The case for neutrinos from SN 1987A

J. M. LoSecco
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556
(Received 15 June 1988; revised manuscript received 25 July 1988)

A case is made for the observation of both neutrinos and antineutrinos in the burst observed from SN 1987A. The angular distribution and energetics support this hypothesis. The experiments have good agreement with each other and with the hypothesis of a 25% forward-scattering component of the form expected from $\nu e \rightarrow \nu e$. The nature of the forward-scattering component can be inferred from known cross sections and experimental expectations. The flux and energy output in various neutrino types can be estimated. A total energy in the range of $5-8\times10^{53}$ ergs seems needed. This result has implications for models of supernova and for neutrino physics itself.

INTRODUCTION

The observation of neutrinos[1-3] from the gravitational stellar collapse in the Large Magellanic Cloud (SN 1987A) is a milestone in neutrino physics and astrophysics. Our detailed understandings of neutrino physics can be used to extract much useful astrophysical information from these data. Many details are revealed in subtleties of the observations. To avoid being led astray, comparison of data from each of the experiments can be used to reduce systematic errors or the probability of statistical fluctuations. The experiments differ in sensitivity and possible systematic errors.

Three groups have reported observations of the neutrino burst from SN 1987A. The Irvine-Michigan-Brookhaven (IMB) group[1] reported eight events in their 6800-ton water Cherenkov detector. The Kamioka group[2] reported 12 events in their 2140-ton water Cherenkov detector. The Kamioka detector has an energy threshold in the vicinity of 5 MeV. This is considerably below the 19-MeV IMB threshold. The Baksan group[3] reported five events in their 200-ton liquid-scintillator detector. Such a device does not record the direction of the interactions. The Baksan threshold is about 10 MeV.

Tables I–III contain most of the information reported by each of these groups.

All groups have reported the time and energy of the interactions. The relative time of the events is better measured than the absolute time so both of these times are listed. Only IMB has an accurate (± 50 ms) absolute time measurement. The burst itself can and has been used to synchronize the experiments. The scattering angle reported by IMB and Kamioka is relative to the forward direction determined by the position of the star SN 1987A at the time of observation.

ANGULAR DISTRIBUTION

Information regarding the types of interactions is limited. Neutrinos of all six types, ν_μ, ν_τ, ν_e, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$, and $\bar{\nu}_e$, are the only known particles that can penetrate the 5000 to 8000 km of earth between the source and detectors, or can escape the interior of the stellar collapse itself. There are only a limited number of reaction channels available to neutrinos on a water or scintillator target at these energies. Some of these reactions can be distinguished by the angular distribution of the recoiling lepton.[4]

In earlier work on this question[5,6] it was concluded that the Kamioka data support the hypothesis of three or four forward-scattering events.[5] This work extends these ideas and incorporates the revised IMB (Ref. 1) and Kamioka[2] data samples and the estimated systematic errors associated with the IMB detector partial malfunction.[1]

The three possible scatterers of the various types of neutrinos and antineutrinos would produce three distinct angular distributions.

Kinematics dictates that scattering from electrons ($\nu e \rightarrow \nu e$) will be strongly peaked forward. Multiple scattering will smear the sharp forward peak over a region of the forward-scattering angle. At the electron energies observed in these experiments multiple scattering[7] is expected to be about $\sigma = 18°$–$22°$. It rises to $\sigma = 30°$ at 10 MeV. This multiple scattering is added to the true scattering angle, which at these energies is as much as 12°–18°. Reconstruction errors will also tend to move the reconstructed track direction away from forward simply due to the larger solid angle available away from the forward direction. The very sharp peak expected for this process by some authors[2] is unrealistic and will not be present. Electron scattering is the only process that can give a forward peak.

Charged-current scattering of electron antineutrinos by protons ($\bar{\nu}_e p \rightarrow e^+ n$) is expected to be nearly isotropic with a very small (10%) backward asymmetry at low energies and forward at high energies. This includes the effects of all known form factors.[8] The momentum transfer at these energies is negligible and so scattering is insensitive to the q^2 dependence of the form factors. In this work the asymmetry parameter (a as in $1+a\cos\theta$) has been calculated from all of the observed interactions using the form factors and energy dependence of Bonetti *et al.*[8] The detectors do not recognize the neutrons resulting from this reaction.

Charged-current nuclear scattering from oxygen

($\nu_e\,{}^{16}O \rightarrow e^-\,{}^{16}F$) or carbon is expected to be predominantly in the backward hemisphere.[9] The oxygen nucleus is tightly bound and requires a large threshold ($E > 15.4$ MeV) to participate in charged-current neutrino or antineutrino reactions.

Since each of these scattering processes give, in principle, distinguishable final states there will be no interference among them. Experimentally the observed angular distribution will be the sum of the contributions of each of them. Often more than one process could account for a specific event so that it is difficult to distinguish types of interactions on an event-by-event basis. For example, a given forward event could result from either electron scattering or charge-current antineutrino scattering on protons. A study of the overall distribution can be used to determine the fraction of events of the several types mentioned. Since the expected distributions do have some small energy dependence it would be best to identify specific events with specific processes. Since this cannot be done we will settle for an average expected distribution using the data as a rough guide.

Figure 1 shows the angular distribution of the IMB observations. Only one of eight events are in the backward hemisphere. Figure 2 shows the angular distribution of the Kamioka observations. The solid curve represents Kamioka events that have energies above the IMB threshold of 19 MeV. Only four of 12 Kamioka events are in the backward hemisphere. Figure 3 shows the angular distribution of the combined sample. Again, the solid curve represents events above the IMB threshold. Only one high threshold event (out of 12) is in the backward hemisphere.

The absence of a particularly strong signal in the backward hemisphere is indicative of the absence of any contribution of charged-current scattering on nuclei. The data can be understood in terms of charged-current reactions of electron antineutrinos on protons and neutrino electron scattering.

A comparison of the raw Kamioka and IMB angular distributions indicates a (Kolmogorov) probability of 81% that they were drawn from the same distribution. This is remarkably good since the different energy thresholds and systematic errors in IMB should produce some differences that are not accounted for in a simple comparison of the raw data.

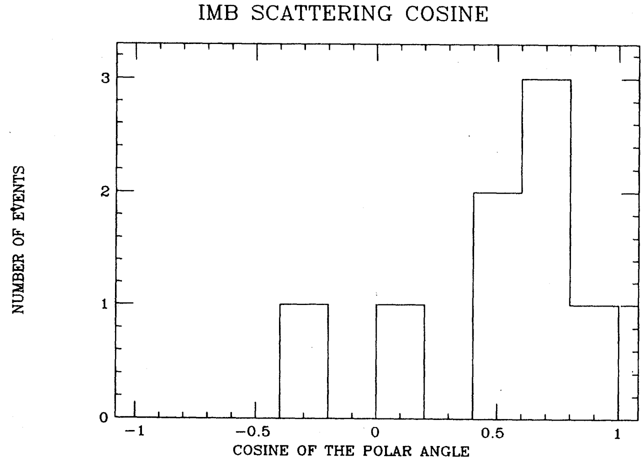

FIG. 1. Histogram of the angular distribution of the eight IMB supernova neutrino events.

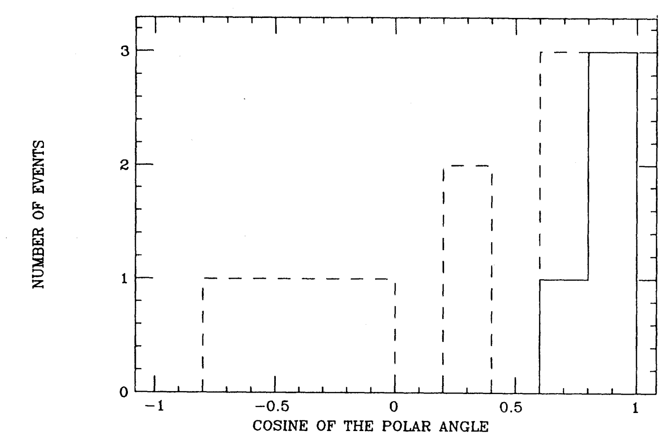

FIG. 2. Histogram of the angular distribution of the 12 Kamioka supernova neutrino events. The dashed curve represents all events. The solid curve represents the four events above the IMB 19-MeV detection threshold.

A reasonable method for studying the angular distribution of a small event sample of this type is to do a maximum-likelihood fit to a minimum number of parameters. This has been done with the scattering angles in Tables I and II using the distribution

$$P(f,x_i)=\frac{1-f}{2}(1+ax_i)+\frac{2f}{\sqrt{2\pi}\theta_0}\exp\left[-\frac{(1-x_i)^2}{2\theta_0^2}\right],$$

f is the fraction of forward scattering (in addition to that

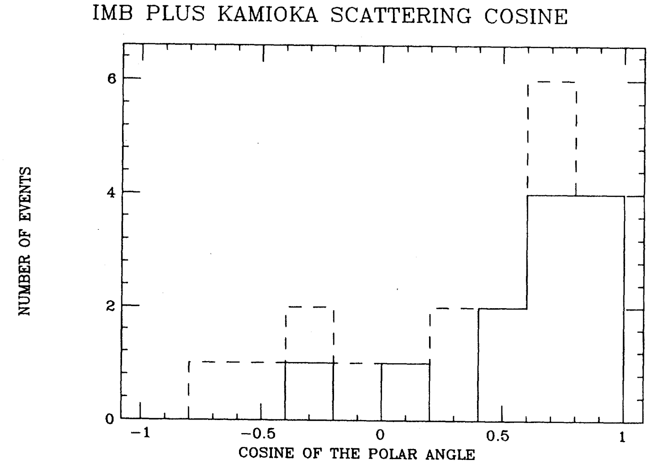

FIG. 3. Histogram of the angular distribution of the 20 supernova neutrino events in the combined IMB and Kamioka sample. The dashed curve represents all events. The solid curve represents the 12 events above the IMB 19-MeV detection threshold.

TABLE I. Summary of IMB events.

Event time	Relative time	Cosine from SN	Angle from SN	Energy (MeV)
7:35:41.374	0.000	0.172	80±10	38±7
7:35:41.786	0.412	0.720	44±15	37±7
7:35:42.024	0.650	0.563	56±20	28±6
7:35:42.515	1.141	0.414	65±20	39±7
7:35:42.936	1.562	0.843	33±15	36±9
7:35:44.058	2.684	0.610	52±10	36±6
7:35:46.384	5.010	0.738	42±20	19±5
7:35:45.956	5.582	−0.246	104±20	22±5

from the approximately isotropic charged-current scattering) present. x is the cosine of the scattering angle. a is the charged-current asymmetry $(1+a\cos\theta=1+ax)$ and is zero at Kamioka energies. a is taken to be 0.23 for IMB data. This includes the energy dependence of the asymmetry (0.13) plus a 10% systematic error resulting from the failed detector components.[1] For the combined sample a is taken as 0.092. θ_0 is the appropriate multiple scattering and other angular dispersion effects. At the energy in question $\theta_0 \approx 0.0875$ which corresponds to $\sigma=22°$. The angular distribution of a forward peak and the angular distribution of charged-current antineutrino scattering are not fit but are imposed on the solution. The best value of f is the most probable fraction of forward scatters present. The fit yields $f=0.23$ for the 20 event combined sample of IMB and Kamioka data.

The maximum-likelihood method permits an even more detailed study of the data. The angles in the tables include an error estimate that accounts for the multiple scattering and reconstruction errors that were put in by hand in using the value $\theta_0 \approx 0.0875$ above. A maximum-likelihood fit has been done using the probability distribution function (PDF) given above but with the forward peak reduced to $\theta_0=0.025$, that is $\sigma \approx 12°$. This is the width appropriate for just the kinematic spread from electron scattering at the highest energies observed. The PDF was sampled at the central values given in Tables I and II but was averaged over a $\pm 1\sigma$ Gaussian. The width of the Gaussian was given by the error quoted for

TABLE II. Summary of Kamioka events.

Event time	Relative time	Cosine from SN	Angle from SN	Energy (MeV)
7:35:35	0.000	0.951	18±18	20.0±2.9
7:35:35	0.107	0.766	40±27	13.5±3.2
7:35:35	0.303	−0.309	108±32	7.5±2.0
7:35:35	0.324	0.342	70±30	9.2±2.7
7:35:36	0.507	−0.707	135±23	12.8±2.9
7:35:36	0.686	0.375	68±77	6.3±1.7
7:35:37	1.541	0.848	32±16	35.4±8.0
7:35:37	1.728	0.866	30±18	21.0±4.2
7:35:37	1.915	0.788	38±22	19.8±3.2
7:35:44	9.219	−0.530	122±30	8.6±2.7
7:35:45	10.433	0.656	49±26	13.0±2.6
7:35:47	12.439	−0.017	91±39	8.9±1.9

TABLE III. Summary of Baksan events.

Event time	Relative time	Energy (MeV)	Internal or external
7:35:11.818	0.000	12±2.4	Internal
7:36:12.253	0.435	18±3.6	Internal
7:36:13.528	1.710	23.3±4.7	External
7:36:19.505	7.687	17±3.4	External
7:36:20.917	9.099	20.1±4.0	External

the specific measured angle. The Gaussian was folded over at the poles when needed. The expression for the likelihood function is

$$L(f)=\prod_{i=1}^{N}\left[\int_{x_i-\sigma_i}^{x_i+\sigma_i} P(f,w(x)) \times \frac{1}{\sigma_i}\exp\left[-\frac{(x-x_i)^2}{2\sigma_i^2}\right]dx\right],$$

where the folding at the poles is accomplished by

$$w(x)=\min(x,2-x), \quad x>0,$$

$$w(x)=\max(x,-2-x), \quad x<0.$$

This fit gives the value $f=0.25\pm0.15$. The result has been estimated two ways. The maximum-likelihood value using all 20 events is $f=0.249$. The variance has been estimated by $\sigma^2(\hat{f})=-[d^2\ln(L)/df^2]^{-1}$ which gives $\sigma=0.147$. A jacknife technique[10] has also been used. In the jacknife technique the mean and variance are estimated by comparing the maximum-likelihood fit for the total 20 event sample with the value obtained from the 20, 19 event subsamples produced by removing one event at a time. The jacknife technique yields $f=0.253\pm0.146$. Figure 4 illustrates the unnormalized maximum-likelihood contour for the combined 20 event sample.

The statistical significance of this result can be measured by comparing the cumulative distribution function[11] (also known as the integral distribution) of the

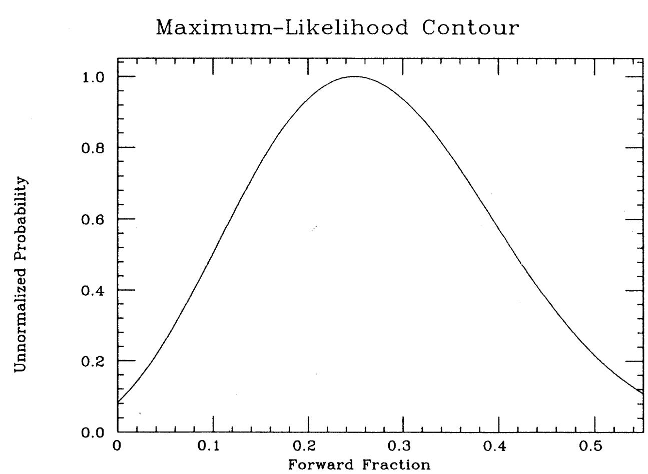

FIG. 4. The maximum-likelihood value calculated with the narrower (12°) forward peak PDF for the 20 event total sample as a function of the forward-scattering fraction.

scattering angle with theoretical distributions containing various fractions of isotropic and forward scattering. The most well-known test is the Kolmogorov-Smirnov test[11] which searches for the maximum absolute difference between the experimental and theoretical curves. The method does not permit the use of the individual scattering angle errors so these are included in the theoretical parameter θ_0, which is taken to be 0.0875, as above. The comparison is made to the integral of our fit function:

$$f\left[1-\operatorname{erf}\left[\frac{1-x}{\sqrt{2}\theta_0}\right]\right]+(1-f)\left[\frac{x+\frac{ax^2}{2}+1-\frac{a}{2}}{2}\right].$$

When this theoretical curve is compared with the data, no forward scattering (i.e., $f=0$) has a Kolmogorov-Smirnov probability of 13% for the Kamioka data and 14% for the IMB data. The combined sample has a probability of only 1.5% of having come from a distribution with no forward-scattering component.

To quantify the significance of our fit for f the data have been compared with distributions with f varying from $f=0$ to $f=0.55$. Figure 5 plots the Kolmogorov-Smirnov probability for each of the two experiments as a function of the forward-scattering fraction. Both experiments have a peak probability at about $f=0.25$. For Kamioka the probability for $f=0.24$ is 84%. For IMB the probability for $f=0.23$ is 53%. For comparison, a χ^2 of eight for eight degrees of freedom has a confidence level comparable to the IMB value quoted here. It is noteworthy that the probability does not drop to as low a value as it had for $f=0$ until $f=0.41$. To put it differently, the probability of having no forward scattering in the sample is comparable to the probability of having 41% forward scattering. The probability for any specific fraction of forward-scattering events may be read off from the figure.

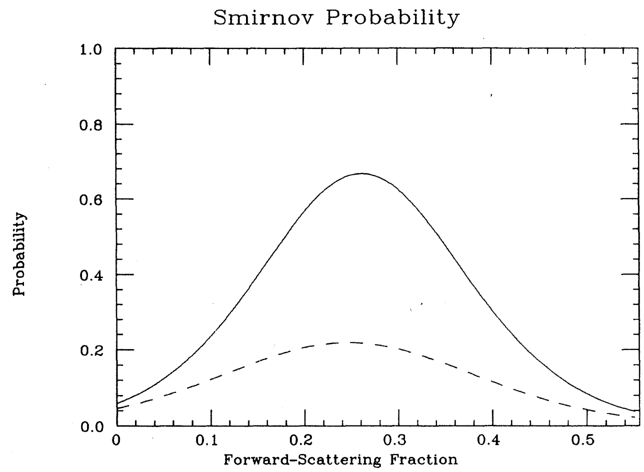

FIG. 6. The Smirnov–Cramer–Von Mises probability as a function of the forward-scattering fraction. The solid curve is for the Kamioka data. The dashed curve is for the IMB data.

The combined sample (20 events) has a peak Kolmogorov-Smirnov probability (of 36%) at $f=0.23$. These tests indicate the most probable value is close to $f=0.25$ for each individual experiment as we have found for the combined sample.

An additional statistical test, the Smirnov–Cramer–Von Mises test has also been used to study the significance of the result. It has the advantage of being an integral test and is, in principle, more sensitive to dispersion of the forward-scattering angle. The probability calculated in this way is plotted in Fig. 6. This test confirms the results already found but with maximum probabilities of 67%, 22%, and 18% for the Kamioka, IMB, and combined samples, respectively. The lower probabilities could indicate that the value $\theta_0=0.0875$ is an underestimate. On the other hand, the lower values for the IMB and combined samples could indicate that

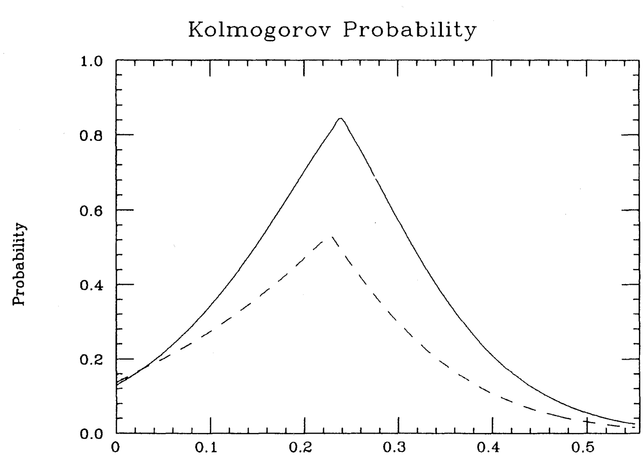

FIG. 5. The Kolmogorov-Smirnov probability as a function of the forward-scattering fraction. The solid curve is the Kamioka data. The dashed curve is for the IMB data.

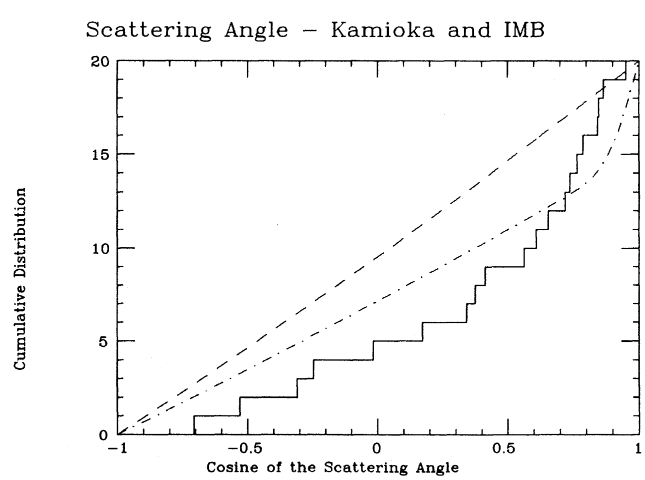

FIG. 7. The cumulative distribution function for the 20 event total sample. The dashed curve is the expected shape in the absence of any forward scattering. The dotted-dashed curve is our best fit with 25% forward scattering.

systematic effects in the IMB experiment[1] have not been completely accounted for.

The value of f found is relatively robust. Since the data at all angles are used in the estimate of f, it is not very sensitive to a range of reasonable values of the multiple scattering used (θ_0). f is determined by the isotropic events in $\approx 80\%$ of the solid angles as well as the forward ones. f is not very sensitive to specific individual events. Variations in the asymmetry a do not seriously effect the result.

Figure 7 is a plot of the cumulative distribution function of the combined data sample with two curves provided for comparison. The (almost) linear dashed curve is the expected distribution in the absence of any forward scattering. The dot-dashed curve is the best fit with 25% forward scattering.

ENERGY CONSIDERATIONS

The conclusion that 25% of the observed events are from forward scattering has followed from a consideration of the angular distribution only. In this section the energy observed in the dominant, isotropic channel will be shown to imply a comparable rate of forward-going events.

The absence of any significant forward-going neutral penetrating background in the sensitive accelerator experiments that observe and measure neutrino electron scattering make it unlikely that a source other than $\nu e \rightarrow \nu e$ need be considered to explain the forward events.

Many authors have calculated the energy output[2,12] in electron antineutrinos above the Kamioka threshold of 8.1 MeV to be about $8\text{–}9\times 10^{52}$ ergs. This result does not change much even if some of the events are attributed to other neutrino types. This is because the charged-current reaction rate varies as the square of the lepton energy. The energy calculation is dominated by the lowest-energy events observed. Table II indicates clearly that the forward-going Kamioka events are the five highest-energy events observed.[5] In general, it is expected that all neutrino types will have comparable luminosities.

To calculate the total energy carried in the pulse, integrate the flux at a given energy times the (anti)neutrino energy times the total area at a distance of 50 kpc ($r = 1.5\times 10^{23}$ cm). This gives

$$E_{T_\nu} = \sum_{i=1}^{N} \frac{E_\nu/\epsilon(E_\nu)}{\sigma(E_\nu)\times N} 4\pi r^2 \ .$$

$\epsilon(E_\nu)$ is the detection efficiency for an event of energy E_ν. $\sigma(E_\nu)$ is the cross section at energy E_ν. N is the number of scatterers available for the process in question.

An energy output of 7×10^{52} ergs in electron neutrinos in *any spectrum* would produce a single electron scattering of the form ($\nu_e e \rightarrow \nu_e e$) in the Kamioka detector [with $\epsilon(E_\nu)=0.9$]. Because the cross section rises linearly with energy this depends only on the total energy output in the form of ν_e's but detection depends on the spectrum being above the energy threshold of about 7 MeV. The forward events seem to satisfy this requirement. The cross sections for electron scattering[13] of other neutrino types are smaller than for ν_e's but the sum of the cross sections for the other five types of neutrinos is comparable to that for ν_e's. If the luminosities of all neutrino types are comparable the number of forward scatterings should double. For a luminosity of 9×10^{52} ergs in each neutrino type one would expect 2.6 forward-scattering events in Kamioka based on energy considerations alone. This 22% is very close to the 25% found from the angular distribution. It has been derived from energy considerations only.

One can also use the observed forward events to check the equal luminosity hypothesis. The three forward-scattering events from Kamioka if considered equally probable as caused by ν_e's and all other neutrino types imply an energy output of about $(11\pm 6.4)\times 10^{52}$ ergs in each neutrino type. (The low statistics make this an impractical way to make an accurate energy measurement but it does confirm the result based on the electron antineutrinos.) The data themselves do not conclusively imply that the forward-going events are produced by a specific mixture of neutrino types but this seems a reasonable hypothesis to pursue. In an earlier paper[5] all of these events had been attributed to $\nu_e e \rightarrow \nu_e e$ scattering. This was an attempt to reduce the total energy output in all neutrino types by assigning all events to the process with the highest cross section. The current approach provides (roughly) equal luminosities in all known type of neutrinos and antineutrinos.

Lepton number conservation implies that the integrated flux of electron antineutrinos must be comparable to the integrated flux of electron neutrinos. The flux can be estimated from the observations but requires far more assumptions than an estimate of the total energy output. In spite of the crudeness of these calculations, note that it is not hard to maintain lepton-number conservation since flux can be hidden below detection threshold. To calculate the flux:

$$F_\nu = \sum_{i=1}^{N} \frac{1/\epsilon(E_\nu)}{\sigma(E_\nu)N} \ .$$

The meaning of the symbols are the same as in the expression for total energy. The value of $\sigma(E_\nu)$ to be used is problematic. The neutrino type is not identified so the cross section is not known. But beyond this basic question the energy dependence of the cross section must be taken into account. On the average the recoiling electron carries 49% of the energy of the incident neutrino in neutrino electron scattering. (If it is antineutrino electron scattering the fraction is smaller.) The angular resolution is far too crude to reconstruct the parent neutrino energy from the recoil electron scattering angle and kinetic energy and the electron scattering events are not uniquely identified anyway.

The most obvious choice is to assume averages for unmeasured quantities. One would expect half of the forward-scattering events to be induced by ν_e's. The average energy of the three most forward tracks in Table II is 25.5 MeV. This implies an average neutrino energy of about 52 MeV, which gives a ν_e flux of $4.5\times 10^{9}\nu_e/\text{cm}^2$ above a minimum energy of about 14 MeV.

This can be compared with an estimated $\bar{\nu}_e$ flux of $2\times10^{10}\bar{\nu}_e/\text{cm}^2$ above 8.1 MeV.

An upper limit on the high-energy (i.e., above threshold) ν_e flux can be estimated by assuming that all of the neutrino energy appears in the recoil energy reported in Table II and that all forward events are ν_e induced. This gives the lowest cross section for ν_e scattering and forces enough flux to explain all the data. This gives a flux of $<1.8\times10^{10}\nu_e/\text{cm}^2$ above a minimum energy of perhaps 20 MeV.

The presence of charged-current scattering channels on oxygen for electron neutrinos and antineutrinos places some (weak) constraints on how high in energy the spectrum can extend before a backward peak should occur from reactions on nuclei.[9] Very crudely, the absence of such events, at say 15 MeV, implies that the high-energy ν_e flux must be considerably below $10^{10}\nu_e/\text{cm}^2$ above about 30 MeV.

A troubling feature[5] of the forward-going events is that their energy is significantly above the average value observed in all other directions. Since the forward-scattering cross section increases linearly with neutrino energy and the isotropic cross section increases quadratically with the antineutrino energy one should expect a larger ratio of forward events to isotropic events at low energy if the spectra of the beams are matched. This is a good reason to doubt the assumption of identical spectra for electron neutrinos and electron antineutrinos.

The electron scattering cross section for electron antineutrinos is large, comparable to the sum of all others except electron neutrinos. One might expect them to make a significant contribution to the observed forward peak. If the events were antineutrino induced it would imply and even higher energy for the antineutrino spectrum than for the neutrino spectrum, since on the average, less energy is carried off by the recoiling electron in antineutrino reactions. This is inconsistent with the electron-antineutrino spectrum found from the energy distribution of the observed isotropic events. The implication is that perhaps the forward events are produced by electron scattering of ν_e, ν_μ, and ν_τ but not $\bar{\nu}_e$.

APPLICATIONS

It is highly probable that forward-scattering events are present in the sample of events collected from the February 23, 1987 supernova. The most reasonable interpretation is that these events are produced by scattering of neutrinos on electrons. The cross sections and y distributions imply that the forward events were induced by ν_e and perhaps other neutrino types.

This implies a total energy output in all forms of neutrinos in the range of $5\text{–}8\times10^{53}$ ergs. This result would follow from considerations based on the majority of events being induced by electron antineutrinos and with equal luminosities in each of six neutrino types. It is corroborated by the elastic scattering events. This energy is considerably above the expected value of 3×10^{53} ergs and would require a remnant mass of from 2.0 to 2.2 solar masses.

The concurrent presence of several neutrino types in a short time period implies a major reduction in limits on the muon and tau-neutrino mass.[14] The similarity in flight times for neutrino and antineutrino implies that gravitation does not distinguish between them. This is an accurate experimental test comparing freely falling matter and antimatter.[15] The simultaneous presence of other neutrino types would also imply that gravity is not sensitive to lepton flavor number.

CONCLUSIONS

These calculations have implications for the nature of the neutrino pulse observed from SN 1987A. These hypotheses do not necessarily rule out many alternative possibilities. The fact that the data seem to have a slightly broader peak than expected from our considerations here may imply some additional physics. But it will be very difficult, with the existing data to distinguish between the various alternatives. At present it seems prudent to clearly present the arguments in support of each hypothesis while bearing in mind the limitations of the data.

The data do not support the hypothesis that the observed events are produced by only the interactions of electron antineutrinos

ACKNOWLEDGMENTS

I would like to thank D. Arnett, A. Burrows, A. Dar, D. Kielczewska, O. Saavedra, D. Schramm, and J. Van der Velde for helpful discussions on these questions. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-87ER40366-A001. I would especially like to thank Amos Yahil who insisted that a proper maximum-likelihood fit be done and who suggested the jacknife method for estimating the central value and error. Part of this work was carried out at the Aspen Center for Physics.

[1]R. M. Bionta *et al.*, Phys. Rev. Lett. **58**, 1494 (1987); C. B. Bratton *et al.*, Phys. Rev. D **37**, 3361 (1988); J. M. LoSecco *et al.*, in Proceedings of the Les Rencontres de Physique de la Vallee d'Aoste, 1988, edited by M. Greco (unpublished).

[2]K. Hirata *et al.*, Phys. Rev. Lett. **58**, 1490 (1987); K. Hirata *et al.*, Phys. Rev. D **38**, 448 (1988), has a revised scattering angle for the second Kamioka event. The revised values are used in this paper.

[3]E. N. Alexeyev *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 751 (1987) [JETP Lett. **45**, 589 (1987)]; E. N. Alexeyev *et al.*, in Proceedings of the ESO Workshop on SN 1987A, Garching, Federal Republic of Germany, 1987 (unpublished).

[4]J. LoSecco, Science **224**, 56 (1984).

[5]J. M. LoSecco, in Proceedings of the TPI Workshop on SN 1987A, Independence, Minneapolis, 1988 (unpublished).

[6]J. M. LoSecco, in Proceedings of the Les Rencontre de Physique de la Vallee d'Aoste (Ref. 1); J. M. LoSecco, in *Proceedings of the Vulcano Workshop on Frontier Objects in Astro-*

physics and Particle Physics, edited by F. Giovannelli [Nuovo Cimento (in press)].

[7]Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).

[8]T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962); S. Bonetti *et al.*, Nuovo Cimento **38**, 260 (1977); C. Llewellyn-Smith, Phys. Rep. **3C**, 261 (1972).

[9]J. B. Langworthy *et al.*, Nucl. Phys. **A280**, 351 (1977); W. C. Haxton, Phys. Rev. D **36**, 2283 (1987).

[10]R. G. Miller, Biometrika **61**, 1 (1974); R. G. Miller, Ann. Math. Statist. **39**, 567 (1968).

[11]W. T. Eadie *et al.*, *Statistical Methods in Experimental Physics* (North-Holland, Amsterdam, 1971).

[12]S. A. Bludman and P. J. Schinder, Astrophys. J. **326**, 265 (1988); I. Goldman *et al.*, Phys. Rev. Lett. **60**, 1789 (1988), and references therein.

[13]B. Kayser *et al.*, Phys. Rev. D **20**, 87 (1979); H. H. Chen *et al.*, in *Neutrino '86: Neutrino Physics and Astrophysics,* proceedings of the 12th International Conference, Sendai, Japan, 1986, edited by T. Kitagaki and H. Yuta (World Scientific, Singapore, 1986).

[14]D. Kielczewska (private communication).

[15]J. M. LoSeccco, Phys. Rev. D **38**, 3313 (1988).