Issue of $\psi\to\gamma\eta,\gamma\eta'$ decays and η-η' mixing

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(Received 29 July 1988)

The issue of the ratio $R=\Gamma(\psi\to\gamma\eta')/\Gamma(\psi\to\gamma\eta)$ and the η-η' mixing angle θ is discussed in an approach on the basis of chiral and large-N_c arguments. Gluonic matrix elements $\alpha_s\langle 0|G\tilde{G}|\eta(\eta')\rangle$ are estimated. $\theta\simeq-20°$ is found to be strongly favored over $\theta\simeq-10°$, and is in agreement with the experimental value of R.

η-η' mixing has been a very interesting problem since SU(3)-flavor symmetry was proposed. Theoretically it is related to the axial U(1) problem and SU(3)-symmetry breaking. Phenomenologically it is involved in various processes such as η $(\eta')\to 2\gamma$, $\psi\to\gamma\eta$ (η'), and many hadronic decays. Over the years the η-η' mixing angle has been taken to be $\theta\simeq-10°$, obtained by using the quadratic Gell-Mann–Okubo mass formula. However, as summarized in Ref. 1, a value of $\theta\simeq-20°$ seems to be consistent with most present evidence. Two decisive pieces of experimental evidence are due to η $(\eta')\to 2\gamma$ decays and $\psi\to\gamma\eta$ (η') decays. The former have recently been studied[2] with one-loop chiral corrections to the decay amplitudes and $\theta=-(23\pm3)°$ is found by fitting the data of $P\to 2\gamma$ $(P=\pi^0,\eta,\eta')$. For the latter define

$$R\equiv\frac{\Gamma(\psi\to\gamma\eta')}{\Gamma(\psi\to\gamma\eta)} \tag{1}$$

and, assuming the decay proceeds through the SU(3)-singlet part of the pseudoscalar meson, one finds[3]

$$R=\left[\frac{k_{\eta'}}{k_\eta}\right]^3\cot^2\theta\ . \tag{2}$$

With the current experimental value[4] of $R_{\rm expt}=4.8\pm0.2$ one finds[1] $\theta\simeq-22°$. This value is consistent with that obtained from $\eta(\eta')\to 2\gamma$ and other sources and is regarded as one of the strongest evidences for $\theta\simeq-20°$.

However, Eq. (2) is based on the applicability of SU(3) symmetry for the decay amplitudes and might be altered rapidly by symmetry-breaking effects. As an extreme, it has even been argued[5] that the mixing formalism cannot be justified *a priori* in this case, and some symmetry-breaking effects on the gluonic matrix element of the η are considered but the η-η' mixing is neglected completely.[5] The theoretical dispute over the ratio R and mixing seems to be very puzzling.

In the following we will discuss this problem in an approach differing from Refs. 1 and 3 and from Ref. 5, by taking account of SU(3)-symmetry-breaking effects and large-N_c (N_c being the number of colors) arguments for the axial U(1) anomaly, and calculating a gluonic matrix element for η (η'). We will show that the data on $\psi\to\gamma\eta(\eta')$ decays still strongly favor $\theta\simeq-20°$ over $\theta\simeq-10°$.

$\psi\to\gamma\eta(\eta')$ decays have been discussed by many authors.[1,3,5,6] These decays proceed primarily through radiation of the photon from the charmed quark or charmed antiquark and through coupling of the charmed quarks to the final pseudoscalar via intermediate gluons. We assume that the transition from charmed quarks to $\eta(\eta')$ is given by the matrix element $\langle 0|\bar{c}i\gamma_5 c|\eta(\eta')\rangle$, which may also be viewed as the mixing of physical η (η') with virtual pseudoscalar $c\bar{c}$ states. The QCD axial anomaly[7] for the charmed-quark operator reads

$$\partial_\mu(\bar{c}\gamma_\mu\gamma_5 c)=2im_c\bar{c}\gamma_5 c+\frac{\alpha_s}{4\pi}G\tilde{G}\ , \tag{3}$$

where $G\tilde{G}=G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$, $G^a_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^a_{\alpha\beta}$, $G^a_{\mu\nu}$ is the gluon field-strength tensor. Define

$$\begin{aligned}\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta\rangle&=if_{c\eta}k_\mu\ ,\\ \langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta'\rangle&=if_{c\eta'}k_\mu\ ,\end{aligned} \tag{4}$$

where $f_{c\eta}$ $(f_{c\eta'})$ is the analogue of decay constant f_η $(f_{\eta'})$. Since η (η') will decouple from $c\bar{c}$ [i.e., the mixing of η (η') with $c\bar{c}$ will vanish] as $m_c\to\infty$, we expect

$$f_{c\eta}=O\left[\frac{m_s}{m_c}f_\eta\right]\ll f_\pi,\quad f_{c\eta'}=O\left[\frac{\Lambda}{m_c}f_{\eta'}\right]\ll f_\pi\ , \tag{5}$$

where Λ is the typical hadronic mass scale [note that theoretically we have $m_u<m_d\ll m_s<\Lambda\ll m_c$ and $m^2_\eta=O(\Lambda m_s)$, $m^2_{\eta'}=O(\Lambda^2)$]. On the other hand,

$$\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta(\eta')\rangle$$

is independent of m_c and is of order $f_\pi m^2_{\eta(\eta')}$. [This will be shown shortly in Eqs. (31) and (32).] Therefore we can neglect

$$\langle 0|\partial_\mu\bar{c}\gamma_\mu\gamma_5 c|\eta(\eta')\rangle=O\left[\frac{m_s}{m_c}f_\pi,\frac{\Lambda}{m_c}f_\pi\right]m^2_{\eta(\eta')}$$

as compared to

$$\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta(\eta')\rangle\ ,$$

and get

$$2m_c\langle 0|\bar{c}i\gamma_5 c|\eta(\eta')\rangle \simeq -\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta(\eta')\rangle \ . \tag{6}$$

Note that because of the nonperturbative nature of the axial anomaly, Eq. (6) holds to any order in α_s. The meaning of Eq. (6) is that coupling of η (η') to charmed quarks [mixing of η (η') with $c\bar{c}$] proceeds indeed through two intermediate gluons. This physical picture is justified on the basis of the QCD axial anomaly, an exact equation for field operators. The decay amplitude for $\psi\to\gamma\eta(\eta')$ is expected to be proportional to the coupling of charmed quarks to η (η'). Therefore we have

$$R=\left|\frac{\langle 0|\bar{c}i\gamma_5 c|\eta'\rangle}{\langle 0|\bar{c}i\gamma_5 c|\eta\rangle}\right|^2\left[\frac{k_{\eta'}}{k_\eta}\right]^3 \simeq \left|\frac{\langle 0|G\tilde{G}|\eta'\rangle}{\langle 0|G\tilde{G}|\eta\rangle}\right|^2\left[\frac{k_{\eta'}}{k_\eta}\right]^3 , \tag{7}$$

where the factor $(k_{\eta'}/k_\eta)^3$ ($=0.813$) is due to the phase space for a P-wave decay.

To estimate the gluonic matrix elements let us assume the physical η and η' states can be written as

$$|\eta\rangle=\cos\theta|\eta_8\rangle-\sin\theta|\eta_0\rangle \ , \tag{8}$$

$$|\eta'\rangle=\sin\theta|\eta_8\rangle+\cos\theta|\eta_0\rangle \ , \tag{9}$$

where η_0 (η_8) is the SU(3) singlet (eighth member of the octet):

$$|\eta_8\rangle=\frac{1}{\sqrt{6}}|u\bar{u}+d\bar{d}-2s\bar{s}\rangle \ , \tag{10}$$

$$|\eta_0\rangle=\frac{1}{\sqrt{3}}|u\bar{u}+d\bar{d}+s\bar{s}\rangle \ . \tag{11}$$

Here we have neglected possible mixing of the η and η' with other pseudoscalars [e.g., $\eta(\eta')$-π^0 mixing, η (η')-glueball mixing, etc.]. The corresponding axial-vector currents and decay constants are defined as

$$A_\mu^8=\frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d-2\bar{s}\gamma_\mu\gamma_5 s) \ , \tag{12}$$

$$A_\mu^0=\frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d+\bar{s}\gamma_\mu\gamma_5 s) \ , \tag{13}$$

$$\langle 0|A_\mu^8|\eta_8\rangle=ik_\mu f_8, \quad \langle 0|A_\mu^8|\eta_0\rangle=ik_\mu f_{80} \ , \tag{14}$$

$$\langle 0|A_\mu^0|\eta_0\rangle=ik_\mu f_0, \quad \langle 0|A_\mu^0|\eta_8\rangle=ik_\mu f_{08} \ . \tag{15}$$

The divergences of the axial-vector currents are

$$\partial_\mu A_\mu^8=\frac{2i}{\sqrt{6}}(m_u\bar{u}\gamma_5 u+m_d\bar{d}\gamma_5 d-2m_s\bar{s}\gamma_5 s) \ , \tag{16}$$

$$\partial_\mu A_\mu^0=\frac{2i}{\sqrt{3}}(m_u\bar{u}\gamma_5 u+m_d\bar{d}\gamma_5 d+m_s\bar{s}\gamma_5 s)+\frac{\sqrt{3}\alpha_s}{4\pi}G\tilde{G} \ . \tag{17}$$

In the chiral theory with nonvanishing and unequal-quark masses ($m_u\neq m_d\neq m_s$), to leading order [i.e., in the SU(3) limit] one has

$$f_8\simeq f_\pi, \quad f_{08}=f_{80}\simeq 0 \ . \tag{18}$$

(Note that to leading order we also have $\langle 0|\bar{u}u|0\rangle=\langle 0|\bar{d}d|0\rangle=\langle 0|\bar{s}s|0\rangle$ and then the Gell-Mann–Okubo mass formula $m_{\eta_8}^2=\frac{4}{3}m_K^2-\frac{1}{3}m_\pi^2$, and to this order due to $m_{u,d}\ll m_s$ the η_8 will mix with η_0.) Therefore,

$$\langle 0|A_\mu^8|\eta\rangle\simeq ik_\mu f_\pi\cos\theta, \quad \langle 0|A_\mu^8|\eta'\rangle\simeq ik_\mu f_\pi\sin\theta \ , \tag{19}$$

$$\langle 0|A_\mu^0|\eta\rangle\simeq -ik_\mu f_0\sin\theta, \quad \langle 0|A_\mu^0|\eta'\rangle\simeq ik_\mu f_0\cos\theta \ . \tag{20}$$

Note that even in the SU(3) limit there is no reason to expect $f_0=f_\pi$, because of the U(1) anomaly effects in the singlet channel. Here we will determine f_0 and θ through diagonalizing the mass-squared matrix

$$\begin{bmatrix} m_{\eta_8}^2 & m_{80}^2 \\ m_{08}^2 & m_{\eta_0}^2 \end{bmatrix} . \tag{21}$$

According to the large-N_c theory,[8] the anomaly contribution is of order $1/N_c$, and therefore in the large-N_c limit ($N_c\to\infty$) the anomaly vanishes and the η_0 would become the ninth Goldstone boson. In this limit we can treat the η_0 on the same footing as the octet Goldstone bosons, and use the PCAC (partial conservation of axial-vector current) hypotheses and current algebra to calculate the mass-squared matrix elements. Moreover, in this limit $f_0=f_\pi$. When a finite N_c ($N_c=3$) is restored the anomaly term $G\tilde{G}$ should be added and f_0 would deviate from f_π [$f_0/f_\pi=1+O(1/N_c)$] (Ref. 9). To leading order in quark masses we then have

$$m_{\eta_0}^0\simeq\left[\frac{f_\pi}{f_0}\right]^2(\tfrac{2}{3}m_K^2+\tfrac{1}{3}m_\pi^2)+\frac{C}{N_c} \ , \tag{22}$$

where the first term is due to the light-quark mass term

$$\frac{2i}{\sqrt{3}f_0}\langle 0|m_u\bar{u}\gamma_5 u+m_d\bar{d}\gamma_5 d+m_s\bar{s}\gamma_5 s|\eta_0\rangle$$

and the second term C/N_c stands for the anomaly contribution, and

$$m_{08}^2=m_{80}^2\simeq-\left[\frac{f_\pi}{f_0}\right]\frac{2\sqrt{2}}{3}(m_K^2-m_\pi^2) \ . \tag{23}$$

As for $m_{\eta_8}^2$ we will take

$$m_{\eta_8}=(0.566\text{–}0.61)\ \text{GeV} \ , \tag{24}$$

a value between the leading-order result [$m_{\eta_8}^2=\frac{4}{3}m_K^2-\frac{1}{3}m_\pi^2=(0.566\ \text{GeV})^2$] and the one with one-loop chiral corrections [$m_{\eta_8}^2\simeq(0.61\ \text{GeV})^2$] (Ref. 2). The physical η and η' states can be obtained by diagonalizing the mass-squared matrix (21). Conversely, using the experimental mass values $m_\eta=0.549$ GeV, $m_{\eta'}=0.958$ GeV, and $m_{\eta_8}=0.566$ (0.61) GeV as input, we can determine the η-η' mixing angle θ and then f_0 and C/N_c:

$$\theta\simeq-10^\circ\ (-20^\circ), \quad f_0\simeq[2.0\ (1.1)]f_\pi \ , \tag{25}$$

$$\frac{C}{N_c} \simeq 0.856\ (0.705)\ \text{GeV}^2\ . \tag{26}$$

We see that not only θ but also f_0 are very sensitive to the value of m_{η_8}. Both values of θ and f_0 are changed by almost a factor of 2, when m_{η_8} has only a small shift of 0.05 GeV from chiral corrections of order of $m_q^2 \ln m_q$. In principle we should also include chiral corrections of the same order into m_{08}^2 and $m_{\eta_0}^2$; for example,

$$m_{08}^2 \simeq -\left[\frac{f_\pi}{f_0}\right]\frac{2\sqrt{2}}{3}[m_K^2 - m_\pi^2 + O(m_s^2 \ln m_s)]\ .$$

This will cause small corrections $\sim 10^{-1}$ to the values of $f_0(\simeq 1.1 f_\pi)$ and C/N_c ($\simeq 0.705$ GeV^2) obtained for $m_{\eta_8} \simeq 0.61$ GeV and $\theta \simeq -20°$. It is interesting to note that our results of $\theta \simeq -20°$ and, in particular, $f_0 \simeq 1.1 f_\pi$ agree (within 10%) with $\theta = -(23\pm 3)°$ and $f_0 = (1.04\pm 0.04) f_\pi$, obtained by fitting the $P \to 2\gamma$ ($P = \pi^0, \eta, \eta'$) decay rates with decay amplitudes incorporating one-loop chiral corrections.[2]

From Eq. (17) we get

$$\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta(\eta')\rangle = \frac{1}{\sqrt{3}}\langle 0|\partial_\mu A_\mu^0|\eta(\eta')\rangle - \frac{2i}{3}\langle 0|m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d + m_s \bar{s}\gamma_5 s|\eta(\eta')\rangle\ , \tag{27}$$

where $\langle 0|\partial_\mu A_\mu^0|\eta\rangle \simeq -f_0 \sin\theta m_\eta^2$ and $\langle 0|\partial_\mu A_\mu^0|\eta'\rangle \simeq f_0 \cos\theta m_{\eta'}^2$, following Eq. (20). The large-N_c approach for η_0 leads to [see (22)]

$$\frac{2i}{\sqrt{3}}\langle 0|m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d + m_s \bar{s}\gamma_5 s|\eta_0\rangle \simeq \frac{f_\pi^2}{f_0}\left(\tfrac{2}{3}m_K^2 + \tfrac{1}{3}m_\pi^2\right)\ . \tag{28}$$

For η_8 we use

$$\begin{aligned} f_\pi\left(\tfrac{4}{3}m_K^2 - \tfrac{1}{3}m_\pi^2\right) &\simeq \langle 0|\partial_\mu A_\mu^8|\eta_8\rangle \\ &= \frac{2i}{\sqrt{6}}\langle 0|m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d - 2m_s \bar{s}\gamma_5 s|\eta_8\rangle \\ &\simeq \frac{2i}{\sqrt{6}}(-2)\langle 0|m_s \bar{s}\gamma_5 s|\eta_8\rangle\ , \end{aligned} \tag{29}$$

where contributions of m_u and m_d are neglected since $m_{u,d} \ll m_s$, and then we get

$$\frac{2i}{\sqrt{3}}\langle 0|m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d + m_s \bar{s}\gamma_5 s|\eta_8\rangle \simeq \frac{2i}{\sqrt{3}}\langle 0|m_s \bar{s}\gamma_5 s|\eta_8\rangle \simeq -\frac{f_\pi}{\sqrt{2}}\left(\tfrac{4}{3}m_K^2 - \tfrac{1}{3}m_\pi^2\right)\ . \tag{30}$$

The corresponding matrix elements for the η and η' can be approximately obtained by using (28) and (30) with (8) and (9). Then substituting them into (27) we obtain

$$\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta'\rangle \simeq \frac{1}{\sqrt{3}}f_0 \cos\theta m_{\eta'}^2 - \frac{1}{\sqrt{3}}\frac{f_\pi^2}{f_0}\left(\tfrac{2}{3}m_K^2 + \tfrac{1}{3}m_\pi^2\right)\cos\theta + \frac{f_\pi}{\sqrt{6}}\left(\tfrac{4}{3}m_K^2 - \tfrac{1}{3}m_\pi^2\right)\sin\theta\ , \tag{31}$$

$$\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta\rangle \simeq -\frac{1}{\sqrt{3}}f_0 \sin\theta m_\eta^2 + \frac{1}{\sqrt{3}}\frac{f_\pi^2}{f_0}\left(\tfrac{2}{3}m_K^2 + \tfrac{1}{3}m_\pi^2\right)\sin\theta + \frac{f_\pi}{\sqrt{6}}\left(\tfrac{4}{3}m_K^2 - \tfrac{1}{3}m_\pi^2\right)\cos\theta\ . \tag{32}$$

Equations (31) and (32) are derived by taking account of the symmetry breaking in quark masses (i.e., $m_{u,d} \ll m_s$), assuming the SU(3)-symmetry limit for octet decay constants (i.e., $f_8 = f_\pi$), and allowing arbitrary singlet decay constant f_0 and η-η' mixing angle. If f_0 and θ are determined through diagonalizing the η_8-η_0 mass matrix, then for $\theta \simeq -10°$ ($-20°$) and $f_0 \simeq [2.0\ (1.1)] f_\pi$ [see (25)] use of Eqs. (31), (32), and (7) will give

$$\begin{aligned} &\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta'\rangle \simeq 0.130\ (0.056)\ \text{GeV}^3\ , \\ &\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta'\rangle \simeq 0.024\ (0.021)\ \text{GeV}^3\ , \end{aligned} \tag{33}$$

$$R \simeq 24\ (5.7)\ . \tag{34}$$

However, in the derivation of $\theta \simeq -20°$ and $f_0 \simeq 1.1 f_\pi$, one-loop chiral corrections have been included in $m_{\eta_8}^2$. To be consistent with these corrections, we may take $f_8 \simeq 1.2 f_\pi$ (see, e.g., Ref. 2 and note that as mentioned before there could also be 10% uncertainty on f_0 due to chiral corrections), and then get

$$\begin{aligned} &\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta'\rangle \simeq 0.055\ \text{GeV}^3\ , \\ &\frac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta\rangle \simeq 0.024\ \text{GeV}^3\ , \end{aligned} \tag{35}$$

$$R \simeq 4.3\ . \tag{36}$$

Comparing (34) and (36) with $R_{\text{expt}} = 4.8\pm 0.2$ (Ref. 4) we see that $\theta \simeq -20°$ is strongly favored over $\theta \simeq -10°$, although there could be some uncertainties on the value of R obtained for $\theta \simeq -20°$, owing to the limited accuracy in our approach.

It is worthwhile to note that even in the limit of $\theta = 0$ we still have nonvanishing $\alpha_s\langle 0|G\tilde{G}|\eta\rangle = O(f_\pi m_\eta^2)$ [see (32)], which is due to SU(3)-symmetry breaking ($m_{u,d} \ll m_s$). Therefore our result for R differs from Eq. (2), which is based on the assumption equivalent to $\langle 0|G\tilde{G}|\eta_8\rangle = 0$. On the other hand, the values of R and $\langle 0|G\tilde{G}|\eta(\eta')\rangle$ are very sensitive to f_0 and θ. This can be seen from (33) and (34) [also note that $(\alpha_s/4\pi)\langle 0|G\tilde{G}|\eta\rangle|_{\theta=0} \simeq 0.017$ GeV^3]. Therefore our result also differs from Ref. 5, where the η-η' mixing is neglected.

In conclusion, the issue about the ratio $R=\Gamma(\psi\rightarrow\gamma\eta')/\Gamma(\psi\rightarrow\gamma\eta)$ and η-η' mixing angle θ may be resolved in present approach on the basis of chiral and large-N_c arguments, and $\theta\simeq-20°$ is strongly favored over $\theta\simeq-10°$ and is in agreement with experiment in R.

This work was supported in part by The Chinese National Science Funds.

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