the stability of our present vacuum. They may, however, play a significant role in the analysis of the vacuum states in supersymmetric theories, for which anti–de Sitter space is the natural vacuum.[13] In a more realistic model, the oscillations in radius of the bubble would be damped by particle creation effects, through gravitational coupling[20] if not through some stronger interaction. It then seems likely that ultimately the bubble would settle down to a static, motionless state at the surface of the star, as this is the only location at which there is no radial acceleration ($d^2R/d\tau^2$) of the bubble wall.

These oscillating solutions are, of course, classical, and hence may not be appropriate to describe the behavior of the quantum fields at the distances comparable to the nucleation size of the bubble. However, it seems likely that the existence of these stationary points in the action will play an important role in the full quantum theory and that it is possible to form real (as opposed to virtual) locally persistent bubbles of true vacuum in this manner.

Region IV. In this region no bubbles can form in the presence of a star (nor in its absence, except for a limited part of this region where a collapsing bubble can form outside a black hole, which is of no lasting consequence).

In summary, it is useful to point out that a solution always exists in which a bubble nucleates in the presence of a star and expands to complete the phase transition if either of the following two conditions are met: (1) $\rho_2 \geq 0$ and Eq. (14) is satisfied or (2) $\rho_2 < 0$ and Eq. (16) is satisfied. These conditions are the same as in the black-hole case.[10] If either condition is violated the decay in the presence of a star is forbidden except in the limited area covered by region III (the shape of which changes as the fluid density is allowed to vary from zero to infinity). Even so, in this last case only oscillating or static bubbles form and, while of interest because they persist indefinitely, they do not expand without bound, and thus cannot complete the phase transition.

IV. BUBBLE NUCLEATION RATES

In this section we will calculate the vacuum decay rate in the case where a bubble of true vacuum forms around a star. In the zero-temperature limit, the rate of formulation of bubbles of the true vacuum has the form

$$\Gamma = A \exp(-B/\hbar)[1+O(\hbar)] , \tag{22}$$

where Γ is the rate of bubble formation per unit volume and A is a coefficient with dimensions of $(\text{length})^{-4}$. The quantity B is the change in the Euclidean action caused by the presence of the bubble, i.e.,

$$B = S_E(\Phi) - S_E(\Phi_+) , \tag{23}$$

where $S_E(\Phi)$ represents the Euclidean action for a bubble solution in which the Higgs field Φ varies between its true-vacuum value at $r=0$ and the false-vacuum value (Φ_+) at infinity, and $S_E(\Phi_+)$ represents the Euclidean action of a purely false-vacuum spacetime.

The determination of A necessarily involves specifying a particular field theory; we will therefore not consider its determination further in this paper. The Euclidean actions involved in the definition of B, however, can be calculated in the thin-wall case once the values of the false- and true-vacuum energy densities and the bubble-wall energy density are specified, without any restriction as to the particle-physics model. In this section we will calculate B for a class of solutions in which a bubble forms around a star and compare the values of B with the equivalent values in the absence of the star.

A general solution in which a bubble of the true vacuum forms around a star and subsequently expands is described by giving the values of the five parameters: ρ_0, ρ_1, ρ_2, σ, and R^*. Since it is difficult to explore the values of B over an entire five-dimensional parameter space and still more difficult to present the results in a coherent fashion, we will restrict our attention here to a particular subset of vacuum decays which previous work has indicated might be of special interest. We will only examine the decays in which the star has a radius R^* less than or equal to $3M$, and in which the star's fluid mass M is equal to the critical mass defined in Ref. 10:

$$M = M_c = 3^{-3/2}\beta^{-1} . \tag{24}$$

From this point on, the mass M will be taken to be equal to the critical mass M_c defined in Eq. (24). Since the star is assumed to have a radius less than or equal to $3M$, a bubble will form in the exterior Schwarzschild metric and its subsequent motion will be defined by Eq. (10). When the Schwarzschild mass is given by Eq. (24), the bubble forms at $R=3M$ and remains there in unstable equilibrium (with this choice of M and with $R^* < 3M$, there is also a second turning point within the star at which a bubble can form; we will not calculate the action of this second solution here). While this particular solution does not expand to infinity, it is the limit of a set of solutions in which the bubbles form at radii greater than $3M$ and then expand to convert the Universe to the true-vacuum state. We have chosen to focus attention on this special case because in the case of a central black hole, the

$R=3M$ solution has the lowest Euclidean action, and thus yields the fastest decay rate for the false vacuum. In addition, it is particularly simple to calculate the Euclidean action for this solution, since the bubble remains outside the star at all times, in either the Lorentzian or Euclidean sectors (in the Euclidean sector the bubble is in stable equilibrium at $R=3M$).

As in the case of a central black hole, it is convenient to divide the Euclidean action calculation up into three separate pieces: the action for the bubble interior $S_E^{\rm int}(\Phi)$, the action for the bubble wall $S_E^{\rm wall}$, and the action for the interior in the false vacuum $S_E^{\rm int}(\Phi_+)$. The exponential decay coefficient B is then

$$B=S_E^{\rm int}(\Phi)+S_E^{\rm wall}-S_E^{\rm int}(\Phi_+) \ . \qquad (25)$$

The contribution of the bubble wall to the Euclidean action is exactly the same as in the central black-hole case; thus,[10]

$$S_E^{\rm wall}=-108\pi^2\sigma M_c^3 \ . \qquad (26)$$

In order to calculate the actions in the interior of the bubble, which includes the interior of the star, it is necessary to know the form of the action for a perfect-fluid solution of the Einstein equations. A number of different variational principles for relativistic perfect fluids have been proposed; here we shall take the action to be given by the integral of the Lagrangian density proposed by Schutz[21] (appropriately recast for a Euclidean signature metric):

$$S_E=\int\left[-p-\frac{R}{16\pi}\right]dV=\int\tfrac{1}{2}(p-\rho)dV \ , \qquad (27)$$

where the second equality follows from the identity $R=8\pi(\rho-3p)$ for a perfect-fluid solution of the Einstein equations. This action has the desirable property that it reduces to the usual result for a perfect fluid with the equation of state of the vacuum, $p_v=-\rho_v$ [see, e.g., Eq. (18) of Ref. 10]. The volume element in (27) is the Euclidean four-volume of the interior of the bubble.

The integration over the Euclidean time coordinate is limited to one period of oscillation of the bubble wall; for the $R=3M$ bubble, this period is[10] $t_0=6\pi(3)^{1/2}M_c\,(1-72\pi\rho_v M_c^2)^{-1/2}$. This period is found by considering solutions with $M=M_c-\epsilon$, where $\epsilon \ll M_c$. Such solutions will, in the Euclidean sector, oscillate in radius about $r=3M$. The period appropriate for the $M=M_c$, $R=3M$ bubble is then found by taking the limit of the period as $\epsilon\to 0$.

The integration over r is divided into two pieces: the interior of the star, for which $0\le r\le R^*$, where $\rho=\rho_0+\rho_v$, and p is given by Eq. (4); and the exterior of the star, $R^*\le r\le 3M$, where $\rho=\rho_v$ and $p=-\rho_v$. The integrals to determine $S_E^{\rm int}(\Phi)$ and $S_E^{\rm int}(\Phi_+)$ may then be performed using Eqs. (27) and (4): if the total energy density in the star is positive, $\rho=\rho_0+\rho_v>0$, then the interior action is

$$S_E^{\rm int}=\frac{12(3)^{1/2}\pi^2 M}{(1-72\pi\rho_v M^2)^{1/2}}\left\{\frac{3\rho_0 R^*}{8\pi\rho}-\tfrac{2}{3}\rho_0 R^{*3}-18\rho_v M^3-\rho_0\left[\frac{3}{8\pi\rho}\right]^{3/2}\left[1-\frac{8\pi\rho R^{*2}}{3}\right]^{1/2}\arcsin\left[\left[\frac{8\pi\rho}{3}\right]^{1/2}R^*\right]\right\} . \qquad (28)$$

If the total energy density (ρ) inside the star is negative, then the interior action is given by an expression which is identical to Eq. (28), except that the last term is replaced by

$$+\rho_0\left[\frac{-3}{8\pi\rho}\right]^{3/2}\left[1-\frac{8\pi\rho R^{*2}}{3}\right]^{1/2}\times\text{arcsinh}\left[\left[\frac{-8\pi\rho}{3}\right]^{1/2}R^*\right] . \qquad (29)$$

The exponential decay coefficient B can now be evaluated by using Eq. (28) with $\rho_v=\rho_1$ to calculate $S_E^{\rm int}(\Phi)$ and with $\rho_v=\rho_2$ to calculate $S_E^{\rm int}(\Phi_+)$, and then combining these results with Eqs. (26) and (25). Note that the variable M ($=M_c$) in Eq. (28) is a function of ρ_1 and ρ_2, defined by Eq. (24). The resulting expression for B applies to any decay centered on a uniform-density star whose radius is less than $3M$, and for which the stellar mass is given by the critical mass of Eq. (24). The coefficient B can then be calculated for this sort of decay for any values of ρ_1,ρ_2,σ, and for values of R^* lying between $3M$ and the minimum radius possible[17] (at which the central pressure diverges) for the given value of ρ_2.

We now restrict our attention to two special cases: first, decay of a positive-energy-density false vacuum ($\rho_2>0$) to the zero-energy true vacuum we occupy today ($\rho_1=0$); second, decay of the present day zero-energy vacuum state ($\rho_2=0$) to a negative-energy-density true vacuum ($\rho_1<0$). In both cases, we will discuss the value of the dimensionless ratio B/B_1, where B_1 is the value of the decay coefficient for the O(4)-symmetric decay calculated by Coleman and De Luccia:[3]

$$B_1=\frac{27\pi^2\sigma^4}{\rho_2(\rho_2+6\pi\sigma^2)^2}\quad(\rho_1=0) \ , \qquad (30)$$

$$B_1=\frac{-27\pi^2\sigma^4}{\rho_1(\rho_1+6\pi\sigma^2)^2}\quad(\rho_2=0) \ . \qquad (31)$$

The ratio B/B_1 is dimensionless and independent of $\hbar$; we take it to be a function of the dimensionless combinations $\rho_1/6\pi\sigma^2$, $\rho_2/6\pi\sigma^2$, and R^*/M, where M is given by Eq. (24). The value of the fluid density ρ_0 is then given by Eq. (6) using the known value of R^* and M. Assuming that the coefficient A is not a strongly varying function of the spacetime geometry, if B/B_1 is greater than one, then the bubble nucleation rate [determined by Eq. (22)] is smaller around a star than in empty space. If B/B_1 is less than one, then the rate is larger, and in this case the star acts as an effective nucleation center for the vacuum phase transition.

In the case of the decay of a positive-energy-density vacuum state into the present zero-energy vacuum, the ratio B/B_1 is plotted as a function of R^*/M for a variety of values of $\rho_2/6\pi\sigma^2$ in Fig. 2. The central pressure of the star increases along the curves as R^*/M is decreased, until the curves terminate (at differing values of R^*/M, dependent on the value of $\rho_2/6\pi\sigma^2$) at the radius where the central pressure diverges. The value of B/B_1 is seen to be substantially less than 1 for all values of R^*/M, indicating that the decay of the vacuum is substantially hastened by the presence of a compact star with the critical mass M_c. Comparing the values of B/B_1 obtained here with the values where the star is replaced by a central black hole,[10] we see that B/B_1 is always less in the case of a central star, for all allowable values of R^*/M. It thus appears that stars, if they exist in the appropriate mass and radius range, will hasten the vacuum phase transition more than the same mass black hole.

The ratio B/B_1 is seen to decrease as the central star becomes more compact and relativistic, i.e., as R^*/M decreases. For fixed values of R^*/M, the ratio is also seen to decrease as ρ_2 is decreased. This variation is of the same sort as the dependence of B/B_1 on ρ_2 in the black-hole case.[10]

Remarkably, for large values of the false-vacuum energy density, and very compact stars, the ratio B/B_1 is seen to change sign and become negative. This indicates that the Euclidean action for the solution with the bubble present is actually less than the action for the pure false-vacuum state. While we cannot take the values of B/B_1 seriously in this case as quantative indicators of the vacuum decay rate, the change in the sign of B clearly shows that the presence of an appropriate mass and radius star will greatly hasten a vacuum phase transition, and will effectively preclude the possibility of supercooling occurring. In the equivalent tunneling calculation in quantum mechanics, the difference in the Euclidean actions (B) vanishes only if the potential barrier vanishes; this suggests that the presence of a star effectively changes the phase transition from being first order to second order for a bubble nucleated around the star. Of course, for a specified model field theory, it may well be that there do not exist stars (or other gravitationally condensed objects) possessing the appropriate masses and radii.

In the second case, our present zero-energy vacuum state is assumed to be a false vacuum, doomed to ultimately decay into a negative-energy density state. The ratio B/B_1 is plotted for a number of choices of true-vacuum energy density, $\rho_1/6\pi\sigma^2$, in Fig. 3, again as a function of the size of the star, R^*/M. In this case, the curves all extend from $R^*/M=2.25$ to $R^*/M=3$. This is because it is the false-vacuum energy density which determines the minimum radius star possible. In this case, the false energy density is zero for all cases, and thus the minimum stellar radius, where the central pressure diverges, is given by $9M/4$, the classic result of general relativity. Again we see that the ratio B/B_1 is always substantially less than 1 in magnitude, and, again is also substantially less than in the case of a central black hole.[10] We thus conclude that compact, relativistic stars within the correct mass and radius range could act as very effective nucleation sites for forming bubbles of a negative-energy-density true vacuum.

In this case, as in the former case, the value of B/B_1 decreases as R^*/M decreases. For fixed R^*/M, B/B_1 decreases as ρ_1 decreases (recall that ρ_1 is now negative, and so decreasing ρ_1 increases the difference in the vacuum energy densities); this is again the same sense of variation as the black-hole case exhibits.[10]

Also as in the former case, for some values of ρ_1 and R^*/M, the ratio B/B_1 takes on negative values. Again, this indicates that the Euclidean action of the bubble solution is actually less than that of the pure false-vacuum state. This implies that stars in this mass and radius range would cause the vacuum to decay immediate-

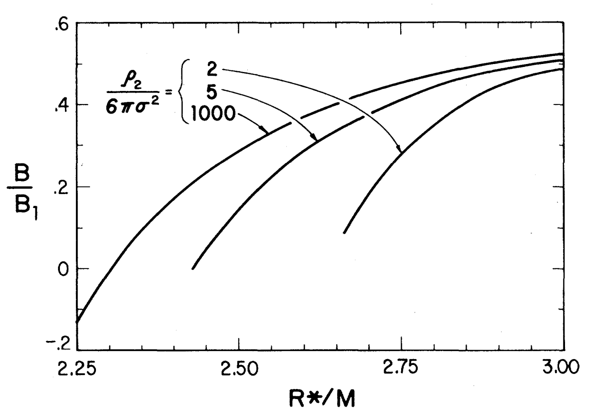

FIG. 2. The ratio B/B_1 for the decay of a positive-energy-density false vacuum into a zero-energy-density true vacuum for the static $R_0=3M_c$ bubble as a function of the stellar radius, R^*/M, for several values of the false-vacuum energy density.

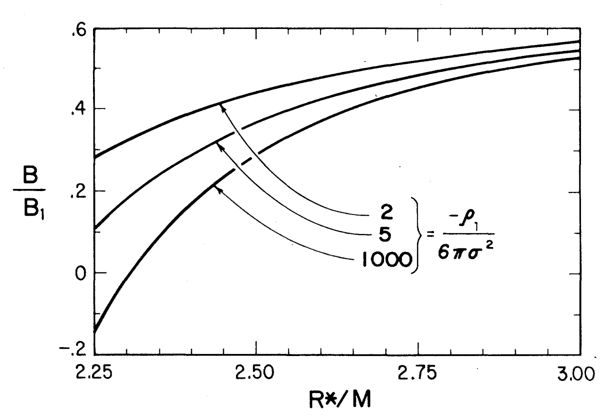

FIG. 3. The ratio B/B_1 for the decay of zero-energy-density false vacuum into a negative-energy-density true vacuum for the static $R_0=3M_c$ bubble as a function of the stellar radius, R^*/M, for several values of the true-vacuum energy density.

ly, and again thus preclude the possibility for supercooling occurring.

Finally, let us consider why the decay coefficients B are smaller (and hence, lead to more rapid decay of the false vacuum) in the case of a central star than in the case of a central black hole. There are two differences between the two cases. First, in the black-hole case, the integral for the Euclidean action interior to the bubble is cut off at the radius of the event horizon, while in the case of the star, the integral extends down to $r=0$. This increase in the interior volume of the bubble tends to decrease the value of the interior action (because the integrand is negative). Second, in the case of the star, there is the positive pressure of the fluid which contributes to the action as described in Eq. (27). This tends to increase the value of the interior action. Of course, both interior actions $S_E^{\rm int}(\Phi)$ and $S_E^{\rm int}(\Phi+)$ are affected; however, evidently, the net effect on B is to substantially decrease it from the value in the black-hole case.

V. DISCUSSION

There are two potential problems in applying these results to realistic vacuum phase transitions. First, the properties of matter are changed in essential ways (e.g., by generating particle masses) in the vacuum phase transitions whose existence and properties we are most confident in our knowledge of (such as in the Weinberg-Salam-Glashow model). The model examined in this paper assumes that the bulk equation of state of the fluid composing the "star" is unchanged by the vacuum phase transition. To the best of our knowledge, the only proposed scheme which could make this realizable is the existence of a "shadow" world, as is proposed in some superstring theories,[22] in which there exist a set of matter fields which interact with ordinary matter *only* via the gravitational interaction. If such "shadow matter" could exist, then the situation examined in this paper could occur either through the existence of "shadow" stars which nucleate an "ordinary" phase transition, or through "ordinary" matter nucleating a phase transition in the "shadow" vacuum state. For other sorts of vacuum phase transitions, in which the bulk equation of state of matter does change significantly, our results can be taken only as an indication that gravitational nucleation effects may be very important in determining the course of the phase transition. Better calculations would necessarily involve restricting attention to the details of a particular model.

The second problem is that the only gravitationally condensed objects in the real Universe which we are currently aware of are immense when compared to microphysical length scales; e.g., neutron stars have radii of some tens of kilometers. Smaller objects, such as rocks or individual nuclei do not possess significant gravitational fields. Despite their large size, it is conceivable that objects as large as neutron stars could nucleate very strongly first-order phase transitions, in which, say, the lifetime of the false vacuum [estimated from the O(4)-symmetric decay] is greater than 10×10^9 yr. If in fact our own vacuum state is false, then our existence tells us that it is extremely long lived; a bubble of the true vacuum formed in or around macroscopic objects such as neutron stars might very well be the avenue by which the false vacuum would begin its decay.

It is also possible that there exist microscopic topologically stable field configurations which are significantly gravitationally bound. The ratio M/R for an elementary particle is of order $M^2/\hbar$; if grand unification occurs near the Planck mass, then monopoles may be significantly gravitationally bound (indeed, if too near the Planck mass the lowest-mass monopole solution will be a black hole). In this case their gravitational field might serve to nucleate a vacuum phase transition.

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