Simple proof of Weil triviality in supersymmetric gauge theories

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Using methods of supermanifold cohomology we give a simple proof of Weil triviality in supersymmetric gauge theories, generalizing previous results of Bonora, Pasti, and Tonin which are relevant to the solution of the anomaly problem for such theories. By considering a supersymmetric gauge theory over an (m,n)-dimensional supermanifold with trivial topology in the odd directions, we prove without imposing constraints on the supercurvature and supertorsion forms the exactness of the forms $P(F^k)$, where F is the curvature form of a connection on the relevant principal super fiber bundle and P is an invariant polynomial on the (super) Lie algebra of the structure (super)group, of degree $k > m/2$. In order to prove the locality of the form X such that $P(F^k)=dX$ we have to use the constraints, but the necessary calculations turn out to be rather easy for any space-time dimension.

I. INTRODUCTION

Differential geometric methods based on the transgression formula of suitable principal bundles have proved to be extremely useful in the analysis of anomalies in Yang-Mills and gravity theories.[1] When trying to generalize these geometric methods to supersymmetric theories,[2] one needs to prove what Bonora and co-workers called *Weil triviality,* i.e., that the form $P(F^k)$ is exact, where F is a curvature form on the relevant principal bundle and P is an invariant polynomial of degree $k=m/2+1$, m being the space-time dimension. Moreover, the superform X such that $P(F^k)=dX$ is required to be local in the field variables.

In Ref. 3 Bonora, Pasti, and Tonin analyzed a cohomology complex whose coboundary operator is determined by the torsion tensor (T cohomology). The constraints on supercurvature and torsion imply conditions on the T cohomology such that the Weil homomorphism is trivial (at the order which is relevant to the anomaly problem) when restricted to the space of gauge-invariant superforms. The actual computations in the case of a high-dimensional space-time involve a heavy and intriguing study of the irreducible representations of the structure group of the frame bundle.

In this paper we show that by using techniques of supermanifold cohomology one can both greatly simplify the proof and generalize the results of Bonora and co-workers. We shall prove that if Q is any principal super fiber bundle on a DeWitt (m,n)-dimensional supermanifold M (Ref. 4) [roughly speaking, an (m,n) supermanifold M is DeWitt if it is a bundle over an m-dimensional ordinary manifold M_0, which physically plays the role of space-time], the Weil homomorphism of the bundle Q is trivial when restricted to invariant polynomials of degree $k>m/2$, i.e., $P(F^k)=dX$ for $k>m/2$, P being an invariant polynomial of degree k.

The assumption that the supermanifold is DeWitt does not seem to be physically restrictive, since so far no physical meaning of a nontrivial topology in the odd directions is known. In contrast with what happens in Ref. 3, in order to prove the exactness of $P(F^k)$ we do not need to take into account constraints on the torsion or curvature, nor do we need to study irreducible representations of any group, nor do we have to consider T cohomology.

All this is shown in Sec. II. If one wants to prove that the form X such that $P(F^k)=dX$ can be chosen so as to be local in the field variables, one must take into account the constraints on the supertorsion and supercurvature. However, locality is proved by means of very simple calculations and the method we are proposing is much simpler for any space-time dimensions than the one described in Ref. 3. In Sec. III we shall sketch this method only for $m=2,4$, but also the computations in higher dimensions can be straightforwardly carried out.

The possibility of proving the exactness of $P(F^k)$ without imposing the standard constraints is intimately connected with our assumption of a trivial topology in the odd directions. An analysis of the global geometry of supersymmetric gauge theories over supermanifolds shows that the result of Bonora and co-workers [i.e., the fact that the standard constraints imply the exactness of $P(F^k)$] holds globally in any topology, at least for a four-dimensional space-time.[5]

II. WEIL TRIVIALITY

Let us give a brief outline of the anomaly problem in supersymmetric gauge theory; to be definite we consider the case of supersymmetric chiral anomalies with external supergravity. Let Q be a principal super fiber bundle over an (m,n) dimensional supermanifold M with structure (super)group G, and suppose on Q there is a connection ω with curvature F. For the sake of simplicity we shall write the relevant equations assuming that the bundle Q is trivial; however, everything can be easily generalized to the case of a nontrivial principal super fiber bundle. (This was done in Ref. 6 in the case of nonsupersymmetric theories.)

In order to write the Becchi-Rouet-Stora-Tyutin (BRST) transformations we need the Faddeev-Popov ghosts c and $\xi=\xi^A\partial_A$ of the group of gauge transformations and of the superdifffeomorphism group, respectively; here A is a collective index $A=(a,\alpha)$, $a=1,\ldots,m$, $\alpha=1,\ldots,n$, and c is locally a mapping from M into Lie(G). Then the BRST transformations are, respectively,

$$\delta_G\omega=-dc-\omega c-c\omega=-D_\omega c, \quad \delta_G c=-c^2\ , \tag{1}$$

and

$$\delta_D\psi=\mathcal{L}_\xi\psi, \quad \delta_D\xi^A=\xi^B\partial_B\xi^A\ , \tag{2}$$

where ψ is any scalar-valued superform and $\mathcal{L}_\xi$ denotes the Lie derivative. The operators δ_G and δ_D are both nilpotent; the operator $\delta=\delta_D+\delta_G$ is nilpotent as well, provided that in addition to (1) and (2) one lets

$$\delta_D\omega=\mathcal{L}_\xi\omega, \quad \delta_D c=\mathcal{L}_\xi c, \quad \delta_G\xi^A=0, \quad \delta_G\psi=0\ .$$

An anomaly $\mathcal{A}$ corresponds to a nontrivial δ-cohomology class modulo d, with ghost number one, so that

$$\delta\mathcal{A}=d\mathcal{B} \quad \text{for some } \mathcal{B}\ , \tag{3a}$$

$$\mathcal{A}\neq\delta\mathcal{A}'+d\mathcal{B}'\ . \tag{3b}$$

If one writes $\mathcal{A}=\mathcal{A}_D+\mathcal{A}_G$, with $\mathcal{A}_D$ and $\mathcal{A}_G$ linear in the corresponding ghost fields, then Eq. (3a) yields

$$[\delta_D\mathcal{A}_D]=[\delta_D\mathcal{A}_G+\delta_G\mathcal{A}_D]=[\delta_G\mathcal{A}_G]=0\ , \tag{4}$$

where [ ] denotes a d-cohomology class.

Now, let us take an invariant polynomial P of order $k=\frac{1}{2}m+1$ on the Lie (super)algebra of G. By using the BRST equations in (1), one proves that

$$P(F^k)=(d+\delta_G)\left[k\int_0^1 dt\, P(\omega',\mathcal{F}_t^{k-1})\right]$$
$$\equiv(d+\delta_G)S\ , \tag{5}$$

where $\omega'=\omega+c$, $\omega'_t=t\omega'$, and

$$\mathcal{F}_t=(d+\delta_G)\omega'_t+\tfrac{1}{2}[\omega'_t,\omega'_t]\ .$$

In Ref. 3 it has been shown that in order to obtain an anomaly $\mathcal{A}$ satisfying Eq. (4) from the descent equations determined by P, one needs the Weil triviality: namely, it must happen that

$$P(F^k)=dX\ , \tag{6a}$$

$$\delta_G X=0\ . \tag{6b}$$

If that is the case, Eq. (5) can be written as

$$(d+\delta_G)\hat{S}=0, \quad \hat{S}=S-X\ . \tag{7}$$

By expanding Eq. (7) according to the ghost number, one obtains the sequence of equations

$$d\hat{S}^0_{2k-1}=0\ , \tag{8a}$$

$$dS^1_{2k-2}+\delta_G\hat{S}^0_{2k-1}=0\ , \tag{8b}$$

$$dS^2_{2k-3}+\delta_G S^1_{2k-2}=0, \quad \ldots\ , \tag{8c}$$

where S^q_{2k-1-q} is a $(2k-1-q)$-superform with ghost number q. If one defines

$$\mathcal{A}_G=S^1_{2k-2}, \quad \mathcal{A}_D=-i_\xi\hat{S}^0_{2k-1}\ , \tag{9}$$

by using Eqs. (8) and (2), and the fact that $\mathcal{L}_\xi=i_\xi d+di_\xi$, one proves that

$$\delta_G\mathcal{A}_G=-dS^2_{2k-3}\ ,$$

$$\delta_D\mathcal{A}_G+\delta_G\mathcal{A}_D=di_\xi S^1_{2k-2}\ ,$$

$$\delta_D\mathcal{A}_D=2di_\xi i_\xi\hat{S}^0_{2k-1}\ ,$$

so that $\mathcal{A}_G$ and $\mathcal{A}_D$ as defined in (9) provide a solution to the anomaly problem (4). Notice that if Eq. (6), and as a consequence Eq. (8a), did not hold it would be impossible to find a partner $\mathcal{A}_D$ of $\mathcal{A}_G$ such that the consistency conditions (4) are satisfied.[3]

Now we give a brief description of the supermanifold techniques we shall use to prove Weil triviality. The fundamental algebraic object is a real exterior algebra $B_L=(B_L)_0\oplus(B_L)_1$ with L generators. One considers the "vector superspace"

$$B_L^{m,n}=(B_L)_0^m\times(B_L)_1^n$$

and defines an (m,n)-dimensional supermanifold as a topological manifold modeled over $B_L^{m,n}$ by means of an atlas whose transition functions satisfy a suitable "supersmoothness" condition. (Actually, a rigorous definition of supermanifolds yielding a viable theory involves some subtleties that are ignored here; see Ref. 7.) A supersmooth function $f:U\subset B_L^{m,n}\to B_L$ has the usual form (superfield expansion)

$$f(x^1\cdots x^m,y^1\cdots y^n)=f_0(x)+\sum_{\alpha=1}^{n}f_\alpha(x)y^\alpha+\cdots$$
$$+f_{1\cdots n}(x)y^1\cdots y^n\ ,$$

where the x's are the even (Grassmann) coordinates, the y's are the odd ones, and the dependence of the coefficient functions $f_{\ldots}(x)$ on the even variables is fixed by their values for real arguments.

Together with the sheaf $\mathcal{G}$ of germs of supersmooth B_L-valued functions on M, one can consider the sheaves Ω^k of germs of k-superforms and an exterior differential $d:\Omega^k\to\Omega^{k+1}$. The cohomology of the differential complex

$$\mathcal{G}(M)\xrightarrow{d}\Omega^1(M)\xrightarrow{d}\Omega^2(M)\xrightarrow{d}\cdots\ ,$$

where $\mathcal{G}(M)$ and $\Omega^k(M)$ are the spaces of global supersmooth functions and k-superforms on M, respectively, is denoted by $H_{\rm SDR}(M)$ (Refs. 8 and 9). It is to be noticed that the analogue of de Rham's theorem

$$H^k_{\rm SDR}(M)\simeq H^k(M,B_L), \quad k\geq 0 \tag{10}$$

[where $H(M,B_L)$ is the Čech cohomology of the constant sheaf B_L on M] in general fails to apply. This is basically due to the fact that, contrary to what happens in the case of ordinary smooth manifolds, the sheaves $\mathcal{G}$ and Ω^k may have nontrivial Čech cohomology in degree higher then zero.[9]

However, the isomorphism (10) holds in a particular case, which is relevant to physical applications. We say that an (m,n)-dimensional supermanifold is DeWitt if it is a locally trivial bundle over an m-dimensional ordinary smooth manifold M_0, called the *body* of M, with fiber the $\mathbb{R}$-vector space $N_L^{m,n}=(N_L)_0^m\times(N_L)_1^m$, N_L being the ideal of nilpotent elements in B_L. It has been shown elsewhere[10] that if M is DeWitt the sheaves $\mathcal{G}$ and Ω^k are cohomologically trivial; this in turn implies that the isomorphism (10) holds, i.e., a de Rham theorem applies provided that the supermanifold is DeWitt.

Since a DeWitt supermanifold is a locally trivial bundle over its body with a contractible fiber, it follows that it is contractible to its body, so that

$$H^k(M,B_L)\simeq H^k(M_0,B_L)\simeq H^k(M_0,\mathbb{R})\otimes_{\mathbb{R}}B_L, \quad k\geq 0 . \tag{11}$$

The isomorphisms (11), together with de Rham's theorem for M_0,

$$H^k_{\mathrm{DR}}(M_0)\simeq H^k(M_0,\mathbb{R}), \quad k\geq 0 ,$$

imply that *if M is a DeWitt supermanifold with body M_0, then*[9]

$$H^k_{\mathrm{SDR}}(M)\simeq H^k_{\mathrm{DR}}(M_0)\otimes_{\mathbb{R}}B_L, \quad k\geq 0 . \tag{12}$$

(This result is the analogue in supermanifold theory of a theorem in graded manifold theory due to Kostant.[11]) From now on we understand that the supermanifold M under consideration is DeWitt. The consequence of the isomorphism (12) that is relevant to our purposes is that any closed k-superform on M is exact if $k>m$, since in that case $H^k_{\mathrm{SDR}}(M)\equiv 0$. Let Q be a principal super fiber bundle on M carrying a connection ω with curvature form F (Ref. 12), and let P be an invariant polynomial of degree k on the Lie superalgebra of the structure group of Q; generalizing classical results, it has been shown in Ref. 12 that $P(F^k)$ determines a closed $2k$-superform on M (that we denote by the same symbol) whose cohomology class $[P(F^k)]\in H^{2k}_{\mathrm{SDR}}(M)$ does not depend on the connection but only on the bundle. If $2k>m$, that cohomology class necessarily vanishes, so that we have the following result:

$$P(F^k)=dX \tag{13}$$

with X a $(2k-1)$-superform on M. When comparing Eqs. (6) and (13) one must consider that the superforms in Eq. (6) are forms on the total space of the bundle, while those in Eq. (13) are forms on the base space M. The X in Eq. (6) is the pullback of the X in Eq. (13), and therefore is invariant under the action of vertical bundle automorphisms, i.e., Eq. (6b) holds.

It is clear from the previous analysis that this result holds for any supersymmetric gauge theory on a supermanifold in any space-time dimension, no matter whether or not the gauge field is coupled to supergravity, and independently of the form of the constraints.

III. LOCALITY OF THE FORM X

In this section we show that, provided the standard constraints on curvature and torsion are taken into account, the form X appearing in Eq. (13) in the case $k=m/2+1$ can be chosen so as to be a polynomial in the components of the field variables. We shall need to consider the operators T and S introduced in Ref. 3; these can be defined in a mathematically consistent way, as the analysis of the geometry of supersymmetric gauge theories done in Ref. 5 showed. Here we shall only sketch the calculations involved in the proof of the locality of X, while the reader interested in mathematical details may refer to Ref. 5.

If the supersymmetric gauge theory under consideration is coupled with supergravity it is necessary to introduce also a principal super fiber bundle Lor(M), which is a subbundle of the superbundle $L(M)$ of frames over M. The structure group of Lor(M) is (the Grassmann generalization of) Spin$(1,m-1)$, the covering group of SO$(1,m-1)$. Let U be an open set in M over which Lor(M) trivializes, and let $\sigma=\{D_i,D_\alpha,\ i=1,\ldots,m,\ \alpha=1,\ldots,n\}$ be a section of Lor(M) over U. We assume on Lor(M) there is a connection satisfying the usual constraints

$$\sigma^*T^i=-\tfrac{1}{2}(C\gamma^i)_{\alpha\beta}\omega^\alpha\omega^\beta, \quad \sigma^*T^\alpha=0 ,$$

where $T^A=\{T^i,T^\alpha\}$ is the torsion form of the connection, the ω's are the coframes dual to the D's, and C is the charge-conjugation matrix.

The even frames $\{D_i\}$ generate over the supersmooth functions on U an involutive rank-$(m,0)$ subbundle $T'U$ of TU, while the odd frames $\{D_\alpha\}$ generate a rank-$(0,n)$ subbundle $T''U$ which is not involutive; indeed the commutator of two D_i's lies in the span of the D_i's, while the anticommutator of two D_α's does not lie in the span of the D_α's. Because of the fact that Spin$(1,m-1)$ acts reducibly on these two subbundles of TU through a block-diagonal representation, they are not mixed by that action but they glue together to yield a global splitting

$$TM=T'M\oplus T''M . \tag{14}$$

Denoting by $T^*M=T^{*'}M\oplus T^{*''}M$ the splitting of the cotangent bundle T^*M dual to (14), we define the super vector bundles of superforms of type (p,q):

$$\Omega^{p,q}=(\wedge^pT^{*'}M)\wedge(\wedge^qT^{*''}M) ;$$

then we have a decomposition

$$\Omega^k=\bigoplus_{p+q=k}\Omega^{p,q}$$

with projections $\pi^{p,q}:\Omega^k\to\Omega^{p,q}$.

A section η of $\Omega^{p,q}$, i.e., a superform of type (p,q) over an open set U in M, is locally written as

$$\eta=\sum\eta_{i_1\cdots i_p,\alpha_1\cdots\alpha_q}\omega^{i_1}\wedge\cdots\wedge\omega^{i_p}\wedge\omega^{\alpha_1}\wedge\cdots\wedge\omega^{\alpha_q} , \tag{15}$$

where the ω^i's are of type (1,0) and the ω^α's of type (0,1).

It was shown in Ref. 5 that the exterior differential d acting on a superform of type (p,q) splits into

$$d=d_0+d_1+T \ , \tag{16}$$

where

$$d_0=\pi^{p+1,q}\circ d, \quad d_1=\pi^{p,q+1}\circ d \ ,$$
$$T=\pi^{p-1,q+2}\circ d \ .$$

Locally T can be written as

$$T\eta=(C\gamma^i)_{\alpha\beta}\omega^\alpha\omega^\beta i_{D_i}\eta \ .$$

We also introduce an operator $S:\Omega^{p,q}\to\Omega^{p+1,q-2}$ locally defined by

$$S\eta=\frac{1}{2n}(\gamma_i C^{-1})^{\alpha\beta}\omega^i i_{D_\alpha} i_{D_\beta}\eta \ ;$$

a simple calculation shows that, if η is of type (p,q), then

$$(TS+ST)\eta=p\eta+\frac{1}{2n}(C\gamma^i)_{\alpha\beta}(\gamma_i C^{-1})^{\mu\nu}\omega^\alpha\omega^\beta i_{D_\mu} i_{D_\nu}\eta \ . \tag{17}$$

We shall describe the procedure we use to prove the locality of X by considering a specific example: namely, the case $m=2$. The physically relevant theories in two dimensions are related to string theory. In that case one cannot assume the standard constraints, i.e., $F_{\alpha\beta}\neq 0$ in general.[13] We have $k=2$ and

$$P(F^2)=P^{2,2}+P^{1,3}+P^{0,4} \ . \tag{18}$$

Any X satisfying

$$P(F^2)=dX \tag{19}$$

is of the form

$$X=X^{2,1}+X^{1,2}+X^{0,3} \ .$$

We wish to show that one can find a two-superform η such that $\bar{X}=X+d\eta$ has no (0,3) component; then of course $P(F^2)=d\bar{X}$, and it will be easy to prove that $\bar{X}$ is local. We must solve the equation

$$\pi^{0,3}(X+d\eta)=0 \tag{20}$$

in the unknown η; setting $\eta=\eta^{2,0}+\eta^{1,1}+\eta^{0,2}$, Eq. (20) gives

$$X^{0,3}+T\eta^{1,1}+d_1\eta^{0,2}=0 \ .$$

This equation is solved if

$$\eta^{0,2}=0 \ , \tag{21a}$$

$$T\eta^{1,1}=-X^{0,3} \ , \tag{21b}$$

$$\eta^{2,0}=0 \ . \tag{21c}$$

Trivial local calculations involving some Dirac matrices algebra show that Eq. (21b) can be solved.

Assuming that $X=X^{2,1}+X^{1,2}$, Eq. (19) can be written

$$P^{2,2}=d_1X^{2,1}+d_0X^{1,2} \ , \tag{22a}$$

$$P^{1,3}=TX^{2,1}+d_1X^{1,2} \ , \tag{22b}$$

$$P^{0,4}=TX^{1,2} \ . \tag{22c}$$

By applying the operator S to Eq. (22c) and using Eq. (17) one determines $X^{1,2}$; the resulting equation can be written down in local components as follows. If $X^{1,2}=X_{i\alpha\beta}\omega^\alpha\omega^\beta\omega^i$ and $SP^{0,4}=P_{i\alpha\beta}\omega^\alpha\omega^\beta\omega^i$, then

$$X_{i\alpha\beta}=P_{i\alpha\beta}-(C\gamma^h)_{\alpha\beta}(\gamma_h C^{-1})^{\mu\nu}P_{i\mu\nu} \ . \tag{23}$$

Applying S to (22b) one obtains

$$X^{2,1}=\tfrac{1}{2}S(P^{1,3}-d_1X^{1,2}) \ . \tag{24}$$

Equations (23) and (24) show that X is local (actually, a polynomial) in the components of the field variables. Aside from notations and conventions, Eq. (24) agrees with the result obtained in Ref. 13 by means of group-theoretical arguments, while in order to transform Eq. (23) into the result of Ref. 13 one needs to plug in the explicit form of $P^{0,4}$, coming from the constraint $F_{\alpha\beta}\propto(C\gamma_5)_{\alpha\beta}R$, where R is the curvature scalar.

In the case $m=4$ one has to use the standard constraints, i.e., $F_{\alpha\beta}=0$. Since $k=3$ we have

$$P(F^3)=F^{4,2}+P^{3,3} \ .$$

It is not hard to see that any X satisfying $P(F^3)=dX$ has only terms of types (4,1), (3,2), and (2,3). With the same reasoning as before we can prove that a four-superform η exists such that $\bar{X}=X+d\eta$ is of type (4,1); i.e., we may assume that X is of type (4,1). Then we have

$$P^{4,2}=d_1X \ , \tag{25a}$$

$$P^{3,3}=TX \ . \tag{25b}$$

Applying the operator S to (25b) one obtains, as a consequence of Eq. (17),

$$X=\tfrac{1}{4}SP^{3,3}$$

so that X is local.

The same computations have been done in the cases $m=6$ and 10; the number of equations one has to solve to determine η does not increase with space-time dimension, so that the procedure comes out to be rather easy in any space-time dimension.

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APPENDIX: INTEGRATING ANOMALIES OVER SPACE-TIME

In supersymmetric gauge theory the anomalies determined in Eq. (9) are usually integrated over space-time by means of a formal procedure, see, e.g., papers by Bonora and co-workers in Ref. 2. We would like to make a few remarks upon a mathematically consistent description of that integration. Whenever we have an (m,n)-dimensional DeWitt supermanifold M whose body M_0 is compact orientable without boundary, and η is an m-superform on M, we can integrate η over M_0 by pulling it back to M_0 by means of a global section of the smooth bundle $M \rightarrow M_0$, which always exists since the fiber of the bundle is diffeomorphic to a vector space. So, if $\sigma : M_0 \rightarrow M$ is such a section, the integral

$$\int_{M_0} \sigma^* \eta \tag{26}$$

is well defined. This is the integral formally defined in Refs. 2 and 3; of course, it will depend also on the section σ. It was shown in Ref. 14 that the integral (26) does not depend upon σ if its integrand is invariant, up to an exact form, under supersymmetry transformations. Thus the integral over space-time of an anomaly is well defined whenever one can choose a supersymmetric representative of the anomaly cohomology class.

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