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P-wave pion scattering in the Los Alamos soliton model

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P-wave pion-nucleon scattering at low and intermediate energies has been calculated in the Los Alamos soliton model that describes the nucleon as a self-consistent soliton state of a dynamic QCD bag or core interacting with its pion field. The *P*-wave phase shifts in the 11 and 33 channels and the inelasticity in the 11 channel are computed within the subspace consisting of at most one physical pion plus one of three soliton states of the dynamic bag or core with its pion field. The soliton states used in the calculation are computed within an approximation that treats only pionic excitations of the self-consistent nontopological soliton ground state corresponding to the nucleon and neglects excited states of the core motion. Within this approximation, the soliton states used are the model's lowest two 11 states and lowest 33 state; these states can be thought of as the nucleon, Roper, and Δ states, where the latter two are in first approximation discrete states in the continuum. The coupling of the excited states to the continuum states gives the states a width in the usual way. The position of the 33 resonance is found to be sensitive to the value of the bag or core radius. The present approximate description of the scattering does not give a resonance in the 11 channel. The implications of these results for future work on the soliton model are discussed.

I. INTRODUCTION

For some time models motivated by the expected features of quantum chromodynamics have provided a useful framework for trying to understand the structure of the nucleon. The first such model was the MIT bag model[1] of the nucleon. This was then improved by including an interaction of the static quantum-chromodynamic bag or core with the pion field;[2] in the resulting cloudy-bag model (CBM), the choice of interaction was motivated by considerations related to chiral current conservation. More recently, the CBM has been modified to include motion of the QCD core. As was shown in Ref. 3, the inclusion of core motion leads to the interpretation of the nucleon as a nontopological soliton ground state of a translation-invariant Hamiltonian; in this picture, the effects of the core motion on the nucleon parameters were shown to be substantial, with a significant decrease in the value of the core radius that gives best overall agreement with the data. This picture of the nucleon as a nontopological-soliton state of a dynamical QCD core interacting with its surrounding pion field is sufficiently different from the cloudy-bag model that we propose to call it the "Los Alamos soliton model" (LASM) of the nucleon. In the CBM the interaction between the static bag or core and the pion field involves transitions between neutron and proton core states as well as transitions to and among isobar core states through pion emission and absorption. The studies so far of the effect of the core motion on the various nucleon parameters were based on a version of the CBM where the isobar core states were not included. We believe that this truncation of the CBM does not compromise the important effects of core motion on observables.

In the LASM, the nucleon consists of a nonstatic core surrounded by its pion field; the core motion and the pion field are determined self-consistently from the Hamiltonian. The core emits and absorbs pion quanta as in the CBM. Like the Skyrme model, the LASM attributes a significant role in nucleon structure to the pion field. In determining the soliton ground state of the system of the nonstatic core with its pion field, the core is treated as a point particle whose motion is determined in a self-consistent way by balancing the kinetic energy of the core motion against the interaction energy of the core with the pion field. In the interaction of the system with the electromagnetic field, the internal structure of the core is included by using a form factor from the MIT bag model corresponding to three massless quarks in their lowest *s*-wave modes, as in the CBM. The spirit of the LASM is thus similar to the nuclear shell model where the nucleons are treated as point particles in the Hamiltonian that determines the wave functions of the various states and treated as extended objects with form factors in their interaction with the electromagnetic field.

The underlying physical picture is that the nucleon has two rather distinct regions. There is an inner region in which the net quark density (quark density minus antiquark density) is substantial and an outer region in which the net quark density is negligibly small. The interesting parameters of this picture are the size or radius R of the inner region and the thickness of the transition between the two regions. In the CBM and the LASM, the transition region is considered to be small enough to be neglected and only the core radius R enters the Hamiltonian. The eventual determination of the nucleon ground state as a particular solution of the QCD Hamiltonian will presumably give the true parameters of the shape of the quark density distribution.

In the approximations that have been used in Ref. 3 to treat the LASM Hamiltonian, only p-wave pions interact with the s-state core. Moreover, in the nucleon ground state of the system, the virtual pions were shown to be in a single (nine isospin-spin substates) p-wave mode, which is called the "internal" pion mode (nine modes). The part of the pion field that is orthogonal to the internal modes is the "external" pion field, and the idea of the separation of the pion field into internal and external parts is that the internal modes interact strongly with the core while the interaction of the external pion field with the core is weak.

In the LASM there are three fundamental modes of excitation of the nucleon state. The first is common to the MIT bag model, the CBM, and the LASM: namely, excitation of the state of the quarks within the core. The second is shared by the CBM and the LASM and consists of excitation of the state of the pion field surrounding the core; this mode of excitation is also treated in the Skyrme soliton model[4] of the nucleon. The third fundamental excitation is unique to the LASM and consists of exciting the quantized motion of the core from its ground state to an excited state.

In this paper we explore some properties of the pionic excitations in the LASM. As has been shown previously,[3] the core radius in the LASM is considerably smaller than that in the CBM, so that it is necessary to recompute the pion-nucleon phase shifts. In the approximations that have been used in Ref. 3 to treat the LASM Hamiltonian, only p-wave pions interact with the s-state core; hence only the p-wave phase shifts are nonzero. Calculation of the s-wave phase shifts will require a more sophisticated treatment of the LASM Hamiltonian; it will be necessary to include p states of the core motion. In addition, the values of the bare pion-nucleon coupling constant are rather large for smaller bag radii. For this reason, we have used a method[5] for the scattering calculation that treats the strongly coupled internal modes of the pion field to all orders while considering only states in which at most a single quantum of the weakly coupled external pion field is present. Within the designated subspace, the calculation is exact and therefore the resulting T matrix is unitary.

The results presented here are limited by several considerations. Most evident are the limitations due to the approximations used in treating the LASM Hamiltonian. Another limitation is the omission of important physical effects from the model in its present state. For example, the effects of the pionic contribution to the nucleon mass have not yet been incorporated in the computations. Preliminary studies indicate that these effects may be substantial and lead to a further decrease in the core radius that gives the best agreement with the data. Also, the core motion has been treated nonrelativistically up to this point; relativistic effects need to be included before a detailed comparison with the experimental data will be sensible. Our interest at present is in the trend of the computed phase shifts, rather than in the detailed comparison with experimental results.

The qualitative questions that can be addressed at this point relate to the P_{33} and P_{11} pion-nucleon phase shifts. The results of the computations described below can be summarized as follows: The P_{33} phase shift at low and medium energies can be described in the LASM as arising from excitation of the pion field around the core; that is, the calculations without the inclusion of quark excitation give a resonance in the P_{33} partial wave at a relatively low energy. Also as in other models, the P_{11} phase shift does not have a resonance at low or medium energy.

The energy of the P_{33} resonance in the LASM is remarkably sensitive to the bag radius. This is quite encouraging, because it may indicate that when the model incorporates the physics that is still lacking, it will be possible to use the 33 partial wave to determine the core radius in the model with considerable accuracy.

For the 11 partial wave, there remains the possibility that the resonance is associated with excitation of the core motion out of its ground state into the $2s$ soliton state. Calculations that might explain the Roper resonance in this way are being pursued.

It is clear from all of the above that the LASM in its present state and with the approximations that have been used to treat it up to the present, is at best a very incomplete description of the nucleon. For this reason, it would be premature to make detailed comparisons with the experimental data available for the pion-nucleon system. Until there is a better understanding of the physics that is responsible for the resonance in the P_{11} partial wave, as well as some quantitative results on the effects of the contribution of the pion field to the nucleon mass and the effects of using relativistic kinematics for the core, such comparisons and the resulting inferences concerning the validity of the model are not meaningful. On the other hand, the model does agree qualitatively with some of the static properties of the nucleon and does give a resonance in the P_{33} partial wave. These results indicate that further research into the properties of the LASM Hamiltonian may well shed light on the physics of the nucleon and possibly lead to calculations that can be confronted with the pion-nucleon data in a meaningful way.

The following section gives the details of the LASM, while Sec. III describes the method for computing phase shifts. Section IV presents the results of the computations and Sec. V summarizes the work.

II. DESCRIPTION OF THE MODEL

The model is formulated in terms of a Hamiltonian that describes a fermion core interacting with a pion field via a Yukawa (single-pion emission and absorption) interaction:

$$H = T_c + H_\pi + H_{\text{Yukawa}} + H^\dagger_{\text{Yukawa}} ,$$
$$T_c = \int \tilde{\Psi}^\dagger(\mathbf{p}) \frac{p^2}{2M} \tilde{\Psi}(\mathbf{p}) d\mathbf{p} ,$$
$$H_\pi = \int \omega(k) a^\dagger_\lambda(\mathbf{k}) a_\lambda(\mathbf{k}) d\mathbf{k} , \tag{2.1}$$
$$H_{\text{Yukawa}} = -\int \frac{a_\lambda(\mathbf{k})}{\sqrt{16\pi^3\omega(k)}} \hat{J}_\lambda(\mathbf{k}) d\mathbf{k} ,$$
$$\hat{J}_\lambda(\mathbf{k}) = \int \tilde{\Psi}^\dagger(\mathbf{p}) \tau_\lambda \boldsymbol{\sigma} \cdot \mathbf{J}_\pi \left[\mathbf{k}, \frac{\mathbf{p}+\mathbf{q}}{2} \right] \tilde{\Psi}(\mathbf{q}) \delta(\mathbf{p}-\mathbf{q} - \mathbf{k}) d\mathbf{p}\, d\mathbf{q} .$$

Here the operator $\Psi(\mathbf{p})$ annihilates the core with momentum $\mathbf{p}$; the core mass is M; in the present calculation M is taken to be the nucleon mass. The operator $a_\lambda(\mathbf{k})$ annihilates a pion with momentum $\mathbf{k}$; $\omega(k) = (k^2+m^2)^{1/2}$ is the energy of the pion; m is taken to be the charged pion mass; the summation convention is used for the pion isospin index λ. For the specific case of the cloudy-bag interaction that has been used in the calculations done so far, the pion-nucleon current operator $\mathbf{J}_\pi$ is taken to be independent of its second argument:

$$\mathbf{J}_\pi(\mathbf{k},\mathbf{K}) = f\frac{\mathbf{k}}{m} c(k), \quad c(k) = \frac{3j_1(kR)}{kR} ; \tag{2.2}$$

the factor i that sometimes appears here has been absorbed into the annihilation operator $a_\lambda(\mathbf{k})$. The factor $c(k)$ is the characteristic form factor that comes from chiral-current continuity arguments.

It has been shown in previous work that this Hamiltonian has a ground state that is a nontopological soliton. In order to set the stage for the scattering computations, it is necessary to describe the procedure used to derive the soliton state. This is like the procedure used in the nuclear shell model, where a localized state is used to generate an approximate ground state of a translation-invariant Hamiltonian. Instead of single-particle states for the nucleons, there are two kinds of modes here: one for the core and one for the pion field.

The states are first restricted to the subspace in which there is a single core present. Then the assumption of a "single" (four isospin-spin substates) s-wave radial function or mode $f(r)$ for the core transforms the original Hamiltonian of (2.1) into a localized-state (LS) Hamiltonian with two parts, the first of which is the expectation value of the kinetic energy of the core in its s state, and the second has the form of a static-source (SS) Hamiltonian for the pion field in which the source density function contains the effect of the core motion:

$$H \simeq H_{\text{LS}} = T_{c,0s} + H_{\text{SS}}, \quad T_{c,0s} = \int \frac{p^2}{2M} |\tilde{f}(\mathbf{p})|^2 d\mathbf{p} ,$$
$$H_{\text{SS}} = H_\pi + \tau_\lambda \int \frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{m} v(k) [a_\lambda(\mathbf{k}) - a^\dagger_\lambda(-\mathbf{k})] d\mathbf{k} , \tag{2.3}$$
$$v(k) = \frac{f c(k) \tilde{\rho}(k)}{\sqrt{16\pi^3\omega(k)}}, \quad \tilde{\rho}(k) = \int e^{-i\mathbf{k}\cdot\mathbf{r}} |f(r)|^2 d\mathbf{r} .$$

This form for H_{LS} means that only p-wave pions can be emitted and absorbed by the source. Therefore, other partial waves are unscattered in this single s-mode approximation and the annihilation operator for the pion field can be replaced by its p-wave part $a^p_\lambda(\mathbf{k})$:

$$a^p_\lambda(\mathbf{k}) = \sum_{j=1}^{3} \left[\frac{3}{4\pi} \right]^{1/2} \frac{k_j}{k^2} a^p_{\lambda j}(k) . \tag{2.4}$$

Then the localized-state Hamiltonian for the p-wave pions is

$$H^p_{\text{LS}} = T_{c,0s} + H^p_\pi - Y^p_{\text{SS}} ,$$
$$H^p_\pi = \int_0^\infty \omega(k) a^{p\dagger}_{\lambda j}(k) a^p_{\lambda j}(k) dk ,$$
$$Y^p_{\text{SS}} = \tau_\lambda \sigma_j \int_0^\infty v_p(k) [a_{\lambda j}(k) + a^\dagger_{\lambda j}(k)] dk , \tag{2.5}$$
$$v_p(k) = \frac{f k^2 c(k) \tilde{\rho}(k)}{\sqrt{12\pi^2\omega(k)}} ,$$
$$\tilde{\rho}(k) = \int e^{-i\mathbf{k}\cdot\mathbf{r}} |f(r)|^2 d\mathbf{r} .$$

This Hamiltonian for the p-wave pion field is the one that has been treated in Ref. 5. The treatment in the rest of this section follows the method developed there. As in that work, the p-wave pion is assumed to be in one of the nine substates of a "single" p-wave pion mode; that is, the pion field annihilation operator $a^p_{\lambda j}(k)$ is written in the form

$$a^p_{\lambda j}(k) = A_{\lambda j} \phi(k) + a^p_{\lambda j \perp}(k) . \tag{2.6}$$

The appropriate form of the internal mode $\phi(k)$ has been determined[5] to be (note that v_p and ϕ are real)

$$\phi(k) = \frac{v_p(k)}{G\omega(k)} , \tag{2.7}$$

where G is a normalization constant,

$$G^2 = \int_0^\infty \frac{v_p^2(k)}{\omega^2(k)} dk = \frac{\beta}{3\pi m^2} \int_0^\infty \frac{[k^2 c(k) \tilde{\rho}(k)]^2}{\omega^3(k)} dk \tag{2.8}$$

and β is the bare coupling constant:

$$\beta \equiv \frac{f^2}{4\pi} . \tag{2.9}$$

The energy of a pion in the internal mode is

$$W = \langle \omega \rangle = \int_0^\infty \omega(k) |\phi(k)|^2 dk = \frac{\beta}{3\pi G^2 m^2} \int_0^\infty \frac{[k^2 c(k) \tilde{\rho}(k)]^2}{\omega^2(k)} dk . \tag{2.10}$$

With these choices, the localized-state Hamiltonian becomes

$$H_{\text{LS}} = H_\parallel + H_{\omega\perp} + H_I + H_I^\dagger . \tag{2.11}$$

The internal Hamiltonian $H_\parallel$ is obtained by neglecting the terms involving the external field operators $a_{\lambda\perp}(\mathbf{k})$ and $a^\dagger_{\lambda\perp}(\mathbf{k})$:

$$H_{\parallel} = T_{c,0s} + WH_A = T_{c,0s} + Vh_A ,$$

$$H_A = G^2 h_A = A^\dagger \cdot A - G\rho\cdot(A^\dagger + A) ,$$

$$= \sum_{\lambda,i=1}^{3} [A^\dagger_{\lambda i} A_{\lambda i} - G\rho_{\lambda i}(A^\dagger_{\lambda i} + A_{\lambda i})] , \quad (2.12)$$

$$\rho_{\lambda i} \equiv \tau_\lambda \sigma_i .$$

The energy operator for the external p-wave mesons is

$$H_{\omega\perp} = \int \omega_\perp(p,q) a^{\rho\dagger}_{\lambda j \perp}(p) a^{\rho}_{\lambda j\perp}(q) dp\, dq ,$$
$$\omega_\perp(p,q) \equiv \omega(p)\delta(p-q) - [\omega(p)+\omega(q)-W]\phi(p)\phi^*(q) \quad (2.13)$$

and the interaction of the core and the internal mesons with the external meson is

$$H_I = \sum_{\lambda j} (A^\dagger - G\rho)_{\lambda j} \int [\omega(k) - W]\phi(k) a^{\rho}_{\lambda j\perp}(k) dk . \quad (2.14)$$

It is clear from the foregoing that the quantities $T_{c,0s}$, W, and G that occur in $H_\parallel$ are all functionals of the core wave function f, so that the ground-state energy ϵ_g of $H_\parallel$ is also a functional of f. Reference 3 showed that the function f_g that minimizes $\epsilon_g\{f\}$ is very nearly an exponential function. The soliton ground state consists of the pionic ground state of H_A surrounding the core with wave function f_g.

III. PHASE SHIFTS IN THE ONE-MESON SECTOR

The operator $H_\parallel$ has a discrete spectrum; its eigenstates are called the "internal" states and denoted $|\alpha\rangle$; the state $|\alpha\rangle$ belongs to the eigenvalue ϵ_α of $H_\parallel$. The computation of the spectrum of $H_\parallel$ has been the subject of considerable work.[6]

The "one-meson sector" consists of all the internal states $|\alpha\rangle$, as well as all the states $a^{\rho\dagger}_{\lambda j\perp}(k)|\alpha\rangle$. The state $a^{\rho\dagger}_{\lambda j\perp}(k)|\alpha\rangle$ is an eigenstate of the asymptotic Hamiltonian $H_\parallel + H_{\omega\perp}$ belonging to the eigenvalue $\epsilon_\alpha + \omega(k)$. The internal state $|\alpha\rangle$ belongs to the eigenvalue ϵ_α of the asymptotic Hamiltonian.

The phase shifts in the one-meson sector are computed by diagonalizing the localized-state Hamiltonian $H_{\rm LS}$ in this sector. This procedure guarantees that the resulting T matrix is unitary. Owing to the conservation of isospin and total angular momentum, the Hamiltonian couples only states with the same values of total isospin I and total angular momentum J. The (I,J) subspace contains states $|\beta\rangle$ with $I_\beta = I$ and $J_\beta = J$, as well as states $a^{\rho\dagger}_{\lambda j\perp}(k)|\gamma\rangle$ for which $|\gamma\rangle$ can couple to a p-wave pion to give total isospin and spin I and J, that is, $I_\gamma = I, I\pm 1$ and $J_\gamma = J, J\pm 1$. In a previous paper,[5] the "one-external-meson" (1EM) approximation to the T matrix was formulated, and the T matrix was evaluated in that approximation. The 1EM approximation includes all graphs in which any number of internal mesons, generated by the $A^\dagger_{\lambda j}$ operators, are present together with at most one external meson, generated by one of the operators $a^\dagger_{\lambda j\perp}$. The details of the method for evaluating the T matrix are given in Ref. 5.

In the one-meson sector of $H_{\rm LS}$, the result for the T matrix in the (IJ) subspace for the transition from the $\pi+\beta$ channel to the $\pi+\gamma$ channel at complex energy λ, is

$$T^{IJ}_{\beta p,\gamma q}(\lambda) = -M^{IJ}_{\beta\gamma}(\lambda)\phi(p)\phi(q) ; \quad (3.1)$$

here M is a matrix:

$$M^{IJ}(\lambda) = \frac{1}{J(\lambda)} - \frac{1}{J(\lambda)} C^{IJ\dagger} \frac{1}{\Delta^{IJ}(\lambda)} C^{IJ} \frac{1}{J(\lambda)} , \quad (3.2)$$

where the notation $1/A$ is used to represent A^{-1} for matrices A. In (3.2) C^{IJ} is the matrix of the current operator $A^\dagger - G\rho$ for absorption of internal mesons:

$$C^{IJ}_{\beta\gamma} = \langle\beta|\{(A^\dagger - G\rho), |\gamma\rangle\}^{I,J} , \quad (3.3)$$

where the curly brackets indicate vector (Clebsch-Gordan) coupling; the C^{IJ} matrix depends only on the internal Hamiltonain $H_\parallel$. The matrix J is diagonal with elements

$$J_{\beta\gamma}(\lambda) = \delta_{\beta\gamma} J_\beta(\lambda) ,$$
$$J_\beta(\lambda) = \int \frac{\phi^2(k)}{\lambda - \epsilon_\beta - \omega(k)} dk = \int \frac{\phi^2[k(\omega)]\frac{dk}{d\omega}}{\lambda - \epsilon_\beta - \omega} d\omega , \quad (3.4)$$

and since $\phi(k)$ is real, it follows that

$$J_\beta(\lambda^*) = J^*_\beta(\lambda) . \quad (3.5)$$

The matrix Δ^{IJ} is given by

$$\Delta^{IJ}_{\beta\gamma}(\lambda) = \delta_{\beta\gamma}(\lambda - \epsilon_\beta) - \sum_\xi C^{IJ}_{\beta\xi}\left[\lambda - \epsilon_\xi - W - \frac{1}{J_\xi(\lambda)}\right] C^{IJ\dagger}_{\xi\gamma} . \quad (3.6)$$

The zeros of $\det(\Delta^{IJ})$ are the discrete-state energies in the IJ subspace in the 1EM approximation.

The first $J^{-1}(\lambda)$ term in the M matrix of Eq. (3.2) can be interpreted as representing the scattering that would occur if all of the C coefficients were zero in $H_{\rm LS}$, that is, it is the scattering matrix generated by the Hamiltonian $H_\parallel + H_{\omega\perp}$. This "orthogonality scattering" is just due to the orthogonality of the external meson field to the internal modes. Mathematically, it takes the form of scattering off of a sum of three separable potentials; this can be seen by carrying out the diagonalization of $H_{\omega\perp}$. As was noted in Ref. 5, in weak coupling some of the C coefficients approach 1. Although some of the C coefficients are of order unity, the 1EM approximation becomes exact in weak coupling, where there is a cancellation of the two terms in M and the T matrix goes to zero. In general, the second term in M gives rise to resonances where the real part of $\det\Delta$ has zeros; between the resonances the "orthogonality scattering" term in M appears to dominate when the coupling is strong, that is, for smaller values of the bag radius. Thus, for strong coupling[7] it is sensible to regard the T matrix as the sum of a background orthogonality-scattering term and a resonant scattering term, while for weak coupling this distinction is no longer valid.

The on-shell conditions for $T^{IJ}_{\beta p,\gamma q}(\lambda)$ are

$$\lambda=\varepsilon_\beta+\omega(p)+i0=\epsilon_\gamma+\omega(q)+i0 \tag{3.7}$$

and the on-shell elastic T matrix for scattering off the α state is

$$T^{IJ}_{\alpha p,\alpha p}[\epsilon_\alpha+\omega(p)+i0]=-M^{IJ}_{\alpha,\alpha}[\epsilon_\alpha+\omega(p)+i0]\phi^2(p)\ . \tag{3.8}$$

The relation of the on-shell T matrix to the complex phase shift $\delta^{IJ}_\alpha(p)+i\eta^{IJ}_\alpha(p)$ for the elastic scattering from the state α in the IJ supspace is given by

$$\begin{aligned} e^{2i\delta^{IJ}_\alpha(p)}e^{-2\eta^{IJ}_\alpha(p)}-1&=2\pi i\frac{dk}{d\omega}T^{IJ}_{\alpha p,\alpha p}[\epsilon_\alpha+\omega(p)+i0] \\ &=2\pi i\phi^2(p)\frac{dk}{d\omega}M^{IJ}_{\alpha,\alpha}[\epsilon_\alpha+\omega(p)+i0]\ . \end{aligned} \tag{3.9}$$

For the phase-shift calculations, it is important to use values of the C coefficients that are as accurate as possible. As was noted in Ref. 6, the diagonal elements $C_{\alpha\alpha}$ are zero for the exact eigenstates of $H_\parallel$. In order to have a convenient method of satisfying this constraint, a new canonical-transformation technique[8] was used to compute the eigenstates of $H_\parallel$ and the C matrix elements.

IV. RESULTS OF THE CALCULATIONS

As in the computation of the static properties of the nucleon in the LASM, the only free parameter in the phase-shift calculations is the bag radius R. The internal states that were used in the computations are the two lowest 11 states and the lowest 33 state. The calculated P-wave phase shifts in the 33 and 11 channels and the inelasticity in the 11 channel for various values of R are shown in Figs. 1, 2, and 3. As was noted in the Introduction, these results are limited both by the fact that some physical effects have not yet been incorporated in the

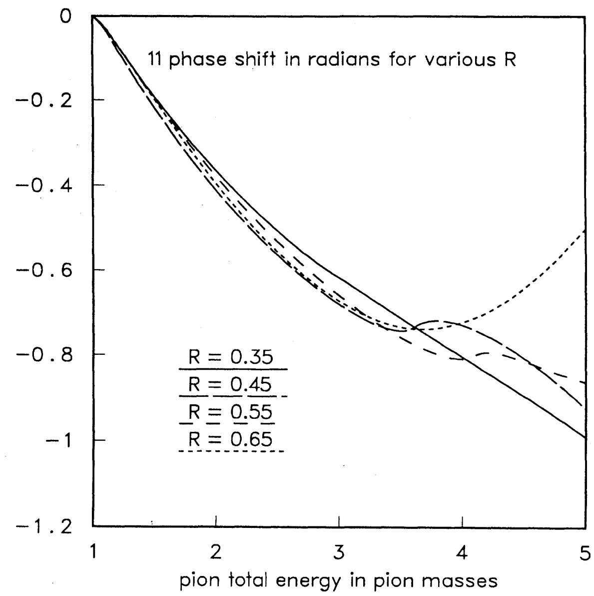

FIG. 2. The P_{11} pion-nucleon phase shift as a function of the total pion energy for various values of the core radius R.

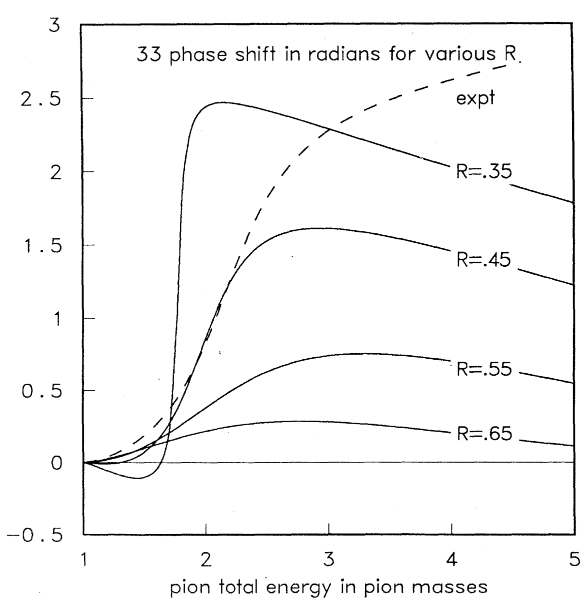

FIG. 1. The P_{33} pion-nucleon phase shift as a function of the total pion energy for various values of the core radius R.

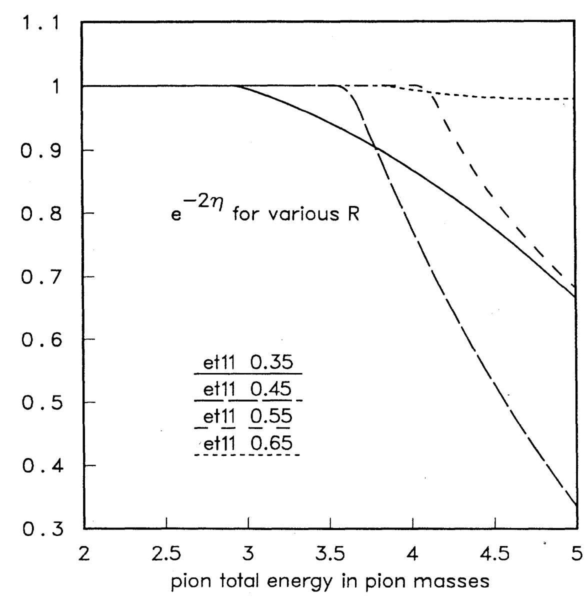

FIG. 3. The P_{11} pion-nucleon inelasticity as a function of the total pion energy for various values of the core radius R.

Hamiltonian as well as by the approximations that have been used to treat the Hamiltonian. The trend of the results for the 33 phase shift is in reasonable agreement with experiment up to pion kinetic energies of approximately two pion masses for a bag radius of about 0.45 fm. With the approximations employed in this paper the prediction for the 11 phase shift compares poorly with the data; it is too large and negative and lacks the resonance that is exhibited by the data. Clearly there is some physics missing from the 11 phase shift and therefore it would be unreasonable to compare predictions for the 11-channel inelasticity with the data.

As was noted in the Introduction, there is a unique feature of the LASM that might lead to a lower-energy resonance in the P_{11} channel like the Roper resonance: namely, the possibility of a 2*s* excitation of the motion of the core. On the other hand, a proper description of the Roper resonance might require an enlargement of the subspace to include states with two physical pions. We are currently exploring the first possibility.

V. SUMMARY

The P_{33} phase shift and the P_{11} phase shift and inelasticity have been computed in the Los Alamos soliton model of the nucleon in the approximation in which the core is restricted to its lowest *s* state and therefore interacts only with *p*-wave pions. The internal pions that dress the core are treated to all orders in the scattering calculation, while the scattering states contain at most one asymptotic pion. The results show that the P_{33} Δ resonance may be due to excitation of the pion field around the quantum-chromodynamic core. In this model the 33 resonance is sensitive to the size of the bag. On the other hand, the P_{11} resonance at 600 meV seems to be of a different nature, possibly involving excitation of the core-motion wave function to a 2*s* state.

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