Limits on right-handed interactions from SN 1987A observations

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We consider the emission of light ($m_{\nu_R} \lesssim 10$ MeV right-handed neutrinos from supernova 1987A via right-handed charged- and neutral-current interactions. By requiring that the neutronization process $e_R^- p \to \nu_R n$ does not carry away most of the energy that can be radiated by the supernova, $E_T \lesssim (4-5)\times 10^{53}$ ergs, we exclude the range of values for the right-handed W-boson mass, $(3.7-6.7)M_{W_L} \le M_{W_R} \le (280-500)M_{W_L}$ for vanishing $W_L - W_R$ mixing parameter ζ, and for $\zeta \neq 0$ we get $(0.4-1.2)\times 10^{-5} \le [\zeta^2 + (M_{W_L}^4/M_{W_R}^4)]^{1/2} \le (0.02-0.07)$. For right-handed W bosons heavier than 300 M_{W_L}, a significant ν_R emission may arise from the neutral-current-induced process $e^+e^- \to \bar\nu_R \nu_R$, as well as $nn \to nn\nu\bar\nu$ mediated by an extra gauge boson Z'. In this case, for an appropriately defined effective mass M_N for Z', we find the excluded region $(2.4-4.3)M_{W_L} < M_N < (30-85)M_{W_L}$, mainly dependent on the core temperature and density profile.

I. INTRODUCTION

The observation of neutrino flux from the supernova 1987A by the Kamiokande[1] and Irvine-Michigan-Brookhaven[2] (IMB) experiments has not only confirmed the standard picture of stellar collapse studied by astrophysicists for years[3] but also has yielded a wealth of information on the properties of the neutrino as well as other physics ideas beyond the standard model. In this paper, we discuss implications for supernova neutrino luminosity if there are right-handed charged- or neutral-current interactions as is the case in left-right-symmetric models of weak interactions[4] or the superstring models.[5] If the right-handed neutrino is light ($m_{\nu_R} \le 10$ MeV) and if it has a charged-current coupling, it can be produced in the core of the supernova via the reaction $e_R^- p \to \nu_R n$. For a certain range of the strength of this charged-current interaction, either of two things can happen. (i) For one range for the strength of this charged-current interaction $(G_R)\nu_R$ has a mean free path larger than the core radius so that it escapes. As G_R increases, the flux of ν_R increases. Since ν_R's are produced in the inner core, they have typical energies of the order of 200–300 MeV. This can cause the integrated ν_R luminosity to saturate the total energy E_T that can be released in the neutron-star formation, thereby putting an upper bound on the strength of the charged-current interactions. (ii) On the other hand, if ν_R has a mean free path smaller than the core radius, it gets trapped and its subsequent thermalization can reduce the luminosity. In order to estimate the ν_R luminosity in this case, one has to calculate the radius of the ν_R-sphere. The ν_R's emitted from the surface of the ν_R-sphere carry away the energy corresponding to its temperature and radius. In turn, if the radius is not big enough, the temperature at the surface of the ν_R-sphere will be high and the energy loss via ν_R emission will dominate. It turns out that the stronger the interaction, the lower the ν_R luminosity. This gives a lower limit on G_R. These considerations therefore rule out a range of the strength of the right-handed current interactions. In terms of the conventional left-right-symmetric models, these limits translate into a range of excluded values for M_{W_R} and the W_L-W_R mixing parameter ζ as follows:

$$(0.4-1.2)\times 10^{-5} \le [\zeta^2 + (M_{W_L}^4/M_{W_R}^4)]^{1/2} \le 0.02-0.07 \tag{1}$$

or, for $\zeta = 0$,

$$(3.7-6.7)M_{W_L} \le M_{W_R} \le (280-500)M_{W_L} . \tag{2}$$

To get these bounds, we assume a maximum total energy $E_T \lesssim 4-5\times 10^{53}$ ergs, which is the gravitational binding energy E_B liberated in the formation of a daughter star of mass $M \simeq (1.6-1.8)\, M_\odot$ by the implosion of the mother star's core. We wish to point out here that our basic strategy is similar to several recent papers which consider bounds on other exotic new physics from SN 1987A observations.[5]

If there are no charged currents but only neutral-current interactions due to the presence of extra light Z' bosons, $\nu_R \bar\nu_R$ emission can take place via the process $e^+e^- \to \nu_R \bar\nu_R$. Again similar considerations as before lead to bounds on the mass of the extra Z' boson depending on the nature of its coupling. If we define

$$\sigma(e^+e^- \to \bar\nu_R \nu_R) \equiv \frac{G_F^2 s}{12\pi} \left[\frac{M_{W_L}}{M_N} \right]^4 \tag{3}$$

(where we have absorbed coupling strengths in the definition of the effective mass M_N and s denotes the center-of-mass energy squared), then we find the region

$$(2.4-4.3)M_{W_L} \le M_N \le (7.5-40)M_{W_L} \tag{4}$$

excluded by the above considerations. The uncertainties in these bounds reflect the uncertainties in the temperature and the density profile of the stellar core after collapse (see below). Unlike the bounds (1) coming from the neutronization process, the bounds (4) from e^+e^- annihilation, which can also be mediated by charged-current interactions, do not depend upon assuming a maximum energy that can be released from the star.

In view of the various uncertainties surrounding the detailed situation in a collapsing core, in what follows, we shall take a simplified picture of the core immediately after collapse.[6,7] For the inner core we shall assume a constant matter density $\rho_C \simeq 8\times10^{14}$ g/cm^3 (corresponding to a total mass $M \simeq 1.4M_\odot$ and a radius $R_C \simeq 10^6$ cm) and a temperature $T_C = 30-70$ MeV. Furthermore, we shall take approximately equal numbers of neutrons, protons, and electrons, with number density $n_C \simeq 2.4\times10^{38}$ cm^{-3}, for $t \lesssim 0.5-1$ sec, before the standard ν_L interactions can become operative in carrying away the star's lepton number.[6] During this time, the electrons are highly degenerate, with a chemical potential $\mu \simeq 300$ MeV. The nucleons on the contrary can be taken as semidegenerate and quasistatic. Outside the constant-density inner core ($R > R_C \simeq 10^6$ cm), for the calculation of the ν_R-sphere, we shall take $\rho(R) = \rho_C(R_C/R)^m$, $m = 3-7$ and correspondingly $T(R) = T_C(R_C/R)^{m/3}$ since thermodynamic considerations imply that $T \propto \rho^{1/3}$.

II. TRAPPING CONDITION

Let us first discuss trapping of the right-handed neutrinos, which occurs if the interactions are strong enough. In turn this sets the lower values of the excluded regions (1) and (4). The most relevant reactions that contribute to trapping are $\nu_R n \to e^- p$ arising from W_R exchange or W_L-W_R mixing, and $\nu_R n \to \nu_R n$, $\nu_R p \to \nu_R p$ arising from Z_R (or Z') exchange. If we define, for both of them,

$$\sigma = A G_F^2 E_\nu^2 \tag{5}$$

we find that, for $A \le 10^{-6}$, the ν_R mean free path l_{ν_R} exceeds the inner core radius $l_{\nu_R} \lesssim R_C$, and no trapping occurs. For $A \gtrsim 10^{-6}$, trapping occurs and to estimate the neutrino luminosity, we have to calculate the radius of the neutrino (ν_R) sphere.[7] Let us define an R-dependent mean free path

$$l_{\nu_R}(R) = \frac{1}{n(R)\sigma(R)} , \tag{6}$$

where $n(R) = n_C(R_C/R)^m$ and $(\langle E_\nu^2\rangle \sim 10T^2)$

$$\sigma(R) \simeq A G_F^2 10 T(R)^2 \simeq A G_F^2 10 T_C^2 (R_c/R)^{2m/3} . \tag{7}$$

The neutrino-sphere radius R_{ν_R} is defined[8] as the value of R at which the "optical depth"

$$\tau_{\nu_R}(R) = \int_R^\infty \frac{dR}{l_{\nu_R}(R)} \tag{8}$$

equals $\frac{2}{3}$, $\tau_{\nu_R}(R_{\nu_R}) = \frac{2}{3}$. This equation sets the following dependence of R_{ν_R} on the parameter A, defined in Eq. (5),

$$R_{\nu_R} \propto A^{3/(5m-3)} \tag{9}$$

and correspondingly, for the temperature T_{ν_R} of the ν_R-sphere,

$$T_{\nu_R} \propto A^{-m/(5m-3)} . \tag{10}$$

Rather than computing the absolute ν_R luminosity $Q(\nu_R)$, we look at its value relative to the standard ν_L luminosity $Q(\nu_L)$, associated with the long-term ν_L-diffusion flux

$$Q(\nu_L) \simeq (5-10)\times10^{51} \text{ ergs/sec} \tag{11}$$

in rough agreement with observation.[1,2] In turn $Q(\nu_L)$ is determined by ν_L trapping and the radius of the ν_L-sphere which in turn are determined by $\nu_L n \to e^- p$, as well as other charged- and neutral-current processes with a cross section also given by (7) with $A \simeq 1$. Since, from the Stephan-Boltzmann law, $Q \propto T^4 R^2$, we get the desired equation

$$\frac{Q(\nu_R)}{Q(\nu_L)} = \left[\frac{T_{\nu_R}}{T_{\nu_L}}\right]^4 \left[\frac{R_{\nu_R}}{R_{\nu_L}}\right]^2 = A^{-(4m-6)/(5m-3)} . \tag{12}$$

Demanding as an efficient ν_R-trapping condition $Q(\nu_R) \lesssim 20 Q(\nu_L)$, we find ($m = 3-7$)

$$A \ge 10^{-3}-10^{-2} . \tag{13}$$

This bound will provide the lower limit on the range of masses of M_{W_R} or $M_{Z_R}(M_{Z'})$ that is excluded by supernova observations.[9]

We arrive at the condition $Q_{\nu_R} \lesssim 20 Q_{\nu_L}$ by requiring that the ν_R luminosity be less than 10^{53} ergs/sec. If $Q(\nu_R) \ge 10^{54}$ ergs/sec, the core would have been cooled in less than a second—in contradiction with observation—whereas ν_R emission at $Q(\nu_R) \le 10^{52}$ ergs/sec would have only had a slight effect. We think that $A > 10^{-3}-10^{-2}$ is a conservative bound, which could perhaps be strengthened by the consideration of core evolution during collapse. On the other hand, this bound will be enough to close the gap of left- to right-handed charged-current interactions by laboratory observations.

III. LIMITS ON RIGHT-HANDED CHARGED-CURRENT INTERACTIONS

Let us write down the charged-current interactions in the left-right-symmetric models:

$$\mathcal{L}_I = \frac{ig}{\sqrt{2}}[W_{1\mu}^+(J_L^\mu \cos\zeta + J_R^\mu \sin\zeta) + W_{2\mu}^+(J_R^\mu \cos\zeta - J_L^\mu \sin\zeta)] + \text{H.c.} , \tag{14}$$

where J_L^μ (J_R^μ) is the left-handed (right-handed) charged current and ζ is the left-right mixing parameter. In what follows, we denote the masses of W_1 and W_2 by M_{W_L} and M_{W_R}, respectively. The presence of the right-handed current will lead to ν_R production in the epoch of neutronization via the reaction $e_R^- p \to \nu_R n$ with cross section

$$\sigma \simeq B\frac{G_F^2 E_e^2}{\pi}(g_V^2 + 3g_A^2) \simeq \frac{6B}{\pi} G_F^2 E_e^2 , \tag{15}$$

where

$$B = \zeta^2 + \left[\frac{M_{W_L}}{M_{W_R}}\right]^4 . \tag{16}$$

The ν_R luminosity by this process, at fixed proton and electron densities n_p and n_e, is given by

$$\dot{Q}(\nu_R)=Vn_p n_e B6\frac{G_F^2}{\pi}\langle E_\nu^3\rangle \ . \tag{17}$$

Using $V=4\times10^{18}$ cm^3, $\langle E_\nu^3\rangle \simeq \langle E_e^3\rangle \simeq (300\ \text{MeV})^3$, and the number densities given in Sec. I, one obtains (we assume that the core density is constant)

$$Q(\nu_R)\simeq 10^{64}B\ \text{ergs/sec} \tag{18}$$

which would naively imply $B \lesssim 10^{-11}$.

One has to take into account however that the overall number of protons is fixed $N_p \simeq 1.4M_\odot/2m_p \simeq 10^{57}$ and is decreasing with time via the neutronization process itself. In order to put the bound on B, rather than the differential luminosity, one has to consider the integrated luminosity on the total energy carried away by the ν_R neutronization process:

$$E_N(t)\simeq -V\int_0^t dt\frac{dn_p}{dt}\langle E_\nu\rangle_t \ . \tag{19}$$

We require that $E_N(t\simeq 1\ \text{sec})$, before the left-handed neutrinos can carry away the star's lepton number, saturates the total energy E_T that can be released

$$E_T\simeq E_B\simeq \frac{3}{5}\frac{GM^2}{R}=3\times10^{53}\ \text{ergs}\left[\frac{10^6\ \text{cm}}{R}\right]\left[\frac{M}{1.4M_\odot}\right]^2 . \tag{20}$$

Using

$$\langle E_\nu\rangle_t \simeq \langle E_e\rangle_t \simeq \mu_e(t)\simeq \mu_e(0)\left[\frac{n_p(t)}{n_p(0)}\right]^{1/3}$$

and

$$\frac{dn_p}{dt}=-6B\frac{G_F^2}{\pi}n_p^2\langle E_e\rangle_t^2 \ , \tag{21}$$

we get

$$E_N(t)=E_N(\infty)\left[1-\left[1+\frac{t}{\tau}\right]^{-4/5}\right] , \tag{22}$$

where

$$E_N(\infty)=\tfrac{3}{4}\mu_e(0)n_p(0)V$$
$$=4\times10^{53}\ \text{ergs}\left[\frac{M}{1.4M_\odot}\right]^{4/3}\left[\frac{10^6\ \text{cm}}{R}\right] \tag{23}$$

and

$$\tau=\frac{3}{5}\frac{\pi}{6BG_F^2 n_p(0)\mu_e^2(0)}\simeq\frac{2.4\times10^{-11}}{B}\ \text{sec} \ .$$

Needless to say, the fact that the $E_N(\infty)$ is even greater than E_T is an artifact of the naive calculation. Nevertheless the same calculation shows that $E_N(1\ \text{sec})$ will saturate E_T, provided, say, $E_N(1\ \text{sec})\lesssim 0.8E_N(\infty)$, which gives in turn $B\geq 1.5\times10^{-10}$. Using Eq. (16) this implies $M_{W_R}\leq 280M_{W_L}$ for $\zeta=0$. On the other hand, one might assume that half the energy escapes from the collapse of the hot shocked mantle; in this case, it is more appropriate to choose $t=2$ sec and $E_N\simeq \frac{1}{2}E_\infty$. This then implies $\tau=1.4$ sec and $M_{W_R}\simeq 500M_{W_L}$. This reflects a basic uncertainty in our estimate of the bound. Furthermore the bound in Eq. (13), using Eqs. (5), (15), and (16), and setting $E_\nu=E_e$ leads to $\zeta>0.07-0.02$ and $M_{W_R}\leq(3.7-6.7)M_{W_L}$, ignoring possible neutral-current effects. These give the excluded ranges of M_{W_R} and ζ stated in Eqs. (1) and (2). Note, however, that present laboratory limits[10,11] from μ^+ decay already rule out the lower limits in Eq. (1) ($M_{W_R}\lesssim 514$ GeV for $\zeta=0$ from μ decay[11]). This combination of supernova observations with laboratory observations would imply $M_{W_R}\geq 23$ TeV and $\zeta<10^{-5}$ for $m_{\nu_R}\lesssim 10$ MeV. These are the most stringent bounds to date on M_{W_R} and ζ.

IV. LIMITS ON NEUTRAL-CURRENT INTERACTIONS

In this section we consider the ν_R production via e^+e^- annihilation. This will be the only production process of right-handed neutrinos in the case of exclusive neutral-current interactions. The same process may also play a role for charged-current interactions, if the bound on the total luminosity, $E_T\leq 4-5\times10^{53}$ ergs, is evaded and a residual significant energy is left after neutronization.

The luminosity for the annihilation process is

$$Q(\nu_R,\bar\nu_R)=Vn(e^+)n(e^-)\times\langle\sigma(e^+e^-\to\nu_R\bar\nu_R)v_{\text{rel}}(E_\nu+E_{\bar\nu})\rangle \tag{24}$$

(where v_{rel} is the relative velocity of e^+ and e^-). Neglecting for the moment the electron chemical potential, one finds, using Eq. (3),

$$Q(\nu_R,\bar\nu_R)=\frac{1}{12\pi}\left[\frac{M_{W_L}}{M_N}\right]^4 G_F^2 V\frac{16}{3\pi^4}T^9F_3F_4 \ , \tag{25}$$

where

$$F_n=\int_0^\infty\frac{y^n dy}{e^y+1}$$

so that

$$Q(\nu_R,\bar\nu_R)=\left[\frac{M_{W_L}}{M_N}\right]43\times(10^{56}-10^{59})\ \text{ergs/sec} \tag{26}$$

depending on the inner core temperature $T_C=30-70$ MeV. A similar range of values is obtained for a nonzero chemical potential ($\mu_e\simeq 300$ MeV for $t\leq 0.5$ sec) since in this case the suppression of the positron density ($e^{-\mu/T}$) is almost completely compensated by the larger electron density.

Requiring $Q(\nu_R,\bar\nu_R)\leq 10^{53}$ ergs/sec gives

$$M_N\lesssim(7.5-40)M_{W_L} \ . \tag{27}$$

Also in the case of pure neutral-current interactions trapping of ν_R's can take place, as discussed in Sec. II via $\nu_R N\to\nu_R N$. Defining, for the relevant cross section, Eq. (5),

$$A\equiv\frac{1}{\pi}\left[\frac{M_{W_L}}{M_N'}\right]^4 \tag{28}$$

we get, from Eq. (13),

$$M_N'\leq(2.4-4.3)M_{W_L} \tag{29}$$

in order to suppress trapping. Notice that M_N, Eq. (3), and M_N', Eq. (28), are in general different, since different vertices are involved in emission and trapping. In turn the different couplings are model dependent. The identification of M_N and M_N' leads to the excluded region (4). Depending on the number of right-handed neutrinos and on their mass, the bound (29) may be in conflict with the cosmological constraint on nucleosynthesis.[12]

After submission of this paper, we came across the paper by Raffelt and Seckel,[13] which also discusses limits on right-handed currents from supernova 1987A observations.

Although obtained employing somewhat different criteria we agree on the upper value of the excluded region for the charged-current interaction strength. On the contrary we disagree on the analogous bound for the neutral currents. This is due to the fact that (i) we consider, as source of right-handed neutrinos, the process $e^+e^- \to \bar{\nu}_R \bar{\nu}_R$, whereas Raffelt and Seckel (RS) consider the "neutrino bremsstrahlung" processes $nn \to nn\bar{\nu}\nu$, $np \to np\bar{\nu}\nu$, and (ii) we take into account the uncertainty in the temperature of the supernova core, $T=30-70$ MeV, whereas RS considered a fixed value of the same temperature, $T=75$ MeV.

Using the matrix element of Friman and Maxwell,[14] we have reevaluated the rate for the bremsstrahlung processes in the nondegenerate limit appropriate to the case of the hot supernova core in its initial stage. (Because of the approximations made by Friman and Maxwell, their calculation, appropriate to the strongly degenerate situation of a cold neutron star, cannot be extrapolated to the supernova situation.[15] As we understand, this extrapolation has been made in Ref. 15.) For the thermally averaged luminosity, we find

$$Q(nn \to nn\bar{\nu}\nu) \approx \frac{Vn_n^2}{2\pi^5} m_n \langle p \rangle g_A^2 G_F^2 \left[\frac{M_{W_L}}{M_N} \right]^4 \frac{T^5}{m_\pi^4} \approx 2\times 10^{60} \left[\frac{M_{W_L}}{M_N} \right]^4 \left[\frac{T}{70\text{ MeV}} \right]^5 \text{ergs/sec} , \tag{30}$$

where $g_A = 1.25$, $\langle p \rangle \simeq 400$ MeV is the average nucleon momentum, and m_n, m_π are the nucleon and pion masses. At $T=70$ MeV, this luminosity is about 20 times smaller than the luminosity extrapolated from the degenerate case,[15] probably due to the overestimation of the nucleon density. However, $Q(nn \to nn\bar{\nu}\nu)$ appears to be larger than $Q(\nu_R, \bar{\nu}_R)$, Eq. (26), from e^+e^- annihilation. Using this result together with the analogous calculation for $Q(np \to np\bar{\nu}\nu)$ and taking into account the uncertainty in the temperature of the supernova core, the upper value for the excluded range of the neutral-current interaction strength, Eq. (27), can be improved to

$$M_N \lesssim (30-85) M_{W_L} .$$

This lower bound is stronger than the one obtained from considerations of e^+e^- annihilation, but weaker than what is claimed in Ref. 13.

The lower values of the excluded regions, both for the charged- and for the neutral-current interaction strengths, that we discuss quantitatively, remain unchanged.

In conclusion, we have excluded a significant range of values for mass of right-handed W_R bosons and W_L-W_R mixing parameter for a light ν_R using the observations of SN 1987A. We also obtain bounds on the masses of extra Z bosons of left-right and superstring models from the same considerations.

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