RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 39, NUMBER 3 1 FEBRUARY 1989

Self-consistent nonperturbative effect of string fragmentation on superstring mass spectra

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(Received 22 February 1988; revised manuscript received 24 October 1988)

Using a simple model for soft self-consistent nonperturbative string fragmentation-recombination loops, we find that, in an open-string theory, ground-state particles necessary for the standard model either become tachyons or acquire large (Planck-scale) masses, making them unacceptable for sub-Planck-scale phenomenology. No comparable difficulties are evident for closed-string theories.

Because of its relative absence of mathematical pathologies and certain features of its ground-state mass spectrum, the superstring[1,2] is widely regarded as the most promising approach to the unification of all the known particle interactions. However, uniqueness continues to be elusive, with several competing[3] open- and[4] closed-string candidate schemes. Moreover, higher-order effects, which involve string fragmentation and recombination, are invariably treated perturbatively, even though superstring theories are believed by many to be intrinsically strongly interacting in nature.[5]

By considering a simple continuum string fragmentation-recombination scheme which is nonperturbative from the beginning, we find that, far from being a negative feature, the strongly interacting nature of string theories may in fact provide a way of helping to select the correct theory. In particular, we find that energy minimization may in some cases necessarily lead to broken-symmetry nonzero ground-state masses at the Planck scale, making the corresponding theory unacceptable as a candidate for the theory underlying sub-Planck-scale phenomenology. So far we have only been able to exclude open-string theories in our scheme, but, eventually, more detailed treatments of this type may perhaps be capable of excluding not only particular closed-string theories, but particular symmetry-breaking and compactification scenarios as well.

Our starting point, or input, will be the (unfragmented) superstring itself, which we assume to continue to give a good description (by itself) of "hard" (short-distance) phenomena. But, as our string is stretched to bigger distances, it becomes energetically more favorable for it to fragment. With further stretching we get further fragmentation and end up with the "jet" production graphs of Figs. 1(a) and 1(b) for open-string theories. Fragmentation implies recombination, and so, if planar graphs dominate, we must have the fragmentation-recombination string-end loop graphs of Fig. 1(c).[6] These are equivalent to the nonperturbative generalized infinite-ladder sum T of Fig. 2 for a given particle (or string) process $12 \rightarrow 34$ in the t channel, where $t \equiv (p_1+p_2)^2 = (\text{energy})^2$; p_i is the ingoing momentum of i, $s \equiv (p_2+p_4)^2$, $u \equiv (p_1+p_4)^2$, $T = T(s,t)$ for appropriate fixed (or averaged-over) values of all the other independent (angular) variables (if there are any), and the final planar-invariant amplitude[7] is a linear combination of $T(s,t)$, $T'(t,u)$, and $T''(u,s)$. In Figs. 2(b), 2(c), . . . , the upper and lower "ladder" exchanges should themselves have the form of the entire sum of Fig. 2.

In an open-string theory, it can be argued that the planar graphs should always dominate over nonplanar ones, even in the presence of fermions.[7] For example, the propagation of string ends for the planar one-loop string-scattering graph of Fig. 3(a) can have N possible internal string-end loops, whereas the corresponding nonplanar Fig. 3(b) is constrained to only one possibility and therefore has a relative $1/N$ suppression factor; N is related to the (large) number of string internal degrees of freedom and is the analogue of the N_{flavor} of mesonic hadron physics. More generally, we have a generalized $1/N$ expansion.[7] Nonplanar (crossed-line) graphs may, of course, be needed to cancel anomalies and restore unitarity when we have zero-mass ground states.[1,2] Planar dominance may then be less obvious. We shall find, however, that energy-minimization leads to situations with broken-symmetry nonzero ground-state masses, for which this difficulty should not arise.

The exchanges $(a, \ldots)$ in Fig. 2(a) have a (t-channel short-range) high-s ($> s_c$) "hard" input unfragmented-string contribution H to the "absorptive part"

$$A[= iT(s-i\epsilon,t) - iT(s+i\epsilon,t)]$$

of Fig. 2, and a (t-channel long-range) low-s ($< s_c$) "soft" contribution L, for which string fragmentation recombination is taken into account self-consistently; similarly for $(b, \ldots)$, $(b', \ldots)$, ...[6] A $\theta(s-s_0)$ factor may be needed for Fig. 2(b) to minimize double counting between Fig. 2(b) and L, where $\theta(x)=1$ for $x>0$, and $\theta=0$ for $x<0$; similarly for Figs. 2(c), ... For moderately high s_c we can make the asymptotic approximation[8]

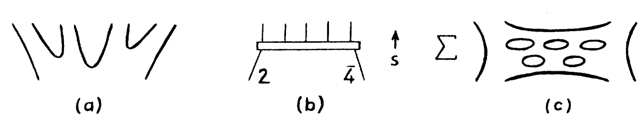

FIG. 1. (a) The propagation of string ends when open strings fragment; (b) the corresponding multiparticle (or multistring) production graph $2\bar{4} \rightarrow$ anything in the s channel; and (c) the fragmentation-recombination string-end loop graphs implied by (a). Our string ends are the analogues of the quarks at the ends of gluon flux tubes in hadron physics.

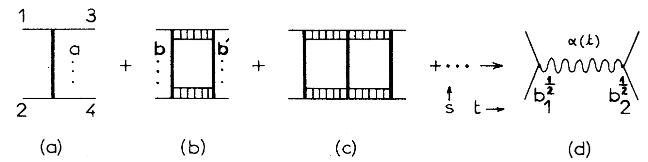

FIG. 2. Generalized infinite ladder sum for $12 \rightarrow 34$ in the t channel generating an "output" Regge trajectory $\alpha(t)$.

$$H(s,t) = b_0(t) \nu^{\alpha_0(t)} \theta(s - s_c) , \quad (1)$$

where $\alpha_0(t)$ is the highest unfragmented-string Regge trajectory, $b_0 = \beta_{12}^0 \beta_{34}^0$ is the corresponding factorizable coupling function, and ν is the familiar crossing-symmetric variable $(s-u)/2$, or

$$\nu = s + \left[t - \sum m_i^2 \right] \Big/ 2 , \quad (2)$$

where the m_i and S_i are the masses and spins of the external particles $i = 1,2,3,4$ of Fig. 2, ordered so $S_1 + S_2 \geq S_3 + S_4$. If we make a single-state $s = s_a (= m_a^2) < s_c$ narrow-peak approximation,

$$L(s,t) \simeq \Gamma(t) \delta(s - s_a) . \quad (3)$$

For a given set of internal quantum numbers, a t-channel partial-wave (or Mellin-transform) j projection of Fig. 2 gives at least two j-plane Regge poles: one at $j = \hat{\alpha}_0$ in the vicinity of the "input" $j = \alpha_0$ of Eq. (1), and one at $j = \alpha$ near the $j = \alpha_L$ pole that would arise if we set $H = 0$ for $(a, \ldots), (b, \ldots), \ldots$ in Fig. 2. If $\alpha_0 \geq \alpha_L$, level repulsion leads to j poles at $\alpha (< \alpha_L)$ and $\hat{\alpha}_0 (> \alpha_0)$; $\hat{\alpha}_0$ gives a tachyon ground state, since the input α_0 gives a zero-mass ground state in a superstring theory. In the event that the corresponding $\hat{\alpha}_0$ residue vanishes, however, the ground state would then arise from an $\alpha < \alpha_0$ j pole, and therefore necessarily has a Planck-scale mass, making it inconsistent with the phenomenological sub-Planck-scale mass spectrum. In either case we would have to reject such a theory.

On the other hand, with $\alpha_0 < \alpha_L$ we again have a tachyon, arising this time from $\alpha (> \alpha_0)$ itself. If its residue vanishes the ground state would then arise from $\hat{\alpha}_0$. In simple models, where α and α_0 are the only poles, this state would have a small positive mass. Since $\hat{\alpha}_0$ is not the highest trajectory, however, this mass could vanish in a more accurate model, particularly if lower-lying Regge "daughter" trajectories and deep string symmetry properties are taken into account. Without a more detailed treatment, there would thus be no reason to reject this kind of theory.

To see whether we have $\alpha_0 \geq \alpha_L$ or $\alpha_0 < \alpha_L$, we need a specific scheme. Now if we set $H \rightarrow 0$ for the moment, so $A \rightarrow A^L$, $\alpha \rightarrow \alpha_L$ and $\sqrt{b_1}\sqrt{b_2} \rightarrow b_L = \beta_{12}^L \beta_{34}^L$ in Fig. 2, we can relate Eq. (3) or Fig. 2(a) to Fig. 2(d) through the "soft"-dynamics finite-energy sum rule (FESR)[6,8,9]

$$\int ds [\Gamma(t) \delta(s - s_a) - b_L(t) \nu^{\alpha_L(t)} \theta(\nu) \theta(\bar{s} - s)] \nu^{\hat{n} - S_1 - S_2} = 0 , \quad (4)$$

where $\hat{n} = 0$ for the lowest-moment planar-graph FESR, which is based on s-plane T analyticity and expresses the average "duality" or equivalence between low-s states such as a and the high-s factorizable Regge $b_L \nu^{\alpha_L}$ extrapolated to low s.[8] We combine (similar) duality (between $b, \ldots ; b', \ldots ; \ldots$ and factorizable Regge behavior) with the usual approximate contractability of the $b, \ldots ; b', \ldots ; \ldots$ exchanges to kinematically factorizable "contact" interactions for (even moderately) high s in general graph theory (see pp. 132–136 of Ref. 10 with $s \leftrightarrow t$). This leads to the factorizable form

$$A_j^L \simeq L_j + L_j' k L_j'' + L_j' k L_j''' k L_j'' + L_j' k L_j''' k L_j''' k L_j'' + \cdots , \quad (5)$$

for the Mellin-transform "partial-wave" projection

$$A_j^L(t) = \int_0^\infty ds\, A^L(s,t) \nu^{-j-1} \quad (6)$$

of Fig. 2(a), 2(b), . . . in our $H \rightarrow 0$ limit (see p. 151–157 of Ref. 10). Since $L_j L_j''' = L_j' L_j''$, Eq. (5) can be summed to give

$$A_j^L = L_j / (1 - \Lambda_j / L_j) , \quad (7)$$

where $\Lambda_j (= L_j' k L_j'')$ is given by Fig. 2(b).

When planar graphs dominate, the usual double-Regge exchange "Amati-Fubini-Stanghellini" (AFS) singularities arising from the Regge behavior of the "ladder" exchanges in Fig. 2(b) are well known to be absent on the physical sheet (see pp. 163–164 of Ref. 10 and p. 102 of Ref. 11). Figure 2(b) then falls off rapidly for large s and $\Lambda(s,t)$ is, therefore, peaked in s. Nonplanar effects would give $\Lambda(s,t)$ a weak $1/N$-suppressed high-s tail.

If $\Lambda_j = L_j$, Eq. (7) will give a j-plane output Regge pole at $j = \alpha_L(t)$ with residue $b_L(t)$, which corresponds to $A^L(s,t) = b_L \nu^{\alpha_L}$ through Eq. (6). If we also use Eq. (4) we can then eliminate all couplings and obtain

$$\frac{\bar{y}^{\alpha_L + 1 - S_1 - S_2}}{\alpha_L + 1 - S_1 - S_2} = \ln y_1 + \frac{g_2}{g_0} y_1^{-2} (\lambda - \alpha_L - \tfrac{3}{2}) , \quad (8)$$

where $y = \nu / \nu_a$, $\bar{y} = \bar{\nu} / \nu_a$, $y_i = \nu_i / \nu_a$, and $\bar{\nu}, \nu_i$ are given by Eq. (2) at $s = \bar{s}, s_i$. We have expanded $y^{\lambda - j - 1}$ about the $s = s_1$ Λ-peak position in $\Lambda_j / L_j = \int dy\, G(y) y^{-j-1}$ and dropped higher $(n > 2)$ finite-width Λ-peak moments $g_n = \int dy\, y^{-\lambda} G(y)(y - y_1)^n$, with λ such that $g_1 = 0$. Since g_2 / g_0 is itself then found to have only a small effect on α_L, even with a fairly large Λ width,[6,12] this should be a reasonable approximation.

Now Figs. 2(a), 2(b), 2(c), . . . , and hence the terms in Eq. (5), correspond to an increasing average number of loops in the s channel (where $\sqrt{s} =$ energy). This in turn is related to an increasing production multiplicity M in

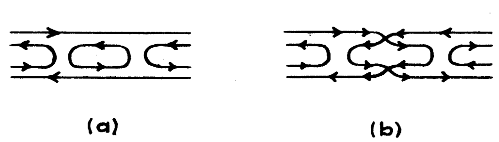

FIG. 3. The propagation of string ends for an (a) planar and (b) nonplanar one-loop string-scattering graph.

the sense that the loop graphs of Figs. 1(c) and 2 were implied by the production graphs of Figs. 1(a) and 1(b), as we have seen. But the inverse of the Mellin transform of Eq. (5) gives peaks at $\nu=\nu_a$, $(\nu_1/\nu_a)\nu_a$, $(\nu_1/\nu_a)^2\nu_a, \ldots$, since L, Λ are peaked at $\nu=\nu_a,\nu_1$.[12,13] If we, therefore, require s_1 (or ν_1) to take on the lowest value capable of consistently giving a solution for α_L from Eq. (8), we then in effect maximize the breaking of our string, since we maximize the average M in any given energy band in the s channel. Since the energy of an unbroken string rises quite rapidly with length and it is energetically more favorable for it to fragment, such maximal breaking corresponds to a minimization of the average energy in the s channel.[6] We then obtain

$$\alpha_L \simeq S_1+S_2-1+\frac{1+x}{\ln\bar{y}}-\frac{g_2/g_0}{e y_1^2(\ln\bar{y})^3}\,, \tag{9}$$

with $y_1=\bar{y}^e+O(|x|)$ and $|x|$ (and, therefore, s_1) as low as possible consistent with other Fig. 2 processes. This result continues to hold even if we add to $L(s,t)$ an $s>\bar{s}(>s_a)$ contribution $b_L\nu^{\alpha_L}\theta(s-\bar{s})\theta(s_0-s)$, which approximates the higher $a,\ldots$ states of Fig. 2(a) in the average-"duality" sense of Eq. (4); but then $y_1=y_0\bar{y}^{e-1}+O(|x|)$.

If we now include a factorizable short-range Eq. (1)-type $H\neq 0$ contribution to $(a,\ldots)$, $(b,\ldots)$, $\ldots$ in Fig. 2, we must modify Eq. (5) by replacing $L\to L+H$, $L'+H'$, etc. If we then sum the resulting series, we must modify Eq. (7) by replacing $1\to L_j'''/(L_j'''+H_j''')$. We then have a $j=\alpha$ Regge j pole when $\Lambda=L-\Lambda H'''/L'''$. Since the original $H=0$ Eq. (7) gave a $j=\alpha_L$ pole when $\Lambda=L$, we obtain, to first order in H,

$$\alpha \simeq \alpha_L + b_L H_{\alpha_L}''''/L_{\alpha_L}''' L_{\alpha_L}\,, \tag{10}$$

where Eq. (1) can be related to the lowest s-channel a_0-spectrum state through an FESR resembling Eq. (4).

Our $12\to 34$ results can be generalized in the usual way to "Reggeized processes" $\alpha_1\alpha_2\to\alpha_3\alpha_4$ in Fig. 2, with $S_i=\alpha_i(m_i^2)$ for $i=1,2,3,4$ and with m_i^2 taking on positive or negative values.

If $x=0$, Eqs. (9) and (10) give an $\alpha(t)$ with spurious branch points at $\nu_a=0$, $s_a-\bar{s}$, s_a-s_1, etc., which arise because our approximations break down in the regions where they occur.[12] Away from these regions, however, our $\alpha(t)$ can be well approximated by the large-$|\nu_a|$ form

$$\alpha(t)\simeq S_1+S_2-\tfrac{1}{2}+2\hat{\alpha}'(\nu_a+x_1) \tag{11}$$

with $x_1=0$, where we have dropped a $p_1\nu_a^{-1}+p_2\nu_a^{-2}+\cdots$ correction. Now the same $\alpha(t)$ arises from an infinite set of "processes" $\alpha_1\alpha_2\to\alpha_3\alpha_4$, with a continuum of m_i^2 values. Then the linear-$\alpha(t)$ Eq. (11) continues to apply even in t regions where the original Eqs. (9) and (10) fail, since it is always possible to find another process (with another $\sum m_i^2$ value) for which Eqs. (9)-(11) are valid in these t regions, and since there is also an overlap t region within which the linear $\alpha(t)$ given by Eq. (11) is valid for both processes and the coefficients of (11) can be made exactly consistent with each other.[12] We conclude that we can always use Eq. (11) for any t. For $x\neq 0$ processes, we can get the same α with $x=x_1\nu_a^{-1}+x_2\nu_a^{-2}+\cdots$ and $x_1\neq 0$ in Eq. (11).

If we now simultaneously minimize $|x_1|$ in Eq. (11) for all possible leading-α_i processes $\alpha_1\alpha_2\to\alpha_3\alpha_4$, we are led to a universal α', since any changes in m_1^2 or m_2^2 are exactly compensated for by changes in S_1 or S_2. Without a universal α', indefinitely large changes in m_1^2 or m_2^2 lead to indefinitely large changes in $|x_1|$, which would therefore not be minimal, as it has to be for $|x|$ to be as low as possible.

Typically, an open-superstring-theory spectrum would have ground-state fermions (f_1,f_3) and bosons (b_0,b_1,b_2) with spins $(\frac{1}{2},\frac{3}{2})$ and (0,1,2), respectively, with the last arising in the usual way from the nonplanar closed-string sector of the theory and, therefore, not participating in the purely planar equations we have been considering (see pp. 146, 188-189, 215-218 of Ref. 2). We now apply Eq. (11) to

$$b_1^r b'\to b_0^r b'',\quad b_1^r b'\to b_1^r b'',\quad b_0^r b'\to b_0^r b'' \tag{12}$$

for a given r, (lowest-mass) internal a, external bosonic b', b'', and output $\alpha=\alpha_b$ in Fig. 2; b_i^r is either a ground-state b_i or a given rth Regge "recurrence" (or orbital excitation) of b_i on the same α, where r can be either positive or negative if we consider $\alpha_1\alpha_2\to\alpha_3\alpha_4$ processes. For $r\neq 0$ it is always possible to have the same internal a in each of the processes in Eq. (12). We then find that there is always some $r=r_0$ for which the maximum $|x_1|$ for Eq. (12) can attain its lowest possible value, and that we must then have

$$m_{b_1}^2-m_{b_0}^2=1/2\alpha'\,. \tag{13}$$

Any deviation from Eq. (13) will also increase the maximum $|x_1|$, either for any given $r(>r_0)$ for which this $x_1>0$, or for any $r(<r_0)$ for which this $x_1<0$, or for both. (If f_3 were to arise in the planar open-string sector, we would also have $m_{f_3}^2-m_{f_1}^2=1/2\alpha'$ from $fb\to fb$; our final conclusions are unchanged.)

Now a complete self-consistent dynamical calculation based on Fig. 2 is always possible with $t\le 0$ and $m_i^2\le 0$; the resulting amplitude can always be analytically continued to $t>0$ and $m_i^2>0$ after completing such a calculation. Therefore, we will make the energy-minimization requirement that, for each $(a,\alpha$-output) combination, the lowest $|x_1|$ be as low as possible for the $\alpha_1\alpha_2\to\alpha_3\alpha_4$ processes with $m_3^2+m_4^2\le 0$. We find that this is attained with positive nonzero $m_{f_1}^2$, $m_{b_0}^2(\simeq 0)$, and $m_{b_1}^2(\simeq 1/2\alpha')$, so that $\alpha(0)\le\alpha_0(0)$ for α_{f_1} and α_{b_1}; $\alpha(0)\ge\alpha_0(0)$ would require much larger $|x_1|$. The small $1/N$-suppressed nonplanar high-s $\Lambda(s,t)$ tails do not change these conclusions. We conclude that open-string theories must be rejected.

In the case of closed-string theories, nonplanar graphs dominate and Fig. 2 must be extended to take crossed-line exchanges into account. We are again led to Eqs. (1)-(7), but this time with modified symmetrized amplitudes T and "signatures" $\hat{n}=0$ or $\hat{n}=1$ in Eq. (4) depending on that symmetry.[9] Since the double-Regge-exchange AFS singularity is now on the physical sheet, a peak approximation is no longer valid for Fig. 2(b) (see pp. 164-168 of Ref. 10 and p. 104 of Ref. 11). We will therefore, instead, approximate it by an average effective double-Regge behavior $\propto \nu^{\alpha_{\rm II}}$, which we assume to persist down to an effective threshold at $s = s_B$,[14] so that we now have

$$G(y) = By^{\alpha_{\rm II}}\theta(y - y_B) \qquad (14)$$

in $\Lambda_j/L_j = \int dy\, Gy^{-j-1}$. Here $j = \alpha_{\rm II}$ may be either the leading (rightmost) AFS j-plane singularity, so $\alpha_{\rm II} = \alpha_{\rm AFS}$, or an average effective singularity with $\alpha_{\rm II} = \alpha_{\rm AFS} - q$ and $q > 0$. Instead of Eq. (8), we now have[15]

$$\frac{\bar{y}^{\alpha_L + 1 + \hat{n} - S_1 - S_2}}{\alpha_L + 1 + \hat{n} - S_1 - S_2} = \ln\left[\bar{y}\frac{\nu_B}{\nu_0}\right] + \frac{1}{\alpha_L - \alpha_{\rm II}}, \qquad (15)$$

where we must have $s_B \gtrsim s_0$ or $\nu_B \gtrsim \nu_0$ if we are to avoid any $L - \Lambda$ double counting. Equation (10) then again gives a small $(\alpha - \alpha_L)$.

If we now require s_B to take on the lowest value consistent with $s_B \gtrsim s_0$ and with other processes, we again maximize the breaking of the string and so minimize the energy in the s channel.

If there are any processes for which the leading AFS singularity dominates Fig. 2(b), $\alpha_{\rm II} \simeq \alpha_{\rm AFS}$ and Eq. (15) gives an approximately linear highest α_L trajectory $\alpha_h(t)$ when $\sum m_i^2$ is such that we can attain $s_B = s_0$. We then have $\alpha_{\rm II}(t) \simeq 2\alpha_h(0) - p + t\alpha_h'/2$, where p depends on the dimensionality of our space; $p = 1$ if $D = 4$.

As in the planar case, we require that the lowest $s_B (= s_0)$, which plays the role that s_1 (or $|x_1|$) played earlier, is attained with $m_i^2 \lesssim 0$. We then find that, if we are to have $\alpha > \alpha_0$, as required for an acceptable theory, we must have $p + q \gtrsim 2$. But, in the absence of a more detailed calculation, there is nothing to exclude any $q \gtrsim 0$. We conclude that we cannot exclude any closed-string theory at this preliminary stage.

This work was supported in part by the U.S. Department of Energy.

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