Dynamical quark-polarization cloud around static quark sources

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First results for the behavior of the virtual quark-antiquark fluctuations around a static-quark charge are presented within lattice QCD. We propose to use the correlation $\langle L(0)\bar{\psi}\psi(r)\rangle$ between a static quark and the fermion condensate as a measure for the quark vacuum occupation number density. We predict a decrease of the quark-polarization cloud in the vicinity of a static quark in agreement with antiscreening effects known from perturbation theory.

I. INTRODUCTION

The great success of the formulation of QCD on space-time lattices was to show that the pure gluon exchange between a static quark and an antiquark yields confinement. This could be proven analytically in a strong-coupling expansion[1] and checked numerically up to the weak-coupling regime. Meanwhile, precise calculations on large lattices allow us to resolve the gluon string between a quark-antiquark pair for separations of more than 1 fm containing an excitation energy of more than 500 MeV (Ref. 2).

Consequently, the question about the influence of the dynamical quark vacuum was posed. Despite the various conceptual and technical difficulties in treating fermions on the lattice, it could be demonstrated during the last couple of years that the creation of virtual quark-antiquark pairs leads to a gradual loss of the string tension.[3,4] One finds a screened potential between a static quark and antiquark both for the Kogut-Susskind and the Wilson definition of lattice fermions. The responsible mathematical mechanism is the explicit breaking of a global Z_3 symmetry through the fermionic action. In order to get more insight into the physical mechanism, one should "measure" the polarization cloud around a quark charge.

The problem of defining an appropriate observable for the dynamical quark-polarization cloud is treated in Sec. II. We suggest using the local chiral condensate $\bar{\psi}\psi(r)$ which represents the fermionic vacuum fluctuations. We work with the finite-temperature formalism using the Polyakov loop to describe a static quark.

To study the quark-antiquark distribution around a static-quark charge we have to compute the correlation function $\langle L(0)\bar{\psi}\psi(r)\rangle$. In Sec. III we present results from a simulation of the full QCD vacuum with dynamical quarks in the Kogut-Susskind prescription taking the fermionic determinant into account by the pseudofermionic method. A comparison with a consistent quenched computation is made. We try to consider the mathematical mechanism within the hopping-parameter expansion. We discuss the physical picture as a remnant of antiscreening being characteristic for the non-Abelian nature of QCD.

The conclusion is drawn in Sec. IV. There we also briefly give some information about intended studies.

II. THEORY

Our goal is to calculate the distribution of virtual quarks and antiquarks in the vicinity of static colored quark sources. Therefore, the main problem for our topic is the choice of a suitable operator corresponding to fermionic pair creation. We suggest studying the fermionic condensate $\bar{\psi}\psi(r)$ around static quarks. As a pointlike propagator, it may be interpreted as the fluctuation of a quark-antiquark pair existing for an infinitely short duration of time. This quantity represents the vacuum density and is related to the sum of particle occupation number plus antiparticle occupation number of the QCD vacuum in the low-momentum limit.[5,6] Another observable of interest would be the charge density $\psi^\dagger\psi$ which counts the particle number minus the antiparticle number of the considered state. In the case of the free Dirac theory, this is a conserved quantity. Since $\psi^\dagger\psi=\bar{\psi}\gamma_0\psi$ is more difficult to be resolved for staggered fermions, we prefer to study $\bar{\psi}\psi$ around static quarks in this first investigation.

The static quark is described by the Polyakov loop $L(r)$. It represents the propagator in the direct time direction for a heavy quark at position r and satisfies the static Dirac equation. Dynamical quarks act as external fields and break the Z_3 symmetry explicitly. $\langle L\rangle$ is still a measure of the quark free energy but it is nonzero for all temperatures T due to vacuum polarization. If antiperiodic boundary conditions are imposed for the fermions, the associated external "magnetic" field will always point in the direction for which $\mathrm{Re}L>0$ (Ref. 7). According to Satz and co-workers[8] $\langle L\rangle\sim\exp(-m_H/2T)$ where m_H is the mass of the lowest quark-antiquark state.

In order to determine the distribution of virtual quarks around a static source we compute the correlation between the static quark $L(0)$ and the local chiral condensate $\bar{\psi}\psi(r)$. The expectation value of the correlation function $L(0)\bar{\psi}\psi(r)$ can be written as a path integral:

$$\langle L(0)\bar{\psi}\psi(r)\rangle=\frac{\int D[U,\bar{\psi},\psi]L(0)\bar{\psi}\psi(r)e^{-(S_G+S_F)}}{\int D[U,\bar{\psi},\psi]e^{-(S_G+S_F)}} . \quad (1)$$

The functional integration extends over all degrees of freedom of the gauge field U and of the fermion fields $\psi,\bar{\psi}$. The total action of the system consists of the gluonic action S_G being the usual plaquette action on the lattice and of the fermionic action S_F for which we choose the Kogut-Susskind discretization[9]

$$\begin{aligned} S_F=\tfrac{1}{4}n_f &\left[\sum_{x,\mu}\tfrac{1}{2}\Gamma_{x,\mu}(\bar{\psi}_x U_{x\mu}^{\dagger}\psi_{x+\hat{\mu}}-\bar{\psi}_{x+\hat{\mu}}U_{x\mu}\psi_x)\right.\\ &\left.+m\sum_x \bar{\psi}_x\psi_x\right]\\ =&\sum_{x,x'}\bar{\psi}_x[D(U)+m]_{xx'}\psi_{x'} . \end{aligned} \quad (2)$$

Here n_f denotes the number of flavors and m the bare mass of the quark field. The $\Gamma_{x,\mu}$ play the role of Dirac matrices and the factor $\frac{1}{4}$ takes the fermion doubling into account. Equation (1) can be integrated analytically over $\bar{\psi}_x$ and ψ_x applying the Matthews-Salam formula:

$$\langle L(0)\bar{\psi}\psi(r)\rangle=\frac{n_f}{4}\frac{\int D[U]L(0)[D(U)+m]_{rr}^{-1}\exp\left[-\left[S_G-\frac{n_f}{4}\mathrm{Tr}\ln(D+m)\right]\right]}{\int D[U]\exp\left[-\left[S_G-\frac{n_f}{4}\mathrm{Tr}\ln(D+m)\right]\right]} , \quad (3)$$

where $\bar{\psi}\psi(r)$ is replaced by the inverse fermion matrix $(D+m)_{rr}^{-1}$ and only the path integration over $U_{x\mu}$ remains. The fermionic determinant $\det(D+m)$ is taken into account by the pseudofermionic method.[9]

III. RESULTS AND DISCUSSION

Now let us turn to the numerical results for $\langle L(0)\bar{\psi}\psi(r)\rangle$. The system was simulated on a $8^3\times 4$ lattice with inverse gluon coupling $\beta=5.2$ in the confinement regime and flavor number $n_f=3$. The problem was treated for virtual quark masses $m=0.1$, 0.2, 0.3, 0.4, and 1.0. We performed 1000 Metropolis Monte Carlo (MC) iterations to equilibrate the pure gauge field. Then we appended a few hundred MC interactions using the pseudofermionic method with a heat-bath algorithm with 50 steps to equilibrate the full QCD vacuum. Subsequently, data were taken over 300 iterations per quark mass.

The correlation function $\langle L(0)\bar{\psi}\psi(r)\rangle$ is displayed in Fig. 1 for the various quark masses. We find the remarkable result that the correlations increase with increasing distance r. This means that polarization effects are suppressed in the near surrounding of a quark. This seems to be the opposite effect to QED regarding fermionic vacuum polarization. The horizontal line in Fig. 1 is the cluster value $\langle L\rangle\langle\bar{\psi}\psi\rangle$ approached for $r\rightarrow\infty$ which is normalized to one. We see that the correlations converge to the cluster value at $r\gtrsim 3$. Beyond this distance the two operators do not feel each other and the quark vacuum takes its undisturbed value driven by chiral-symmetry breaking. From Fig. 1 we recognize that the correlations become flatter with increasing quark mass. This indicates that the heavier the quark sea, the less it is influenced by an external charge.

In order to be sure that the described effect actually originates in the virtual quark cloud we made quenched calculations with the same set of parameters. Switching off the fermionic determinant in Eq. (3) we obtain the correlation function $\langle L(0)\bar{\psi}\psi(r)\rangle$ shown in Fig. 2. We observe a completely different picture than in Fig. 1. One clearly sees that the correlations stay numerically constant for all distances. Since in the quenched approximation the cluster value $\langle L\rangle\langle\bar{\psi}\psi\rangle$ is zero we conclude that $\langle L(0)\bar{\psi}\psi(r)\rangle$ vanishes. This symbolizes that there are no polarization effects around an external charge in the quenched case.

At this point we want to try to achieve some analytical explanation for the computed effect. It seems remarkable that increasing correlations can be resolved in a numerical simulation. For this purpose, we simply write the fermion condensate $\bar{\psi}\psi(r)$ in the hopping-parameter expansion[8,10]

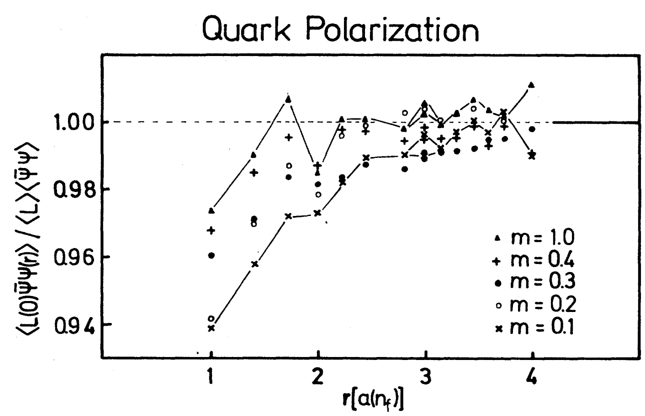

FIG. 1. Correlation $\langle L(0)\bar{\psi}\psi(r)\rangle$ between the Polyakov loop $L(0)$ of a static quark and the local fermion condensate $\bar{\psi}\psi(r)$ as a measure for quark vacuum fluctuations (see text). The distance r is given in units of the lattice constant a. The system was simulated for gluon coupling $\beta=5.2$ and bare quark masses $m=0.1$, 0.2, 0.3, 0.4, and 1.0 lying in the confinement regime. The asymptotic line is expected from the cluster theorem and is normalized to one. The envelope was drawn to guide the eye. The error bars are of the size of the spatial anisotropy effects.

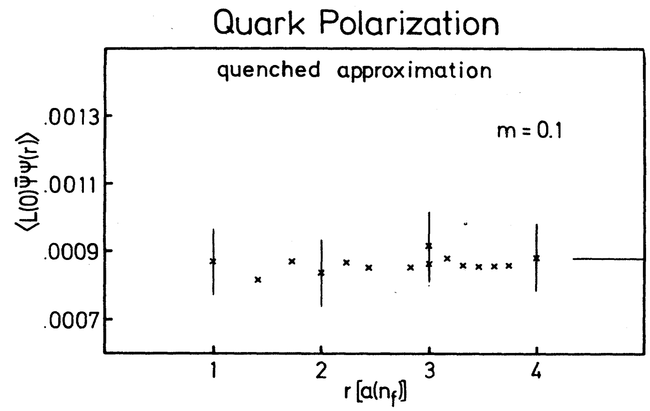

FIG. 2. Correlations $\langle L(0)\bar{\psi}\psi(r)\rangle$ between the Polyakov loop $L(0)$ and the local fermion condensate $\bar{\psi}\psi(r)$. The system is studied in the quenched approximation for $\beta=5.2$ and $m=0.1$. No polarization effects can be resolved in contrast to Fig. 1. A few error bars are inserted.

$$\bar{\psi}\psi(r)=\frac{1}{m}\left[a-\frac{b}{m^{N_t}}\mathrm{Re}L(r)-\frac{c}{m^4}\mathrm{Re}\Box(r)+O(m^{-6})\right], \tag{4}$$

which is a series over all closed loops weighted with the inverse mass according to their lengths. The leading terms of this expansion consist for temporal extension $N_t \leq 4$ of the Polyakov loop L and of the plaquette $\Box$ (Ref. 11). It is important to note that these terms contribute with the same (negative) sign to series (4). Now we insert the hopping-parameter expansion into the correlation function (1):

$$\langle L(0)\bar{\psi}\psi(r)\rangle=\frac{1}{m}\left[a\langle L\rangle-\frac{b}{m^{N_t}}\langle L(0)\mathrm{Re}L(r)\rangle-\frac{c}{m^4}\langle L(0)\mathrm{Re}\Box(r)\rangle\right]. \tag{5}$$

The correlations between Polyakov loops show an exponential decrease to the cluster value and the same behavior is expected for correlations between a Polyakov loop and a plaquette.[4] Being aware of the minus signs in Eq. (5) we can estimate the shape of $\langle L(0)\bar{\psi}\psi(r)\rangle$ as a function of the distance r: *It will increase with increasing distance until it saturates to a constant value corresponding to the cluster theorem.* That is what has been observed in Fig. 1.

It has to be noted that the same functional form holds true for the hopping-parameter expansion (5) in the quenched approximation where one has to correlate Polyakov loops and plaquettes coming from quenched gauge field configurations. The main difference to full QCD vacuum is that the Polyakov loop $\langle L\rangle$ is zero. Thus the correlations $\langle L(0)\bar{\psi}\psi(r)\rangle$ have to vanish at larger r. Reflecting a density expectation value they cannot become negative and must be zero for finite r. As a matter of fact we found numerically in Fig. 2 that the correlations practically vanish for all distances.

Let us now turn to the physical explanation of the underlying result. The main question is why are the quark-antiquark fluctuations suppressed near a quark source? Inserting one or more quark sources into the empty QCD vacuum the system tries to become locally colorless by creation of virtual gluon strings. This is in analogy to forcing a hypothetical magnetic monopole into a superconductor where magnetic flux tubes are produced ending at an antimonopole or leaving the system at the boundary.[12] The creation of massless gluons is favored in comparison to quark-antiquark pairs having a finite mass. These gluons carry the color charge away from the static quark. Because of their charge they are themselves sources of further gluons amplifying the central charge. When the production of such gluon strings requires too much energy they end in virtual antiquark-quark pairs. A more detailed analysis should exhibit that quark creation is suppressed in comparison to antiquark creation in the immediate neighborhood of a static charge. The distance of maximum pair creation is identical with the screening length found in hadronization studies of static quark-antiquark systems.[4] In the quenched approximation, we find no effect since due to the omission of the fermionic determinant there are no virtual quarks around. The abundance of virtual gluons around quark sources is known from perturbation theory and renormalization-group equation as antiscreening being a characteristic of QCD quite different from QED. It is the antiscreening effect of the color-charged gluons which gives rise to the asymptotic freedom and the running coupling constant.[13] The gluonic antiscreening mechanism seems to survive from the regime of high momentum transfer to nucleonic distances.

IV. CONCLUSION

To conclude, we have demonstrated that the vacuum fluctuations decrease in the vicinity of an external quark source compared with the fluctuations of the dynamical quark sea in empty space. This mechanism confirms the basic concept of the MIT bag model.[14] There one assumes that the "true" QCD vacuum is partially destroyed inside the bag volume by the presence of quarks carrying color. The bag can be treated by methods of perturbation theory and is often called "perturbative" vacuum. Furthermore, we would like to note that the fermion condensate is the order parameter for chiral-symmetry breaking. Our results demonstrate that chiral-symmetry breaking is a local effect. In the vicinity of a color source chiral symmetry is partially restored.

In the near future, we plan to perform a corresponding full computation with Wilson fermions. Since Wilson fermions break chiral symmetry explicitly, one has to deal with the problem of subtracting the corresponding contribution from the chiral condensate.[15] As a matter of fact, the chiral condensate embraces both quark fluctuations plus antiquark fluctuations. Thus, it would be very desirable to separate the vacuum pair fluctuations into the occupation number densities due to virtual quark charges and antiquark charges, respectively. For this purpose one has to study the charge density $\psi^\dagger\psi$ in addition to $\bar{\psi}\psi$. Moreover, a complete measurement of the chromoelectric field and of the quark sea in the environment of a static quark-antiquark system is a great challenge.[16] Further, a study of polarization phenomena in lattice QED could bring more understanding of the different mechanisms of screening and antiscreening.

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