Tests of flavor and nonet symmetry in the decays of charmed mesons into two pseudoscalars

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The standard model leads to specific flavor-symmetry properties for the decays of charmed mesons into two members of the pseudoscalar nonet. We derive linear relations amongst the amplitudes for specific decay modes from these properties together with the additional assumption of nonet symmetry, and we use them to set bounds on the branching ratios for various decays. Large violations of these bounds would imply large nonet-symmetry-breaking effects in charm decay, and possibly large flavor-symmetry violations as well. There are experimental indications for such breaking from D^0 and D_s decays.

In the standard model of quarks and leptons, the Cabibbo-allowed decays of charmed mesons into noncharmed hadrons are endowed with well-defined flavor-symmetry properties.[1] The interaction behaves as a vector in isospace and in U-spin space, and it transforms as an admixture of the **6*** and **15** representations under flavor SU(3). These transformation properties lead to linear sum rules among amplitudes for different decay modes, and the sum rules, in turn, are expected to be valid to the same extent that strong interactions respect the flavor symmetries.

From past experience with strange-particle decays,[2] we expect the isospin selection rules to hold to a high degree of accuracy because they are broken only at the level of electromagnetic interactions; and we expect the flavor-SU(3) rules to hold at the level of 10%, which represents the degree of flavor-SU(3) breaking. To find out whether this is indeed the case, we use the rich predictions of the standard model for the two-body decays of charmed mesons as tools with which to test flavor symmetry in charm decay. We should emphasize that we are dealing only with the flavor-symmetry properties of Cabibbo-allowed charm decay and not with dynamical models, as have been discussed by several other authors.[3]

In addition to flavor-SU(3) selection rules, we shall make use of the additional assumption of nonet symmetry. Nonet symmetry is inspired by the almost ideal mixing of the ϕ and ω mesons, but it has been extended to pseudoscalar mesons as well. It has the effect of relating the coupling constants associated with SU(3)-singlet mesons to those associated with the corresponding octets, and it has been used in all the flavor-SU(3) analyses with which this author is familiar.[1,4] Nonet symmetry will therefore yield fewer coupling constants than the most general SU(3) analysis of charm decay.

Here we consider decays into two pseudoscalar mesons. There are altogether nine possible decay modes of the triplet of charmed mesons into two members of the pseudoscalar nonet and, as is well known,[4] only three independent coupling constants allowed by the standard model and nonet symmetry. Consequently we obtain six independent relations among the decay modes, and we use them to set bounds on the branching ratios for various modes. In particular we discuss decays of the D^0 and the D_s into final states containing η and η' mesons; these modes are the most sensitive to nonet symmetry and its possible breaking.

The two pseudoscalar mesons are in an S wave in the final state of the decay, and so the flavor-SU(3) quantum numbers of the final state must correspond to the symmetric product of two nonets; that is, a symmetric octet $\mathbf{8}_S$ and a 27-plet.[1,4] If nonet symmetry is broken, there will be an additional octet final state $\mathbf{8}_B$ formed from the product of the singlet in one nonet times the octet component of the other. When we combine the triplet $\bar{D}_i^4$ $(i=1,2,3)$ representing the charmed mesons with $\mathbf{8}_S$, we can form one **6*** representation and one **15**, but when we combine it with the **27** we can only form a **15**. There are therefore only three independent amplitudes[4] in the case of nonet symmetry; when the nonet-breaking final state $\mathbf{8}_B$ is included, there will be one additional **6*** amplitude and one extra **15**. The expressions for each decay mode in terms of these amplitudes are shown in Table I.

We can derive the relations between various decay modes either directly from Table I, or by considering specific selection rules. From the isospin selection rule $\Delta T=1$, we obtain two sum rules:

$$A(D^0\to\pi^+K^-)+\sqrt{2}A(D^0\to\pi^0\bar{K}^0)=A(D^+\to\pi^+\bar{K}^0) \tag{1}$$

and

$$A(D_s\to\pi^+\pi^0)=0\ , \tag{2}$$

where $A(D\to XY)$ denotes the amplitude for decay into pseudoscalar mesons $X+Y$. The first sum rule is a well-known consequence of the $\Delta T=1$ rule and it has already been used to show that there are significant final-state interaction phase shifts in the $\pi\bar{K}$ decay modes.[5] The second sum rule is a simple consequence of the fact that the isovector combination of two pions is the vector product of two vectors in isospace, and therefore antisymmetric.

From the U-spin selection rule $\Delta U=1$, we obtain two more sum rules:

TABLE I. Amplitudes and phase-space factors for specific decay modes of charmed mesons into two pseudoscalar mesons. The symbols S and T denote the symmetric **8** and **27** final states, respectively, in the nonet symmetry scheme and the subscripts refer to the overall SU(3) transformation properties of the particular term in the effective Hamiltonian; B refers to the nonet-breaking octet final state. The phase-space factor is proportional to the center-of-mass momentum.

Mode	Phase-space factor	S_6	S_{15}	T_{15}	B_6	B_{15}
$D^+\to\pi^+\bar{K}^0$	0.92	0	0	2	0	0
$D^0\to\pi^+K^-$	0.92	1	1	$\frac{4}{5}$	0	0
$D^0\to\pi^0\bar{K}^0$	0.92	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$3\sqrt{2}/5$	0	0
$D^0\to\eta_8\bar{K}^0$	0.83	$-1/\sqrt{6}$	$-1/\sqrt{6}$	$\sqrt{6}/5$	0	0
$D^0\to\eta_1\bar{K}^0$	0.61	$2/\sqrt{3}$	$2/\sqrt{3}$	0	1	1
$D_s\to\pi^+\pi^0$		0	0	0	0	0
$D_s\to K^+\bar{K}^0$	0.86	-1	$+1$	$\frac{4}{5}$	0	0
$D_s\to\pi^+\eta_8$	0.92	$-\sqrt{2/3}$	$+\sqrt{2/3}$	$-2\sqrt{6}/5$	0	0
$D_s\to\pi^+\eta_1$	0.76	$2/\sqrt{3}$	$2/\sqrt{3}$	0	-1	1

$$A(D_s\to K^+\bar{K}^0)-(\tfrac{3}{2})^{1/2}A(D_s\to\pi^+\eta_8)=A(D^+\to\pi^+\bar{K}^0) \quad (3)$$

and

$$\sqrt{3}\,A(D^0\to\bar{K}^0\eta_8)-A(D^0\to\bar{K}^0\pi^0)=0\ . \quad (4)$$

Here η_8 is the pure octet isosinglet member of the nonet and Eq. (4) is the direct U-spin analogue of Eq. (2) for D_s decay into pions. The sum rules in Eqs. (1)–(4) are independent of nonet symmetry and hold for all values of the nonet breaking amplitudes in Table I.

We can establish tests for nonet symmetry in these decays by considering final states which belong to the **27** representation and which depend upon only one independent amplitude in the symmetry limit. We thus obtain another two sum rules which hold only when the nonet breaking amplitudes of Table I vanish:

$$A(D_s\to\pi^+\eta_1)-\sqrt{2}\,A(D_s\to\pi^+\eta_8)=\frac{2\sqrt{3}}{5}A(D^+\to\pi^+\bar{K}^0) \quad (5)$$

and

$$2\sqrt{2}\,A(D^0\to\bar{K}^0\eta_8)+A(D^0\to\bar{K}^0\eta_1)=\frac{2\sqrt{3}}{5}(D^+\to\pi^+\bar{K}^0)\ , \quad (6)$$

where η_1 is the pure SU(3)-singlet member of the pseudoscalar nonet.

Equations (1)–(6) contain the full set of nonet symmetry and flavor-SU(3) predictions for the Cabibbo-allowed decays of charmed mesons into two pseudoscalars, and they hold for all admixtures of the **6*** and **15** representations in the effective Hamiltonian. Therefore, if they are not satisfied experimentally we will have to conclude either that the effects of flavor-SU(3) breaking in the decays of charmed mesons are much greater than expected, or that nonet symmetry is not valid. Let us now see what are the consequences of these sum rules.

As noted above, the first sum rule has been used on conjunction with the experimental branching ratios for the modes of $D\to\pi\bar{K}$ to show that there are large phase shifts engendered by final-state interactions in the $\pi\bar{K}$ system.[5] This suggests that we should treat the decay amplitudes as complex numbers and the sum rules as triangular relations in the Argand diagram. As for Eq. (2), the decay mode of the D_s into $\pi^+\pi^0$ has not been observed, and we presume that the amplitude does indeed vanish.

The four remaining sum rules involve decays into the η_8 and η_1, and they provide us with bounds on these modes. We use the canonical octet-singlet mixing for the physical mass eigenstates η and η',

$$\eta=\eta_8\cos\theta-\eta_1\sin\theta\ ,$$
$$\eta'=\eta_8\sin\theta+\eta_1\cos\theta\ , \quad (7)$$

and consider the values of $-10°$ and $-20°$ for the angle θ (Ref. 6). From Eqs. (4), (6), and (7) we then find that

$$A(D^0\to\bar{K}^0\eta)=\frac{1}{\sqrt{3}}(\cos\theta+\sqrt{8}\sin\theta)A(D^0\to\bar{K}^0\pi^0)-\frac{\sqrt{12}}{5}A(D^+\to\pi^+\bar{K}^0)\ , \quad (8)$$

$$A(D^0\to\bar{K}^0\eta')=\frac{1}{\sqrt{3}}(\sin\theta-\sqrt{8}\cos\theta)A(D^0\to\bar{K}^0\pi^0)+\frac{\sqrt{12}}{5}A(D^+\to\pi^+\bar{K}^0)\ . \quad (9)$$

To extract bounds on the η and η' decay modes from Eqs. (8) and (9), we make use of the central values of the measured branching ratios for[7,8]

$$B(D^0\to\pi^0\bar{K}^0)=1.89\pm0.2\pm0.2\%\ ,$$
$$B(D^+\to\pi^+\bar{K}^0)=3.2\pm0.5\pm0.2\%\ , \quad (10)$$

together a ratio of 2.5 for the D^+ to D^0 lifetimes[7,8] and the phase-space factors shown in Table I. From the triangular inequalities implied by Eqs. (8) and (9), we then find that

$$0.06\ (0.05)\% \le B(D^0 \to \bar{K}^0\eta) \le 0.25\ (0.08) \quad \text{for } \theta=-10^\circ\ (-20^\circ) \tag{11}$$

and

$$1.7\ (1.8)\% \le B(D^0 \to \bar{K}^0\eta') \le 6.5\ (6.5)\ , \quad \text{for } \theta=-10^\circ\ (-20^\circ)\ . \tag{12}$$

The measured branching ratio for $D^0 \to \bar{K}^0\eta$ is[7,8] $(1.5\pm0.7)\%$ and is hardly consistent with Eq. (11). In order to raise the upper bound in Eq. (11) without violating flavor SU(3), we must include the nonet-breaking combination B_6+B_{15} in the effective interaction (see Table I). Since it will increase the upper bound on the η branching ratio through the η_1 component, this combination will yield a proportionally larger change in the bounds on the η' mode in Eq. (12). The branching ratio for $D^0 \to \bar{K}^0\eta'$, which has yet to be measured, will therefore be an interesting test for this manner of nonet-symmetry breaking.

Turning to the decays of the D_s, we find that we can use Eqs. (3) and (5) in two different ways. One is to eliminate the amplitude for $D^+ \to \pi^+\bar{K}^0$ and obtain a relation among D_s decay modes alone; this will yield bounds on the relative branching ratios of these modes. The other way is to use the two equations in conjunction with Eq. (7) to set bounds on the absolute branching ratios of these modes via the measured branching ratio for the D^+ mode.

Eliminating the D^+ amplitude from Eqs. (3) and (5), and using Eq. (7), we obtain the sum rule

$$\left[\cos\theta-\frac{\sqrt{8}}{5}\sin\theta\right] A(D_s \to \pi^+\eta') = \left[\frac{\sqrt{8}}{5}\cos\theta+\sin\theta\right] A(D_s\to\pi^+\eta) + \frac{\sqrt{12}}{5} A(D_s \to K^+\bar{K}^0)\ . \tag{13}$$

With the aid of the phase-space factors in Table I we can now use this sum rule to obtain a triangular inequality for the decay mode $D_s \to \pi^+\eta'$ in terms of the other two D_s modes:

$$\left[0.91\left[\frac{\sqrt{8}}{5}\cos\theta+\sin\theta\right][B(D_s\to\pi^+\eta)]^{1/2}-0.65[B(D_s\to K^+\bar{K}^0)]^{1/2}\right]^2 \le \left[\cos\theta-\frac{\sqrt{8}}{5}\sin\theta\right]^2 B(D_s\to\pi^+\eta') \le \left[0.91\left[\frac{\sqrt{8}}{5}\cos\theta+\sin\theta\right][B(D_s\to\pi^+\eta)]^{1/2}+0.65[B(D_s\to K^+\bar{K}^0)]^{1/2}\right]^2 . \tag{14}$$

The experimental data on D_s decays suggest that[7-9]

$$B(D_s \to K^+\bar{K}^0) \approx 0.3 B(D_s\to\pi^+\eta)\ , \tag{15}$$

and so we obtain the following bounds on the ratio of the $\pi^+\eta'$ to $\pi^+\eta$ final states:

$$0\ (0.03) \le \frac{B(D_s\to\pi^+\eta')}{B(D_s\to\pi^+\eta)} \le 0.43\ (0.22) \quad \text{for } \theta=-10^\circ\ (-20^\circ)\ . \tag{16}$$

Recent data from Mark II (Ref. 10) indicate that the empirical ratio is significantly greater than the upper bound. To change the upper bound without violating flavor symmetry, we must include the combination B_{15}-B_6 in the effective Hamiltonian (see Table I); since this combination affects only the η_1 amplitudes, it can be chosen so as to have a greater impact on the η' decay mode than on the η mode.

Now let us consider bounds on the absolute values of these branching ratios. From Eqs. (3), (5), and (7), we obtain expressions for the D_s decay amplitudes into $\pi^+\eta$ and $\pi^+\eta'$ final states:

$$A(D_s\to\pi^+\eta) = (\tfrac{2}{3})^{1/2}(\cos\theta-\sqrt{2}\sin\theta) A(D_s\to K^+\bar{K}^0) - (\tfrac{2}{3})^{1/2}\left[\cos\theta-\frac{\sqrt{8}}{5}\sin\theta\right] A(D^+\to\pi^+\bar{K}^0)\ , \tag{17}$$

$$A(D_s\to\pi^+\eta') = (\tfrac{2}{3})^{1/2}(\sin\theta+\sqrt{2}\cos\theta) A(D_s\to K^+\bar{K}^0) - (\tfrac{2}{3})^{1/2}\left[\sin\theta+\frac{\sqrt{8}}{5}\cos\theta\right] A(D^+\to\pi^+\bar{K}^0)\ . \tag{18}$$

Using the central value of the $D^+\to\pi^+\bar{K}^0$ branching ratio in Eq. (10) together with a D^+ to D_s lifetime ratio of 2.5 and the phase-space factors of Table I, we find the following triangular inequalities for the branching ratios of the D_s decay modes:

$$\{[B(D_s\to\pi^+\eta)]^{1/2}-0.84(\cos\theta-\sqrt{2}\sin\theta)[B(D_s\to K^+\bar{K}^0)]^{1/2}\}^2$$
$$\le 0.27\left[\cos\theta-\frac{\sqrt{8}}{5}\sin\theta\right]^2 B(D^+\to\bar{K}^0\pi^+)=0.086\left[\cos\theta-\frac{\sqrt{8}}{5}\sin\theta\right]^2\%$$
$$\le\{[B(D_s\to\pi^+\eta)]^{1/2}+0.84(\cos\theta-\sqrt{2}\sin\theta)[B(D_s\to K^+\bar{K}^0)]^{1/2}\}^2 \quad (19)$$

and

$$\{[B(D_s\to\pi^+\eta')]^{1/2}-0.77(\sin\theta+\sqrt{2}\cos\theta)[B(D_s\to K^+\bar{K}^0)]^{1/2}\}^2$$
$$\le 0.22\left[\sin\theta+\frac{\sqrt{8}}{5}\cos\theta\right]^2 B(D^+\to\pi^+\bar{K}^0)=0.07\left[\sin\theta+\frac{\sqrt{8}}{5}\cos\theta\right]^2\%$$
$$\le\{[B(D_s\to\pi^+\eta')]^{1/2}+0.77(\sin\theta+\sqrt{2}\cos\theta)[B(D_s\to K^+\bar{K}^0)]^{1/2}\}^2 . \quad (20)$$

Using Eq. (15), we can turn Eq. (19) into a pair of bounds on the $\pi^+\eta$ and $K^+\bar{K}^0$ branching ratios

$$\begin{aligned} &0.41\ (0.40)\% \le B(D_s\to\pi^+\eta)\le 5.3\ (9.2)\% ,\\ &0.12\ (0.12)\% \le B(D_s\to K^+\bar{K}^0)\le 1.6\ (2.8)\% , \end{aligned} \quad (21)$$

for $\theta=-10°\ (-20°)$, respectively.

Since the $D_s\to K^+\bar{K}^0$ branching ratio is roughly equal to that for $D_s\to\pi^+\phi$ (Refs. 7 and 8), the bounds placed on it in Eq. (21) are not too far out of line with our previous prediction of a small branching ratio for this parity-violating mode under the assumptions of nonet and flavor-SU(3) symmetry.[11] However, the symmetry bounds on both the $\pi\phi$ and the $\pi\eta$ [see Eq. (21)] modes of D_s decay are smaller than the experimental values discussed in the literature,[8,10] and so it would again seem that nonet symmetry is broken in the decays of charmed mesons. Whether we must also break flavor SU(3) remains to be seen; it will depend upon whether the two additional amplitudes B_6 and B_{15} will be sufficient to account for all the branching ratios of the D^0 and the D_s into final states containing η and η' mesons.

We can summarize our discussion by saying that the branching ratios of D_s decay relative to one another are not consistent with nonet symmetry, and neither are their absolute values. In the case of the D^0 meson, nonet symmetry implies a set of bounds on the decay modes $\bar{K}^0\eta$ and $\bar{K}^0\eta'$ [Eqs. (11) and (12)]; of these bounds only that for $D^0\to\bar{K}^0\eta$ [Eq. (11)] can be tested at present, and it too appears to be inconsistent with existing data.

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