PHYSICAL REVIEW D VOLUME 38, NUMBER 3 1 AUGUST 1988

Simple supersymmetric strongly coupled preon model

S. Fajfer
Institut za Fiziku, University of Sarajevo, Sarajevo, Yugoslavia

D. Tadić
Zavod za Teorijsku Fiziku, Prirodoslovno-Matematički Fakultet, University of Zagreb, Zagreb, Croatia, Yugoslavia
(Received 29 February 1988)

This supersymmetric-SU(5) composite model is a natural generalization of the usual strong-coupling models. Preon superfields are in representations $\mathbf{5}^*$ and $\mathbf{10}$. The product representations $\mathbf{5}^*\times\mathbf{10}$, $\mathbf{5}\times\mathbf{10}$, $\mathbf{5}\times\mathbf{5}$, and $\mathbf{5}^*\times\mathbf{5}$ contain only those strongly hypercolor bound states which are needed in the standard electroweak theory. There are no superfluous quarklike states. The neutrino is massless. Only one strongly hypercolor bound singlet ($\mathbf{10}\times\mathbf{10}^*$) can exist as a free particle. At higher energies one should expect to see a plethora of new particles. Grand unification happens at the scale $M\sim10^{14}$ GeV. Cabibbo mixing can be incorporated by using a transposed Kobayashi-Maskawa mixing matrix.

I. INTRODUCTION

It is possible to construct a very simple, in a sense minimal, preon model[1–6] which describes a quark-lepton generation by using only two superfields. This model is closely connected with the fermion-boson model proposed by Abbott and Farhi[5] and with the model of Maalampi and Pulido[7] which unifies hypercolor (HC), color (C), and electromagnetic (E) forces within supersymmetric SU(5).

The basic superfields of our model do not contain any redundant particles besides those needed to construct experimentally seen quarks and leptons. The model is also free of anomalies that emerge quite naturally from the unique choice of the basic superfields. In a sense this model is a minimal supersymmetric (SUSY) version of the Abbott-Farhi models.[5]

As often suggested[5–7] weak interactions are understood as some kind of van der Waal forces mediated by the composite $W^\pm$ and Z bosons. The fundamental forces are unified within the supersymmetric gauge group SU(5). At low energies the "world" is described by the direct-product gauge group

$$G_P=\mathrm{SU}(2)_{\mathrm{HC}}\times\mathrm{SU}(3)_C\times\mathrm{U}(1)_E\ ,\qquad G_P\subset\mathrm{SU}(5)\ . \tag{1.1}$$

The G_P classification of the basic preons needed to build up one family of left-handed quarks and leptons is shown in Table I.

Here α and β denote chiral G_P superfields, which are contained in the appropriate [see Eq. (1.5) below] SU(5) supermultiplets. Their quantum numbers are quite similar to those of preons x and y introduced by Ref. 7. However, this paper utilizes the fact that 2 and 2* representations of SU(2) are equivalent.[5] Thus the left-handed quarks and leptons can be constructed from only two superfields, as follows:

$$u_L\in(\alpha\beta^*),\quad e_L\in(\beta\beta)\ ,\qquad d_L\in(\alpha\beta),\quad \nu_L\in(\beta\beta^*)\ . \tag{1.2}$$

This fixes two model features in a unique way: The HC group has to be SU(2); the preon charges have to be those which are shown in Table I.

These statements can be further qualified by comparison with Ref. 7, whose model can work for any $\mathrm{SU}(N)_{\mathrm{HC}}$ gauge group. Its general preon assignment could be[8]

$$\alpha(N,1,\tfrac{1}{2}+\delta),\quad \beta(N,1,-\tfrac{1}{2}+\delta)\ ,\qquad x(\bar N,3,\tfrac{1}{6}-\delta),\quad y(\bar N,1,-\tfrac{1}{2}-\delta)\ . \tag{1.3}$$

Here we have indicated the $\mathrm{SU}(N)_{\mathrm{HC}}$ and $\mathrm{SU}(3)_C$ multiplets and the electric charge, which includes an arbitrary quantity δ. The choice $N=2$ and $\delta=0$ corresponds to the preons used by Ref. 7. With $N=4$ one obtains the preons used by Ref. 8.

The preons from Table I are included in the fundamental 5 and 10 representations of the SU(5) group. Those representations have the following G_P-based decompositions:

$$\begin{aligned}&\square=(\square,\bullet)\,(\bullet,\square)\ ,\\ &5=(2,1,\tfrac{1}{2})+(1,3,-\tfrac{1}{3})\ ,\\ &\boxminus=\left[\boxminus,\bullet\right]+(\square,\square)+\left[\bullet,\boxminus\right]\ ,\\ &10=(1,1,1)+(2,3,\tfrac{1}{6})+(1,3^*,-\tfrac{2}{3})\ .\end{aligned} \tag{1.4}$$

TABLE I. Classification of preons.

Preon	$\mathrm{SU}(2)_{\mathrm{HC}}$	Multiplets $\mathrm{SU}(3)_C$	$\mathrm{U}(1)_E$, charge
α	2	3	$\frac{1}{6}$
β	2	1	$-\frac{1}{2}$

Besides the preons these multiplets contain only the left-handed charge conjugates of the quarks and the electron:

$$\begin{aligned} &\mathbf{10}:\ \alpha(2,3,\tfrac{1}{6})+u_L^c(1,3^*,-\tfrac{2}{3})+e_L^c(1,1,1)\ ,\\ &\mathbf{5}^*:\ \beta(2,1,-\tfrac{1}{2})+d_L^c(1,3^*,\tfrac{1}{3})\ . \end{aligned} \tag{1.5}$$

This decomposition justifies the opening statement that our model is in some sense a minimal model.

The representations (1.5) are not only without the redundant exotic fields, they also automatically lead to the anomaly-free theory. The combination of SUSY-SU(5) representations **10**+**5*** is indeed anomaly-free.[9,10]

The problems with unification are the same as the ones already discussed in Ref. 7. At the HC interaction scale q_H the HC coupling constant α_H has to be larger than the color coupling constant α_C:

$$\alpha_H(q_H)>\alpha_C(q_H)\ . \tag{1.6}$$

Relative magnitudes of the coupling constants α_G $(G=H,C)$ are determined by b_N factors $[N\sim SU(N)]$ which appear in the one-loop evaluation equations:

$$\begin{aligned} &\frac{1}{\alpha_G(q)}=\frac{1}{\alpha_G(q_G)}+\frac{b_N}{2\pi}\ln\frac{q}{q_G}\ ,\\ &b_N=3N-\sum n_R T(R)\ . \end{aligned} \tag{1.7}$$

Here n_R denotes the number of representations [i.e., SU(2) doublets or SU(3) triplets] and $T(R)$ is the corresponding Casimir invariant. [In all our cases $T(R)=\frac{1}{2}$.] The condition (1.6) implies $b_H>b_C$. This is not satisfied by the representations (**5***, **10**). One finds

$$(\mathbf{5}^*,\mathbf{10}),\quad b_H=4,\quad b_C=7\ . \tag{1.8}$$

As a remedy, Ref. 7 proposed that the model contains some additional mass-split multiplets. In order to account for that splitting one can introduce additional couplings to the representations **75**, **50**, **24**, and **15**. These representations must contain supermassive particles only, which do not contribute to b factors. The coupling **75**×**50**×**5*** (**75**×**50***×**5**) (Refs. 11 and 12) tends to make the SU(3) triplet d_L^c in **5** (**5***) heavier. The $SU(2)_{HC}$ doublet component α (α^*) in **10** (**10***) can be made superheavy through coupling **24**×**15***×**10** (**24**×**15**×**10***). Both of those breakings of SU(5) symmetry go in the right direction and help to explain why quarks are more massive than leptons.

Such symmetry breaking is also needed in a model which contains several generations. In the theory with three unbroken multiplets $(\mathbf{5}_i^*,\mathbf{10}_i)$ corresponding to three generations, the b factor for hypercolor interaction vanishes

$$b_H=6-\tfrac{1}{2}(12)=0\ . \tag{1.9}$$

Any additional generation would lead to a negative b_H, thus destroying the asymptotic freedom. (However, it would be premature to use this as an explanation for the observed number of generations.)

In a model with three generations, one can assume that only the third generation, where quarks are rather massive, ought to be mass-split: i.e.,

$$2\times(\mathbf{5}^*,\mathbf{10})_{1,2}+(\mathbf{5}^*,\widetilde{\mathbf{10}})_3+3\times(\widetilde{\mathbf{10}},\widetilde{\mathbf{10}}^*)_{r,s} \tag{1.10a}$$

which gives

$$b_H=\tfrac{3}{2},b_C=1\ . \tag{1.10b}$$

Here, the tilde (i.e., $\widetilde{\mathbf{10}}$) denotes the mass-split multiplet. All anomalies are canceled between generation multiplet pairs $(\mathbf{5}^*,\mathbf{10})_i$, or between additional decouplets $(\mathbf{10},\mathbf{10}^*)_r$. The unification scale M is determined by the equation[8]

$$M=q_H^{b_H/(b_H-b_C)}q_C^{b_C/(b_C-b_H)}\ . \tag{1.11}$$

For the case (1.10) with $q_C=0.1$ GeV and $q_H=3.10^4$ GeV one finds the standard value

$$M=2.7\times10^{15}\ \text{GeV}\ . \tag{1.10c}$$

If one uses the lower q_H value (which might be more compatible with the experimental W and Z masses):

$$q_H=3\times10^2\ \text{GeV},\quad q_c=0.1\ \text{GeV}$$

one finds

$$M=2.7\times10^9\ \text{GeV}\ . \tag{1.10d}$$

With that value one can hardly explain, even with the arguments of Ref. 7, the experimental data, or better the absence of data, for the proton decay.

Fortunately one can do quite a bit better with an alternative choice:

$$\sum_i(\tilde{\mathbf{5}}_i^*,\widetilde{\mathbf{10}}_i)+\sum_{\alpha,\beta=1}^{k}(\widetilde{\mathbf{10}}_\alpha,\widetilde{\mathbf{10}}_\beta^*) \tag{1.12a}$$

for which one finds, with $k=4$,

$$\begin{aligned} &b_H=\tfrac{9}{2},\quad b_k=\tfrac{15}{2}-k\ ,\\ &b_4=\tfrac{7}{2}\ . \end{aligned} \tag{1.12b}$$

With $q_C=0.1$ GeV and $q_H=3\times10^2$ GeV one obtains

$$M=4.4\times10^{14}\ \text{GeV}\ . \tag{1.12c}$$

This is almost a large enough value to be in agreement with the experimental limits on proton decay. With that value the arguments of Ref. 7 are much more convincing.

Obviously the value of the unification mass depends very much on the symmetry-breaking pattern. Only the heavily mass-split **5*** and **10** multiplets lead to an acceptable M value. In such a situation quarks ought to be considerably more massive than leptons.

The relation (1.11) was used to unify hypercolor with color forces. That does not automatically lead to the unification with the electromagnetic $[u(1)_E]$ forces. However, if one assumes that unification, then the strength of the electromagnetic interaction is fixed by the magnitude of the universal coupling constant

$$\alpha_H(M)=\alpha_C(M)=\alpha_M\ . \tag{1.13}$$

This constant is determined by

$$\frac{1}{\alpha_M}=1+\frac{b_x}{2\pi}\ln\frac{M}{q_x} ,$$
$$b_x=b_H,b_C, \quad q_x=q_H,q_C , \tag{1.14}$$
$$\alpha_H(q_H)=\alpha_C(q_C)=1 .$$

Once α_M is known, one can find $\alpha_E(q)$ for any q. This follows from the relation[7]

$$\frac{1}{\alpha_E(q)}=\frac{5}{3}\left[\frac{1}{\alpha_M}\right]-\frac{b_E}{2\pi}\ln\frac{M}{q} ,$$
$$b_E=-\text{tr}Q^2 . \tag{1.15}$$

With the model (1.10a) and with the value of M given by (1.10d) one finds

$$\alpha_M^{-1}=4.82, \quad b_E=-\tfrac{143}{6} ,$$
$$\alpha_E^{-1}(100\text{ GeV})=72.9 . \tag{1.16}$$

The selected value of q is, hopefully, large enough for a reasonable numerical accuracy. The obtained coupling constant is about 32% too large.

With the model (1.12), and with M (1.12c), one finds

$$\alpha_M^{-1}=21.06, \quad b_E=-\tfrac{163}{6} ,$$
$$\alpha_E^{-1}(100\text{ GeV})=161.0 . \tag{1.17}$$

The charge coupling constant is now about 13% too small, which seems quite encouraging. By suitable changes of q_H and q_C one can fine-tune the theory in order to produce the required α_E value. Unfortunately, this leads to smaller unification scale M, as, for example,

$$q_H=2\times10^2\text{ GeV}, \quad q_C=0.35\text{ GeV} ,$$
$$M=8.92\times10^{11}\text{ GeV} ,$$
$$\alpha_M^{-1}=16.91, \quad b_E=-\tfrac{163}{6} , \tag{1.18}$$
$$\alpha_E^{-1}(100\text{ GeV})=127.24 .$$

Some other choices of the mass-split multiplets, which can lead to much higher M values, are discussed in the Appendix. There is no doubt that the unification mass value is the weakest point in the whole model, which otherwise can account for all weak phenomena.[6,13]

II. CLASSIFICATION OF THE COMPOSITES

The composite quarks and leptons are assumed to be in the SUSY-SU(5) **1**, **5**, or **10*** representations:

$$u\sim(\alpha\beta^*)\to\mathbf{10}\times\mathbf{5}=\underline{\mathbf{10}}^*+\mathbf{40} ,$$
$$d\sim(\alpha\beta)\to\mathbf{10}\times\mathbf{5}^*=\underline{\mathbf{5}}+\mathbf{45}^* ,$$
$$e\sim(\beta\beta)\to\mathbf{5}^*\times\mathbf{5}^*=\underline{\mathbf{10}}^*+\mathbf{15}^* , \tag{2.1}$$
$$\nu\sim(\beta\beta^*)\to\mathbf{5}^*\times\mathbf{5}=\mathbf{1}+\mathbf{24} .$$

The $G_P=\text{SU}(2)_{\text{HC}}\times\text{SU}(3)_C\times\text{U}(1)_E$ classification of representations **10** and **5** has already been given in (1.5).

For other multiplets from (2.1) one finds

$$\mathbf{45}: \ (1,3,-\tfrac{4}{3})+(2,1,-\tfrac{1}{2})+(2,8,-\tfrac{1}{2})+(2,3,\tfrac{7}{6})$$
$$+(1,6,\tfrac{1}{3})+(3,3^*,\tfrac{1}{3})+(1,3,\tfrac{1}{3}) ,$$
$$\mathbf{40}: \ (1,8,-1)+(2,1,\tfrac{3}{2})+(2,3^*,-\tfrac{1}{6})$$
$$+(1,3,\tfrac{2}{3})+(2,6,-\tfrac{1}{6})+(3,3,\tfrac{2}{3}) , \tag{2.2}$$
$$\mathbf{15}: \ (1,6,-\tfrac{2}{3})+(3,1,1)+(2,3,\tfrac{1}{6}) ,$$
$$\mathbf{24}: \ (2,3,-\tfrac{5}{6})+(1,8,0,)+(2,3^*,\tfrac{5}{6}) \ +(1,1,0)+(3,1,0) .$$

The left-handed quarks and electrons must be in the representations underlined in (2.1) if one requires that the left-handed composite field from (2.1) has the same classification as the right-handed "elementary" partner from (1.5). For example,

$$d_R\sim\mathbf{5}, \quad d_L\sim\mathbf{5} ,$$
$$u_R\sim\mathbf{10}^*, u_L\sim\mathbf{10}^* . \tag{2.3}$$

Some typical Yukawa couplings are

$$\lambda_U(\alpha\beta^*)_L u_R, \quad \lambda_d\overline{(\alpha\beta)}_L d_R ,$$
$$\lambda_e\overline{(\beta\beta)}_L e_R . \tag{2.4}$$

Here the coupling constants λ_x are determined[7] by the quark and lepton masses.

In this model all neutrinos quite naturally stay massless. The representations **5*** and **10** contain no ν_R states so that one cannot make Yukawa couplings containing the composite ν_L neutrino. Even if neutrinos acquire masses though some further symmetry breaking, it would be reasonable to assume that they should be smaller than quark and charged-lepton masses.

A complete discussion of all composites, which can be contained in (3.1) is quite instructive. These objects are listed in Tables II–V, which give their $\text{SU(2)}_{\text{HC}}\times\text{SU}(3)_C\times\text{U}(1)_E$ classifications. The capital letter after the preon combination [i.e., HC after $(\alpha\beta)$ in Table II] indicates the interaction which acts among constituents. In each table there is only one hypercolor-bound combination. It always corresponds to one of the known fermions or bosons (including their SUSY partners, i.e., the left-handed quark, the corresponding s

TABLE II. Content of the product $\mathbf{10}\times\mathbf{5}^*$.

Combination	SU(2)_{HC}	SU(3)_C	Charge	SU(5)
$(\alpha\beta)$; HC	1,3	3	$-\frac{1}{3}$	**5,45***
(αd_L^c); C	2	1,8	$\frac{1}{2}$	**5,45***
$(u_L^c\beta)$; E	2	3*	$-\frac{7}{6}$	**45***
$(u_L^c d_L^c)$; C	1	3*,6*	$-\frac{1}{3}$	**45***
(e_L^+/β); E	2	1	$\frac{1}{2}$	**5,45***
$(e_L^+ d_L^c)$; E	1	3*	$\frac{4}{3}$	**45***

TABLE III. Content of the product $\mathbf{10}\times\mathbf{5}$.

Combination	$SU(2)_{HC}$	$SU(3)_C$	Charge	SU(5)
$(\alpha\beta^*)$, *HC*	**1,3**	**3**	$\frac{2}{3}$	$\mathbf{10^*,40}$
(αd_L^{*C}); *C*	**2**	$\mathbf{3^*,6}$	$-\frac{1}{6}$	$\mathbf{10^*,40}$
$(u_L^c\beta^*)$; *E*	**2**	$\mathbf{3^*}$	$-\frac{1}{6}$	$\mathbf{10^*,40}$
$(u_L^c d_L^{*c})$;*C*	**1**	**1,8**	-1	$\mathbf{10^*,40}$
$(e_L^c\beta^*)$; *E*	**2**	**1**	$\frac{3}{2}$	**40**
$(e_L^c d_L^{*C})$; *E*	**1**	**3**	$\frac{2}{3}$	$\mathbf{10^*,40}$

quark, etc.) All other exotic combinations feel either color interaction (*C*) or relatively weak electromagnetic-like (*E*) field.

Among listed exotics there are only two, in Tables III and V, which are color scalars and, therefore, in principle, experimentally directly visible. If one includes the $\mathbf{10^*}\times\mathbf{5^*}$ combination (which is analogous to $\mathbf{10}\times\mathbf{5}$ shown in Table III), there must exist a triplet of such exotic states with charges $+1,-1$, and 0. Their spatial vector components would look very much like a ρ-meson triplet. More interesting are their fermion components which should appear as SUSY partners of the vector composites. At the moment one can only assume that their respective masses are large and that they might become visible when new accelerators become operational.

Besides the new particles listed in (2.1) one should also expect some exotics made out of two ten-dimensional SU(5) representations. There are two distinct possibilities:

$$\mathbf{10}\times\mathbf{10}=\mathbf{5^*}+\mathbf{45}+\mathbf{50} \tag{2.5}$$

and

$$\mathbf{10}\times\mathbf{10^*}=\mathbf{1}+\mathbf{24}+\mathbf{75}\ . \tag{2.6}$$

A third combination $\mathbf{10^*}\times\mathbf{10^*}$ follows trivially from (2.5).

The G_P decomposition of the representations **75** and **50** is

$$\begin{aligned}\mathbf{75}:\ &(3,8,0)+(1,8,0)+(2,6,\tfrac{5}{6})+(2,6^*,-\tfrac{5}{6})\\ &+(1,3,\tfrac{5}{6})+(2,3,-\tfrac{5}{6})+(1,3^*,\tfrac{5}{6})+(2,3^*,\tfrac{5}{6})\\ &+\underline{(1,1,0)_{HC}}\ ,\\ \mathbf{50}:\ &(2,8,-\tfrac{1}{2})+(1,6^*,-\tfrac{4}{3})+(3,6,\tfrac{1}{3})\\ &+(1,3^*,\tfrac{1}{3})+(2,3,\tfrac{7}{6})+(1,1,2)_E\ .\end{aligned} \tag{2.7}$$

TABLE IV. Content of the product $\mathbf{5^*}\times\mathbf{5^*}$

Combination	$SU(2)_{HC}$	$SU(3)_C$	Charge	SU(5)
$(\beta\beta)$; HC	**1,3**	**1**	-1	$\mathbf{10^*,15^*}$
(βd_L^c); *E*	**2**	$\mathbf{3^*}$	$-\frac{1}{6}$	$\mathbf{10^*,15^*}$
$(d_L^c d_L^c)$; *C*	**1**	$\mathbf{3,6^*}$	$\frac{2}{3}$	$\mathbf{10^*,15^*}$

TABLE V. Content of the product $\mathbf{5^*}\times\mathbf{5}$.

Combination	$SU(2)_{HC}$	$SU(3)_C$	Charge	SU(5)
$(\beta\beta^*)$; HC	**1,3**	**1**	0	**1,24**
(βd_L^{*C}); *E*	**2**	**3**	$-\frac{5}{6}$	**24**
$(d_L^c\beta^*)$; *E*	**2**	$\mathbf{3^*}$	$\frac{5}{6}$	**24**
$(d_L^c d_L^{c*})$; *C*	**1**	**1,8**	0	**24**

Here a "visible" exotic state can only be the state from **75**,

$$(1,1,0)_{HC}\ , \tag{2.8}$$

which can be made from, for example, hypercolor bound $(\alpha\alpha^*)$ combination. An analogous state exists also in the representation **24** in (2.2). Experimentally such states should appear as neutral singlets at, presumably, high enough energies.

The dilepton state with charge 2 in the **50** (with index *E*) could have been made out only of the combination $(e_L^c e_L^c)$ which interacts electromagnetically. However, their charges are equal and the force is repulsive.

III. FERMIONS AND BOSONS

As the theory is supersymmetric one has to assume that dynamics is such as to make SUSY partners massive enough to stay unobservable at the presently available energies. Other dynamical assumptions are analogous and/or practically identical to those listed in Refs. 6 and 13.

The natural structure of the known quarks and leptons would be, for example,

$$(\alpha_{Fi}\beta_{S1});(\beta_{Fi}\beta_{S1})\ . \tag{3.1}$$

Here we have indicated generation (i) and spin (F or S). The choice of β_{S1} for the scalar component is the one which leads to the observed universality of the weak interactions, as it will be discussed below. We will call it "the first choice."

There also exists "the second choice" which corresponds to the replacement

$$\beta_{S1}\to\sum_i\beta_{Si} \tag{3.2}$$

in the combinations (3.1) and also in the proposed structure of the intermediate vector bosons (IVB's), which will be discussed below.

Intermediate vector bosons are made out of scalar components in SUSY β's. Moreover, if there are several generations, meaning, for example, three combinations $(\mathbf{5^*},\mathbf{10})_i$ $(i=1,2,3)$, then W bosons are made out of β_{S1} and β_{S1}^* fields only (or alternatively out of the combinations $\sum_i\beta_{Si}$ and $\sum_i\beta_{Si}^*$ corresponding to the second choice). The "isoscalar" combination has to be heavier than the three "isovector " combinations.[14] The bound states made out of β_{Si},β_{Si}^* $(i=2,3)$ must also be quite heavy.

The last dynamical assumption is needed if one uses several generations and if the first choice has been made. It is not necessary if the second and third generation of quarks and leptons are understood as excited bound states.

The assumption that IVB's are made out of scalars β_S and β_S^* was made in all related models.[5-7,13] In our case it is necessary in order to have the massless left-handed neutrino

$$\nu_L \sim (\beta_{\rm LF}\beta_S^*) \ . \tag{3.3}$$

Here LF means a left-handed fermion. If one assumed that IVB's are made out of fermion components β_F, than the combination which corresponds to a neutral lepton would be right handed:

$$(\beta_S\beta_{\rm LF}^*) \sim \nu_R \ . \tag{3.4}$$

Related problems would emerge with the up quark (1.2) where one would find

$$(\alpha_S\beta_{\rm LF}^*) \sim u_R \ . \tag{3.5}$$

However, combinations (3.4) and (3.5) can lead to the occurrence of the right-handed interactions of van der Waal's type, which would be either superweak, or observable at much higher energies [assuming that the bosons made out of (β_F,β_F) and (β_F,β_F^*) are very massive].

As in other models,[6] the combination

$$\begin{bmatrix}\beta_S\\ \beta_S^*\end{bmatrix}, \quad I_W=\tfrac{1}{2} \tag{3.6}$$

can be classified as a weak-isospin (I_W) doublet. Then the intermediate vector bosons

$$\begin{aligned} &W_\mu^+ \sim (\beta_S^*\partial_\mu\beta_S^*), \quad W_\mu^- \sim (\beta_S\partial_\mu\beta_S) \ , \\ &W_\mu^0 \sim (\beta_S\partial_\mu\beta_S^* - \beta_S^*\partial_\mu\beta_S) \end{aligned} \tag{3.7}$$

are the members of the isotriplet $I_W=1$, the neutral one, W^0, mixes quite naturally with the $U(1)_E$ field a_μ from the SU(5) gauge group (1.1). The mixing is discussed at great length in Refs. 15–17. The physical photon A and Z^0 fields are given by

$$\begin{aligned} &A_\mu = a_\mu + kW_\mu^0 \ , \\ &Z_\mu^0 = (1-k^2)^{1/2}W_\mu^0 \ , \\ &k=\sin^2\theta_W, \quad m_{Z^0}=m_W/(1-k^2)^{1/2} \ . \end{aligned} \tag{3.8}$$

The weak-isoscalar ($I_W=0$) state

$$V_\mu^0 \sim (\beta_S\partial_\mu\beta_S^* + \beta_S^*\partial_\mu\beta_S) \tag{3.9}$$

either cannot create a spin-1 particle from the vacuum[6] or its mass has to be larger than 480 GeV (Ref. 14).

This brief discussion summarizes the usual dynamical assumptions.[6,15,16]

The mixing of generations can be described in two ways.

(a) The recurrent generations can be understood as the excited states of the basic fundamental combinations of preons. Their mixing is a consequence of the binding HC dynamics, which will be, hopefully, explained sometime in the future.

(b) For each generation, there exists a corresponding set of preons, labeled by index i in (3.1). The mixing of generations is then associated with the mixing of the corresponding preons. In our model this can be achieved by assuming a symmetry-breaking interaction which mixes $SU(3)_C$ triplets contained in the SU(5) decouplets. The mixing matrix must be the transposed Kobayashi-Maskawa (KM) mixing matrix.

For the sake of clarity, this will be first illustrated for the two generations only. One has

$$\begin{aligned} &10_1\colon\ \alpha_1(2,3,\tfrac{1}{6})+u_L^c(1,3^*,-\tfrac{2}{3})+e_L^c(1,1,1) \ , \\ &5_1^*\colon\ \beta_1(2,1,-\tfrac{1}{2})+d_L^c(1,3^*,\tfrac{1}{3}) \ , \\ &10_2\colon\ \alpha_2(2,3,\tfrac{1}{6})+c_L^C(1,3^*,-\tfrac{2}{3})+\mu_L^C(1,1,1) \ , \\ &5_2^*\colon\ \beta_2(2,1,-\tfrac{1}{2})+s_L^C(1,3^*,\tfrac{1}{3}) \ . \end{aligned} \tag{3.10}$$

The preon mixing is determined by

$$\begin{bmatrix}\alpha_d\\ \alpha_s\end{bmatrix} = \begin{bmatrix}c & -s\\ s & c\end{bmatrix}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}, \quad c=\cos\theta_C, \quad s=\sin\theta_C \ , \tag{3.11a}$$

$$\begin{bmatrix}\hat u_L^C\\ \hat c_L^c\end{bmatrix} = \begin{bmatrix}c & -s\\ s & c\end{bmatrix}\begin{bmatrix}u_L^C\\ c_L^C\end{bmatrix} \ . \tag{3.11b}$$

Comparison with the usual Cabibbo mixing

$$\begin{bmatrix}\tilde d_L\\ \tilde s_L\end{bmatrix}_{\rm stand} = \begin{bmatrix}c & s\\ -s & c\end{bmatrix}\begin{bmatrix}d_L\\ s_L\end{bmatrix}_{\rm stand} \tag{3.12}$$

shows that one has a transposed matrix in (3.11). Left-handed bound states are

$$\begin{aligned} &\hat u_L = c(\alpha_1\beta_1^*) - s(\alpha_2\beta_1^*) \ , \\ &\hat c_L = s(\alpha_1\beta_1^*) + c(\alpha_2\beta_1^*) \ , \\ &\hat d_L = c(\alpha_1\beta_1) - s(\alpha_2\beta_1) \ , \\ &\hat s_L = s(\alpha_1\beta_1) + c(\alpha_2\beta_1) \ . \end{aligned} \tag{3.13}$$

The mass eigenstates are determined by the Yukawa couplings of the type

$$\overline{(\alpha_1\beta_1^*)_L}u_R = \bar u_L u_R \ . \tag{3.14}$$

Taking into account the mixing (3.11) one finds

$$(c\bar{u}_L - s\bar{c}_L)(cu_R - sc_R)+(s\bar{u}_L + c\bar{c}_L)(s\bar{u}_R + cc_R)+(c\bar{d}_L - s\bar{s}_L)d_R + (s\bar{d}_L + c\bar{s}_L)s_R$$
$$= \bar{u}_L u_R + \bar{c}_L c_R + (c\bar{d}_L - s\bar{s}_L)d_R + (s\bar{d}_L + c\bar{s}_L)s_R \ . \qquad (3.15)$$

The obvious interpretation is that the left-handed composite physical quark states are determined by the following preon combinations:

$$u_L^P = (\alpha_1\beta_1^*), \quad c_L^P = (\alpha_2\beta_1^*) ,$$
$$d_L^P = c(\alpha_1\beta_1) - s(\alpha_2\beta_1) , \qquad (3.16)$$
$$s_L^P = s(\alpha_1\beta_1) + c(\alpha_2\beta_1) .$$

If the effective weak interactions are mediated by the composite W bosons, the states (3.16) lead to the usual Cabibbo-suppressed strangeness-changing decays which would be, in the standard electroweak model, described by using (3.12). This is graphically illustrated in Fig. 1. The decay shown in Fig. 1 is Cabibbo suppressed because only α_1 from s_L^P (3.16) can be combined with α_1 in u_L^P. In a suitable shorthand notation this can be written as

$$\bar{u}_L^P S_L^P = \overline{(\alpha_1\beta_1^*)}[s(\alpha_1\beta_1) + c(\alpha_2,\beta_1)]$$
$$= s(\bar{u}_L s_L)_{\text{stand}} . \qquad (3.17)$$

There are no problems with neutral currents. Using the same notation as in (3.17) one can write

$$\bar{s}_L^P d_L^P = sc\overline{(\alpha_1\beta_1)}(\alpha_1\beta_1) - sc\overline{(\alpha_2\beta_1)}(\alpha_2\beta_1)$$
$$\rightarrow 0 . \qquad (3.18)$$

Proceeding in the same way, one can recover all results of the standard electroweak theory.

With three generations, one must use the transposed Kobayashi-Maskawa (KM) matrix

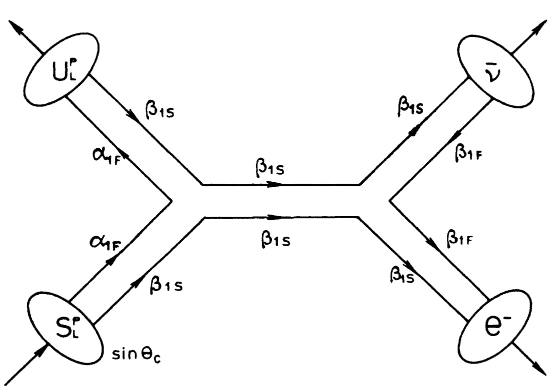

FIG. 1. Cabibbo-suppressed ($\sin\theta_C$) leptonic decay of the strange quark. Solid lines correspond to either scalar (S) or fermion (F) preons. Ovals (i.e., u_L) symbolize the bound preon states (3.1), (3.3), and (3.15).

$$U^T = \begin{bmatrix} U_{ud} & U_{cd} & U_{td} \\ U_{us} & U_{cs} & U_{ts} \\ U_{ub} & U_{cb} & U_{tb} \end{bmatrix} \qquad (3.19)$$

and the following physical left-hand composites:

$$u_L^P = (\alpha_1\beta_1^*), \quad c_L^P = (\alpha_2\beta_1^*), \quad t_L^P = (\alpha_3\beta_1^*) ,$$
$$\begin{bmatrix} d_L^P \\ s_L^P \\ b_L^P \end{bmatrix} = U^T \begin{bmatrix} d_L \\ s_L \\ b_L \end{bmatrix} . \qquad (3.20)$$

IV. OVERVIEW AND CONCLUSION

Although the basic symmetry of this model differs from the SU(5) grand-unified-theory (GUT) symmetry, the model's physical properties are similar to the ones contained in the minimal GUT. The basic representations of the model 5 and 10, lead to four "observable" exotic states only; a hypercolor bound singlet ($10\times10^*$) and three color bound states (10×5, $10^*\times5$, $5^*\times5$). Among the combinations listed in Tables II–V all hypercolor bound states correspond to known quarks or leptons, the same ones which one encounters in 5 and 10 representations of the minimal GUT.

In this model quark and lepton charges emerge as SU(5) determined property. This parallels the situation encountered in the SU(5)-GUT model where the charge operator is also one of the group's generators. In view of all that, we felt justified to associate the attribute "minimal" with the present preon model.

In the SUSY-SU(5) model the neutrino is quite naturally massless. This is again similar to the minimal GUT-SU(5) model.

The model shows more or less unique dynamical properties. As it was already discussed IVB's have to be made out of the scalar components of the β superfields. One has to use either the first generation β_1 (the first choice) or the linear combination $\sum_{k=1}^{3}\beta_k$ (the second choice) if one is to reproduce the observed electroweak universality. Moreover either β_{S1} or $\sum\beta_{Sk}$ has to be contained in all quarks and leptons.

With the second choice there are just IVB's and the three generations of quarks and leptons with their exotics and excited states.

If the first choice is the one which is realized in nature, the particle spectrum is richer. One might say that all generations of quarks and leptons, together with IVB's and corresponding exotics and resonances, belong to the first "genus," or group.

The second genus (group) is obtained by the replacement

$$\beta_{S1}(\beta^*_{S1})\rightarrow\beta_{S2}(\beta^*_{S2}) \ . \tag{4.1}$$

Obviously, there must also exist a third genus.

Some of the exotics, as, for example, the color bound triplet (5×10) would be repeated in each genus. There exists also a number of the hypercolor bound singlet, $(10\times 10^*)$ whose contents are $(\alpha_1\alpha_1^*)$, $(\alpha_1\alpha_2^*)$, $(\alpha_2\alpha_2^*)$, etc.

Additional new particles would emerge from various combinations of superfield scalar and fermion components. Thus, for example, besides the established up-quark state

$$(\alpha_F\beta_S) \ , \tag{4.2a}$$

one should also expect the state

$$(\alpha_S\beta_F) \tag{4.2b}$$

with the same spin and flavor, but at, hopefully much higher energy.

The quark, lepton, and exotic spectra, discussed in the third section, must be repeated twice more, presumably with larger and larger masses. At the moment theory is not capable of any more detailed prediction.

Introduction of the KM mixing angles (see Sec. III) was in keeping with the "minimal" character of this model. It can be convincingly argued that the usage of the transposed KM matrix is almost unavoidable if one wants to preserve model features such as quark-lepton universality and to generate the quark masses through Yukawa couplings.

In this model the right-handed states (i.e., u_R, d_R, e_R, etc.) are fundamental pointlike objects. It is possible to speculate[7] that their compositeness might be also revealed at some different, subpreonic, level. These new, different preons, say subpreons, could again cause a plethora of much more massive fermion and boson states.

With some imagination one can also envisage a mixing of states from massive genuses with the heavy subpreonic composites. However, at present, all such speculations would be a utopianism.

The basic ingredient of this model is the SU(5) classification of preonic flavors. This classification retains its usefulness even if one does not interpret SU(5) as a gauge group which unifies all interactions. The grand unification is the weakest feature of the model.

SUSY features of the model introduce all needed fermion and scalar preons in the most natural way. Both scalar (β_S) and fermion (β_F) components of the β superfield must appear:

$$\beta=\begin{bmatrix}\beta_S\\ \beta_F\end{bmatrix} \ . \tag{4.3}$$

Without supersymmetry, one would have to introduce two new fields

$$\begin{aligned} x_S &\sim \beta_S \ ,\\ y_F &\sim \beta_F \ . \end{aligned} \tag{4.4}$$

In the SUSY-SU(5) model all anomalies are quite easily and naturally canceled.

An abundance of new particles at high energies would be discovered even if there was only one basic $(5^*,10)$ preon combinations and if all successive generations corresponded to the excitations of the preon bound states. Naturally, the characteristics of the corresponding particle spectrum would be somewhat different.

The main physical prediction of the model is that instead of a desert one should find an abundance of new particles appearing in high- and very-high-energy experiments.

ACKNOWLEDGMENTS

This work has been supported by joint Yugoslav-American NSF Project Nos. JFT-526 and JFP-683. One of us (S.F.) would also like to thank the Max-Planck-Institut für Physik and Astrophysik in Munich, where part of this work was completed.

APPENDIX

This appendix contains some additional speculations about generations and about SU(5)-symmetry breaking and grand unification.

Second and third generations could be, in principle, understood as excited bound states made out of one set of fundamental multiplets $(5^*,10)$. This leads to some problems with the unification scale, as it is illustrated in the following few examples:

$$\begin{aligned} &(5^*,10)+4\times(\tilde{10},\tilde{10}^*) \ ,\\ &b_H=4,\quad b_C=3,\quad b_F=-22 \ ,\\ &M=q_H^4 q_C^{-3}=8.1\times 10^{12}\ \text{GeV} \ ,\\ &(q_H=3\times 10^2\text{GeV},\quad q_C=0.1\ \text{GeV} \ ,\\ &\alpha_M^{-1}=16.29,\quad \alpha_E^{-1}(100\ \text{GeV})=115.1\\ &\qquad\qquad (\Delta g_E\sim +5\%) \ ; \end{aligned} \tag{A1}$$

$$\begin{aligned} &(\tilde{5}^*,\tilde{10})+4\times(\tilde{10},\tilde{10}^*) \ ,\\ &b_H=\tfrac{11}{2},\quad b_C=5,\quad b_E=-\tfrac{43}{2} \ ,\\ &M=q_H^{11}q_C^{-10}\rightarrow\text{post-Planckian region} \end{aligned} \tag{A2}$$

$$\begin{aligned} &(\tilde{5}^*,\tilde{10})+5\times(\tilde{10},\tilde{10}^*) \ ,\\ &b_H=\tfrac{11}{2},\quad b_C=4,\quad b_F=-\tfrac{157}{6} \ ,\\ &M=q_H^{11/3},\quad q_C^{-8/3}=5.61\times 10^{11}\ \text{GeV} \ ,\\ &\alpha_M^{-1}=19.69,\quad \alpha_E^{-1}(100\ \text{GeV})=126.3\\ &\qquad\qquad (\Delta g_E\sim +0.3\%) \ . \end{aligned} \tag{A3}$$

These examples illustrate that the answer depends very much on the details of the model. It is obvious that the model (A3) can be easily adjusted (by minute changes of q_H and q_C values) to give perfect α_E. As far as the unification is concerned, the one-generation model (A3) is as acceptable as the three-generation model (1.18).

The jump, which happens when going from 4 (A2) to $5\times(\tilde{10},\tilde{10}^*)$ combinations (A3) exists for the three-generation case also. For example,

$$3\times(5^*,\widetilde{10})+2\times(\widetilde{10},\widetilde{10}^*)\ ,$$

$$b_H=\tfrac{9}{2},\quad b_C=4,\quad b_E=-\tfrac{113}{6}\ , \tag{A4}$$

$$M=q_H^9 q_C^{-8}\rightarrow \text{post-Planckian region}\ ;$$

$$3\times(5^*,\widetilde{10})+3\times(\widetilde{10},\widetilde{10}^*)\ ,$$

$$b_H=\tfrac{9}{2},\quad b_C=3,\quad b_E=-\tfrac{27}{2}\ ,$$

$$M=q_H^3,\quad q_C^{-2}=2.7\times10^9\ \text{GeV}\ ,$$

$$\alpha_M^{-1}=12.47,\quad \alpha_E^{-1}(100\ \text{GeV})=57.55\quad(\Delta g_E\sim+50\%)\ . \tag{A5}$$

very poor α_E value, which is even larger than (1.16), shows that the unification of the $U(1)_E$ does not at all follow automatically in our preon models. This unification requirement can be used to select the most acceptable model variants.

[1]L. Lyons, Part. Nucl. Phys. **10**, 227 (1983).
[2]O. W. Greenberg and J. Sucher, Phys. Lett. **99B**, 639 (1981).
[3]H. Fritzsch and G. Mandelbaum, Phys. Lett. **102B**, 312 (1981).
[4]J. C. Pati, Phys. Lett. **144B**, 375 (1984).
[5]L. F. Abbott and E. Farhi, Phys. Lett. **101B**, 69 (1981); Nucl. Phys. **B189**, 547 (1981).
[6]M. Claudson, E. Farhi, and R. L. Jaffe, Phys. Rev. D **34**, 873 (1986).
[7]J. Maalampi and J. Pulido, Phys. Lett. **165B**, 85 (1985).
[8]S. Fajfer and D. Tadić, Phys. Rev. D **35**, 361 (1987).
[9]Tung-sheng Tu, Report No. BIHEP-TH-84-21, 1984 (unpublished).
[10]G. 't Hooft, in *Recent Developments in Field Theory,* edited by G. 't Hooft *et al.* (Plenum, New York, 1980), p. 135.
[11]B. Grinstein, Nucl. Phys. **B206**, 387 (1982).
[12]H. P. Nilles, Phys. Rep. **110**, 1 (1984).
[13]S. A. Devyanin and R. L. Jaffe, Phys. Rev. D **33**, 2615 (1986).
[14]R. D. Peccei, in *Selected Topics in Electroweak Interations,* proceedings of the Lake Louise Winter Institute, Lake Louise, Canada, 1987, edited by J. M. Cameron, B. A. Campbell, A. Kamal, F. Khanna, J. Ng, and J. Poutissou (World Scientific, Singapore, 1987), p. 564.
[15]J. D. Bjorken, Phys. Rev. D **13**, 335 (1978).
[16]P. Q. Hung and J. J. Sakurai, Nucl. Phys. **B143**, 81 (1978).
[17]K. Ahama, in Physics at TeV Energy Scale, KEK, Ibaraki, Japan, 1987 (unpublished).