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Interpretation of the wave function of the Universe

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A probabilistic interpretation of the wave function of the Universe is proposed in which "probability" and "unitarity" are inherently approximate concepts. Their validity is limited by the accuracy of the semiclassical approximation for the entire Universe. This approach (which is a logical extension of DeWitt's) defines a positive-semidefinite probability distribution and satisfies the correspondence principle with classical cosmology and with quantum mechanics. In particular, one recovers the standard probabilistic interpretation of the wave function for a small subsystem of the Universe.

I. INTRODUCTION

In quantum cosmology the Universe is described by a single wave function ψ. This wave function can be found by solving the functional differential equations derived by Wheeler[1] and DeWitt[2] with appropriate boundary conditions. However, the meaning of the cosmological wave function is not well understood.

In conventional quantum mechanics, a quantum system described by coordinates q_i is characterized by a wave function $\psi(q_i,t)$. The probability to find the system in configuration-space element $d\Omega_q$ at time t is given by

$$dP=|\psi(q_i,t)|^2 d\Omega_q \ . \tag{1}$$

If ψ is well behaved at infinity, then the integral of $|\psi|^2$ taken over the whole configuration space is independent of time and can be normalized to one:

$$\int |\psi(q_i,t)|^2 d\Omega_q = 1 \ . \tag{2}$$

This follows immediately from the fact that $|\psi|^2$ is a time component of a conserved probability current. It also follows from Eq. (1) that $dP\geq 0$.

In quantum cosmology, the wave function of the Universe is a functional, defined on superspace,[3] which is the space of all three-dimensional metrics, $h_{ij}(\mathbf{x})$, and all matter field configurations $\phi_A(\mathbf{x})$. Hence, we can write

$$\psi[h_{ij}(\mathbf{x}),\phi_A(\mathbf{x})] \ .$$

The most puzzling thing about this wave function is that it does not depend on time. This leads to difficulties, since time plays such a central role in quantum mechanics.

At first one might think that the time independence of ψ means that the Universe is static. However, this need not be the case. In general relativity time is an arbitrary label, and physics should be independent of it. A physically meaningful time can be defined using some geometric or matter variables. In other words, clocks, being parts of the Universe, are also described by the wave function of the Universe.[2]

The question we shall be mainly concerned with in this paper is that of the probabilistic interpretation of ψ. The question is what is the analogue of Eq. (1) in quantum cosmology? A satisfactory answer to this question should comply with the correspondence principle: one should be able to recover Eq. (1) for a small subsystem of the Universe. Moreover, in the classical limit, $\hbar\to 0$, quantum cosmology should reduce to classical cosmology, and the probability distribution defined by the wave function of the Universe should describe an ensemble of classical universes. These are obvious requirements, but they seem to have been overlooked in most previous discussions of the problem. In Sec. II, I shall briefly review the previous proposals for the interpretation of ψ and indicate the difficulties they are facing. My own approach is presented in Secs. III and IV. In Sec. III it is developed in the classical limit, when all the variables describing the Universe can be treated semiclassically, and in Sec. IV it is extended to the case when some of the variables are essentially quantum. Unlike other proposals, this approach defines a positive-semidefinite probability distribution and satisfies the correspondence principle. The price one has to pay is that the definition of the probability density becomes inherently approximate, its accuracy being limited by the accuracy of the semiclassical approximation for those degrees of freedom which are nearly classical (e.g., the scale factor of the Universe). The unitarity condition similar to Eq. (2) is also satisfied, albeit only approximately.

II. PREVIOUS PROPOSALS

To simplify the discussion, I shall concentrate on homogeneous minisuperspace models,[2,4] in which the geometric and matter variables are independent of $\mathbf{x}$. The action for such a model is

$$S=\int dt\{p_\alpha \dot{h}^\alpha - N[g^{\alpha\beta}p_\alpha p_\beta + U(h)]\} \ , \tag{3}$$

where h^α is a unified notation for superspace variables (e.g., h_{ij} and ϕ_A), p_α is the momentum conjugate to h^α, $N(t)>0$ is the lapse function, and $g_{\alpha\beta}$ is the superspace metric with signature $(+,-,\dots,-)$. The "superpotential" $U(h^\alpha)$ is given by

$$U = h^{1/2}[V(\phi) - {}^{(3)}R] \,, \tag{4}$$

where $h = |\det h_{ij}|$, $V(\phi_A)$ is the potential energy of matter fields, and ${}^{(3)}R$ is the curvature of three-space with metric h_{ij}.

The Wheeler-DeWitt (WD) equation corresponding to the action (3) is

$$(\nabla^2 - U)\psi = 0 \,, \tag{5}$$

where

$$\nabla^2\psi = \nabla_\alpha\nabla^\alpha\psi = \frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}\, g^{\alpha\beta}\partial_\beta)\psi \,,$$

$g = |\det g_{\alpha\beta}|$, $\partial_\alpha = \partial/\partial h^\alpha$, and ∇_α is a covariant derivative in metric $g_{\alpha\beta}$. The factor ordering in Eq. (5) has been chosen so that it is invariant under general coordinate transformations in superspace. This guarantees that the form of the equation is the same for all choices of superspace variables. (For example, we could use $h^{\alpha\beta}\sqrt{h}$ instead of $h_{\alpha\beta}$.) We can still add a term ξR to the operator ∇^2 in Eq. (5), where R is the curvature of superspace. Misner,[4] and more recently Halliwell,[5] have argued that the parameter ξ should be chosen as $\xi = (n-2)/4(n-1)$, where n is the minisuperspace dimensionality. With this choice, the Wheeler-DeWitt equation is invariant under scale transformations

$$g_{\alpha\beta} \to f g_{\alpha\beta}, \quad \psi \to f^{1-n/2}\psi, \quad U \to f^{-1}U \,, \tag{6}$$

where $f(h^\alpha)$ is an arbitrary function. These transformations correspond to redefinitions of the lapse function, $N \to fN$.

We now turn to the definition of probability in superspace. The most straightforward extension of Eq. (1) is[6,7]

$$dP = |\psi(h^\alpha)|^2\sqrt{g}\; d^n h \,. \tag{7}$$

Although this equation looks similar to Eq. (1), it is in fact very different, since "time" is now included among the variables h^α, and so $\sqrt{g}\,d^n h$ corresponds to $d\Omega_q dt$. It is not clear how one can recover the conservation of probability and the standard interpretation of quantum mechanics for small subsystems in this approach. Another problem with Eq. (7) is that in all models considered so far the integral of $|\psi|^2$ over the whole superspace diverges,

$$\int |\psi(h^\alpha)|^2\sqrt{g}\; d^n h = \infty \,, \tag{8}$$

and so the distribution (7) is not normalizable. The origin of this divergence is easily understood if we note that Eq. (8) is analogous to

$$\int |\psi(q_i,t)|^2 d\Omega_q dt = \infty \,, \tag{9}$$

which follows directly from Eq. (2).

An alternative approach to the interpretation of ψ, first suggested by DeWitt,[2] is based on the conserved current

$$\nabla_\alpha j^\alpha = 0 \,, \tag{10}$$

$$j^\alpha = -\frac{i}{2} g^{\alpha\beta}(\psi^*\nabla_\beta\psi - \psi\nabla_\beta\psi^*) \,. \tag{11}$$

[Note that the WD equation (5) is just an n-dimensional Klein-Gordon equation with a variable mass, and the current (11) is the corresponding Klein-Gordon current.] The probability distributions are defined on $(n-1)$-dimensional surfaces, which play a role similar to that of constant-time surfaces in conventional quantum mechanics. The probability to find the Universe represented by a point in a surface element $d\Sigma_\alpha$ is

$$dP = j^\alpha d\Sigma_\alpha \,. \tag{12}$$

The conservation of probability is ensured by the conservation of the current. Another attractive feature of Eq. (12) is[4,5] that it is invariant under scale transformations (6). The problem with this approach is that dP defined by Eq. (12) can be negative. For example, if dP is positive for a certain wave function ψ, then it is negative for the complex-conjugate wave function $\psi' = \psi^*$. This is the old problem of negative probabilities in the Klein-Gordon equation. DeWitt's[2] has shown, in a minisuperspace model, that this difficulty can be avoided by an appropriate choice of equal-time surfaces. However, it remained unclear how his approach could be extended to a more general situation.

In the following sections I would like to suggest an interpretation of ψ which is also based on the current (11), but does not suffer from the negative-probability problem.[8] My approach is a logical continuation of DeWitt's.[2] The main new ingredient is that the superspace variables are divided into two classes: classical and quantum. As we shall see, the presence of classical variables is crucial for assuring the positive semidefiniteness of the probability.

III. CLASSICAL UNIVERSES

The point that has not been much discussed in the context of quantum cosmology and that I would like to emphasize here is the essential role played by classical measuring devices in the interpretation of quantum mechanics. All realistic devices are not exactly classical, but have some quantum uncertainty. Taking a clock as an example, there is always a nonzero probability that the clock will run backwards. The bigger the clock is, the smaller the quantum fluctuations are; in this sense the best clock is the entire Universe. The size of the Universe imposes a bound on the accuracy of quantum measurements, and in particular on the accuracy of clocks.[2] In a small universe, subject to large quantum fluctuations, the concept of time cannot be introduced, and it appears that the wave function for such a Universe cannot be given any meaningful interpretation. Hence, we shall try to interpret the wave function of the Universe only in the domain where at least some of the variables h^α are semiclassical.

In this section we shall consider the case when all the variables are semiclassical. Then ψ is a superposition of terms of the form

$$\psi = A(h)e^{iS(h)} \,, \tag{13}$$

where the classical action $S(h^\alpha)$ is assumed to be a real function of h^α. The semiclassical character of the variables h^α is manifested in the existence of a small dimensionless parameter λ, proportional to $\hbar$, so that the WKB expansion is an expansion in powers of λ. The superpotential $U(h^\alpha)$ is of the order λ^{-2} and the action $S(h^\alpha)$ is of the order λ^{-1}. Substituting ψ from Eq. (13) into the Wheeler-DeWitt equation, we find, in the lowest order in λ, the Hamilton-Jacobi equation for S:

$$g^{\alpha\beta}(\nabla_\alpha S)(\nabla_\beta S)+U=0 \ . \tag{14}$$

In the next order we obtain an equation for the amplitude A,

$$2\nabla A\cdot\nabla S+A\nabla^2 S=0 \ , \tag{15}$$

which expresses the conservation of the current (11):

$$j^\alpha=|A^2|\nabla^\alpha S \ . \tag{16}$$

The action $S(h)$ describes a congruence of classical trajectories; there is a trajectory through each point in superspace (except in classically forbidden regions where S becomes complex). The momentum on the trajectory at point h^α is $p_\beta=\nabla_\beta S(h^\alpha)$, and the "velocity" is

$$\dot{h}^\alpha=2N\nabla^\alpha S \ . \tag{17}$$

In order to find $\dot{h}^\alpha$, one has to specify the lapse function $N(t)$. The trajectories can begin or end at the boundaries of superspace which represent singular three-geometries and matter field configurations. Closed trajectories without ends are not possible if $S(h)$ is a single-valued function on superspace. We shall assume that the superspace variables are chosen so that S is single valued. The trajectories can also begin or end at points where $\nabla S=0$ and the semiclassical approximation breaks down. For example, the wave function specified by "tunneling" boundary conditions[9,10] defines a congruence in which all trajectories begin on a surface with $\nabla S=0$, at the boundary between classically allowed and classically forbidden regions.

We shall define probability distributions on $(n-1)$-dimensional surfaces, which play the role of equal-time surfaces. We can choose any family of surfaces such that each surface is crossed once and only once by all the trajectories of the congruence, as illustrated in Fig. 1(a). It is clear from the figure that all the surfaces are crossed by the trajectories in the same direction. The mathematical expression of this fact is that $\dot{h}^\alpha d\Sigma_\alpha$ has the same sign for all surface elements $d\Sigma_\alpha$. The choice of this sign is arbitrary (since the sign of $d\Sigma_\alpha$ is arbitrary), and we shall choose

$$\dot{h}^\alpha d\Sigma_\alpha>0 \ . \tag{18}$$

The probability density is given by Eq. (12), and from Eqs. (16)–(18) we see that it is positive semidefinite. The normalization of ψ should be chosen so that

$$\int j^\alpha d\Sigma_\alpha=1 \ . \tag{19}$$

It should be emphasized that the positive semidefiniteness of the probability is due to the inequality (18) which follows from the fact that the trajectories are not allowed to recross the equal-time surfaces. If a surface Σ were crossed more than once by some trajectories of the congruence, as illustrated in Fig. 1(b), then $dP=j^\alpha d\Sigma_\alpha$ would have opposite signs in regions where Σ is crossed by the trajectories in opposite directions (e.g., in regions AB and BC).

An example of a possible choice of equal-time surfaces is given by the surfaces of constant S which are orthogonal to the congruence of trajectories. We can also make a coordinate transformation in superspace choosing $h_n=t$ as one of the coordinates. Then Eq. (10) takes the form

$$\frac{\partial\rho}{\partial t}+\partial_a J^a=0 \ , \tag{20}$$

where $J^a=\rho\dot{h}^a$ and the index a takes values from 1 to $(n-1)$. We can interpret $\rho(h^a,t)$ as the distribution function for an ensemble of classical universes. The

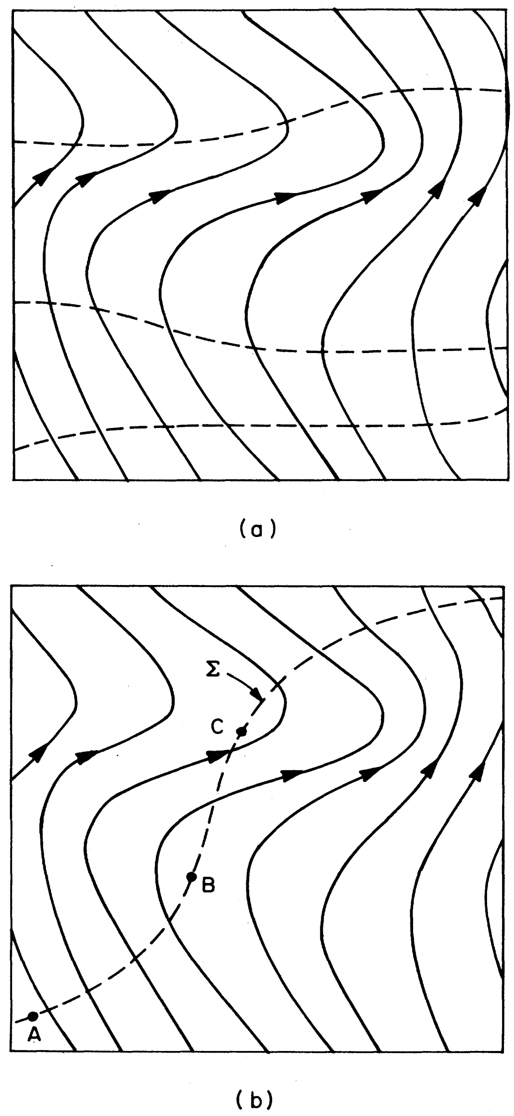

FIG. 1. In this figure solid lines represent classical trajectories in superspace and dashed lines represent $(n-1)$-dimensional surfaces. Figure (a) illustrates a possible choice of equal-time surfaces for a congruence of trajectories. In (b), Σ is not a legitimate equal-time surface, since some trajectories cross it more than once.

phase-space distribution function is given by[11–13]

$$f(h^a,p_a,t)=\rho(h^a,t)\delta^{(n-1)}(p_a-\partial_a S) \tag{21}$$

and satisfies the continuity equation

$$\frac{\partial f}{\partial t}+\frac{\partial f}{\partial h^a}\dot{h}^a+\frac{\partial f}{\partial p_a}\dot{p}_a=0 . \tag{22}$$

The $(n-1)$ superspace coordinates h^a can be chosen so that they remain constant on the trajectories defined by S. Then $J^a=0$ and $\partial\rho/\partial t=0$. This choice of coordinates is similar to comoving coordinates in general relativity: the variables h^a are used as labels for trajectories, and $\rho(h^a)$ gives the probability density for the Universe to evolve along the trajectory labeled by h^a.

We now consider the case when ψ is a superposition of several WKB-type terms:

$$\psi=\sum_k \psi_k, \quad \psi_k=A_k e^{iS_k} . \tag{23}$$

All functions $S_k(h^\alpha)$ are solutions of the Hamilton-Jacobi equation (14) with different integration constants. We shall assume that all sets of constants are sufficiently different, so that the corresponding trajectories are macroscopically distinct. Suppose first that the congruences of trajectories defined by the actions S_k admit a common family of equal-time surfaces (this will not, in general, be the case). Then the distribution function $\rho(h^a,t)$ defined by the total wave function ψ can be written as

$$\rho=\sum_k \rho_k+(\text{cross terms}), \tag{24}$$

where ρ_k is the probability distribution corresponding to ψ_k. All cross terms contains rapidly oscillating factors of the form $\exp[i(S_k-S_k')]$ and average out to zero when ρ is integrated with any smooth function. [One expects integrals to be $\sim\exp(-b/\lambda)$ with $b\sim 1$ (Refs. 14 and 15).] Within the accuracy of the semiclassical approximation these terms can be dropped. In other words, for classical universes the interference between different WKB terms in (23) is unimportant, and the difference between pure and mixed states disappears. The distribution functions ρ and f are given by the sums

$$\begin{aligned}\rho(h^a,t)&=\sum_k \rho_k(h^a,t) , \\ f(h^a,p_a,t)&=\sum_k f_k(h^a,p_a,t) .\end{aligned} \tag{25}$$

If the actions S_k do not admit a common family of equal-time surfaces, we can use a different choice of surfaces for each S_k. In fact, this may be more convenient even if there is a family of common surfaces. For example, one may want to consider the probability distribution for homogeneous cosmologies as a function of the proper time τ. Then the constant τ surfaces would of course be different for different S_k. The total probability distributions can still be defined by the sums (25). These distributions describe an ensemble of universes consisting of subensembles described by ρ_k and f_k. The probability for the Universe to belong to the kth subensemble is $\int j_k^\alpha d\Sigma_{k\alpha}$, where $j_k^\alpha=|A_k|^2\nabla^\alpha S_k$ is the current constructed from the wave function ψ_k. For the distributions to be properly normalized, we should require

$$\sum_k \int j_k^\alpha d\Sigma_{k\alpha}=1 . \tag{26}$$

The total probability is conserved simply because it is conserved for each subensemble separately.

IV. SMALL QUANTUM SUBSYSTEMS

We now turn to a more general situation, where not all the superspace variables are semiclassical. We shall preserve the notation h^α $(\alpha=1,\dots,n-m)$ for the semiclassical variables and introduce the notation q^ν $(\nu=1,\dots,m)$ for the quantum variables. Indices from the beginning and from the middle of the greek alphabet will be used for the classical and quantum subspaces, respectively. We shall assume that the effect of q^ν on the dynamics of h^α is negligible; in this sense the variables q^ν correspond to a small subsystem of the Universe. The Wheeler-DeWitt equation (5) can be written as

$$(\nabla_0^2-U_0-H_q)\psi=0 . \tag{27}$$

The operator $H_0=\nabla_0^2-U_0(h)$ is the part of the Wheeler-DeWitt operator obtained by neglecting all quantum variables q^ν and their momenta, $p_\nu=-i\partial/\partial q^\nu$. The smallness of the subsystem is mathematically manifested in the existence of a small, dimensionless parameter ϵ, such that $H_q\psi/H_0\psi=O(\epsilon)$. Here, we shall assume that $\epsilon\sim\lambda$, so that $H_q=O(\lambda^{-1})$. This is a reasonable assumption, since the semiclassical character of the Universe and the smallness of the subsystem are both due to the fact that the Universe is large. However, there may be cases where one has to distinguish between the two parameters.

The superspace metric tensor can be expanded in powers of λ. We shall assume that the variables h^α and q^ν are normalized so that the leading terms of the metric are of zeroth order in λ. The metric tensor components with indices in the classical subspace can be written as

$$g_{\alpha\beta}(h,q)=g_{\alpha\beta}^{(0)}(h)+O(\lambda) , \tag{28}$$

where $g_{\alpha\beta}^{(0)}\sim 1$. The Laplacian ∇_0^2 in Eq. (27) is constructed using the metric $g_{\alpha\beta}^0(h)$. It will be assumed that the subspaces defined by h^α and q^ν are approximately orthogonal, that is, $g_{\alpha\nu}=O(\lambda)$. This condition appears to be necessary for a clean division of variables into classical and quantum.

The wave function of the Universe can be written as

$$\psi(h,q)=\sum_k \psi_k(h)\chi_k(h,q) , \tag{29}$$

where ψ_k are given by Eq. (23). Let us first consider one term of this sum:

$$\psi=A(h)e^{iS(h)}\chi(h,q)\equiv\psi_0\chi . \tag{30}$$

The wave function $\psi_0(h)=Ae^{iS}$ satisfies the equation

$$(\nabla_0^2-U_0)\psi_0=0 \tag{31}$$

and we obtain, like in the previous section, Eqs. (14) and

(15) for S and A. The equation for the wave function of the subsystem χ has the form

$$\nabla_0^2\chi+2[\nabla_0(\ln A)]\nabla_0\chi+2i(\nabla_0 S)\nabla_0\chi-H_q\chi=0 \ . \quad (32)$$

The first two terms are of higher order in λ than the third term and can be neglected; this gives

$$2i(\nabla_0 S)\nabla_0\chi=H_q\chi \ . \quad (33)$$

Note that this equation is consistent with the assumption that $H_q=O(\lambda^{-1})$.

Using the classical relation (17), we can rewrite Eq. (33) as

$$i\frac{\partial\chi}{\partial t}=NH_q\chi \ . \quad (34)$$

This is the Schrödinger equation for the subsystem in the background defined by $h^{a}(t)$. [Note that due to the time-reparametrization invariance dt can appear only in combination $N(t)dt$. This explains the presence of $N(t)$ in Eq. (34).] The derivation of the Schrödinger equation from the Wheeler-DeWitt equation has been discussed in Refs. 2, 16–18, and 13.

To find the probability distribution defined by the wave function (30), we note that the leading term in the WKB expansion of the current (11) is

$$j^{\alpha}=|\chi|^2|A|^2\nabla_0^{\alpha}S\equiv j_0^{\alpha}\rho_\chi \quad (35)$$

for the components in the classical subspace and

$$j^{\nu}=-\frac{i}{2}|A|^2(\chi^*\nabla^{\nu}\chi-\chi\nabla^{\nu}\chi^*)\equiv\tfrac{1}{2}|A|^2 j_\chi^{\nu} \quad (36)$$

for the components in the quantum subspace. Here, $j_0^{\alpha}=|A|^{\alpha}\nabla_0^{\alpha}S$ is the classical probability current for the variables h^{α}. Using the conservation of the total current $\nabla_\alpha j^{\alpha}+\nabla_\nu j^{\nu}=0$ and of the classical current $\nabla_{0\alpha}j_0^{\alpha}=0$, we obtain[19]

$$\partial\rho_\chi/\partial t+N\nabla_\nu j_\chi^{\nu}=0 \ . \quad (37)$$

The probability distribution corresponding to Eq. (35) can be written as

$$\rho(h,q,t)=\rho_0(h,t)|\chi(q,h(t),t)|^2 \ . \quad (38)$$

Here, $\rho_0(h,t)$ is the classical probability distribution for the variables h^{a} and $\rho_\chi=|\chi|^2$ is the probability distribution for quantum variables q^{ν} on the classical trajectories $h^{a}(t)$. If we represent the surface element $d\Sigma$ on the equal-time surfaces as $d\Sigma=d\Sigma_0 d\Omega_q$, where $d\Sigma_0$ is the surface element in the subspace defined by h^{α}, then $\rho_0(h,t)$ is normalized by

$$\int\rho_0 d\Sigma_0=1 \quad (39)$$

and $\chi(q,h,t)$ can be normalized by

$$\int|\chi|^2 d\Omega_q=1 \ . \quad (40)$$

Here, $d\Omega q=|\det g_{\mu\nu}|^{1/2}d^{m}q$. Hence, we have recovered the standard interpretation of the wave function for a small subsystem of the Universe. I find this a strong argument in favor of using the definition of probability based on the current (11).

If ψ is a superposition of the form (29), then, following the lines of the previous section, we obtain

$$\rho(h,q,t)=\sum_k\rho_{0k}(h,t)|\chi_k(q,h,t)|^2 \ . \quad (41)$$

The division of variables into classical and quantum needs not be the same in different regions of superspace or for different terms in the sum (29) (Ref. 20). Because of various instabilities, quantum fluctuations in some of the variables can grow with time, so that at late times these variables become semiclassical with a high accuracy. A good example of this kind of behavior is given by inflationary scenarios, where the dynamics of the inflation field ϕ is initially dominated by quantum fluctuations, then it becomes essentially classical, and eventually has a significant effect on the dynamics of the scale factor. When one of the quantum variables q^{ν} becomes semiclassical, the corresponding terms in (29) turn into sums:

$$\psi_k(h)\chi_k(h,q)\to\sum_{k'}\psi_{kk'}(h')\chi_{kk'}(h',q') \ . \quad (42)$$

Here, the set of quantum variables $\{q'\}$ contains one variable less than $\{q\}$, and the set $\{h'\}$ contains one classical variable more than $\{h\}$. The solution of the Hamilton-Jacobi equation $S_{kk'}(h')$ also has an extra integration constant compared to $S_k(h)$, and different terms in Eq. (42) correspond to different values of that constant. Physically, the transition (42) corresponds to branching of each classical trajectory $h^{\alpha}(t)$ into many trajectories $h'^{\alpha}(t)$ with different initial conditions for the new classical variable.

The unitarity condition

$$\int j_k^{\alpha}d\Sigma_{k\alpha}=\sum_{k'}\int j_{kk'}^{\alpha}d\Sigma_{kk'\alpha} \quad (43)$$

is satisfied, as long as the cross terms can be neglected. Here, the surface Σ_k crosses the trajectories before branching and the surfaces $\Sigma_{kk'}$ cross the corresponding congruences after branching. Since the relation (43) is only approximate, unitarity is an approximate concept in quantum cosmology.[9,21]

V. CONCLUSIONS

The interpretation of the wave function of the Universe suggested in this paper is approximate by its nature. It holds that the probability for the Universe (or its part) to be in a certain state can be calculated only approximately, with an accuracy not exceeding the accuracy of the semiclassical approximation for the entire Universe.[22] (The actual accuracy can be much smaller than this bound. It is determined by the accuracy of the WKB approximation for all the classical variables h^{α} and by the accuracy of the assumption that the quantum variables q^{ν} have no effect on h^{α}.) For a Universe of Planckian size, the semiclassical approximation breaks down, and probabilities cannot be calculated. In this approach, unitarity is also an approximate concept. Obviously, the probabilities cannot (at least should not) add up to one with a greater accuracy than they themselves are defined.

I do not think that the approximate nature of this approach is a great disadvantage. In fact, it agrees well with the standard interpretation of quantum mechanics, in which an essential role is played by classical measuring devices.

To calculate probabilities in a domain where the Universe is semiclassical, one has first to represent the wave function in the form

$$\psi=\sum_k \psi_k \chi_k=\sum_k A_k(h)e^{iS_k(h)}\chi_k(h,q) , \tag{44}$$

where the semiclassical variables h^α are described by WKB wave functions. $\chi_k(h,q)$ are the wave functions for the quantum variables q^ν in the background of h^α, and it is assumed that the effect of q^ν on the dynamics of h^α is negligible. The probability distributions are defined separately for each term in the sum (44). The distribution for the kth term is defined on a family of hypersurfaces $\Sigma_k(t)$ which are crossed by all classical trajectories of the action S_k in the positive direction [see Eq. (18)]. These hypersurfaces play the role of constant time surfaces. The probability for the variables h^α, q^ν to have values within a surface element $d\Sigma_{k\alpha}$ is

$$dP_k=j_k^\alpha d\Sigma_{k\alpha} , \tag{45}$$

where

$$j_k^\alpha=|A_k|^2\nabla^\alpha S_k|\chi_k|^2\equiv j_{0k}^\alpha|\chi_k|^2 \tag{46}$$

is the current (11) calculated for the wave function $\psi_k\chi_k$. With $d\Sigma_{k\alpha}=d\Sigma_{0k\alpha}d\Omega_{kq}$, Eq. (45) takes the form

$$dP_k=dP_{0k}|\chi_k|^2 d\Omega_{kq} . \tag{47}$$

Here, dP_{0k} is the classical probability distribution for the variables h^α and $|\chi_k|^2$ is the usual quantum probability density. Equation (47) is clearly in agreement with the correspondence principle.

We note that it is not necessary to include *all* the macroscopic variables into h^α. For example, Schrödinger's cat can be described by quantum variables q^ν, as long as the effect of the cat on the dynamics of the variables h^α can be neglected. The division of superspace variables into h^α and q^ν is, to a large extent, arbitrary.

Before one can calculate probabilities using Eq. (45), the wave function (44) should be properly normalized. The normalization condition is

$$\sum\int j^\alpha d\Sigma_\alpha=1 , \tag{48}$$

where the summation is over all independent congruences of classical trajectories. To avoid double counting, one should not include trajectories resulting from branching (see the end of the previous section) if the original, prebranching congruence has been included.

As an example, let us consider normalization of the wave function defined by the "tunneling" boundary conditions.[9,10] The classical trajectories corresponding to this wave function begin in the vicinity of the hypersurface $U_0(h^\alpha)=0$ with $\nabla S\approx 0$, so that the WKB approximation breaks down. The dominant part of the trajectories begin in a small part of this hypersurface corresponding to nearly spherical three-geometries with matter fields near the maximum of $V(\phi)$. If $V(\phi)$ is sufficiently flat near the maximum, then the initial parts of the trajectories describe inflationary universes. These universes start out with a single classical degree of freedom—the scale factor, with the remaining degrees of freedom describing quantum fields in de Sitter-invariant states. In that region of superspace the wave function is given by a single term of the form (30), and the normalization condition is simply

$$\int j^\alpha d\Sigma_\alpha=1 . \tag{49}$$

All the classical trajectories describing later evolution branch off the initial congruence of inflationary universes. The classical trajectories end at the boundaries of superspace corresponding to singular geometries or matter fields. (There may also be some trajectories ending on the surface $U_0=0$ with $\nabla S\approx 0$. Such trajectories may represent universes undergoing quantum tunneling to another classically allowed configuration, possibly with a different topology.)

It should be emphasized that the analysis in this paper involves a number of simplifying assumptions. The most important one is the restriction to homogeneous minisuperspace models. An extension to the inhomogeneous case is not trivial and deserves separate study. There are several other issues that need clarification.

(1) We have assumed that the classical trajectories corresponding to different terms of the sum (44) are macroscopically distinct, so that the cross terms in the probability distribution can be neglected. This is not always satisfied (for example, when the sum is replaced by an integral over a continuous parameter). It appears that one has to divide the trajectories into macroscopically distinct classes. This would be similar to coarse graining in statistical physics.

(2) The assumption that q^ν have no effect on the dynamics of h^α can be relaxed by using a self-consistent approach similar to the Hartree-Fock approximation. Such an approach has been discussed in Refs. 13, 23, and 24, where it has been shown that, with some additional assumptions, it is equivalent to using the expectation value of $T_{\mu\nu}$ as the source in classical Einstein equations.

(3) It would be interesting to study the general properties of the congruences of classical trajectories defined by actions $S(h)$ and to investigate possible topological obstructions for the existence of a family of equal-time surfaces.

(4) Finally, I should mention the semiphilosophical issues arising when one attempts to apply a probabilistic theory to the Universe, of which one has only a single copy. Here I made no attempt to deal with these issues and took a simple-minded approach that the theory describes an ensemble of Universes. For a recent discussion and references see Ref. 25.

[1]J. A. Wheeler, in *Battelle Rencontres,* edited by C. DeWitt and J. A. Wheeler (Benjamin, New York, 1968).

[2]B. S. DeWitt, Phys. Rev. **160**, 1113 (1967).

[3]The term "superspace" is now in use in the supersymmetry theory, where its meaning is quite different. The term was originally introduced by J. A. Wheeler in quantum gravity [in *Relativity, Groups and Topology,* edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1964)] and later by A. Salam and J. Strathdee in the context of supersymmetry [Phys. Rev. D **11**, 1521 (1975)]. Although quantum gravity has priority, the supersymmetry usage appears to be well entrenched, and one might consider switching to "hyperspace." (This proposal came up in a discussion with Murray Gell-Mann, and if it is ever followed, he is entitled to half of the credit.) This paper has no connection with supersymmetry, and since no confusion can arise, I shall stick to the old terminology.

[4]C. W. Misner, in *Magic Without Magic,* edited by J. R. Klauder (Freeman, San Francisco, 1972).

[5]J. J. Halliwell, Phys. Rev. D **38**, 2468 (1988).

[6]S. W. Hawking and D. N. Page, Nucl. Phys. **B264**, 185 (1986).

[7]I. G. Moss and W. A. Wright, Phys. Rev. D **29**, 1067 (1983).

[8]A preliminary version of this interpretation has been discussed in Ref. 9.

[9]A. Vilenkin, Phys. Rev. D **33**, 3560 (1986).

[10]A. Vilenkin, Phys. Rev. D **37**, 888 (1988).

[11]A. H. Guth and S.-Y. Pi, Phys. Rev. D **32**, 1899 (1985).

[12]For semiclassical variables this expression is essentially equivalent to a Wigner function; see Ref. 13.

[13]J. J. Halliwell, Phys. Rev. D **36**, 3626 (1987).

[14]A. B. Migdal, *Qualitative Methods in Quantum Theory* (Benjamin, Reading, MA, 1977).

[15]It is possible that the difference between S_k and S_k' is characterized by a parameter λ'^{-1}, such that $1 \ll \lambda'^{-1} \ll \lambda^{-1}$. Then integrals of the cross terms are of the order $\exp(-b/\lambda')$ with $b \sim 1$.

[16]V. Lapchinsky and V. A. Rubakov, Acta Phys. Pol. B **10**, 1041 (1979).

[17]T. Banks, Nucl. Phys. **B249**, 332 (1985); T. Banks, W. Fischler, and L. Susskind, *ibid.* **B262**, 159 (1985).

[18]J. J. Halliwell and S. W. Hawking, Phys. Rev. D **31**, 1777 (1985).

[19]In order to compare different terms arising in expressions such as $\nabla_\alpha j^\alpha$ it is useful to note that $\nabla_\nu = O(\lambda^{-1/2})$. [The kinetic terms of H_q contain second derivatives with respect to q^ν with coefficients of zeroth order in λ. Assuming that these terms are non-negligible, we obtain $\nabla_\nu = O(\lambda^{-1/2})$.]

[20]In fact, it is not even necessary for the division into classical and quantum variables to be the same within a single equal-time surface. If it is not the same, then the probability distributions should be defined on overlapping parts of the surface. Note that the boundaries between regions with different sets of classical and quantum variables are not sharply defined and that near such a boundary the representations (29) corresponding to the neighboring regions should both be valid.

[21]The approximate nature of unitarity has been discussed, from a different viewpoint, by J. Hartle, Phys. Rev. D **37**, 2818 (1988); **38**, 2985 (1988); University of California at Santa Barbara report (unpublished).

[22]An alternative point of view that can be taken is that the approximate nature of our interpretation is due to the fact that quantum gravity is itself a low-energy approximation to a more fundamental theory. In that theory it may still be possible to enforce unitarity and to make a rigorous definition of probability. Then we would expect that the interpretational scheme of the fundamental theory should reduce, in the semiclassical limit, to the interpretation suggested in this paper.

[23]P.D. D'Eath and J. J. Halliwell, Phys. Rev. D **35**, 1100 (1987).

[24]J. B. Hartle, in *Gravitation in Astrophysics,* edited by B. Carter and J. B. Hartle (Plenum, New York, 1987).

[25]M. Gell-Mann, J. B. Hartle, and V. Telegdi (unpublished).