Analytical studies on single W- and Z-boson production in electron-positron collisions

Mituaki Katuya
Laboratory of Computer Science, University of Shizuoka, Yada 395, Shizuoka 422, Japan
(Received 12 July 1988)

Analytical formulas for the differential and total cross sections of single W-boson and Z-boson production in electron-positron collisions are given by using the equivalent-photon approximation.

I. INTRODUCTION

The standard $SU(2)\times U(1)$ gauge theory of the weak and electromagnetic interactions[1-3] is in excellent agreement with all the existing neutral-current data as well as the observed properties[4,5] of the W and Z bosons. As in the $\bar{p}p$ collider experiments where the discoveries of the W and Z bosons have been made, they will be observed also in electron-positron collider experiments, where more precise data about them will be supplied.

We are going to discuss electron-positron collider experiments at higher, TeV energies, which will probably be available by using a linear collider. In TeV energies the s-channel photon and Z-boson contributions will become small, and the contributions from the t- and u-channel photon exchanges will become dominant. Extensive studies on the production of such heavy particles as the W and Z bosons through the t- and u-channel photon exchanges will hence become necessary for the future electron-positron collider physics at TeV energies, because these heavy bosons have many decay channels and will become important backgrounds to be explored.

In this paper we study the single W-boson and Z-boson production processes:

$$e^+e^-\to e^+W^-\nu_e(W^+\bar{\nu}_e e^-)\ , \tag{1.1}$$

$$e^+e^-\to e^+e^-Z\ . \tag{1.2}$$

We give analytical formulas for the cross sections of these processes which are invariant under the Lorentz transformations along the beam direction. These formulas are derived at the tree level within the real-photon approximation.[6-8]

The real-photon approximation allows us to express the cross-section formulas for processes (1.1) and (1.2) by estimating the contributions from the subprocesses

$$\gamma e^-(\gamma e^+)\to W^-\nu_e(W^+\bar{\nu}_e)\ , \tag{1.3}$$

$$\gamma e^-(\gamma e^+)\to Ze^-(Ze^+)\ . \tag{1.4}$$

Since processes (1.3) and (1.4) are apparently quite similar to the Compton scattering, $\gamma e^-\to\gamma e^-$, we will call these processes SU(2) Compton scattering in this paper. Process (1.3), and, hence, process (1.1) contain a three-gauge-boson $WW\gamma$ coupling at the tree level whereas processes (1.4) and (1.2) have not. The three-point vector-boson coupling is characteristic of non-Abelian gauge theories, therefore, the comparison of the two SU(2) Compton processes (1.3) and (1.4) should be interesting.

This paper is organized as follows. In Sec. II our notations are explained. The SU(2) Compton cross sections are studied in detail in Sec. III and the analytical formulas for processes (1.1) and (1.2) are obtained in Sec. IV. Two Appendixes which may be useful in deriving the formulas in Secs. III and IV are complemented. A summary is given in Sec. V.

II. NOTATIONS

In this paper we shall use the following data and notations:

$$m_e=0.511\times10^{-3}\ \text{GeV (electron mass)}\ , \tag{2.1}$$

$$M_W=81.8\ \text{GeV}\ (W\text{-boson mass})\ , \tag{2.2}$$

$$M_Z=92.6\ \text{GeV}\ (Z\text{-boson mass})\ , \tag{2.3}$$

$$\alpha=e^2/4\pi\ , \tag{2.4}$$

$$\beta=M_Z/\sqrt{M_Z^2-M_W^2}\ , \tag{2.5}$$

$$\xi_V=\tfrac{1}{2},\quad \xi_A=-\tfrac{1}{2}\ , \tag{2.6}$$

$$\zeta_V=\tfrac{1}{2}-2/\beta^2,\quad \zeta_A=-\tfrac{1}{2}\ , \tag{2.7}$$

$$x_W=s/M_W^2\ , \tag{2.8}$$

$$x_Z=s/M_Z^2\ , \tag{2.9}$$

where s is the square of the center-of-mass energy for processes (1.1) and (1.2). The four-momenta of the particles appearing in processes (1.3) and (1.4) are defined as in Fig. 1. As for the momenta W and Z of the W and Z bosons, respectively, we define the following:

$$W_T(Z_T):\ \text{transverse momentum of } W(Z) \text{ boson}\ , \tag{2.10}$$

$$W_L(Z_L):\ \text{longitudinal momentum of } W(Z) \text{ boson}\ , \tag{2.11}$$

$$T_W=\frac{M_W^2+W_T^2}{M_W^2},\quad T_Z=\frac{M_Z^2+Z_T^2}{M_Z^2}\ . \tag{2.12}$$

Corresponding to the variables in (2.8) and (2.9), the invariant variables for processes (1.3) and (1.4) are defined as

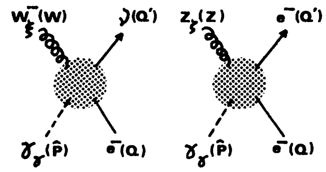

FIG. 1. Definition of the four-momenta. The momenta of photon, electron, weak boson, Z boson, final electron, and neutrino are shown in parentheses of their corresponding particles. Indices ξ, ζ, and γ denote the polarization vectors of the W boson, Z boson, and photon, respectively.

$$\hat{x}_W=\hat{s}/M_W^2 \ , \tag{2.13}$$

$$\hat{x}_Z=\hat{s}/M_Z^2 \ , \tag{2.14}$$

where $\hat{s}$ is the square of the c.m. system for processes (1.3) and (1.4),

$$\hat{s}=(\hat{P}+Q)^2 \ . \tag{2.15}$$

Moreover, we shall use the notation

$$\hat{u}=(\hat{P}-Q')^2 \ , \tag{2.16}$$

$$\hat{t}=(Q-Q')^2 \tag{2.17}$$

for the invariant variables and

$$V_{abc}(A,B,C)=(A-B)_c g_{ab}+(B-C)_a g_{bc} +(C-A)_b g_{ca} \ , \tag{2.18}$$

for the vertex function of the three-point gauge fields.

III. SU(2) COMPTON SCATTERING

The amplitudes for the SU(2) Compton scatterings (1.3) and (1.4) are written as

$$T_W^{\xi\gamma}=\frac{\beta e^2}{\sqrt{2}}\left[\bar{u}(Q')\gamma^\xi(\xi_V+\xi_A\gamma_5)\frac{1}{\hat{\not{P}}+\not{Q}}\gamma^\gamma u(Q)-\bar{u}(Q')\gamma_\alpha(\xi_V+\xi_A\gamma_5)u(Q)\frac{1}{t-M_W^2}V^{\gamma\xi\alpha}(\hat{P},-W,W-\hat{P})\right] \ , \tag{3.1}$$

$$T_A^{\zeta\gamma}=\frac{\beta e^2}{2}\left[\frac{M_Z}{M_W}\right]\left[\bar{u}(Q')\gamma^\zeta(\zeta_V+\zeta_A\gamma_5)\frac{1}{\hat{\not{P}}+\not{Q}}\gamma^\gamma u(Q)+\bar{u}(Q')\gamma^\gamma\frac{1}{\not{Q}-\not{Z}-m_e}\gamma^\zeta(\zeta_V+\zeta_A\gamma_5)u(Q)\right] \ , \tag{3.2}$$

where the gauge-boson wave functions are truncated.

The following two points which discriminate between these two processes should be noticed. (1) The three-point vertex of the gauge fields contributes only to the W-production process (1.3), even though we call the W- and Z-production processes in one lump as the SU(2) Compton scattering. (2) Most of the electron mass can be neglected for the SU(2) Compton processes, because the electron mass is small enough in comparison with the weak-boson mass. But the electron mass in the pole term $1/(\not{Q}-\not{Z}-m_e)$ in Eq. (3.2) cannot be neglected because this term diverges in the backward scattering if the electron mass is neglected. Thus some electron masses must be retained carefully when the SU(2) Compton Z production is studied.

Since the electromagnetic U(1) symmetry still holds exactly, we can see the exact gauge invariance for these two amplitudes (3.1) and (3.2) as

$$\hat{P}_\gamma T_W^{\xi\gamma}=\hat{P}_\gamma T_Z^{\zeta\gamma}=0 \ , \tag{3.3}$$

if the equation of motion for the spinors and the on-shell condition for the massive gauge field are used.

The squared amplitudes for the SU(2) Compton scatterings averaged over initial and summed over final spins are

$$\frac{1}{4}\sum_{\text{pol}}|T_W^{\xi\gamma}|^2=e^4\beta^2(\xi_V^2+\xi_A^2)\hat{x}_W f_W\left[\frac{\hat{t}-M_W^2}{\hat{s}},\hat{x}_W\right] \tag{3.4}$$

and

$$\frac{1}{4}\sum_{\text{pol}}|T_Z^{\zeta\gamma}|^2=\frac{1}{2}e^4\beta^2(\zeta_V^2+\zeta_A^2)\hat{x}_W f_Z\left[\frac{\hat{u}-m_e^2}{\hat{s}-m_e^2},\hat{x}_Z\right] \ , \tag{3.5}$$

where

$$f_W\left[\frac{\hat{t}-M_W^2}{\hat{s}},\hat{x}_W\right]=\left[\frac{\hat{t}-M_W^2}{\hat{s}}\right](\hat{x}_W^{-1})+(3\hat{x}_W^{-1}+2\hat{x}_W^{-2})+\left[\frac{\hat{t}-M_W^2}{\hat{s}}\right]^{-1}(4\hat{x}_W^{-1}+2\hat{x}_W^{-2}+2\hat{x}_W^{-3}) + \left[\frac{\hat{t}-M_W^2}{\hat{s}}\right]^{-2}(2\hat{x}_W^{-1}+2\hat{x}_W^{-3}) \tag{3.6}$$

and

$$f_Z\left[\frac{\hat{u}-m_e^2}{\hat{s}-m_e^2},x_Z\right]=\left[\frac{\hat{u}-m_e^2}{\hat{s}-m_e^2}\right](-\hat{x}_Z^{-1})+(2\hat{x}_Z^{-2})+\left[\frac{\hat{u}-m_e^2}{\hat{s}-m_e^2}\right]^{-1}(-\hat{x}_Z^{-1}+2\hat{x}_Z^{-2}-2\hat{x}_Z^{-3}) \ . \tag{3.7}$$

As shown in Appendix A, the transverse-momentum distribution for the SU(2) Compton scatterings which are invariant under the Lorentz transformations along the beam direction are given as

$$\frac{d\hat{\sigma}_W}{dT_W}=\pi\alpha^2\beta^2(\xi_V^2+\xi_A^2)\frac{1}{M_W^2}g_W(T_W,\hat{x}_W) \tag{3.8}$$

and

$$\frac{d\hat{\sigma}_Z}{dT_Z}=\frac{1}{2}\pi\alpha^2\beta^2(\xi_V^2+\xi_A^2)\frac{1}{M_W^2}g_Z(T_Z,\hat{x}_Z)\ , \tag{3.9}$$

where

$$\begin{aligned} g_W(T_W,\hat{x}_W)=\frac{1}{\xi_L}\Bigg[&(5\hat{x}_W^{-1}+3\hat{x}_W^{-2})+\left[\frac{1}{\hat{x}_W+1+\xi_L}+\frac{1}{\hat{x}_W+1-\xi_L}\right](-8-4\hat{x}_W^{-1}-4\hat{x}_W^{-2})\\ &+\left[\frac{1}{(\hat{x}_W+1+\xi_L)^2}+\frac{1}{(\hat{x}_W+1-\xi_L)^2}\right](8\hat{x}_W+8\hat{x}_W^{-1})\Bigg]\ , \end{aligned} \tag{3.10}$$

$$\xi_L=\sqrt{(\hat{x}_W+1)^2-4T_W\hat{x}_W}\ , \tag{3.11}$$

$$g_Z(T_Z,\hat{x}_Z)=\frac{1}{\zeta_L}\left[(\hat{x}_Z^{-1}+3\hat{x}_Z^{-2})+\left[\frac{1}{\hat{x}_Z-1+\epsilon+\zeta_L}+\frac{1}{\hat{x}_Z-1+\epsilon-\zeta_L}\right](2-4\hat{x}_Z^{-1}+4\hat{x}_Z^{-2})\right]\ , \tag{3.12}$$

$$\zeta_L=\sqrt{(\hat{x}_Z+1-\epsilon)^2-4T_Z\hat{x}_Z}\ , \tag{3.13}$$

and

$$\epsilon=m_e^2/M_Z^2\ . \tag{3.14}$$

It should be noted that the distributions for the W- and Z-boson productions have remarkable differences in their behavior at $T_W=T_Z=1$. This can be seen as

$$g_W(T_W=1,\hat{x}_W)=2-2\hat{x}_W^{-1}+\hat{x}_W^{-2} \tag{3.15}$$

and

$$g_Z(T_Z=1,\hat{x}_Z)=\frac{1}{\epsilon}(\hat{x}_Z^{-1}-2\hat{x}_Z^{-2}+2\hat{x}_Z^{-3})+\cdots\ . \tag{3.16}$$

We can see the very large cross section at $T_Z=1$ which comes from the electron pole $1/(\not{Q}-\not{Z}-m_e)$.

Finally, the total cross sections are given as

$$\hat{\sigma}_W=\pi\alpha^2\beta^2(\xi_V^2+\xi_A^2)\frac{1}{M_W^2}h_W(\hat{x}_W) \tag{3.17}$$

and

$$\hat{\sigma}_Z=\tfrac{1}{2}\pi\alpha^2\beta^2(\xi_V^2+\xi_A^2)\frac{1}{M_W^2}h_Z(\hat{x}_Z)\ , \tag{3.18}$$

where

$$h_W(\hat{x}_W)=(1-\hat{x}_W^{-1})(2+\tfrac{5}{2}\hat{x}_W^{-1}+\tfrac{7}{2}\hat{x}_W^{-2})-\frac{\ln\hat{x}_W}{\hat{x}_W}(4+2\hat{x}_W^{-1}+2\hat{x}_W^{-2})\ , \tag{3.19}$$

$$h_Z(\hat{x}_Z)=\zeta_0\hat{x}_Z^{-1}(\tfrac{1}{2}\hat{x}_Z^{-1}+\tfrac{3}{2}\hat{x}_Z^{-2})+\ln\frac{\hat{x}_Z-1+\epsilon+\zeta_0}{\hat{x}_Z-1+\epsilon-\zeta_0}(\hat{x}_Z^{-1}-2\hat{x}_Z^{-2}+2\hat{x}_Z^{-3})\ , \tag{3.20}$$

and

$$\zeta_0=\sqrt{(\hat{x}_Z-1-\epsilon)^2-4\epsilon}\ . \tag{3.21}$$

The factor ζ_0 vanishes at the threshold energy $\hat{s}=(M_Z+m_e)^2$. It is useful in practice to express $h_Z(\hat{x}_Z)$ at $(\hat{x}_Z-1)\gg\epsilon$ as[9]

$$h_Z(\hat{x}_Z)=(1-\hat{x}_Z^{-1})(\tfrac{1}{2}\hat{x}_Z^{-1}+\tfrac{3}{2}\hat{x}_Z^{-2})+\ln\frac{(\hat{x}_Z-1)^2}{\epsilon\hat{x}_Z}(\hat{x}_Z^{-1}-2\hat{x}_Z^{-2}+2\hat{x}_Z^{-3})\ . \tag{3.22}$$

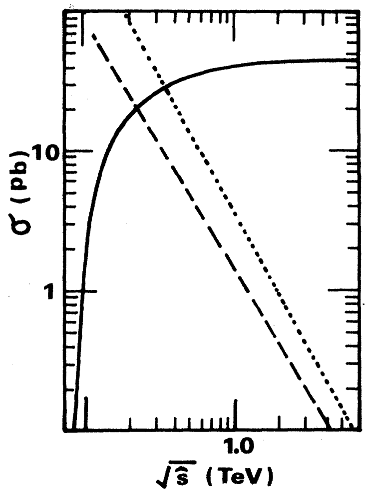

FIG. 2. Total cross section for the SU(2) Compton scattering. The total cross sections for $\gamma e^- \to W^- \nu_e$ and $\gamma e^- \to Z e^-$ are shown in pb unit, by a solid curve and dashed lines, respectively. The total cross section for the original Compton $\gamma e^- \to \gamma e^-$ is shown by the dotted line.

We will illustrate the energy dependence of the total cross sections for the SU(2) Compton scatterings in Fig. 2. The total cross section at high energy for the Z production behaves as

$$\hat{\sigma}_Z(\hat{s} \to \infty) = \tfrac{1}{2}\pi\alpha^2\beta^2(\xi_V^2+\xi_A^2)\frac{M_Z^2}{M_W^2}\frac{\ln(\hat{s}/m_e^2)}{\hat{s}} . \tag{3.23}$$

On the contrary, the total cross section for the W production increases and approaches the following asymptotic value:

$$\hat{\sigma}_W(\hat{s} \to \infty) = \pi\alpha^2\beta^2(\xi_V^2+\xi_A^2)\frac{2}{M_W^2} \simeq 44 \text{ pb} . \tag{3.24}$$

It may be useful to append the cross-section formula at high energy for the original Compton scattering process[10] in our notation:

$$\hat{\sigma}_\gamma(\hat{s} \to \infty) = 2\pi\alpha^2\frac{\ln(\hat{s}/m_e^2)}{\hat{s}} . \tag{3.25}$$

IV. CROSS-SECTION FORMULAS FOR PROCESSES (1.1) AND (1.2)

In the previous section the transverse-momentum distributions and the total cross sections for the SU(2) Compton scatterings which are invariant under the Lorentz transformation along the beam direction have been derived in the center-of-mass system for the photon and electron (positron).

Now we discuss the single W-boson and Z-boson productions in the electron-positron scatterings by assuming the validity of the real-photon approximation.[6–8] It is shown in Refs. 11 and 12 that this approximation leads to reliable results for high-energy behavior of the cross section for electroproduction of heavy particles such as the W boson and top quark as well as the ordinary hadrons such as the pion and kaon. The equivalent-photon approximation reads

$$d\sigma_{e^+e^- \to e^+X} = \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{\omega} N(\omega)\, d\hat{\sigma}_{\gamma e^- \to X'} \tag{4.1}$$

with

$$N(\omega) = \frac{\alpha}{\pi}\left[\ln\frac{E}{m_e}\right]\left[1+\left[1-\frac{\omega}{E}\right]^2\right] + \cdots , \tag{4.2}$$

$$X = W^-\nu_e \quad \text{or} \quad Ze^- , \tag{4.3}$$

where E is the initial beam energy of the positron (electron) which emits the photon with energy ω in the rest frame of electron (positron).

As shown in Appendix B the formulas of the transverse-momentum distributions for processes (1.1) and (1.2) at high energies, $T_W - 1 \gg \epsilon$ and $T_Z - 1 \gg \epsilon$, are given as

$$\frac{d\sigma_W}{dT_W} = \left[\ln\frac{E}{m_e}\right]\alpha^2\beta^2(\xi_V^2+\xi_A^2)\frac{1}{M_W^2}G_W(T_W,x_W) , \tag{4.4}$$

where

$$G_W(T_W,x_W) = \sqrt{(x_W+1)^2-4T_Wx_W}\, G_W^{(1)} - \frac{1}{x_W}\ln\left|\frac{2\sqrt{T_W(T_W-1)}}{x_W+1-2T_W+\sqrt{(x_W+1)^2-4T_Wx_W}}\right| G_W^{(2)}$$
$$+\ln\left|\frac{2x_W\sqrt{T_W(T_W-1)}}{x_W+1-2T_Wx_W+\sqrt{(x_W+1)^2-4T_Wx_W}}\right| G_W^{(3)} , \tag{4.5}$$

$$G_W^{(1)} = T_W(22x_W^{-1}) + (-33x_W^{-1}-\tfrac{7}{3}x_W^{-2}) + T_W^{-1}(\tfrac{34}{3}x_W^{-1}-2x_W^{-2}+\tfrac{4}{3}x_W^{-3}) + T_W^{-2}(-\tfrac{19}{3}x_W^{-1}+\tfrac{7}{3}x_W^{-2}-\tfrac{4}{3}x_W^{-3}) , \tag{4.6}$$

$$G_W^{(2)} = (x_W^{-1}) + T_W^{-1}(8-2x_W^{-1}) + T_W^{-2}(4x_W-4+2x_W^{-1}) , \tag{4.7}$$

$$G_W^{(3)} = T_W^2(-44) + T_W(88+12x_W^{-1}) + (-52-20x_W^{-1}-x_W^{-2}) + T_W^{-1}(8x_W^{-1}-2x_W^{-2}) + T_W^{-2}(4-4x_W^{-1}+2x_W^{-2}) , \tag{4.8}$$

and, similarly,

$$\frac{d\sigma_Z}{dT_Z}=\left[\ln\frac{E}{m_e}\right]\alpha^3\beta^2(\xi_V^2+\xi_A^2)\frac{1}{M_W^2}G_Z(T_Z,x_Z)\ , \tag{4.9}$$

where

$$G_Z(T_Z,x_Z)=\sqrt{(x_Z+1)^2-4T_Zx_Z}\,G_Z^{(1)}-\frac{1}{x_Z}\ln\left|\frac{2\sqrt{T_Z(T_Z-1)}}{x_Z+1-2T_Z+\sqrt{(x_Z+1)^2-4T_Zx_Z}}\right|G_Z^{(2)}$$
$$+\ln\left|\frac{2x_Z\sqrt{T_Z(T_Z-1)}}{x_Z+1-2T_Zx_Z+\sqrt{(x_Z+1)^2-4T_Zx_Z}}\right|G_Z^{(3)}\ , \tag{4.10}$$

$$G_Z^{(1)}=(T_Z-1)(22x_Z^{-1})+(5x_Z^{-1}-\tfrac{7}{3}x_Z^{-2})+(T_Z-1)^{-1}(\tfrac{4}{3}x_Z^{-1}+\tfrac{7}{3}x_Z^{-2}+\tfrac{4}{3}x_Z^{-3})\ , \tag{4.11}$$

$$G_Z^{(2)}=(3x_Z^{-1})+(T_Z-1)^{-1}(-2-2x_Z^{-1})\ , \tag{4.12}$$

$$G_Z^{(3)}=(T_Z-1)^2(-44)+(T_Z-1)(-32+12x_Z^{-1})+(-4-x_Z^{-2})+(T_Z-1)^{-1}(2x_Z^{-1}+2x_Z^{-2})\ . \tag{4.13}$$

When $T_W=T_Z=1$ ($W_T^2=Z_T^2=0$), we obtain the following formulas simply if Eqs. (3.15) and (3.16) are used instead of Eqs. (3.10) and (3.12):

$$G_W(T_W=1,x_W)=\ln x_W(4+4x_W^{-1}+x_W^{-2})+(1-x_W^{-1})(-6-2x_W^{-1}) \tag{4.14}$$

and

$$G_Z(T_Z=1,x_Z)=\frac{1}{\epsilon}\left[(1-x_Z^{-1})(\tfrac{4}{3}+\tfrac{7}{3}x_Z^{-1}+\tfrac{4}{3}x_Z^{-2})-\frac{\ln x_Z}{x_Z}(2+2x_Z^{-1})\right]+\cdots\ . \tag{4.15}$$

Again we have a remarkable feature at $T_Z=1$ due to the term $1/\epsilon$ for the Z-boson production which traces back to the electron pole for the SU(2) Compton Z production. We show the transverse-momentum distributions for processes (1.1) and (1.2) at several energies in Fig. 3.

The total-cross-section formulas for the W-boson and Z-boson production processes (1.1) and (1.2) can be derived simply by performing the integration (4.1) where the total-cross-section formulas on the SU(2) Compton scatterings (3.17) with (3.19) and (3.18) with (3.22) are used. The results are given as

$$\sigma_W=\left[\ln\frac{E}{m_e}\right]\alpha^3\beta^2(\xi_V^2+\xi_A^2)\frac{1}{M_W^2}H_W(x_W)\ , \tag{4.16}$$

where[11]

$$H_W(x_W)=(1-x_W^{-1})(-\tfrac{115}{9}+\tfrac{83}{9}x_W^{-1}-\tfrac{34}{9}x_W^{-2})+x_W^{-1}(\ln x_W)[4x_W+3-x_W^{-1}+\tfrac{4}{3}x_W^{-2}+(\ln x_W)(4-x_W^{-1})] \tag{4.17}$$

and

$$\sigma_Z=\left[\ln\frac{E}{m_e}\right]\alpha^3\beta^2(\xi_V^2+\xi_A^2)\frac{1}{M_W^2}H_Z(x_Z)\ , \tag{4.18}$$

where

$$H_Z(x_Z)=(1-x_Z^{-1})[(-\tfrac{4}{9}-\tfrac{4}{3}\ln\epsilon)+(-\tfrac{7}{9}-\tfrac{7}{3}\ln\epsilon)x_Z^{-1}+(-\tfrac{22}{9}-\tfrac{4}{3}\ln\epsilon)x_Z^{-2}]$$
$$+(1-x_Z^{-1})[\ln(1-x_Z^{-1})](\tfrac{8}{3}+\tfrac{14}{3}x_Z^{-1}+\tfrac{8}{3}x_Z^{-2})+x_Z^{-1}(\ln x_Z)[(-2+2\ln\epsilon)+(-3+2\ln\epsilon)x_Z^{-1}-\tfrac{4}{3}x_Z^{-2}]$$
$$+x_Z^{-1}(\ln x_Z)^2(-1-x_Z^{-1})+x_Z^{-1}[\mathrm{Sp}(1)-\mathrm{Sp}(x_Z^{-1})](4+4x_Z^{-1})\ . \tag{4.19}$$

Here Sp(y) is the Spence function defined by

$$\mathrm{Sp}(y)=-\int_0^y\frac{dx}{x}\ln(1-x)\ , \tag{4.20}$$

$$\mathrm{Sp}(1)=\pi^2/6\ . \tag{4.21}$$

Also the total cross sections for processes (1.1) and (1.2) are shown in Fig. 4 from $\sqrt{s}=0.2$ TeV to $\sqrt{s}=2$ TeV.

V. SUMMARY

The $\bar{p}p$ collider experiments at CERN (Refs. 4 and 5) confirmed the existence of the massive gauge fields, W and Z bosons which seem to be produced through the Drell-Yan mechanism;[13,14] however, these weak bosons are still fascinating to physicists because the non-Abelian gauge theory predicts the new interaction, $WW\gamma$ cou-

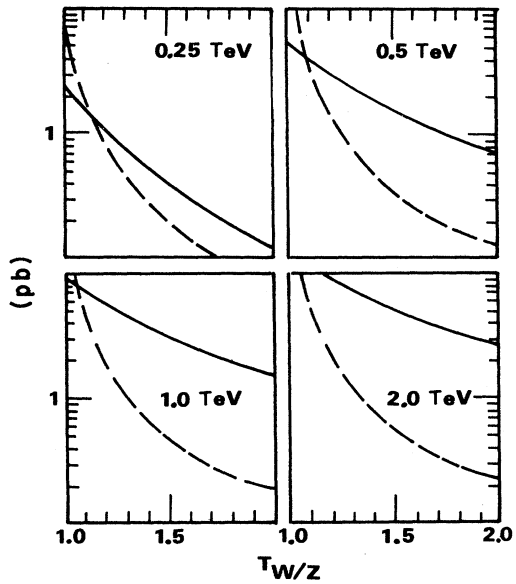

FIG. 3. Transverse-momentum distribution for single W and Z boson production in electron-position scattering. The T_W and T_Z distributions are illustrated in pb units at $\sqrt{s}=0.25$, 0.5, 1.0, and 2.0 TeV. The solid and dashed curves are figured out by using our formulas for process (1.1) and process (1.2).

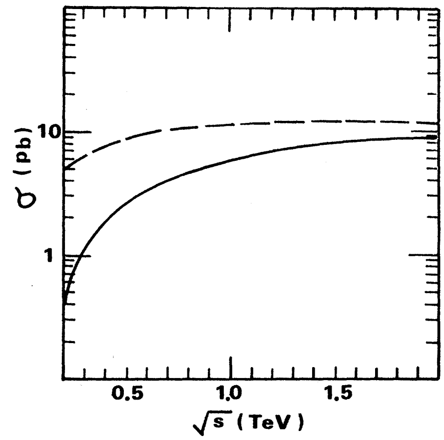

FIG. 4. Total cross section for single W- and Z-boson production in electron-positron scattering. The same as Fig. 3.

pling among the gauge fields, and even its glimpse is not caught at the present $\bar{p}p$ collider experiments.

Since the Glashow-Weinberg-Salam gauge theory has been proposed, the production of a single W boson in electron-positron collider experiments has been studied by many authors[11,15] below the threshold of the pair production of weak bosons, where the magnetic moment of the weak boson is pointed out to give an important role to the production cross sections. But it came out that the magnitude of the cross section for a single W boson is too small to be observed if the magnetic moment has the value predicted in the standard theory. As for the Z boson, the interesting and abundant data will be published in the electron-positron collider experiment at $\sqrt{s}=M_Z$; however, it seems very hard to extract the data about the magnetic moment of the W boson from this experiment.

The amplitude for the weak-boson production in the electron-nucleon collision is discussed by various authors,[16] who considered

$$\gamma u \rightarrow W^+ d \tag{5.1}$$

and its crossed channels to show how kinematics and quark charge assignment determine the occurrence of a zero in the helicity amplitude. Also Cortes, Hagiwara, and Herzog[17] discussed how to test the $WW\gamma$ coupling of the standard model at $\bar{p}p$ collider experiments at the Fermilab Tevatron collision energy.

If the electron-positron collider experiment at 200 GeV becomes available, the measurement of the magnetic moment of the W boson is easily done through the pair production of the W boson.[18,19] The cross section for the pair production of the W bosons decreases as $1/s$ and becomes negligibly small at TeV energy, where the contributions from the t- and u-channel photon exchanges will become dominant.

In this paper we derived the cross-section formulas on the single weak boson (W and Z) production in the electron-positron collisions by assuming the validity of the equivalent-photon approximation and the standard theory.

Since the cross-section formulas are written down in the invariant form under the Lorentz transformation along the beam direction, they can be applied also to the eN collision where the target nucleon is at rest, if the coupling constants are modified suitably.

ACKNOWLEDGMENTS

The author is grateful for many useful suggestions and discussions by Y. Shimizu, T. Munehisa, and K. Hagiwara at KEK. The calculations were done at the KEK Computer Center where the REDUCE 3.2 (Ref. 20) is used for the analytical computations in the text.

APPENDIX A

In this appendix the transverse-momentum distributions for the SU(2) Compton scatterings (3.8)–(3.14) are given. By using the squared amplitudes (3.6) and (3.7) averaged over initial and summed over final spins, the differential cross sections are written down in the Lorentz-invariant form:

$$W_0 \frac{d\hat{\sigma}_W}{d^3 w} = \alpha^2 \beta^2 (\xi_V^2 + \xi_A^2) \frac{1}{M_W^2} f_W \left[\frac{\hat{t}-M_W^2}{\hat{s}}, \hat{x}_W \right]$$
$$\times \delta((\hat{P}+Q-W)^2) \tag{A1}$$

and

$$Z_0\frac{d\hat{\sigma}_Z}{d^3z}=\frac{1}{2}\alpha^2\beta^2(\zeta_V^2+\zeta_A^2)\frac{1}{M_W^2}f_Z\left[\frac{\hat{u}-m_e^2}{\hat{s}-m_e^2},\hat{x}_Z\right]\delta((\hat{P}+Q-Z)^2-m_e^2)\ . \tag{A2}$$

The delta functions in the c.m. system read

$$\delta[(\hat{P}+Q-W)^2]=\frac{W_0}{2\sqrt{\hat{s}}\,|W_L|}\left[\delta\left[W_L-\frac{\sqrt{\hat{s}}}{2}\xi_L\hat{x}_W^{-1}\right]+\delta\left[W_L+\frac{\sqrt{\hat{s}}}{2}\xi_L\hat{x}_W^{-1}\right]\right] \tag{A3}$$

and

$$\delta[(\hat{P}+Q-Z)^2-m_e^2]=\frac{Z_0}{2\sqrt{\hat{s}}\,|Z_L|}\left[\delta\left[Z_L-\frac{\sqrt{\hat{s}}}{2}\zeta_L\hat{x}_Z^{-1}\right]+\delta\left[Z_L+\frac{\sqrt{\hat{s}}}{2}\zeta_L\hat{x}_Z^{-1}\right]\right]\ , \tag{A4}$$

where ξ_L and ζ_L are defined in Eqs. (3.11) and (3.13), respectively. Notice that

$$\frac{\hat{t}-M_W^2}{\hat{s}}=-\frac{1}{2}\hat{x}_W^{-1}\left[(\hat{x}_W+1)-\frac{2}{\sqrt{\hat{s}}}\hat{x}_W W_L\right]\ , \tag{A5}$$

for the SU(2) Compton W production and

$$\frac{\hat{u}-m_e^2}{\hat{s}-m_e^2}=-\frac{1}{2}\hat{x}_Z^{-1}\left[(\hat{x}_Z-1+\epsilon)-\frac{2}{\sqrt{\hat{s}}}\hat{x}_Z Z_L\right]\ , \tag{A6}$$

for the SU(2) Compton Z production, and

$$\frac{d^3p}{P_0}=\frac{\pi}{P_0}dP_T^2dP_L\ , \tag{A7}$$

where $p\,(P_0,P_T,P_L)$ is the magnitude of the momentum (energy, transverse momentum, longitudinal momentum) for W or Z boson.

Now it is easy to get the transverse-momentum distributions for the SU(2) Compton scatterings (3.8)–(3.14) if the integrations over the longitudinal momenta of W and Z are done.

The total cross sections (3.17)–(3.21) are obtained by notifying the upper and lower limits on the transverse momenta of the W boson and Z boson as

$$0\le W_T^2\le\frac{(\hat{s}-M_W^2)^2}{4\hat{s}} \tag{A8}$$

and

$$0\le Z_T^2\le\frac{(\hat{s}+M_Z^2-m_e^2)^2}{4\hat{s}}-M_Z^2\ . \tag{A9}$$

APPENDIX B

If the integration variable y is defined by

$$y=\frac{\omega}{E}\ , \tag{B1}$$

Eq. (4.1) leads to

$$d\sigma_{e^+e^-\to e^+X}=\int_{y_{\min}}^{1}\frac{dy}{y}N(y)d\sigma_{\gamma e^-\to X}\ , \tag{B2}$$

where $Y_{\min}$ is the minimal value of y for the reaction and

$$N(y)=\frac{\alpha}{\pi}\left[\ln\frac{E}{m_e}\right][1+(1-y)^2]\ . \tag{B3}$$

In the rest frame of the electron (positron) we have

$$y=\frac{\hat{s}}{s}\ ; \tag{B4}$$

then we get

$$\frac{d\sigma_{e^+e^-\to e^+W^-\nu}}{dT_W}=\left[\ln\frac{E}{m_e}\right]\alpha^3\beta^2(\xi_V^2+\xi_A^2)\frac{1}{M_W^2}G_W(T_W,x_W) \tag{B5}$$

and

$$\frac{d\sigma_{e^+e^-\to e^+e^-Z}}{dT_Z}=\left[\ln\frac{E}{m_e}\right]\alpha^3\beta^2(\zeta_V^2+\xi_A^2)\frac{1}{M_W^2}G_Z(T_Z,x_Z)\ , \tag{B6}$$

where

$$G_W(T_W,x_W)=\int_{y_{\min}}^{1}\frac{dy}{y}[1+(1-y)^2]g_W(T_W,yx_W)\ , \tag{B7}$$

$$y_{\min}=x_W^{-1}[(2T_W-1)+2\sqrt{T_W(T_W-1)}]\ , \tag{B8}$$

for the W-boson production process, and

$$G_Z(T_Z,x_Z)=\int_{y_{\min}}^{1}\frac{dy}{y}[1+(1-y)^2]g_Z(T_Z,yx_Z)\ , \tag{B9}$$

$$y_{\min}=x_Z^{-1}[(2T_Z-1+\epsilon)+2\sqrt{T_Z(T_Z-1+\epsilon)}]\ , \tag{B10}$$

for the Z-boson production process.

The following formulas for the y integrations are helpful in deriving Eqs. (4.4)–(4.13):

$$A\equiv\int\frac{1}{\sqrt{ay^2+by+c}}dy=\frac{1}{\sqrt{a}}\ln\left|\sqrt{a}\,y+\sqrt{ay^2+by+c}+\frac{b}{2\sqrt{a}}\right|\ , \tag{B11}$$

$$B\equiv\int\frac{y^{-1}}{\sqrt{ay^2+by+c}}dy$$

$$=\frac{1}{\sqrt{c}}\ln\left|\frac{y}{\sqrt{c}+\sqrt{ay^2+by+c}+\frac{b}{2\sqrt{c}}y}\right| . \quad \text{(B12)}$$

$$\int\frac{y^2}{\sqrt{ay^2+by+c}}dy$$

$$=\frac{1}{4a^2}(2ay-3b)\sqrt{ay^2+by+c}+\frac{3b^2-4ac}{8a^2}A , \quad \text{(B13)}$$

$$\int\frac{y}{\sqrt{ay^2+by+c}}dy=\frac{1}{a}\sqrt{ay^2+by+c}-\frac{b}{2a}A , \quad \text{(B14)}$$

$$\int\frac{y^{-2}}{\sqrt{ay^2+by+c}}dy=\frac{-1}{cy}\sqrt{ay^2+by+c}-\frac{b}{2c}B , \quad \text{(B15)}$$

$$\int\frac{y^{-3}}{\sqrt{ay^2+by+c}}dy$$

$$=\frac{3bc-2c}{4c^2y^2}\sqrt{ay^2+by+c}+\frac{3b^2-4ac}{8c^2}B , \quad \text{(B16)}$$

and

$$\int\frac{y^{-4}}{\sqrt{ay^2+by+c}}dy=\frac{1}{24c^3y^2}[(16ac-15b^2)y^2+10bcy-8c^2]\sqrt{ay^2+by+c}-\frac{b(5b^2-12ac)}{16c^3}B . \quad \text{(B17)}$$

[1]S. L. Glashow, Nucl. Phys. **22**, 579 (1961).

[2]S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[3]A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8),* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[4]UA1 Collaboration, Phys. Lett. **122B**, 103 (1983); UA2 Collaboration, *ibid.* **122B**, 476 (1983).

[5]UA1 Collaboration, Phys. Lett. **126B**, 398 (1983); UA2 Collaboration, *ibid.* **129B**, 130 (1983).

[6]E. Fermi, Z. Phys. **29**, 315 (1924); C. Weizsacker and E. J. Williams, *ibid.* **88**, 244 (1934).

[7]R. B. Curtis, Phys. Rev. **104**, 211 (1956); R. H. Dalitz and D. R. Yennie, *ibid.* **105**, 1598 (1957).

[8]S. J. Brodsky, K. Kinoshita, and H. Terazawa, Phys. Rev. D **4**, 1532 (1971).

[9]E. M. Renard, in *Photon-Photon Collisions,* proceedings of the 7th International Workshop, Paris, France, 1986, edited by A. Courau and P. Kessler (World Scientific, Singapore, 1987).

[10]O. Klein and Y. Nishina, Z. Phys. **52**, 853 (1929).

[11]H. Neufeld, Z. Phys. C **17**, 145 (1983); O. Cheyette, Phys. Lett. **137B**, 432 (1984).

[12]M. Katuya, J. Morishita, T. Munehisa, and Y. Shimizu, Prog. Theor. Phys. **75**, 92 (1986).

[13]S. Drell and T. M. Yan, Ann. Phys. (N.Y.) **66**, 578 (1971).

[14]S. Geer and W. J. Stirling, Phys. Lett. **152B**, 373 (1985); E. Eichten *et al.*, Rev. Mod. Phys. **56**, 579 (1984); G. Altarelli *et al.*, CERN Report No. TH.4015/84, 1984 (unpublished).

[15]J. Ellis and M. K. Gaillard, CERN Yellow Report No. 76-18, p. 21, 1976; W. Alles, C. Boyer, and A. J. Buras, Nucl. Phys. **B119**, 125 (1977); A. Donnachie and K. J. F. Gaemmers, Z. Phys. C **4**, 37 (1980); M. Katuya and T. Niuya, *ibid.* **26**, 255 (1984); M. Katuya, *ibid.* **28**, 509 (1985).

[16]K. O. Mikaelian, M. A. Samuel, and D. Sahdev, Phys. Rev. Lett. **43**, 746 (1979); K. O. Mikaelian, Phys. Rev. D **25**, 66 (1982); S. J. Brodsky and R. W. Brown, Phys. Rev. Lett. **49**, 966 (1982); R. W. Brown, K. L. Kowalski, and S. J. Brodsky, Phys. Rev. D **28**, 624 (1983).

[17]J. Cortes, K. Hagiwara, and F. Herzog, Nucl. Phys. **B278**, 26 (1986).

[18]K. J. F. Gaemers and G. J. Gounaris, Z. Phys. C **1**, 259 (1979); O. P. Sushkov, V. V. Flambaum, and I. B. Khripolovich, Yad. Fiz. **20**, 1016 (1974) [Sov. J. Nucl. Phys. **20**, 537 (1975)].

[19]F. Bletzacker and H. T. Nieh, Nucl. Phys. **B124**, 511 (1977); R. W. Brown and K. O. Mikaelian, Phys. Rev. D **19**, 922 (1979).

[20]A. C. Hearn, Report No. CPT-76-MARSELLE, 821, 1976 (unpublished).