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Predictions of the standard model for $B_c^{\pm}$ weak decays

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The semileptonic (inclusive and some exclusive) and nonleptonic (inclusive and two-body) $B_c^{\pm}$ decays are discussed in detail. All these decay widths are estimated by use of spectator-model dominance. The most promising two-body decay channels for searching for $B_c^{\pm}$ are also discussed.

I. INTRODUCTION

Weak decays of heavy flavors have been discussed extensively in the literature.[1] Up to now there has been no detailed discussion about $B_c^{\pm}$ ($\bar{b}c$ and $b\bar{c}$ bound states) decays. The reason for this delay is twofold: first, people thought that there would be no experimental data on $B_c^{\pm}$ available in recent years; second, the detection of $B_c^{\pm}$ decays is more complicated because of the small branching ratios and some of the final states of $B_c^{\pm}$ decays involve charmed and bottom mesons D, F, $B_d B_s$, $B_u^{\pm}$, etc., and these mesons themselves are already difficult to detect. But, recent proposals of B-meson factories at the Fermilab Tevatron (thousands of thousands $B_c^{\pm}$'s are produced every minute), University of California at Los Angeles, and KEK in Japan raised the hope for measuring $B_c^{\pm}$ decays in the near future. For future use we had better get the standard-model predictions on $B_c^{\pm}$ decays beforehand. On the other hand, $B_c^{\pm}$ physics has its own characteristics. For instance, $B_c^{\pm}$ carry both charm and bottom quantum numbers and are the heaviest mesons with explicit charm and bottom quantum numbers. In addition, in $B_c^{\pm}$, both c (or $\bar{c}$) quark and $\bar{b}$ (or b) quark decay, and the lifetime of the c quark is shorter than that of the $\bar{b}$ quark. But, in D, F, B_d, B_s, $B_u^{\pm}$, only the heavy quark decays. Although the s quark is not stable, the lifetime of the s quark is much longer than that of c and b quarks; so in fact, the s quark inside F, B_s, does not decay. This difference between $B_c^{\pm}$ and other heavy mesons is worth studying. For $B_c^{\pm}$ decays both $\bar{b}$ and c can be spectators. This is also an interesting place for testing the validity of the spectator-model dominance.

There are also many other properties of the $B_c^{\pm}$ decays which need to be explored. In this paper we first discuss the weak decays of $B_c^{\pm}$ by taking B_c^{+} as an example and totally ignore CP violation. We shall leave if for discussion elsewhere. In Sec. II, we first give a brief account for semileptonic decays. Section III deals with inclusive nonleptonic decays. In Sec. IV we discuss all possible nonleptonic two-body decays caused by c- and $\bar{b}$-quark decays. In Sec. V, we give a short discussion and conclusion.

II. SEMILEPTONIC B_c^{+} DECAYS

The spectator model has been justified by the charm-decay data for exclusive nonleptonic two-body and semileptonic decays.[2] People also applied this model to study B_d, $B_u^{\pm}$ decays.[2] There is a common belief that the heavier the mesons are, the better the spectator model applies. So, we assume spectator model dominance throughout this paper and leave its justification to the future experimental tests. Here in B_c^{+} decays, both c quark and $\bar{b}$ quark can be spectators.

According to Rückl,[3] the inclusive semileptonic widths, including QCD corrections, mass corrections, and radiative corrections, are the following. For c decay ($\bar{b}$ as a spectator),

$$\Gamma_{\rm SL}^{(c)}=\sum_{\substack{l=e,\mu\\ q=s,d}}|V_{cq}|^2 I\left[\frac{m_q^2}{m_c^2},\frac{m_l^2}{m_c^2},0\right]\left[1-\frac{2\alpha_s(m_c)}{3\pi}f\left[\frac{m_q^2}{m_c^2}\right]\right]\frac{G_F^2 m_c^5}{192\pi^3}\ . \tag{1}$$

For $\bar{b}$ decay (c as a spectator),

$$\Gamma_{\rm SL}^{(\bar{b})}=\sum_{\substack{l=e,\mu,\tau\\ q=c,u}}|V_{bq}|^2 I\left[\frac{m_q^2}{m_b^2},\frac{m_l^2}{m_b^2},0\right]\left[1-\frac{2\alpha_s(m_b)}{3\pi}f\left[\frac{m_q^2}{m_b^2}\right]\right]\frac{G_F^2 m_b^5}{192\pi^3}\ , \tag{2}$$

where

$$I(x,y,z)=12\int_{(x+y)^2}^{(1-z)^2}\frac{d\xi}{\xi}(\xi-x^2-y^2)(1+z^2-\xi)[\lambda(1,z^2,\xi)\lambda(\xi,x^2,y^2]^{1/2}\ ,$$

$$\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc,\quad I(x,0,0)\equiv g(x)=1-8x^2-24x^2\ln x+8x^6-x^8\ ,$$

$f(x)$ is a correction function. Both $g(x)$ and $f(x)$ can be found in the table in Ref. 4.

For the numerical results we need to know the Kobayashi-Maskawa (KM) matrix elements and quark masses. We take

$$s_1=0.225\ ,\quad c_1=0.974\ ,\quad s_2\sim s_1^2,\ \ s_3\sim 0.5 s_2\ . \tag{3}$$

For the value of $|V_{cb}|^2$, we do not have the precise number. But several numbers extracted from the B semileptonic decays are available[5,6] at present. In Ref. 5, $|V_{cb}|=0.041\pm0.004\pm0.005$. In Ref. 6, $|V_{cb}|=0.043$ or 0.052. Here we use its average value

$$|V_{cb}|\simeq 0.045,\quad |V_{cb}|^2\simeq 2.03\times 10^{-3}\ . \tag{4}$$

For $|V_{ub}|^2$ we take $|V_{ub}|^2=s_1^2 s_3^2\approx 0.25 s_1^6\approx 3.24\times 10^{-5}$. All these values are not "precise"; there are still some uncertainties, but the order of magnitudes is correct.

Because the semileptonic decay widths are sensitive to quark masses, we use here two sets of different quark masses. One set is for the current-quark masses used in Ref. 7. The other is constituent-quark masses used in Ref. 8. We refer to them as (A) and (B), respectively:

$$(A)\quad m_u=5\ \text{MeV},\quad m_d=7.5\ \text{MeV}\ ,\quad m_s=150\ \text{MeV},\quad m_c=1.4\ \text{GeV}\ ,\quad m_b=4.6\ \text{GeV}\ ,\quad \alpha_s(m_c)\sim 0.39,\quad \alpha_s(m_b)\sim 0.26\ ;$$

$$(B)\quad m_u=m_d=0.35\ \text{GeV},\quad m_s=0.55\ \text{GeV}\ ,\quad m_c=1.7\ \text{GeV},\quad m_b=4.9\ \text{GeV}\ ,\quad \alpha_s(m_c)\sim 0.35,\quad \alpha_s(m_b)\sim 0.256\ .$$

We also include the corresponding values of α_s in each case. Again, there are also uncertainties for the quark masses.

From all these parameters we can calculate the inclusive semileptonic decay widths by Eqs. (1) and (2). We can also calculate the decay widths for any specific channel, for example, $c\to e^+\nu_e X$, $c\to\mu^+\nu_\mu X$, $\bar{b}\to e^+\nu_e X,\ldots$, etc., from individual terms of $l=e$, or μ, or τ in Eqs. (1) and (2). We list all these results in Table I(a). The total B_c^+ semileptonic decay width is

$$\Gamma_{\rm SL}(B_c^+)=\Gamma_{\rm SL}^{(c)}+\Gamma_{\rm SL}^{(\bar{b})}=\begin{cases}57.58\times 10^{10}\ \text{s}^{-1} & \text{for case } (A)\ ,\\ 117.76\times 10^{10}\ \text{s}^{-1} & \text{for case } (B)\ .\end{cases} \tag{5}$$

From Table I(a) we can also see that the contribution of c-quark decay to $\Gamma_{\rm SL}(B_c^+)$ is the same order of magnitude as that of $\bar{b}$-quark decay.

In addition, theory and experiment suggest that[5,6] the main exclusive semileptonic decay channels are $B_c^+\to\eta_c e^+\nu_e$, $B_c^+\to\psi e^+\nu_e$ for $\bar{b}\to\bar{c}$ decay and $B_c^+\to B_s^0 e^+\nu_e$, $B_c^+\to B_s^{*0}e^+\nu_e$ for $c\to s$ decay. In order to calculate these decay widths we need to know some form factors:[8]

$$\langle\eta_c|\bar{b}\gamma_\mu(1-\gamma_5)C|B_c^+\rangle=\left[P_{B_C}+P_{\eta_C}-\frac{m_{B_C}^2-m_{\eta_C}^2}{q^2}\right]_\mu F_1(q^2)+\frac{m_{B_C}^2-m_{\eta_C}^2}{q^2}q_\mu F_0(q^2),\quad q=P_{B_C}-P_{\eta_C}=P_e+P_{\nu_e}\ , \tag{6}$$

$$\begin{aligned}\langle\psi|\bar{b}\gamma_\mu(1-\gamma_5)C|B_C^+\rangle=&\frac{2}{m_{B_C}+m_\psi}\epsilon_{\mu\nu\rho\sigma}\epsilon^\nu P_{B_C}^\rho P_\psi^\sigma V(q^2)\\&+i\left[\epsilon_\mu(m_{B_C}+m_\psi)A_1(q^2)-\frac{\epsilon\cdot q}{m_{B_C}+m_\psi}(P_{B_C}+P_\psi)_\mu A_2(q^2)-\frac{\epsilon\cdot q}{q^2}q_\mu 2m_\psi A_3(q^2)\right]\\&+i\frac{\epsilon\cdot q}{q^2}2m_\psi q_\mu A_0(q^2)\ .\end{aligned} \tag{7}$$

We have a similar expression for $B_c^+\to B_s^0$, $B_c^+\to B_s^{*0}$ transitions. All the pole masses of the form factors can be found in Refs. 2 and 8. The corresponding overlap integrals $h_0\equiv h_1$, h_V, h_{A_1}, h_{A_2}, h_{A_0}, etc., can be found in Table II.

Neglecting the electron mass and taking ν_e massless, we can compute the widths of $B_C^+\to\eta_C e^+\nu_e$, $B_C^+\to\psi e^+\nu_e$, $B_C^+\to B_s^0 e^+\nu_e$, and $B_C^+\to B_s^{*0}e^+\nu_e$. We list them in Table I(b) for three cases of $\omega=0.4$, 0.6, 0.8, respectively. Again, we take $|V_{cb}|\sim 0.045$.

III. INCLUSIVE NONLEPTONIC B_c^+ DECAYS

Both c- and $\bar{b}$-quark decays can contribute to the B_c^+ nonleptonic decay width. According to Ref. 3 we can use heavy-quark hadronic decay width to approximate the in-

TABLE I. (a) Semileptonic decay widths of B_c^+ meson for both current-quark-mass and constituent-quark-mass inputs. ($|V_{cb}|=0.045$.) (b) Some exclusive semileptonic decay widths of B_c^+ for constituent-quark-mass input. (For case B only, $|V_{cb}|\sim 0.045$.)

(a)

Decay width	Case (A) (10^{10} s^{-1})	Case (B) (10^{10} s^{-1})
$\Gamma(c\to e^+\nu_e X)$	13.07	37.68
$\Gamma(c\to \mu^+\nu_\mu X)$	13.07	37.68
$\Gamma_{SL}^{(c)}=\sum_{l=e,\mu}\Gamma(c\to l^+\nu_l X)$	26.13	75.36
$\Gamma(\bar b\to e^+\nu_e X)$	11.08	14.84
$\Gamma(\bar b\to \mu^+\nu_\mu X)$	11.08	14.84
$\Gamma(\bar b\to \tau^+\nu_\tau X)$	9.29	12.72
$\Gamma_{SL}^{(\bar b)}=\sum_{l=e,\mu,\tau}\Gamma(b\to l^+\nu_l X)$	31.45	42.40
$\Gamma(B_c^+\to e^+\nu_e X)$	24.15	52.52
$\Gamma(B_c^+\to \mu^+\nu_\mu X)$	24.15	52.52
$\Gamma(B_c^+\to \tau^+\nu_\tau X)$	9.29	17.72
$\Gamma_{SL}^{(B_c^+)}=\Gamma_{SL}^{(c)}+\Gamma_{SL}^{(\bar b)}$	57.58	117.76

(b)

Process	$\Gamma(\omega=0.4)$ (10^{10} s^{-1})	$\Gamma(\omega=0.6)$ (10^{10} s^{-1})	$\Gamma(\omega=0.8)$ (10^{10} s^{-1})
$B_c^+\to\eta_c e^+\nu_e$	0.12	0.72	1.38
$B_c^+\to\psi e^+\nu_e$	0.30	2.10	4.38
$B_c^+\to B_s^0 e^+\nu_e$	0.72	3.18	4.57
$B_c^+\to B_s^{*0} e^+\nu_e$	1.52	4.16	6.26

clusive nonleptonic decay width: namely,

$$\Gamma_{NL}(Q\to\text{hadrons})=\sum_{q_1,q_2,q_3}|V_{Qq_1}|^2|V_{q_2q_3}|^2 I\left[\frac{m_{q_1}^2}{m_Q^2},\frac{m_{q_2}^2}{m_Q^2},\frac{m_{q_3}^2}{m_Q^2}\right]\times\frac{2c_+^2+c_-^2}{3}\left[1+\frac{2\alpha_s(m_Q)}{3\pi}h\right]3\frac{G_F^2m_Q^5}{192\pi^3}\,, \tag{8}$$

where Q represents a heavy quark (c or $\bar b$). The factor[3]

$$\frac{2c_+^2+c_-^2}{3}\left[1+\frac{2\alpha_s(m_Q)}{3\pi}h\right]\approx\begin{cases}1.60 & \text{for charm decay}\,,\\ 1.18 & \text{for bottom decay}\,,\end{cases} \tag{9}$$

is not sensitive to quark masses, and the summation in Eq. (8) should go through all possible final-quark states.

It is easy to compute the inclusive nonleptonic widths for both c and $\bar b$ decays. We find

$$\Gamma_{NL}(c\to\text{hadrons})\approx 4.8\frac{G_F^2m_c^5}{12\pi^3}\,, \tag{10}$$

$$\Gamma_{NL}(\bar b\to\text{hadrons})\approx 3.54\frac{G_F^2m_b^5}{192\pi^3}|V_{cb}|^2\left[I\left[\frac{m_c^2}{m_b^2},0,0\right]+I\left[\frac{m_c^2}{m_b^2},0,\frac{m_c^2}{m_b^2}\right]\right]. \tag{11}$$

The total inclusive nonleptonic decay width of B_c^+ is

$$\Gamma_{NL}(B_c^+\to\text{hadrons})=\Gamma_{NL}(c\to\text{hadrons})+\Gamma_{NL}(\bar b\to\text{hadrons})\,. \tag{12}$$

All these numerical results are presented in Table III. Again, we give the results for the case of current-quark mass and that of constituent-quark mass. The widths are sensitive to quark masses, and the contribution from c-quark decay is also the same order of magnitude as that from $\bar b$ decay.

IV. NONLEPTONIC TWO-BODY DECAYS

We shall follow Refs. 2 and 8 to discuss nonleptonic two-body decays. For Cabibbo-allowed c-quark decay we use the effective Hamiltonian

$$H_{\text{eff}}=\frac{G_F}{\sqrt2}V_{cs}^*V_{ud}[a_1(m_c)(\bar u\Gamma_\mu d)_H(\bar s\Gamma^\mu c)_H+a_2(m_c)(\bar u\Gamma_\mu c)_H(\bar s\Gamma^\mu d)_H]\,, \tag{13}$$

where $\Gamma_\mu=\gamma_\mu(1-\gamma_5)$ is the left-handed chirality projector. For the values of $a_1(m_c)$ and $a_2(m_c)$ we use QCD calculated values.[3] That is,

$$a_1(m_c)\approx c_1(m_c)\approx 1.21\,,\quad a_2(m_c)\approx c_2(m_c)\approx -0.42\,. \tag{14}$$

These values are very close to that used by Refs. 2 and 8.

We omit all other effective Hamiltonians for all c of $\bar b$ decays. But for $\bar b$ decay we again use the QCD value for the parameters $a_1(m_b)$, $a_2(m_b)$:

$$a_1(m_b)\approx c_1(m_b)\approx 1.1\,,\quad a_2(m_b)\approx c_2(m_b)\approx -0.24\,. \tag{15}$$

All the definitions of the constants a_1,a_2,c_1,c_2 can be found in Refs. 2 and 3.

In order to compute the decay amplitudes, we need to know the relevant meson form factors and overlap integrals of meson wave functions. We use the same method as in Ref. 8, namely, by use of relativistic quark wave functions in the infinite-momentum frame.

For the orbital part of the meson wave function, we take[8]

$$\phi_m(\mathbf{P}_T,x)=N_m\sqrt{x(1-x)}\exp(-\mathbf{P}_T^2/2\omega^2)\exp\left[-\frac{m_2}{2\omega^2}\left[x-\frac12-\frac{m_{q_1}^2-m_{q_2}^2}{2\omega^2}\right]\right], \tag{16}$$

TABLE II. Form factors at $q^2=0$ for $P\to P$ and $P\to V$ transitions with ω in units of GeV.

Transition	Overlap integral	$\omega=0.4$	$\omega=0.5$	$\omega=0.6$	$\omega=0.7$	$\omega=0.8$	$\omega=0.9$	$\omega=1.0$
$B_c^+\to B_s^0$	h_0	0.340	0.585	0.715	0.801	0.859	0.898	0.925
$B_c^+\to B_u^+$	h_0	0.320	0.516	0.662	0.763	0.831	0.878	0.91
$B_c^+\to\eta_c$	h_0	0.170	0.305	0.420	0.551	0.583	0.640	0.687
$B_c^+\to F^+$	h_0	0.0190	0.0793	0.172	0.273	0.370	0.455	0.582
$B_c^+\to D^+$	h_0	0.0140	0.0667	0.154	0.255	0.352	0.439	0.515
$B_c^+\to B_s^{0*}$	h_{A_0}	0.432	0.611	0.734	0.815	0.869	0.906	0.931
	h_V	3.296	4.697	5.657	6.263	6.612	6.782	6.833
	h_{A_1}	0.478	0.682	0.821	0.909	0.959	0.984	0.992
	h_{A_2}	1.057	1.560	1.909	2.079	2.086	1.965	1.754
$B_c^+\to B_u^{+*}$	h_{A_0}	0.349	0.542	0.682	0.777	0.842	0.885	0.916
	h_V	2.890	4.516	5.690	6.446	6.887	7.109	7.187
	h_{A_1}	0.370	0.578	0.729	0.826	0.882	0.910	0.920
	h_{A_2}	0.602	0.978	1.240	1.350	1.321	1.183	0.973
$B_c^+\to\psi$	h_{A_0}	0.156	0.290	0.408	0.502	0.576	0.635	0.684
	h_V	0.222	0.417	0.591	0.738	0.860	0.965	1.059
	h_{A_1}	0.156	0.293	0.416	0.519	0.605	0.679	0.745
	h_{A_2}	0.156	0.298	0.431	0.551	0.660	0.762	0.862
$B_c^+\to D^{+*}$	h_{A_0}	0.0149	0.0686	0.156	0.257	0.354	0.440	0.516
	h_V	0.0227	0.105	0.224	0.405	0.568	0.772	0.864
	h_{A_1}	0.0135	0.0628	0.145	0.242	0.338	0.430	0.515
	h_{A_2}	0.0122	0.0574	0.134	0.227	0.324	0.420	0.513
$B_c^+\to F^{+*}$	h_{A_0}	0.0199	0.0808	0.173	0.274	0.370	0.455	0.529
	h_V	0.0296	0.121	0.264	0.424	0.582	0.730	0.866
	h_{A_1}	0.0185	0.0758	0.165	0.265	0.363	0.456	0.540
	h_{A_2}	0.0170	0.0708	0.156	0.255	0.356	0.456	0.552

where we use the constituent-quark masses of case (B). $\langle \mathbf{P}_T^2\rangle=\omega^2$ is the average transverse quark momentum squared. All other parameters are explained in Ref. 8 where $\omega=0.40$ GeV is taken according to the hint from D-meson decay data. They also apply $\omega=0.40$ GeV to $B_d, B_u^{\pm}$ decays. But in our case of $B_c^{\pm}$ decay, there are no data available at the present time and we do not know whether we should take $\omega=0.40$ GeV for $B_c^{\pm}$ decays. But we definitely have some reasoning to take bigger ω instead of $\omega=0.40$ GeV. The reason is the following. For the decays of $B_c^+\to B_s^0\pi^+$ or $B_c^+\to B_u^+\bar{K}^0$, the overlap integral of the wave functions of $B_c^+-B_s^0$, or of $B_c^+-B_u^+$ should be close to unity because both mesons are very heavy and have close masses. In other words, these integrals, i.e., $h_0\equiv h_1$, should be close to the result using the nonrelativistic harmonic-oscillator wave functions:

$$h_0\equiv h_1\simeq\left[\frac{2m_{B_s}m_{B_c}}{m_{B_c}^2+m_{B_s}^2}\right]^{1/2}\simeq 0.99\ . \tag{17}$$

In our case of relativistic harmonic-oscillator wave func-

TABLE III. Inclusive nonleptonic B_c^+ decay widths for both current-quark-mass and constituent-quark-mass inputs.

Decay width	Case (A) (10^{10} s^{-1})	Case (B) (10^{10} s^{-1})
$\Gamma_{\rm NL}(c\to{\rm hadrons})$	89.64	236.55
$\Gamma_{\rm NL}(\bar{b}\to{\rm hadrons})$	92.72	119.34
$\Gamma_{\rm NL}(B_c^+\to{\rm hadrons})$	182.36	355.89

TABLE IV. Widths of Cabibbo-allowed c decays in units of 10^{10} s^{-1}.

Process	$\Gamma(\omega=0.4)$	$\Gamma(\omega=0.6)$	$\Gamma(\omega=0.8)$
$B_s^0\pi^+$	2.71	12.00	17.32
$B_u^+\bar{K}^0$	0.563	2.41	3.80
$B_s^0\rho^+$	1.63	7.19	10.38
$B_u^+\bar{K}^{0*}$	0.206	0.882	1.39
$B_s^{0*}\pi^+$	3.65	10.54	14.78
$B_u^{+*}\bar{K}^0$	0.450	1.51	2.64
$B_u^{+*}\bar{K}^{0*}$	2.01	7.74	11.35
$B_s^{0*}\rho^+$	18.99	55.91	76.42
Total	30.209	98.182	138.08

TABLE V. Widths of Cabibbo-suppressed c decays in units of 10^{10} s^{-1}.

Process	$\Gamma(\omega=0.4)$	$\Gamma(\omega=0.6)$	$\Gamma(\omega=0.8)$
$B_d^0\pi^+$	0.175	0.751	1.18
$B_s^0K^+$	0.197	0.873	1.26
$B_s^{*0}K^+$	0.187	0.542	0.760
$B_u^+\pi^0$	1.05×10^{-2}	4.53×10^{-2}	7.13×10^{-2}
$B_u^+\phi$	3.43×10^{-4}	1.47×10^{-3}	2.31×10^{-3}
$B_s^0K^{+*}$	7.72×10^{-3}	3.31×10^{-2}	5.21×10^{-2}
$B_d^0\rho^+$	0.195	0.837	1.32
$B_u^+\rho^0$	1.17×10^{-2}	5.04×10^{-2}	7.93×10^{-2}
$B_u^+\omega$	1.10×10^{-2}	4.71×10^{-2}	7.41×10^{-2}
$B_d^{0*}\pi^+$	0.178	0.682	1.04
$B_u^{+*}\pi^0$	1.07×10^{-2}	4.11×10^{-2}	6.25×10^{-2}
$B_d^{0*}\rho^+$	1.05	4.07	5.99
$B_u^{+*}\rho^0$	6.32×10^{-2}	0.245	0.361
$B_u^{+*}\omega$	6.36×10^{-2}	0.246	0.363
Total	2.157	8.453	12.61

tions, the corresponding overlap integral of Eq. (21) for $\omega=0.80$ is

$$h_0 \equiv h_1 \simeq 0.86 \ . \tag{18}$$

All the values of overlap integrals $h_0=h_1$, h_V, h_{A_1}, h_{A_2}, $h_{A_0}=h_{A_3}$ for different ω for the relativistic case are listed in Table II. We shall compute the decay widths for the three cases of $\omega=0.40$, 0.60, 0.80 GeV. Some widths are sensitive to the values of ω because of the overlap integrals. But when $\omega>0.80$ GeV, the changes of h_0, h_{A_1}, h_{A_2}, h_V are not large.

We define the decay constants of pseudoscalar and vector mesons as usual:

$$\langle 0|A^\mu|P\rangle = if_P k_P^\mu \ , \qquad \langle 0|J^\mu|V\rangle = F_V\epsilon^\mu m_V \ . \tag{19}$$

In our numerical computation we use the same values of $f_{\eta_c}=f_K=0.162$ GeV.

We list the decay widths of the Cabibbo-allowed c decays in Table IV, the Cabibbo-suppressed c-quark decays in Table V, the Cabibbo-suppressed $\bar{b}$ decays in Table VI and the Cabibbo-double-suppressed $\bar{b}$ decays in Table VII, respectively.

V. DISCUSSION AND CONCLUSION

From Table II we see that the values of the overlap integrals h_0, h_V, h_{A_1}, h_{A_2}, h_{A_0}, etc., increase with increasing ω. When $\omega<0.7$ GeV, they change quite dramatically. But $\omega\geq0.8$ GeV, they change slowly. If we argue that the overlap integral h_0 for $B_c^+\to B_s^0$ or $B_c^+\to B_u^+$ transition should be close to unity, then $\omega=0.80$ GeV is a reasonable choice. The calculated widths depend on these overlap integrals and increase when ω increases. But after $\omega=0.8$ GeV, the calculated widths should increase slowly. So we take the calculated widths at $\omega=0.8$ as the preferred value. Then looking at Tables IV–VII we can see that the most promising processes of nonleptonic two-body B_c^+ decays are, for Cabibbo-allowed c decays,

$$B_c^+\to B_s^{0*}\rho^+,\ B_s^0\pi^+,\ B_s^{0*}\pi^+,\ B_u^{+*}\bar{K}^{0*},\ B_s^0\rho^+ \ ,$$

TABLE VI. Widths of Cabibbo-suppressed $\bar{b}$ decays in units of 10^{10} s^{-1}.

Process	$\Gamma(\omega=0.4)$	$\Gamma(\omega=0.6)$	$\Gamma(\omega=0.8)$
$\eta_c\pi^+$	2.85×10^{-2}	0.174	0.336
$\bar{D}^0D^+$	2.47×10^{-5}	2.98×10^{-3}	1.56×10^{-2}
η_cF^+	3.69×10^{-2}	0.184	0.301
$\eta_c\rho^+$	7.36×10^{-2}	0.448	0.863
$\psi\pi^+$	5.41×10^{-3}	3.72×10^{-2}	7.42×10^{-2}
$\bar{D}^{0*}D^+$	3.66×10^{-5}	4.42×10^{-3}	2.32×10^{-2}
$\bar{D}^0D^{+*}$	2.12×10^{-5}	2.35×10^{-3}	3.58×10^{-2}
$\psi\rho^+$	7.04×10^{-2}	0.485	0.977
$\bar{D}^{0*}D^{+*}$	8.06×10^{-5}	9.01×10^{-3}	4.78×10^{-2}
$F^{+*}\eta_c$	4.03×10^{-1}	0.215	0.374
ψF^+	1.56×10^{-2}	6.46×10^{-2}	0.080
ψF^{+*}	9.64×10^{-2}	0.445	0.641
Total	0.368	2.08	3.80

TABLE VII. Widths of Cabibbo-double suppressed $\bar{b}$ decays in units of 10^{10} s^{-1}. (For case B only, $|V_{cb}|=0.045$.)

Process	$\Gamma(\omega=0.4)$	$\Gamma(\omega=0.6)$	$\Gamma(\omega=0.8)$
$B_c^+ \to K^+\eta_c$	2.24×10^{-3}	1.36×10^{-2}	2.63×10^{-2}
$\bar{D}^0F^+$	2.30×10^{-6}	1.88×10^{-4}	8.69×10^{-4}
$K^{+*}\eta_c$	3.86×10^{-3}	2.36×10^{-2}	4.55×10^{-2}
$\bar{D}^{0*}F^+$	3.36×10^{-6}	2.76×10^{-4}	1.32×10^{-3}
$K^+\psi$	5.34×10^{-4}	3.66×10^{-3}	7.30×10^{-3}
$\bar{D}^0F^{+*}$	1.90×10^{-6}	1.43×10^{-4}	6.47×10^{-4}
$K^{+*}\psi$	3.93×10^{-3}	2.73×10^{-2}	5.26×10^{-2}
$\bar{D}^{0*}F^{+*}$	1.28×10^{-4}	9.90×10^{-3}	4.66×10^{-2}
$D^+\eta_c$	2.02×10^{-3}	1.02×10^{-2}	1.63×10^{-2}
$D^{+*}\eta_c$	2.35×10^{-3}	1.26×10^{-2}	2.19×10^{-2}
$D^+\psi$	9.45×10^{-4}	3.97×10^{-3}	4.74×10^{-3}
$D^{+*}\psi$	5.23×10^{-3}	2.45×10^{-2}	3.36×10^{-2}
$D^0\pi^+$	5.03×10^{-6}	6.09×10^{-4}	3.18×10^{-3}
$D^0\rho^+$	1.36×10^{-5}	1.66×10^{-3}	8.61×10^{-3}
$D^{0*}\pi^+$	5.44×10^{-6}	6.02×10^{-4}	3.05×10^{-3}
$D^{0*}\rho^+$	1.66×10^{-5}	1.88×10^{-3}	9.57×10^{-3}
$D^+\pi^0$	1.20×10^{-7}	1.45×10^{-5}	7.57×10^{-5}
$D^+\eta$	5.96×10^{-8}	7.20×10^{-6}	3.77×10^{-5}
$D^+\eta'$	5.81×10^{-8}	7.02×10^{-6}	3.68×10^{-5}
$D^+\rho^0$	3.24×10^{-7}	3.95×10^{-5}	2.05×10^{-4}
$D^+\omega^0$	3.24×10^{-7}	3.92×10^{-5}	2.05×10^{-4}
$D^{+*}\pi^0$	1.29×10^{-7}	1.43×10^{-5}	7.26×10^{-5}
$D^{+*}\eta$	6.31×10^{-8}	7.00×10^{-6}	3.56×10^{-5}
$D^{+*}\eta'$	6.14×10^{-8}	6.81×10^{-6}	3.46×10^{-5}
$D^{+*}\rho^0$	3.95×10^{-7}	4.47×10^{-5}	2.28×10^{-4}
$D^{+*}\omega^0$	3.95×10^{-7}	4.47×10^{-5}	2.28×10^{-4}
F^+D^0	$9.10\times10^{-2}\lvert V_{ub}\rvert^2$	$1.46\times10\lvert V_{ub}\rvert^2$	$8.15\times10\lvert V_{ub}\rvert^2$
$F^{+*}D^0$	$1.39\times10^{-1}\lvert V_{ub}\rvert^2$	$2.29\times10\lvert V_{ub}\rvert^2$	$1.29\times10^2\lvert V_{ub}\rvert^2$
F^+D^{0*}	$3.45\times10^{-2}\lvert V_{ub}\rvert^2$	$5.79\lvert V_{ub}\rvert^2$	$3.25\times10\lvert V_{ub}\rvert^2$
$F^{+*}D^{0*}$	$1.44\times10^{-1}\lvert V_{ub}\rvert^2$	$2.16\times10\lvert V_{ub}\rvert^2$	$1.25\times10^2\lvert V_{ub}\rvert^2$

for Cabibbo-suppressed c decays,

$$B_c^+ \to B_d^{0*}\rho^+,\ B_d^0\rho^+,\ B_d^0\pi^+,\ B_d^{0*}\pi^+,\ B_s^0K^+ ,$$

and for Cabibbo-suppressed $\bar{b}$ decays,

$$B_c^+ \to \psi\rho^+,\ \eta_c\rho^+,\ \psi F^{+*} .$$

Especially, $B_c^+ \to \psi\rho^+$ has a very clear signal of $\psi \to l^+l^-$ and $\rho^+ \to \pi^+\pi^0$.

The total B_c^+ nonleptonic two-body decay width from Tables IV–VII is about 154×10^{10} s^{-1}. But from Table III, the inclusive nonleptonic decay width is about 356×10^{10} s^{-1} (both numbers are calculated for constituent-quark mass). About 60% nonleptonic B_c^+ decays are missing. Of course B_c^+ nonleptonic decays are not only two-body decays. But the estimate for the inclusive nonleptonic decay width by use of the spectator model is not trustable. This is because of final-state interactions and the uncertainty of the quark masses. The estimates for the exclusive nonleptonic two-body decays are more reliable. Because we cannot give a good estimation of the lifetime of B_c^+ for the above reason, we cannot estimate the branching ratios in all the decay channels.

The total width of the two-body $\bar{b}$ nonleptonic decays is only about 4×10^{10} s^{-1}. That means that the $\bar{b}$ quark prefers to decay into multibody final states due to the heavier mass.

The results for the inclusive semileptonic decays should be reasonable. For the case of constituent-quark mass the total inclusive semileptonic width is about 120×10^{10} s^{-1} [see Table 1(a)]. The total sum of the exclusive semileptonic decay width listed in Table 1(b) is about 20×10^{10} s^{-1} for $\omega=0.8$. It is $\sim17\%$ of the inclusive one.

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