Complete formalism for charge-symmetry tests in n-p elastic scattering

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A complete set of tests of charge symmetry in n-p elastic scattering is given in a formalism in which the number of tests, the types of observables appearing in the tests, and the characteristics of the spin-amplitudes parameters utilized in each test are easily identifiable and displayable. The results allow one to pinpoint the particular test which is theoretically the most interesting and experimentally the most advantageous.

Tests of symmetry laws have been of central concern in subatomic physics. Polarization measurements have always played a pivotal role in such tests, partly because, in some cases, they can provide null experiments as tests of symmetries whereas the unpolarized differential cross sections cannot do that, and partly because such polarization experiments can discern both the magnitude and the phase of the symmetry-breaking amplitude, while the unpolarized differential cross section can measure only magnitude squares. For the same reason symmetry-breaking effects in unpolarized differential cross sections are second order in the small symmetry-breaking amplitudes, while they can be of first order in the polarization observables.

In particular, a number of papers have recently studied charge-symmetry breaking in n-p elastic scattering, partly motivated by a very clever experimental method[2] which has been used to measure such an effect. A recent review[3] has summarized some of the considerations and results.

The present paper contributes to the investigation of charge-symmetry breaking in n-p elastic scattering by differing from previous papers in the following respects.

(a) The optimal transversity formalism is used to analyze the reaction.

(b) Emphasis is placed on testing the symmetry breaking on the amplitude level rather than on the observable level.

(c) The amplitude-observable structure is displayed in its complete generality and simplicity, thus allowing one instantly to determine the number of tests, the nature of the observables appearing in each test, and the particular parameters of the symmetry-breaking amplitude which is being tested.

In the aspects in which the present paper overlaps with previous ones, the results agree. The present paper, however, gives perhaps a deeper "explanation" of why the particular tests have the form they have and what the limitations are in searching for such tests.

The treatment can be given very concisely since it can be based on features of the optimal formalism that have been amply explained and demonstrated in the literature.[4] Since for any parity-conserving reaction the phenomenological determination of the amplitudes is the simplest and most accurate (i.e., containing minimal off-diagonal elements of the merror matrix) in the transversity system, we use that system in this paper. Using for the amplitudes the usual notation in which the four arguments of the complex reaction amplitude D refer, in this order, to the first final, the first initial, the second final, and the second initial particle, the reaction containing four-spin-$\frac{1}{2}$ particles, when parity conservation and time-reversal invariance are imposed, is described by the six complex reaction amplitudes $D(++,++)=\alpha$, $D(--,--)=\beta$, $D(++,--)=\gamma_1$, $D(--,++)=\gamma_2$, $D(+-,+-)=\delta$, $D(+-,-+)=\xi$, where $+$ denotes $+\frac{1}{2}$ for the spin projection in the quantization directions (in this case, the normal to the scattering plane), and $-$ similarly denotes a spin projection of $-\frac{1}{2}$. When charge symmetry holds, we have $\gamma_1=\gamma_2$, so our task will be to measure the difference between γ_1 and γ_2.

The next task is to write out the relationship between the bilinear products of the above six amplitudes and the experimental observables. The latter will be denoted by four arguments in parentheses, the order of the arguments being the first initial particle, the second initial particle, the first final-state particle, and the second final-state particle. We will write the observables directly in the so-called secondary observable notation, that is, in one in which unpolarized and simply polarized particles appear in the observables, since these are then immediately identifiable with the actual experimental situations. The optimal formalism, evolved for treating reactions containing particles with arbitrary spins, uses for the secondary observables here the arguments A, Δ, R, and I, standing for unpolarized (averaged) particles, for particles simply polarized in the quantization direction (in this case in the direction normal to the reaction plane), for particles simply polarized in the longitudinal direction, and for particles simply polarized in the planar transverse direction. Thus the last three of the above four arguments can be identified with the indices N, L, and S more conventionally used for nucleon-nucleon scattering, except that there are some subtleties of signs which are specified in the Appendix.

In order to make the basis in which we work more explicit, we can write the M matrix in the form

$$M=\alpha\rho_1+\beta\rho_2+\gamma_1\rho_3+\gamma_2\rho_4+\delta\rho_5+\epsilon\rho_6\ ,$$

where the six spin tensors assume the form

$$\rho_1=\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\quad \rho_2=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix},$$

$$\rho_3=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\quad \rho_4=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix},$$

$$\rho_5=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&1&0&0\end{bmatrix},\quad \rho_6=\begin{bmatrix}0&0&0&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{bmatrix}$$

if we use the convention for the labeling of the matrix elements that the four rows correspond to $(A,B)=(+,+)$, $(+,-)$, $(-,+)$, and $(-,-)$, and the four columns correspond to $(C,D)=(+,+)$, $(+,-)$, $(-,+)$, and $(-,-)$.

We know that, in general, the bilinear products of the reaction amplitudes and the observables are connected to each other by submatrices which are of size 1×1, 2×2, 4×4, and 8×8 for any four-particle reaction, regardless of the values of the spins of the particles. In particular, for the reaction involving four spin-$\frac{1}{2}$ particles, when Lorentz invariance alone is imposed, we have 16 amplitudes, and thus 256 bilinear products and that many linearly independent observables, and these products and observables are connected to each other by two 8×8 matrices, sixteen 4×4 matrices, forty-eight 2×2 matrices, sixty-four 1×1 matrices, and another sixteen 1×1 matrices involving magnitude squares (these are denoted by 1). This adds up to $(2\times8)+(16\times4)+(48\times2)+(64\times1)+(16\times1)=256$. This holds for the primary observables. The secondary observables arrange themselves into thirty 8×8 matrices and one 16×16 matrix.

The imposition of the parity conservation and time-reversal invariance reduces the 16 independent amplitudes to 6, and hence the 256 independent bilinear amplitude products and the 256 linearly independent observables to 36. As we know from the general theory of the optimal formalism, in the transversity formalism the 4×4 and the 1×1 submatrices (except for the 1_M types) vanish for the primary observables of this reaction, and the other remaining matrices simplify. In particular, for the secondary observables we are left with 15 submatrices: One 6×6, containing magnitude squares (originating from 1); nine 4×4, one of them coming from the 8×8 submatrices of the primary observables and the remaining eight from the 2×2 submatrices for the primary observables; two 3×3, coming from 2×2 submatrices for the primary observables; and one 2×2, coming from the 8×8 submatrices of the primary observables. This relationship between the observables and the bilinear products of amplitude is given in Table I.

The identification of the origin of these submatrices in terms of the submatrices for the primary observables is important because we know from the general theory of the optimal formalism that, if we denote a primary observable (nD), if n particles in it have off-diagonal arguments (i.e., arguments pointing in directions other than the quantization direction), then the observables in the 8×8, 4×4, 2×2, 1×1, and 1_M type submatrices for the primary observables are $(4D)$, $(3D)$, $(2D)$, $(1D)$, and $(0D)$ types. In the corresponding submatrices for the secondary observables this rule translates into the following result. We will now denote a secondary observables (nP) if n particles in it are in a specific polarization state and hence are not unpolarized. Using this notation we can then state that matrices coming from 8×8 matrices for the primary observables contain observables of the type $(4P)$: from 4×4 contain observables of types $(4P)$ and $(3P)$; from 2×2 contain observables of types $(4P)$, $(3P)$, and $(2P)$; from 1×1 contain observables of types $(4P)$, $(3P)$, $(2P)$, and $(1P)$; and from 1_M contain observables of types $(4P)$, $(3P)$, $(2P)$, $(1P)$, and $(0P)$. Thus we can clearly and simply typify the observables occurring in the submatrices of Table I.

Similarly, we can also typify the bilinear products of amplitudes that occur in these submatrices. Again drawing on the general theory of the optimal formalism, we can immediately state that the submatrices coming from 8×8's of the primary observables contain products of two amplitudes which differ from each other in all four arguments of the amplitudes. Similarly the number of indices in which the two amplitudes forming the products differ are three for 4×4 matrices of the primary observables, two for 2×2 matrices of the primary observables, one for 1×1 matrices of the primary observables except the 1_M type, and finally zero for the 1_M matrices of the primary observables.

All these regularities are reflected in Table I.

Now we can turn to the tests of charge symmetry. As mentioned above, the imposition of charge symmetry, in addition to parity conservation and time-reversal invariance, reduces the number of independent amplitudes to five by making $\gamma_1=\gamma_2$. Thus we will have 25 rather than 36 linearly independent observables. This means that we have 11 independent tests of charge symmetry.

We know from the general theory of optimal formalisms that it is impossible[5] to construct a dynamics-independent, null experiment for charge-symmetry testing. This result was actually proven for time-reversal invariance and not charge symmetry, but the structures of the two symmetries are so similar in the optimal formalism that the theorem can be directly taken over for charge symmetry also. Thus we know that all of our charge-symmetry tests will involve at least two observables which then have to be compared.

In fact, we know even more details about these tests from the general theory of the optimal formalism. We know that for charge symmetry, just as for time-reversal invariance, the tests will be of two kinds: Some will be "mirror relations," in which two observables, with particle arguments interchanged, will become equal if the symmetry holds, while some others (in general a smaller number of them) will be of nonmirror type and will involve more than two observables. Indeed, one can deter-

TABLE I. The relationship between observables and bilinear products of amplitudes in the transversity optimal formalism. For the notation, see the text.

1_M

	$\lvert\alpha\rvert^2$	$\lvert\sigma\rvert^2$	$\lvert\gamma_1\rvert^2$	$\lvert\epsilon\rvert^2$	$\lvert\gamma_2\rvert^2$	$\lvert\beta\rvert^2$
$(AA;AA)=(\Delta\Delta;\Delta\Delta)$	+1	+2	+1	+2	+1	+1
$(\Delta A;AA)=(A\Delta;\Delta\Delta)=(\Delta\Delta;A\Delta)=(AA;\Delta A)$	+1	0	+1	0	−1	−1
$(A\Delta;AA)=(\Delta A;\Delta\Delta)=(\Delta\Delta;\Delta A)=(AA;A\Delta)$	+1	0	−1	0	+1	−1
$(\Delta\Delta;AA)=(AA;\Delta\Delta)$	+1	+2	−1	−2	−1	+1
$(\Delta A;\Delta A)=(A\Delta;A\Delta)$	+1	−2	−1	+2	−1	+1
$(\Delta A;A\Delta)=(A\Delta;\Delta A)$	+1	−2	−1	+2	−1	+1

8_1

	$\mathrm{Re}\alpha\beta^*$	$\mathrm{Re}\gamma_1\gamma_2^*$	$\lvert\delta\rvert^2$	$\lvert\epsilon\rvert^2$
$(RR;RR)=(II;II)$	+1	+1	+1	+1
$-(RR;II)=-(II;RR)$	+1	−1	+1	−1
$(RI;RI)=(IR;IR)$	+1	+1	−1	−1
$(RI;IR)=(IR;RI)$	+1	−1	−1	+1

8_1

	$\mathrm{Im}\alpha\beta^*$	$\mathrm{Im}\gamma_1\gamma_2^*$
$-(RR;RI)=(II;IR)=-(RI;RR)=-(IR;II)$	+1	−1
$-(RR;IR)=(II;R,I)=-(RI;II)=(IR;RR)$	+1	+1

2_1

	$\mathrm{Re}\alpha\delta^*$	$\mathrm{Re}\gamma_1\epsilon^*$	$\mathrm{Re}\gamma_2\epsilon^*$	$\mathrm{Re}\beta\delta^*$
$(AA;RR)=-(\Delta\Delta;II)$	+1	+1	+1	+1
$-(AA;II)=(\Delta\Delta;RR)$	+1	−1	−1	+1
$(A\Delta;RR)=-(\Delta A;II)$	+1	−1	+1	−1
$(A\Delta;II)=(\Delta A;RR)$	+1	+1	−1	−1

2_2

	$\mathrm{Re}\alpha\gamma_1^*$	$\mathrm{Re}\delta\epsilon^*$	$\mathrm{Re}\beta\gamma_2^*$
$(AR;AR)=(\Delta I;\Delta I)$	+1	+2	+1
$(AI;AI)=(\Delta R;\Delta R)$	+1	−2	+1
$(AR;\Delta R)=(\Delta I;AI)=(AI;\Delta I)=(\Delta R;AR)$	+1	0	−1

2_3

	$\mathrm{Re}\alpha\epsilon^*$	$\mathrm{Re}\gamma_1\delta^*$	$\mathrm{Re}\gamma_2\delta^*$	$\mathrm{Re}\beta\epsilon^*$
$(AR;RA)=(\Delta I;I\Delta)$	+1	+1	+1	+1
$(AI;IA)=(\Delta R;R\Delta)$	+1	−1	−1	+1
$(AR;R\Delta)=(\Delta I;IA)$	+1	−1	+1	−1
$(AI;I\Delta)=(\Delta R;RA)$	+1	+1	−1	−1

2_4

	$\mathrm{Re}\alpha\epsilon^*$	$\mathrm{Re}\gamma_2\delta^*$	$\mathrm{Re}\gamma_1\delta^*$	$\mathrm{Re}\beta\epsilon^*$
$(RA;AR)=(I\Delta;\Delta I)$	+1	+1	+1	+1
$(IA;AI)=(R\Delta;\Delta R)$	+1	−1	−1	+1
$(RA;\Delta R)=(I\Delta;AI)$	+1	−1	+1	−1
$(IA;\Delta I)=(R\Delta;AR)$	+1	+1	−1	−1

2_5

	$\mathrm{Re}\alpha\gamma_2^*$	$\mathrm{Re}\delta\epsilon^*$	$\mathrm{Re}\beta\gamma_1^*$
$(RA;RA)=(I\Delta;I\Delta)$	+1	+2	+1
$(IA;IA)=(R\Delta;R\Delta)$	+1	−2	+1
$(RA;R\Delta)=(I\Delta;IA)=(IA;I\Delta)=(R\Delta;RA)$	+1	0	−1

TABLE I. (*Continued*).

	2_6			
	$\mathrm{Re}\alpha\delta^*$	$\mathrm{Re}\gamma_1\epsilon^*$	$\mathrm{Re}\gamma_2\epsilon^*$	$\mathrm{Re}\beta\delta^*$
$(RR;AA)=-(II;\Delta\Delta)$	+1	+1	+1	+1
$-(II;AA)=(RR;\Delta\Delta)$	+1	−1	−1	+1
$(RR;A\Delta)=-(II;\Delta A)$	+1	−1	+1	−1
$-(II;A\Delta)=(RR;\Delta A)$	+1	+1	−1	−1

	2_1			
	$\mathrm{Im}\alpha\delta^*$	$\mathrm{Im}\gamma_1\epsilon^*$	$\mathrm{Im}\gamma_2\epsilon^*$	$\mathrm{Im}\beta\delta^*$
$-(AA;RI)=-(\Delta\Delta;IR)$	+1	−1	+1	−1
$-(AA;IR)=-(\Delta\Delta;IR)$	+1	+1	−1	−1
$-(A\Delta;RI)=-(\Delta A;IR)$	+1	+1	+1	+1
$-(A\Delta;IR)=-(\Delta A;RI)$	+1	−1	−1	+1

	2_2		
	$\mathrm{Im}\alpha\gamma_1^*$	$\mathrm{Im}\delta\epsilon^*$	$\mathrm{Im}\beta\gamma_2^*$
$-(AR;AI)=(\Delta I;\Delta R)=(AI;AR)=-(\Delta R;\Delta I)$	+1	0	−1
$-(AR;\Delta I)=(\Delta I;AR)$	+1	+2	+1
$(AI;\Delta R)=-(\Delta R;AI)$	+1	−2	+1

	2_3			
	$\mathrm{Im}\alpha\epsilon^*$	$\mathrm{Im}\gamma_1\delta^*$	$\mathrm{Im}\gamma_2\delta^*$	$\mathrm{Im}\beta\epsilon^*$
$-(AR;IA)=(\Delta I;R\Delta)$	+1	+1	−1	−1
$(AI;RA)=-(\Delta R;I\Delta)$	+1	−1	+1	−1
$-(AR;I\Delta)=(\Delta I;RA)$	+1	−1	−1	+1
$(AI;R\Delta)=-(\Delta R;IA)$	+1	+1	+1	+1

	2_4			
	$\mathrm{Im}\alpha\epsilon^*$	$\mathrm{Im}\gamma_2\delta^*$	$\mathrm{Im}\gamma_1\delta^*$	$\mathrm{Im}\beta\epsilon^*$
$-(RA;AI)=(I\Delta;\Delta R)$	+1	+1	−1	−1
$(IA;AR)=-(R\Delta;\Delta I)$	+1	−1	+1	−1
$-(RA;\Delta I)=(I\Delta;AR)$	+1	−1	−1	+1
$(IA;\Delta R)=-(R\Delta;AI)$	+1	+1	+1	+1

	2_5		
	$\mathrm{Im}\alpha\gamma_2^*$	$\mathrm{Im}\delta\epsilon^*$	$\mathrm{Im}\beta\gamma_1^*$
$-(RA;IA)=(I\Delta;R\Delta)=(IA;RA)=-(R\Delta;I\Delta)$	+1	0	−1
$-(RA;I\Delta)=(I\Delta;RA)$	+1	+2	+1
$(IA;R\Delta)=-(R\Delta;IA)$	+1	−2	+1

	2_6			
	$\mathrm{Im}\alpha\delta^*$	$\mathrm{Im}\gamma_1\epsilon^*$	$\mathrm{Im}\gamma_2\epsilon^*$	$\mathrm{Im}\beta\delta^*$
$(RI;AA)=(IR;\Delta\Delta)$	+1	−1	+1	−1
$(IR;AA)=(RI;\Delta\Delta)$	+1	+1	−1	−1
$(RI;A\Delta)=(IR;\Delta A)$	+1	+1	+1	+1
$(IR;A\Delta)=(RI;\Delta A)$	+1	−1	−1	+1

mine, from the general theory, also the numbers of each kind, and obtain, for our case, that there will be ten mirror relations and one nonmirror relation.

We can also draw conclusions from the general theory about the type of observables occurring in these tests. The only submatrix in which $(1P)$ observables occur is 1_M, and since this submatrix reduces from 6×6 to 5×5, there will be one single test occurring in this submatrix which, as we said, contains only magnitude squares of the amplitudes. Thus we conclude that the experimentally simplest test, involving two observables in each of which only one of the four particles is polarized, will be one involving the difference of the magnitude squares of γ_1 and of γ_2.

Similarly, since, as explained earlier, we know what types of amplitude products and what types of observables appear in each submatrix, we can determine the structure of each of the 11 tests.

TABLE II. The 11 tests of charge symmetry in *np* elastic scattering. For the notation, see the text.

Polarization type	$\mathrm{Re}\gamma_1,\gamma_2$	$\mathrm{Re}(\alpha+\beta)$	$\mathrm{Re}(\alpha-\beta)$	$\mathrm{Re}\delta$	$\mathrm{Re}\epsilon$
(1P)	$(\Delta A;AA)-(A\Delta;AA)$ $=2(\lvert\gamma_1\rvert^2-\lvert\gamma_2\rvert^2)$ (1_M)				
(2P)			$(AR;AR)-(RA;RA)$ $=(AI;AI)-(IA;IA)$ $=\mathrm{Re}(\gamma_1-\gamma_2)(\alpha-\beta)^*$ $(2_2$ and $2_5)$		
(3P)		$(AR;\Delta R)-(RA;R\Delta)$ $=\mathrm{Re}(\gamma_1-\gamma_2)(\alpha+\beta)^*$ $(2_2$ and $2_5)$		$(AI;I\Delta)-(AR;R\Delta)$ $=(RA;\Delta R)-(IA;\Delta I)$ $=2\,\mathrm{Re}(\gamma_1-\gamma_2)\delta^*$ $(2_3$ and $2_4)$	$(A\Delta;II)-(A\Delta;RR)$ $=-(II;A\Delta)-(RR;A\Delta)$ $=2\,\mathrm{Re}(\gamma_1-\gamma_2)\epsilon^*$ $(2_1$ and $2_6)$
(4P)	The only nonmirror relation: $(RR;RR)+(RR;II)$ $+(RI;RI)-(RI;IR)$ $=4\,\mathrm{Re}\gamma_1\gamma_2^*$				

Polarization type	$\mathrm{Im}\gamma_1,\gamma_2$	$\mathrm{Im}(\alpha+\beta)$	$\mathrm{Im}(\alpha-\beta)$	$\mathrm{Im}\delta$	$\mathrm{Im}\epsilon$
(2P)		$-(AR;AI)+(RA;IA)$ $=2\,\mathrm{Im}(\alpha+\beta)(\gamma_1-\gamma_2)^*$ $(2_2$ and $2_5)$		$-(AR;IA)+(AI;RA)$ $=(IA;AR)+(RA;AI)$ $=2\,\mathrm{Im}(\gamma_1-\gamma_2)\delta^*$ $(2_3$ and $2_4)$	$-(AA;RI)+(AA;IR)$ $=(RI;AA)-(IR;AA)$ $=-2\,\mathrm{Im}(\gamma_1-\gamma_2)\epsilon^*$ $(2_1$ and $2_6)$
(3P)			$-(AR;\Delta I)+(RA;I\Delta)$ $=(AI;\Delta R)-(IA;R\Delta)$ $=2\,\mathrm{Im}(\alpha-\beta)(\gamma_1-\gamma_2)^*$ $(2_2$ and $2_3)$		
(4P)	$-(RR;RI)+(RR;IR)$ $=-2\,\mathrm{Im}\gamma_1\gamma_2^*$				

What needs to be emphasized is that virtually the complete specification of the 11 tests could thus be obtained from the general theory of the optimal formalism, without writing down any specifics, without calculating traces, or without doing any algebra at all. To be sure, Table I gave us the actual relationships between each observable and the bilinear products, but that table has so far not been utilized at all in deducing the set of conclusions enumerated above, concerning the nature of the tests.

In particular, we could determine through these general considerations the following features of the tests.

(1) The number of independent tests.

(2) How many observables would appear in each test.

(3) How many particles are unpolarized in each of these tests.

(4) In what direction the particles are polarized in each of these tests.

(5) With what other amplitude the charge-symmetry-breaking amplitude is multiplied in each of these tests.

(6) Whether the real or the imaginary part of the amplitude product enters each test.

It is only when we are interested in conceptually inessential (though of course in practice important) details, such as the coefficients and signs in the submatrices and hence in the expressions for the tests that a modicum of algebraic calculation has to be carried out.

For the reaction in question, *np* elastic scattering, it may not make a large difference whether we generate the tests through this general knowledge of the polarization structure of particles or through the straightforward, routine, and laborious calculations of traces and alike, since the reaction involving four spin-$\frac{1}{2}$ particles has received so much attention in the past 35 years that the necessary calculations have in fact been carried out. When, however, similar considerations are applied to spinwise even just a little more complicated reactions, the advantages of using the general method become very evident.

The 11 tests are tabulated in Table II, according to the number of polarized particles required and according to the kind of product in which the symmetry-breaking amplitude appears. From Table II it is easy to generate a practical procedure for exploring charge-symmetry breaking. I will describe it for the case when some model calculations for the contribution to the symmetry-breaking amplitude have been made. One should note that such calculations generally offer contributions from a number of different mechanisms (for example, different meson exchanges). Since the different contributions will generally have different magnitudes and phases, a set of experiments offers the possibility of checking these contributions separately. The procedure is therefore as follows.

(1) For each contribution the magnitude and phase of the resulting addition to the symmetry-breaking amplitude should be calculated. Since an overall phase of the set of reaction amplitudes is arbitrary, it is important to determine, in comparison with the convention used in calculating these contributions, as to what the relative phase is between the calculated amplitudes and the phenomenologically obtained amplitudes.

(2) From a phenomenological analysis of *np* elastic scattering (assuming charge symmetry), the magnitudes and phases of the five reaction amplitudes $\alpha, \beta, \gamma_1=\gamma_2, \delta,$ and ϵ should be determined as a function of energy and angle.

(3) From Table II it should be determined, for the case of the calculated and determined amplitudes, which observables, and at what energies and angles, are optimal for testing the various contributions to charge-symmetry breaking.

(4) The tests should be carried out, if possible, at various energies and angles, since the various mechanisms for symmetry breaking also vary (and vary differently from each other) as functions of energy and angle. Such a reliance on the type of experiment, on the energy, and on the angle will help greatly in checking the various contributions to symmetry breaking separately from each other.

(5) The choice of measurements naturally must also include a consideration of the experimental constraints. Indeed, such factors may play a dominant part. For example, the already existing measurement[2] utilized a clever trick by choosing as the kinematic point (energy and angle) for the experiment one at which a dynamical accident could make the experiment a null experiment and thus could hope to measure something of the order of 10^{-3} or so in amplitudes. Once one knows which observables offer tests at all, however, other similar tricks may be discovered.

It might be noted that if, for reasons of experimental feasibility, we consider only $(1P)$- and $(2P)$-type experiments, we still have products of the symmetry-violating amplitude with all four other amplitudes, but in some cases only the real parts and in some other cases only the imaginary parts. This possibly leaves room for discrete ambiguities in the solutions for the symmetry-breaking amplitude.

Finally, it should be mentioned that the above analysis and procedure holds for any ("class")[6] of charge-symmetry breaking of the four listed in Ref. 6. For all classes, the number of amplitudes increases from 5 to 6, and the extra term will have the same spin structure in any of the four cases. Distinctions among the classes can be made either theoretically or by studying also reactions other than *np* elastic scattering. Another handle to make distinctions may be the comparison between the reaction observables at a given angle and at its complementary angle.

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APPENDIX

The list below gives the correspondence between the notation of observables in the optimal formalism and the traditional notation as used, for example, in Ref. 7:

$$(A,A;A,A)=\sigma_0,\quad (A,\Delta;A,A)=P(=P_N)\ ,$$

$$(\Delta,\Delta;A,A)=C_{NN},\quad (R,R;A,A)=C_{LL}\ ,$$

$$(I,I;A,A)=C_{SS},\quad (I,R;A,A)=C_{SL}\ ,$$

$$(A,\Delta;A,\Delta)=D_{NN},\quad (A,R;A,R)=-D_{LL}\ ,$$

$$(A,I;A,I)=-D_{SS},\quad (A,R;A,I)=-D_{LS}\ ,$$

$$(\Delta,A;A,\Delta)=K_{NN},\quad (R,A;A,R)=-K_{LL}\ ,$$

$$(I,A;A,R)=-K_{SS},\quad (I,A;A,R)=-K_{SL}\ ,$$

$$(I,I;A,\Delta)=H_{SSN},\quad (I,\Delta;A,I)=-H_{SNS}\ ,$$

$$(I,\Delta;A,R)=-H_{SNL},\quad (I,R;A,\Delta)=H_{SNL}\ ,$$

$$(\Delta,I;A,I)=-H_{NSS},\quad (\Delta,I;A,R)=-H_{NSL}\ ,$$

$$(\Delta,R;A,I)=-H_{NLS},\quad (R,I;A,\Delta)=H_{LSN}\ ,$$

$$(R,\Delta;A,I)=-H_{LNS},\quad (I,I;I,I)=-H_{SSSS}\ ,$$

$$(I,I;I,R)=-H_{SSSL}\ .$$

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[1]See, for example, M. J. Iqbal and J. A. Niskanen, Phys. Rev. C **38**, 2259 (1988).

[2]See, for example, R. Abegg *et al.*, J. Phys. Soc. Jpn. Suppl. **55**, 369 (1986).

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