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Quantum field theory of the Universe

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As is well known, the wave function of the Universe dictated by the Wheeler-DeWitt equation has a difficulty in its probabilistic interpretation. In order to overcome this difficulty, we explore a theoretical possibility of the second quantization of the Universe, following the same passage historically taken for the Klein-Gordon particles and the Nambu-Goto strings. It turns out that multiple production of universes is an inevitable consequence even if the initial state is *nothing*. The problematical interpretation of the wave function of the Universe is circumvented by introducing an internal comoving model detector, which is an analogue of the DeWitt-Unruh detector in quantum field theory in curved space-time.

I. INTRODUCTION

Since the fundamental paper by DeWitt,[1] quantum geometrodynamics has been investigated by many people. Recently, Hawking and his collaborators[2] and Vilenkin[3] revived the interest in this field in the context of quantum cosmology. Despite the promising features (e.g., inflation) of the wave function of the Universe discovered in various semiclassical solutions of the Wheeler-DeWitt equation, there remains a serious difficulty in its probabilistic interpretation. Because the Wheeler-DeWitt equation is a hyperbolic second-order differential equation, there are no such conserved quantities that are positive definite. This is a familiar situation with which we once encountered in the case of the Klein-Gordon equation in particle physics. More fundamentally, we do not understand even the meaning of *probability* for the very Universe in which we live; different universes are impossible for us to recognize.

In this paper we would like to explore the possibility of the second (third?) quantization of the wave function of the Universe. Our motivation is simple: by proceeding to the quantum field theory of the Universe, we can overcome the difficulty of negative probability just as we succeeded in the case of the Klein-Gordon equation. (This idea of reinterpretation of the wave function may not be new, e.g., see Ref. 4.)

One of the advantages of our approach is that there is a privileged state *nothing* where even the Universe does not exist. Suppose our state is this empty state *nothing*. Then how can our Universe emerge from nothing? The Wheeler-DeWitt equation, which is reinterpreted as a field equation, will tell us how.

To be specific, let us consider an isotropic homogeneous spatially flat universe. For a homogeneous scalar field, the Wheeler-DeWitt equation takes the form, up to the operator-ordering ambiguity,

$$[\partial_\alpha^2-\partial_\phi^2+U(\alpha,\phi)]\Psi(\alpha,\phi)=0, \quad U=8\pi^4 e^{6c\alpha}V(\phi)\ , \tag{1.1}$$

where $\alpha=c^{-1}\ln a$, and $c=\sqrt{4\pi G/3}$. Hereafter we take the unit $c=1$. As we remarked before, the partial differential equation (1.1) is hyperbolic and α plays the role of time in the Klein-Gordon equation. The big difference of Eq. (1.1) from the ordinary Klein-Gordon equation is the presence of the time-dependent potential $U(\alpha,\phi)$ which is not an *ad hoc* quantity introduced by hand but an integral part of the Einstein gravity. It is well known, in ordinary quantum field theory, that the particles can be created from vacuum if the external potential is time dependent.[5] In the same way in our field theory of the Universe, many universes can be created from "nothing." They are not *a priori* causally disconnected objects; they may coherently affect our observation of the Universe. Here we are forced to reconsider the basic assumptions of classical cosmology. For example, do we live in one and only one universe? Is the Universe always countable? (More details will be given in Sec. III.) The idea of multiple universes is hardly new. For example, Sato, Kodama, Sasaki, and Maeda discussed the multiple production of universes in the inflationary universe scenario. They are essentially leftover wormholes on the occasion of the inflationary phase transition. Recently Hawking discussed the path integral over all the topologies of space-time.[2] His approach is analogous to the Polyakov[7] approach in string theory while ours corresponds to the field theory of a string in the manner of Kaku and Kikkawa.[8]

However, a natural question may arise: what is the observational consequence of the theories of multiple universes? We will attempt to answer this question in the context of our quantum field theory of the Universe.

The plan of this paper is as follows. In Sec. II we give a brief review of the Wheeler-DeWitt equation and summarize the present status of quantum cosmology from our point of view. We are going to develop the formalism of our second-quantized field theory of the Universe in Sec. III. A specific model is investigated in Sec. IV and the number of universes, though unobservable, is estimated there. We discuss in Sec. V the issue of the detector in more detail. The final section is devoted to a summary and discussions.

II. QUANTUM MECHANICS OF THE UNIVERSE: REVIEW OF THE WHEELER-DeWITT EQUATION

This section is devoted to a brief review of the quantum mechanics of the Universe in order to introduce notation and to clarify our motivation for the second quantization of the Universe.

We start from the Einstein action of gravity and the action of a one-component real scalar field:

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R+\text{boundary terms}+\int d^4x\sqrt{-g}\,[\tfrac{1}{2}(\partial_\mu\phi)^2-V(\phi)]\,, \tag{2.1}$$

where R is a scalar curvature and V is a matter potential which includes a possible cosmological constant effectively. Since the full argument on all the gravitational variables is so hard, we restrict our consideration to the homogeneous and isotropic metric:

$$ds^2=N^2(t)dt^2-a^2(t)d\sigma^2\,. \tag{2.2}$$

Here the lapse function N represents the general time-coordinate transformation freedom. On the other hand, the shift function which represents the general space-coordinate transformation freedom is neglected here since it is trivial for the dynamics in general. The three-space metric is given by

$$d\sigma^2=d\chi^2+f^2(\chi)(d\theta^2+\sin^2\theta\,d\phi^2) \tag{2.3}$$

where the "radius" $f(\chi)$ is expressed as

$$f(\chi)=\begin{cases}\sin\chi & \text{if } k=+1\quad(0\le\chi<2\pi),\\ \chi & \text{if } k=0\quad(0\le\chi<\infty),\\ \sinh\chi & \text{if } k=-1\quad(0\le\chi<\infty),\end{cases}$$

where k is the signature of the spatial curvature. For the restricted metric Eq. (2.2), the total action becomes

$$S=\frac{V}{16\pi G}\int dt\,Na^3\left\{\frac{6}{a^2}\left[k+\frac{\dot a^2}{N^2}+\frac{a}{N}\left[\frac{\dot a}{N}\right]^{\cdot}\right]-2\Lambda\right\}+\text{boundary terms}+V\int dt\,Na^3\left[\frac{1}{2}\frac{\dot\phi^2}{N^2}-V(\phi)\right]. \tag{2.4}$$

We have introduced the spatial volume V of the homogeneous region we consider. This volume becomes $2\pi^2$ for $k=1$; however, for other values of k, this must be some properly fixed finite constant. The Lagrangian is read from the above action as

$$L=VN\left[\frac{3}{8\pi G}a\left[k-\frac{\dot a^2}{N^2}\right]+\frac{1}{2}a^3\frac{\dot\phi^2}{N^2}-a^3V(\phi)\right]. \tag{2.5}$$

The canonical momenta for the variables N, a, and ϕ are given, respectively, by

$$p_N=\frac{\partial L}{\partial\dot N}=0,\quad p_a=\frac{\partial L}{\partial\dot a}=-\frac{3V}{4\pi GN}a\dot a\,,\qquad p_\phi=\frac{\partial L}{\partial\dot\phi}=\frac{Va^3}{N}\dot\phi\,. \tag{2.6}$$

The first equation merely constrains the variable (primary constraint). The total Hamiltonian is constructed as

$$H=p_N\dot N+N\mathcal{H}\,, \tag{2.7}$$

where

$$\mathcal{H}=p_a\dot a+p_\phi\dot\phi-L=-\frac{2\pi GN}{3Va}p_a^2+\frac{N}{2Va^3}p_\phi^2-\frac{3VkN}{8\pi G}a+NVa^3V(\phi)\,. \tag{2.8}$$

The temporal evolution of any dynamical variables is generated by this Hamiltonian. For the compatibility of the constraint in the first line of Eq. (2.6) and the dynamics generated by the Hamiltonian of Eq. (2.7), the following equation must hold:

$$\mathcal{H}=0\,, \tag{2.9}$$

which constrains the dynamics of our system (secondary constraint). There are also constraints for the shift variables which are related to the spatial coordinate transformation invariance. However, they have nothing to do with dynamics.

Now we proceed to the quantum mechanics from the above classical description. We introduce the wave function of the Universe, Ψ. The constraint equations [the first equation of (2.6) and (2.9)] must be imposed as restrictions on the states:

$$p_N\Psi=0,\quad \mathcal{H}\Psi=0\,. \tag{2.10}$$

These constraints nullify all the dynamical evolution generated by the total Hamiltonian Eq. (2.7). A commutator of any operator and the total Hamiltonian becomes zero if it is evaluated for the above constrained states. The disappearance of time seems disappointing; however, it is a proper consequence of the invariance of general coordinate transformation in general relativity. The first equation of the above restrictions merely says that the wave function Ψ does not depend on the lapse function N. Therefore we expect that the second equation of the above restrictions may contain the information of dynamics, if any. The equation is expressed, in the representation that the variables a and ϕ are diagonal, as

$$\left[\partial_a^2+\frac{p}{a}\partial_a-\frac{1}{a^2}\partial_\phi^2+U_1(a,\phi)\right]\Psi(a,\phi)=0\,,\qquad U_1=2V^2V(\phi)a^4-kV^2a^2\,, \tag{2.11}$$

where we have taken the unit $c=\sqrt{4\pi G/3}=1$, and p represents a part of ambiguity in operator ordering. This equation is also expressed, for the new variable $\alpha=\ln a$, as

$$[\partial_\alpha^2+(p-1)\partial_\alpha-\partial_\phi^2+U_2(\alpha,\phi)]\Psi(\alpha,\phi)=0\ ,$$
$$U_2=2V^2e^{4\alpha}\left[e^{2\alpha}V(\phi)-\frac{3k}{8\pi G}\right]\ . \tag{2.12}$$

Equations (2.11) and (2.12) are called the Wheeler-DeWitt equations. Recovery of a dynamical time is possible if we introduce an appropriate clock variable into our system. The correlation between the clock variable and the other variables establishes an internal time. Actually the time evolution is derived by using the WKB approximation assuming the semiclassical nature of the clock variable.

Because the Wheeler-DeWitt equation is a hyperbolic second-order partial differential equation, it is impossible to construct a conserved (with respect to a time) probability current whose zeroth component is positive definite. Actually, a square of the absolute value of the wave function $|\Psi|^2$ is not a zeroth component of any conserved current. Moreover, the conserved current $j=(j_a,j_\phi)$, $\partial_a j_a-\partial_\phi j_\phi=0$ is given by

$$j_a=ia^p(\Psi^*\partial_a\Psi-\Psi\partial_a\Psi^*)\ ,$$
$$j_\phi=-ia^{p-2}(\Psi^*\partial_\phi\Psi-\Psi\partial_\phi\Psi^*)\ , \tag{2.13}$$

however, it does not have positive-definite zeroth component j_a. This problem cannot be solved if we do not proceed to the quantum field theory.

III. QUANTUM FIELD THEORY OF THE UNIVERSE

In this section we second quantize the Wheeler-DeWitt equation reviewed in the previous section. Before that it is probably instructive to recall the case of a relativistic point particle, which is very similar to our gravitational case. The classical action is given by

$$S_1=-m\int d\tau\left[\left[\frac{dx^\mu(\tau)}{d\tau}\right]^2\right]^{1/2}\ , \tag{3.1}$$

where m, τ, and $x^\mu(\tau)$ are, respectively, the mass, proper time, and position of the particle. This action has the well-known invariance under the reparametrization $\tau\to\tau'=f(\tau)$. In this sense the action Eq. (3.1) is an analogue of the gravitational Einstein action which is invariant under the general coordinate transformation. The canonically conjugate momenta

$$p^\mu\equiv\frac{\partial L}{\partial\dot{x}^\mu}=-m\frac{dx^\mu}{d\tau}\left[\frac{dx^\nu}{d\tau}\frac{dx_\nu}{d\tau}\right]^{-1/2} \tag{3.2}$$

are manifestly constrained as

$$p_\mu^2-m^2=0\ , \tag{3.3}$$

which is nothing but the mass-shell condition. When we go over to quantum mechanics, the constraint Eq. (3.3) is replaced by the condition on the state vector ϕ:

$$\left[\left[-i\frac{\partial}{\partial x^\mu}\right]^2-m^2\right]\phi(x)=0\ , \tag{3.4}$$

which is the Klein-Gordon equation. As is well known, since the time component of the conserved current is not positive semidefinite, we get into trouble in probabilistic interpretation at this level. A standard procedure to cure this difficulty is to reinterpret the state vector ϕ as an operator. Then the action for the quantum field ϕ is now given by

$$S_2=\tfrac{1}{2}\int d^4x\left[\left[\frac{\partial\phi}{\partial x^\mu}\right]^2-m^2\phi^2\right]\ , \tag{3.5}$$

which reproduces the operator equation (3.4).

Almost the same line of argument was made in Ref. 7 for the string theory and opened up the road to field theory of strings.

We will follow the same passage also in general relativity and will see what the consequence will be. Although our idea should go through in full quantum gravity, we are going to present the formalism of the second quantization of the Universe in the minisuperspace model. (In full quantum gravity, there remains a difficulty in the identification of time coordinate in superspace.) The action for the first-quantized theory was given in Eq. (2.1) with the reduced metric Eq. (2.2), as in Eq. (2.4). The constraint equation was

$$[\partial_\alpha^2-\partial_\phi^2+U_2(\alpha,\phi)]\Psi(\alpha,\phi)=0\ ,$$
$$U_2=2V^2e^{4\alpha}\left[e^{2\alpha}V(\phi)-\frac{3k}{8\pi G}\right]\ , \tag{3.6}$$

where we have chosen the operator ordering so that $p=1$ in Eq. (2.12). As we have previously remarked, the logarithm of the scale factor α plays the role of time while the scalar field ϕ does that of space. $\Psi[\alpha,\phi]$ is now regarded as a universe field in the minisuperspace.

The action which reproduces Eq. (3.6) is constructed to be

$$S_2=\tfrac{1}{2}\int d\alpha\, d\phi\left[\left[\frac{\partial\Psi}{\partial\alpha}\right]^2-\left[\frac{\partial\Psi}{\partial\phi}\right]^2-U_2\Psi^2\right]\ . \tag{3.7}$$

Here we have assumed that the Universe is *neutral scalar* though other possibilities cannot be excluded. At the moment we do not consider the higher polynomial terms in Ψ which represent the interaction of universes. (See Sec. V for the discussion of the branching of universes.)

It is straightforward to canonically quantize the system Eq. (3.7). We impose the canonical commutation relations

$$\left[\frac{\partial\Psi[\alpha,\phi]}{\partial\alpha},\Psi[\alpha,\phi']\right]=-i\delta(\phi-\phi')\ ,$$
$$\left[\frac{\partial\Psi[\alpha,\phi]}{\partial\alpha},\frac{\partial\Psi[\alpha,\phi']}{\partial\alpha}\right]=0\ , \tag{3.8}$$
$$[\Psi[\alpha,\phi],\Psi[\alpha,\phi']]=0\ .$$

Let $\{u_p[\alpha,\phi]\}$ be a complete set of normalized positive-frequency solutions of Eq. (3.6). The normalization condition should be

$$i\int d\phi\left[u_p^*\frac{\partial}{\partial\alpha}u_q-u_q\frac{\partial}{\partial\alpha}u_p^*\right]=\delta_{pq}\ ,\qquad(3.9)$$

the precise meaning of which will be fully discussed later. Here the subscript p labels the mode function. In full quantum gravity, presumably, p stands for all the characteristics of the Universe: energy density, electron number, etc. We expand the Universe field in these normal modes,

$$\Psi[\alpha,\phi]=\sum_p(c_pu_p[\alpha,\phi]+c_p^\dagger u_p^*[\alpha,\phi])\ ,\qquad(3.10)$$

where c_p and $c_p^\dagger$ are annihilation and creation operators, respectively, whose commutation relations are deduced from Eq. (3.8) as follows:

$$[c_p,c_q^\dagger]=\delta_{pq},\ \ [c_p,c_q]=[c_p^\dagger,c_q^\dagger]=0\ .\qquad(3.11)$$

The Universe Fock space is spanned by the vectors

$$\{c_{p_1}^\dagger c_{p_2}^\dagger\cdots|0\rangle\}\ ,\qquad(3.12)$$

with the ground state $|0\rangle$ being defined by

$$c_p|0\rangle=0\quad\text{for all } p\ .\qquad(3.13)$$

The state $|0\rangle$ represents the state of nothing, where even space-time does not exist.

In the following sections, we apply this formalism to the calculation of the average number of the created universes due to the potential term in Eq. (3.6), and to the response of a detector of the Universe field.

IV. CREATION OF THE UNIVERSE FROM NOTHING

In the previous section, we constructed the quantum field theory of the Universe along the standard procedure of the second quantization. One of the most conspicuous features of the field theory is that the particle number can change. The Universe can also be created and annihilated according to the law of the field theory. In this section we try to apply this theory to the creation of our Universe from nothing, the state of literally nothing, where even space-time does not exist. From the standpoint of the classical theory of general relativity, there is no reason for one special space-time to appear from nothing.

Quantum field theory of the universe is very similar to ordinary quantum field theory in curved space-time where, in general, the global vacuum state cannot be defined. Therefore in general particles (in our case universes) are observed to be created due to the discrepancy between the initial vacuum state and the final one. This is indeed true in our case. Since the potential term in Eq. (2.12) is obviously *time* (α) and *space* (ϕ) dependent, universes are created from nothing. Moreover there is no room to replace the form of the potential U_2 in Eq. (2.12); it is strictly fixed from the Einstein action which we have chosen at the beginning of our consideration. Thus the creation of universes from nothing is an inevitable consequence of our theory. The production rate is explicitly obtained by calculating the Bogoliubov transformation coefficients between in fields and out fields. In the equation for the universe field, Eq. (2.12), we notice that the potential term U_2 decreases asymptotically in the *past:* $\alpha\to-\infty$. Thus it is possible to set up a natural in-vacuum state by defining the positive-frequency mode function as proportional to $\exp(ip\phi-i\omega\alpha)$.

The out vacuum is also uniquely set up by defining the positive-frequency mode function as outgoing. Note that this is also an eigenstate of the momentum operator $p_\alpha=-i\partial/\partial\alpha$ with negative eigenvalue. This mode classically corresponds to the expanding universe as is seen from Eq. (2.6). We have to stress that the creation of the universe is not caused from genuine interaction of the universe field but just from the mismatch of the in and out vacuums.

For simplicity of demonstration, we are going to use the model of spatial flat minisuperspace and of finite positive cosmological constant in this section. (The negative spatial curvature model goes essentially in the same way as the present calculation.)

The field equation becomes in this case

$$(\partial_\alpha^2-\partial_\phi^2+2V^2e^{6\alpha}V_0)\Psi(\alpha,\phi)=0\ ,\qquad(4.1)$$

where $\alpha=\ln a$, and V_0 is a constant part of the ϕ field potential. The normal-mode function $u[\alpha,\phi]$ is expressed in terms of the Bessel function by

$$u[\alpha,\phi]=\mathcal{N}Z_\nu\left[\frac{V}{3}\sqrt{2V_0}e^{3\alpha}\right]e^{ip\phi}\ ,\qquad(4.2)$$

where $\nu=-i|p|/3$. The positive-frequency in-mode functions are defined to be

$$u_p^{\rm in}[\alpha,\phi]=\left[\frac{\pi}{6}\right]^{1/2}\left[\sinh\frac{\pi|p|}{3}\right]^{-1/2}e^{ip\phi}\times J_\nu\left[\frac{V}{3}\sqrt{2V_0}e^{3\alpha}\right]\ ,\qquad(4.3)$$

where J is the Bessel function of the first class. This form is chosen because it reduces, for $\alpha\to-\infty$, to

$$\to\frac{1}{\sqrt{2|p|}}e^{i(p\phi-|p|\alpha)}\ ,\qquad(4.4)$$

which is a natural positive-frequency mode function as an out-going mode. The normalization is determined by Eq. (3.9). Thus the universe field Ψ is expanded as

$$\Psi[\alpha,\phi]=\sum_p(c_p^{\rm in}u_p^{\rm in}[\alpha,\phi]+\text{H.c.})\ .\qquad(4.5)$$

Accordingly the in vacuum $|0,\text{in}\rangle$ is defined by

$$c_p^{\rm in}|0,\text{in}\rangle=0\quad\text{for } p\in\mathbf{R}\ .\qquad(4.6)$$

On the other hand, the positive frequency out-mode functions are given by

$$u_p^{\rm out}[\alpha,\phi]=\frac{1}{2}\left[\frac{\pi}{3}\right]^{1/2}e^{-\pi|p|/6}e^{ip\phi}H_\nu^{(2)}\left[\frac{V}{3}\sqrt{2V_0}e^{3\alpha}\right]\ ,\qquad(4.7)$$

where $H^{(2)}$ is the Hankel function. This form is chosen because it reduces, for $\alpha \to +\infty$, to

$$\propto \exp\left[-\frac{3\alpha}{2} - ie^{3\alpha}\right], \tag{4.8}$$

which is asymptotically an out-going mode. Note that it is also a negative eigenstate of the momentum operator: $p_\alpha = -i\partial/\partial\alpha$. Thus, from Eq. (2.6), this mode classically corresponds to the purely expanding universe. The normalization is also determined by Eq. (3.9). In the same way as in the previous in-modes, the Universe field Ψ is alternatively expanded as

$$\Psi[\alpha,\phi] = \sum_p (c_p^{\text{out}} u_p^{\text{out}}[\alpha,\phi] + \text{H.c.}) . \tag{4.9}$$

Accordingly the out vacuum $|0,\text{out}\rangle$ is defined by

$$c_p^{\text{out}}|0,\text{out}\rangle = 0 \quad \text{for } p \in \mathbf{R} . \tag{4.10}$$

Now we calculate the Bogoliubov transformation coefficients between these in and out fields. This is an easy task if we use the following relation between Bessel functions:

$$H_\nu^{(2)}(z) = \frac{-i}{\sin\pi\nu}[e^{i\pi\nu}J_\nu(z) - J_{-\nu}(z)] . \tag{4.11}$$

Then the Bogoliubov coefficients $c_i(p,q)$ defined by

$$u_p^{\text{out}}[\alpha,\phi] = \sum_q \{c_1(p,q)u_q^{\text{in}}[\alpha,\phi] + c_2(p,q)u_q^{*\text{in}}[\alpha,\phi]\} \tag{4.12}$$

are given by

$$\begin{aligned} c_1(p,q) &= \left[\frac{1}{1-e^{-2\pi|p|/3c}}\right]^{1/2} \delta_{pq} , \\ c_2(p,q) &= \left[\frac{1}{e^{2\pi|p|/3c}-1}\right]^{1/2} \delta_{pq} . \end{aligned} \tag{4.13}$$

They surely satisfy the probability conservation condition: $|c_1|^2 - |c_2|^2 = 1$.

From the above, we can calculate the average number of the produced universes from nothing (the in vacuum). The average number of universes in the pth mode N_p is defined to be

$$N_p = \langle 0,\text{in}|c_{\text{out}}^\dagger(p)c_{\text{out}}(p)|0,\text{in}\rangle , \tag{4.14}$$

which becomes

$$N_p = |c_2(p,p)|^2 = (e^{2\pi|p|/3c} - 1)^{-1} . \tag{4.15}$$

Note that this form precisely coincides with the Planckian distribution of temperature proportional to $c = \sqrt{8\pi G/3}$.

Now we consider the meaning of the mode functions labeled by p. Suffix p is an eigenvalue of the momentum operator p_ϕ, which is classically given by Eq. (2.6). Thus p is related to the matter energy by

$$E_m = \frac{p^2}{2V^2a^3} . \tag{4.16}$$

The classical Universe with constant p satisfies the equation

$$\dot{a}^2 - \frac{8\pi G}{3}a^2\left[\frac{p^2}{2V^2a^6} + V_0\right] = 0 . \tag{4.17}$$

The asymptotic solutions are

$$a(t) = \begin{cases} \left[\dfrac{12\pi G p^2}{3V}\right]^{1/6} t^{1/3} & \text{for } a \ll \left[\dfrac{p^2}{2VV_0}\right]^{1/2} , \\[2ex] a_0 \exp\left[\left[\dfrac{8\pi G V_0}{3}\right]^{1/2} t\right] & \text{for } a \gg \left[\dfrac{p^2}{2VV_0}\right]^{1/6} . \end{cases} \tag{4.18}$$

Thus the distribution Eq. (4.15) of created universes represents the average number of these universes which are labeled essentially by the matter density.

V. INTERACTION AND DETECTION

So far we have studied a free field theory of the Universe with a scale-factor-dependent potential but without any self-interaction of universes. In the previous section we have found the multiple production of universes. However a natural question may arise: how can we observe them? To answer this question we have to take account of interaction of universes: namely, the interaction vertex at which the Universe branches off. At present, however, our technology of field theory of the Universe is too primitive to fully formulate interaction terms. (Remember, even in a much simpler case of string field theory, vertex construction is highly nontrivial.)

Roughly speaking, the interaction may have a form Ψ^3. We just assume as a not-yet-justified approximation that our mother Universe is classical and only the baby universe is quantized. Accordingly, the interaction Ψ^3 reduces to $\Psi^2_{\text{mother}}\Psi_{\text{baby}}$ and the factor Ψ^2_{mother} is replaced by 2×2 matrix corresponding to universes of detector states "up" and "down." Namely, we consider a hypothetical device which clicks when the baby universe plunges into our mother Universe.

Precisely, let the trajectory of our mother Universe in the minisuperspace be $a(t)$ and $\phi(t)$ with t being the proper time of a comoving observer. The observer is supposed to be equipped with a device which can observe the baby universe field Ψ_{baby} at the point $a(t)$, $\phi(t)$ in the minisuperspace. The device is similar to the DeWitt-Unruh detector in the quantum field theory in curved space-time.[9] Namely, the detector, which consists of a two-level atom, counts the rate of transition from the ground state to the excited state of the atom whenever a universe is observed. The simplest interaction which mimics this sort of detector will be

$$S^{\text{det}} = \int dt\, K(t)\Psi(a(t),\phi(t)) . \tag{5.1}$$

Here the operator K which characterizes the detector is the 2×2 matrix with the following elements:

$$\langle 1|K(t)|0\rangle = \langle 0|K(t)|1\rangle^* = g e^{-iEt} ,$$
$$\langle 0|K(t)|0\rangle = \langle 1|K(t)|1\rangle = 0 , \tag{5.2}$$

with E being the energy difference of the two levels $|1\rangle$ and $|0\rangle$, g the small coupling constant.

Let us compute the transition rate in the model discussed in Sec. IV, a spatially flat model with a cosmological constant. The total transition probability is simply the absolute square of the amplitude:

$$F_{\rm tot}(E) = g^2 \int \frac{dp}{2\pi} \left| \int dt\, e^{-iEt} u_p^{*\rm in}(X(t)) \right|^2 , \tag{5.3}$$

where $X(t)$ collectively stands for $a(t)$ and $\phi(t)$. The incoming base $u_p^{\rm in}$ is in our present case given by

$$u_p^{\rm in} = \left[\frac{\pi}{6}\right]^{1/2} \left[\sinh \frac{\pi|p|}{3}\right]^{-1/2} e^{ip\phi} \times J_{-i|p|/3}\left[\frac{V}{3}\sqrt{2V_0}\, e^{3\alpha}\right] . \tag{5.4}$$

Introducing the mean time $t_c = (t+t')/2$ and $t_\Delta = t - t'$, we can rewrite (5.3) as

$$F_{\rm tot}(E) = \int dt_c F(E, t_c) , \tag{5.5}$$

where $F(E,t_c)$, the transition rate at the cosmic time t_c is given by

$$F(E,t_c) = g^2 \int dt_\Delta \int \frac{dp}{2\pi} e^{-iEt_\Delta} u_p^{\rm in}(X(t_c + t_\Delta/2)) \times u_p^{*\rm in}(X(t_c - t_\Delta/2)) . \tag{5.6}$$

Let us assume that E is sufficiently large so that the important contribution comes from the small-t_Δ region in the t_Δ integration and that $X(t)$, namely $a(t)$ and $\phi(t)$, are slowly varying functions of time for a large value of t_c. We may use the asymptotic (WKB) form for the Bessel function in Eq (5.4) to get

$$u_p^{\rm in} \approx \left[\frac{\pi}{6}\right]^{1/2} \left[\sinh \frac{\pi|p|}{3}\right]^{-1/2} \times e^{ip\phi} \frac{1}{p^2 + 2V_0 V^2 e^{6\alpha(t)}} \times \cos\left[\int^{\alpha(t)} d\alpha (p^2 + 2V_0 V^2 e^{6\alpha})^{1/2} + i\frac{\pi p}{6} - \frac{\pi}{4}\right] . \tag{5.7}$$

When we apply Eq. (5.7) to Eq. (5.6), we make a further approximation

$$\alpha(t_c \pm t_\Delta/2) \approx \alpha(t_c) \pm \frac{t_\Delta}{2}\dot{\alpha}(t_c) ,$$

$$\phi(t_c \pm t_\Delta/2) \approx \phi(t_c) \pm \frac{t_\Delta}{2}\dot{\phi}(t_c) .$$

We obtain

$$F(E,t_c) = g^2 \int \frac{dp}{2\pi} \frac{1}{e^{2\pi|p|/3} - 1} \frac{1}{2(p^2 + 2V_0 V^2 e^{6\alpha(t)})^{1/2}} \times [2\pi\delta(E - \dot{\alpha}(p^2 + 2V_0 V^2 e^{6\alpha(t)})^{1/2} - p\dot{\phi}) + 2\pi\delta(E + \dot{\alpha}(p^2 + 2V_0 V^2 e^{6\alpha(t)})^{1/2} - p\dot{\phi}) e^{2\pi|p|/3}] . \tag{5.8}$$

In Eq. (5.8), $\alpha(t)$ and $\phi(t)$ give a classical trajectory of our Universe and satisfy the equation of motion

$$\dot{\alpha} = \left[\frac{q^2}{V^2 \alpha^6} + 2V_0\right]^{1/2} ,$$
$$\dot{\phi} = -\frac{q}{V^2 a^3} \quad (q > 0) . \tag{5.9}$$

Here we have chosen the expanding phase for the scale factor a and the decreasing phase of the scalar field ϕ. Hence the second term inside the square brackets in Eq. (5.8) vanishes. A straightforward computation gives us the transition rate per unit volume:

$$F(E,t_c)/Va^3 = g^2 \frac{1}{e^{2\pi|p*|/3} - 1} \frac{1}{2E^*} \theta(E - 2V_0 Va^3) , \tag{5.10}$$

where

$$E^* = (E^2 - 4V_0^2 V^2 a^6)^{1/2} ,$$
$$p^* = \epsilon q/m^2 + \sqrt{q^2 + m^2 V^2 a^6}\sqrt{\epsilon^2/m^4 - 1} ,$$

with ϵ and m^2 being the energy density E/Va^3 and $2V_0$, respectively. In particular, for the de Sitter mother universe ($q=0$), we have

$$F(E,t_c)/Va^3 = g^3 \frac{1}{e^{E^*/T} - 1} \frac{1}{2E^*} , \tag{5.11}$$

where T is an effective temperature $3\sqrt{2V_0}/2\pi$, three times the standard Hawking temperature. To summarize we have essentially obtained the Planckian distribution of baby universes.

Needless to say, our detector so far discussed is a hypothetical one. The threshold energy may be gigantic. It is likely, however, that we can find a completely different kind of detector, which mimics a telescope and does not detect other universes directly but may exhibit an indirect consequence of the quantum era of our mother Universe.

VI. SUMMARY AND DISCUSSION

In order to overcome the problem of probabilistic interpretation of the Wheeler-DeWitt equation in quantum gravity, we have proposed the second quantization of the Universe. In a minisuperspace model, we have explicitly demonstrated that there is multiple production of universes, even if we start from *nothing*. Moreover the universe production is an inevitable consequence of the field theory of the Universe since the gravity couples to everything and therefore the matter part of the Hamiltonian acts as a *time-dependent potential*. For a spatially flat model with a positive cosmological constant, we obtained a Planckian distribution for the produced universes. Subsequently, we have introduced a hypothetical detector which can count the baby universe which plunges into our mother Universe. There, only the baby universe is second quantized. For the above model, we have obtained the Planckian distribution of baby universes with almost the Hawking temperature.

It is conceptually straightforward to go over to the second quantization of full quantum gravity from our minisuperspace model. For this purpose, we have to develop techniques such as the Becchi-Rouet-Stora-Tyutin formalism in string field theories to write down the action of field theory of the Universe. Here lessons we have learned from string field theory will be valuable.

The reader may be mystified by the words *second quantization of the Universe.* Are we forced to swallow a fancy theory which is completely different from the ordinary well-studied quantum field theory? This is not the case; if we confine ourselves to our single Universe in a short time period, our theory is equivalent to the ordinary quantum field theory.

Let us roughly sketch the idea. If we ignore the branching off of universes, the action becomes a bilinear form:

$$S=\int \mathcal{D}g\, \mathcal{D}\phi\, \Psi[g,\phi]\mathcal{H}\Psi[g,\phi]\ , \tag{6.1}$$

where $\mathcal{H}$ is the Wheeler-DeWitt Hamiltonian and the functional integrals are over three-geometry g and over the spatial configuration of matter field ϕ. Let us closely look at the effect of the interaction term in the $\mathcal{H}$, say $f\phi^3$, in the usual sense of field theory. The interaction term in our action is

$$S_{\text{int}}=\int \mathcal{D}g\, \mathcal{D}\phi\, \Psi[g,\phi]f\phi^3\Psi[g,\phi]\ . \tag{6.2}$$

We expand the Universe field Ψ in normal modes which may look like

$$\Psi=c(|0\rangle)\Psi(|0\rangle)+\sum_{p}c(|p\rangle)\Psi(|p\rangle)+\sum_{p_1,p_2}c(|p_1,p_2\rangle)\Psi(|p_1,p_2\rangle)+\cdots+\text{H.c.} \tag{6.3}$$

Note that the expansion is over the Fock space in the ordinary quantum field theory. For example, $c(|0\rangle)$ annihilates a universe which is in the vacuum state. $c^{\dagger}(|p\rangle)$ creates a universe which contains a single particle with momentum p. The normal modes are configuration representations of the vectors in the Fock space which functionally depend on the three-geometry. Therefore the interaction reduces to

$$S_{\text{int}}=f\sum_{p_1,p_2}c^{\dagger}(|p_1+p_2\rangle)c(|p_1,p_2\rangle)+\cdots$$

if the dominant three-geometry is the flat space. This equation reads that the universe which contains a single ϕ particle changes to the one with two ϕ particles. We may convince ourselves that we can compute the S matrix in this way. We have not yet proved that the result is identical to the standard one if the geometry is effectively flat. It seems very plausible though.

During the course of our study, the papers by Hawking,[10] by Coleman,[11] and by Giddings and Strominger[12] came to our attention. It seems that all of them point to the second quantization of the Universe. However, their emphasis is on the effect of baby universe through the wormhole instanton.

After completion of this paper, we became aware of the recent papers by Banks[13] and Giddings and Strominger.[14]

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