Leptonic decay constant f_B of the $B(b\bar{d})$ meson and the b-quark mass

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We determine the value of the current b-quark mass using the existing detailed experimental information in moment and Borel-transformed sum rules for the vector b-quark current $\bar{b}\gamma_\mu b$. The resulting value ($m_b=4.17\pm0.02$ GeV at $p^2=-m_b^2$) is used for calculating the decay constant f_B and the mass of the B meson via two sum rules: one constructed from the correlation function of two pseudoscalar currents and the other from the correlation function of a pseudoscalar and an axial-vector current. We find $f_B=170\pm20$ MeV.

I. INTRODUCTION

In view of the recent measurement of $B\bar{B}$ mixing[1] the value of the leptonic decay constant f_B of the $B(b\bar{d})$ meson [$M_B=5.27$ GeV (Ref. 2)] is of considerable significance. In the theoretical evaluation of this mixing, one of the major uncertainties is the estimate of the matrix element

$$\langle \bar{B}^0 \,|\, [\bar{b}\gamma_\mu(1-\gamma_5)d]^2 \,|\, B^0\rangle = \tfrac{8}{3}Bf_B^2M_B^2 \ . \tag{1.1}$$

A low value of f_B (together with a low value of B) would make it hard to reconcile the measured value of the $B\bar{B}$ mixing with a low mass for the top quark.[3] However, the range of values of f_B existing in the literature is large.[4] Even the values obtained from various types of QCD sum rules differ by more than a factor of 2. In such a framework f_B has been determined for the first time in Ref. 5 using moment sum rules, which led to a high value of f_B due to an improper treatment of continuum contributions. This was corrected in Ref. 6, where Borel-transformed sum rules resulted in a low value for f_B ($f_B \sim f_\pi$). Subsequently, it has been pointed out in Ref. 7 that the value of the on-shell b-quark mass used in Ref. 6 is too high and has led to a serious underestimate of f_B. The value for f_B quoted in Ref. 7,

$$f_B=190\pm30 \text{ MeV} , \tag{1.2}$$

has recently been confirmed in Refs. 8 and 9. (It should be noted that our definition of f_B is $\sqrt{2}$ larger than in Refs. 8 and 9.) Actually the Borel sum rules employed in obtaining the value (1.2) are very sensitive to the value of the b-quark mass (this fact has also been observed in Ref. 9).

In the present paper we shall furnish further evidence in favor of the value (1.2) in two ways. First (in Sec. II), we shall reduce the uncertainty in f_B by using the existing data on the vector polarization function in the $b\bar{b}$ system (the Υ resonance and its radial excitations) to obtain a very accurate determination of the b-quark mass. The better experimental information available at present makes it possible to improve the result of Ref. 10. Second (Sec. III), we shall construct a novel sum rule from the correlation function of the pseudoscalar current

$$P(x)=\bar{d}(x)i\gamma_5 b(x) \tag{1.3}$$

with the axial-vector current

$$A_\mu(x)=\bar{d}(x)\gamma_\mu\gamma_5 b(x) \ . \tag{1.4}$$

Both currents couple to the B meson with coupling $f_BM_B^2/m$ and $-if_Bp_\mu$, respectively. Here, m is the mass of the b quark (we shall use this notation throughout the paper). The resulting sum rule is used in combination with the sum rule employed in Refs. 6 and 7, and makes it possible to have better control over the parameters of the method. In particular, the continuum threshold s_0 will be fixed unambiguously. An extra check is provided by using the new sum rule for determining M_B. This is carried out in Sec. IV. In Sec. V we summarize the results and state our conclusions.

II. DETERMINATION OF THE b-QUARK MASS m_b

In this section we shall determine the value of the b-quark mass using moments of the polarization function of the vector current $j_\mu(x)=\bar{b}(x)\gamma_\mu b(x)$:

$$M_n=\frac{1}{n!}\left[-\frac{d}{dQ^2}\right]^n \Pi(Q^2)\Bigg|_{Q^2=0} =\frac{1}{\pi}\int\frac{ds}{s^{n+1}}\text{Im}\Pi(s) \ , \tag{2.1}$$

where $Q^2=-q^2$ and

$$(q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2) = i\int d^4x\, e^{iqx}\langle 0 \,|\, T[j_\mu(x)j_\nu(0)] \,|\, 0\rangle \ . \tag{2.2}$$

Taking into account the first-order α_s correction and the gluon condensate contribution to $\Pi(Q^2)$ we obtain, for M_n, the expression[11]

$$M_n^{\text{theor}}=A_n(1+a_n\alpha_s - b_n\phi_b) \ . \tag{2.3}$$

Explicit formulas for a_n and b_n can be found in Ref. 11, while

$$A_n=\frac{3\times 2^n(n+1)(n-1)!}{4\pi^2(2n+3)!!}\,\frac{1}{(4m^2)^n} \ . \tag{2.4}$$

Although the actual values of a_n and b_n are of minor concern to us here, we have listed them for completeness in Table I. It is only important to realize that for $n \geq 9$ the α_s correction in (2.3) becomes too large for the first-order approximation to be reliable, and that, for the $b\bar{b}$ system the value of ϕ_b,

$$\phi_b = \frac{4}{9}\pi^2 \left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle \Big/ (4m^2)^2 \simeq 1.5\times 10^{-5} \quad (2.5)$$

is so small ($m \sim 4.2$ GeV) that for $n \leq 10$ the gluon condensate contribution in (2.3) can safely be neglected (for more details see Ref. 10).

It can also be seen from (2.4) that the moments M_n are very sensitive to the value of m. So by comparing M_n^{theor} with a phenomenological expression calculated by using the detailed experimental information available in the $b\bar{b}$ vector channel an accurate determination of m can be made. Experimentally, six resonances have been identified in this channel. Their mass values and electronic widths are[12]

$$\begin{aligned}
&\Upsilon(1S):\ M = 9.4600\ \text{GeV},\quad \Gamma(e^+e^-) = 1.33\pm 0.06\ \text{keV}\ ,\\
&\Upsilon(2S):\ M = 10.0234\ \text{GeV},\quad \Gamma(e^+e^-) = 0.60\pm 0.04\ \text{keV}\ ,\\
&\Upsilon(3S):\ M = 10.3555\ \text{GeV},\quad \Gamma(e^+e^-) = 0.43\pm 0.03\ \text{keV}\ ,\\
&\Upsilon(4S):\ M = 10.5770\ \text{GeV},\quad \Gamma(e^+e^-) = 0.24\pm 0.05\ \text{keV},\\
&\Upsilon(5S):\ M = 10.8650\ \text{GeV},\quad \Gamma(e^+e^-) = 0.31\pm 0.07\ \text{keV}\ ,\\
&\Upsilon(6S):\ M = 11.0190\ \text{GeV},\quad \Gamma(e^+e^-) = 0.13\pm 0.03\ \text{keV}\ .
\end{aligned} \quad (2.6)$$

The $\Gamma(e^+e^-)$ values of the first three resonances include the latest (surprisingly large) corrections to the e^+e^- widths.[12,13] Using the relation (assuming δ-function resonances)

$$\text{Im}\Pi(s) = \sum_R \frac{27}{4\alpha^2} M_R \Gamma_R(e^+e^-)\delta(s - M_R^2) + \text{continuum contributions}\ , \quad (2.7)$$

we can calculate the experimental moments M_n^{expt}. The error in M_n^{expt} is determined by the error in $\Gamma(e^+e^-)$, which we take to be 5%.

Before going over to a comparison of M_n^{theor} and M_n^{expt} a few remarks are in order. First, we shall not assume a simple θ-function behavior for the continuum contribution as is done in Ref. 10, but instead subtract the integral from the continuum threshold s_0 to infinity in the theoretical moments, i.e., M_n^{theor} becomes

$$M_n^{\text{theor}} = \frac{1}{\pi}\int_{4m^2}^{s_0} \frac{ds}{s^{n+1}} \text{Im}\Pi^{\text{theor}}(s)\ . \quad (2.8)$$

This implies that the a_n and A_n in (2.3) become functions of s_0 as well. Second, we shall not use α_s as an independent variable, but calculate it via the formula

$$\alpha_s(4m^2) = \frac{12\pi}{25}\frac{1}{\ln(4m^2/\Lambda^2)} \quad (2.9)$$

with $\Lambda = 100-150$ MeV. Third, we have to note that the renormalization point of m is $p^2 = -m^2$ (in the Landau gauge). The choice of a different normalization point would affect the values of a_n in (2.3). The normalization point $p^2 = -m^2$ has been chosen[11] to ensure that the α_s corrections in (2.3) are small in order that the approximation makes sense. Having determined $m\,(p^2 = -m^2)$ it is

TABLE I. The values of the coefficients a_n and b_n in Eq. (2.3) for $n = 1-10$; the resonance contribution to the experimental moments for the resonance set (2.6) and to the theoretical moments for the parameter set (2.11) and $\Lambda = 150$ MeV.

n	a_n	b_n	$10^{2n+1}M_n^{\text{theor}}$	$10^{2n+1}M_n^{\text{expt}}$
1	0.734	3.4	0.126	0.125
2	0.706	13.3	0.128	0.128
3	0.508	32.7	0.132	0.133
4	0.216	64.6	0.138	0.139
5	−0.140	112.0	0.146	0.146
6	−0.543	177.9	0.156	0.155
7	−0.981	265.2	0.167	0.166
8	−1.448	377.1	0.180	0.178
9	−1.938	516.5	0.192	0.193
10	−2.449	686.4	0.202	0.209

easy to transform to the gauge-invariant on-shell quark mass $m(p^2=+m^2)$ via the formula[11]

$$m(p^2=+m^2)=m(p^2=-m^2)\left[1+\frac{2\alpha_s}{\pi}\ln 2\right] . \qquad (2.10)$$

We can now compare the experimental moments M_n^{expt} calculated by substituting the resonance part of (2.7) into (2.1) with the theoretical moments M_n^{theor} which are given by (2.8). The parameters to be determined are s_0 and $m(p^2=-m^2)$ while Λ can vary between 100 and 150 MeV. For $\Lambda=150$ MeV and for several values of m and s_0 we have plotted in Fig. 1 the theoretical moments against the experimental ones (for $\Lambda=100$ MeV we get a similar picture), and in Table I we have listed the experimental and theoretical moments for $m(p^2=-m^2)=4.17$, $\sqrt{s_0}=11.6$ GeV, and $\Lambda=150$ MeV, which give the best fit. In fact, it turns out that the range of values for $m(p^2=-m^2)$ which reproduce all moments within the relatively large error of 5% is extremely narrow reflecting the strong dependence of (2.4) on m. We conclude

$$m(p^2=-m^2)=4.17\pm0.02 \text{ GeV} ,$$
$$\sqrt{s_0}=11.6\pm0.2 \text{ GeV} , \qquad (2.11)$$

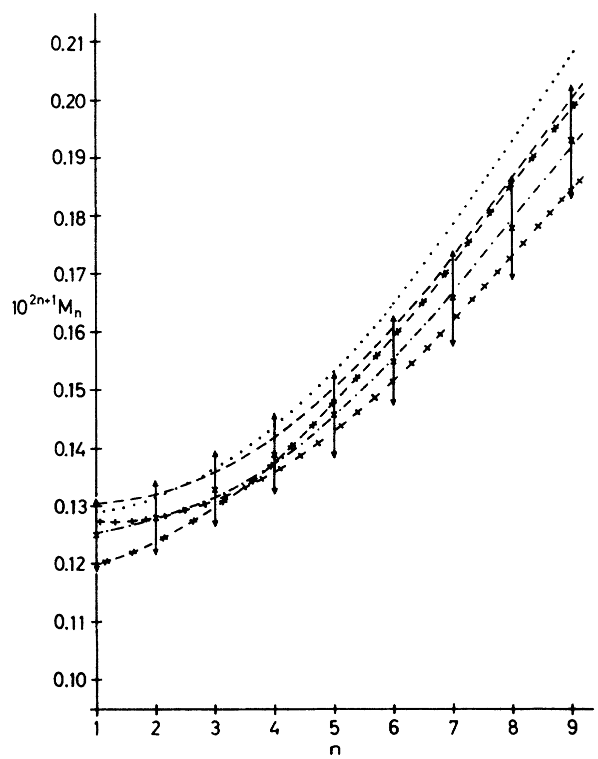

FIG. 1. The experimental and theoretical resonance contributions to the moments M_n for various values of $m(p^2=-m^2)$ and $\sqrt{s_0}$ and for $\Lambda=150$ MeV. The bars denote a 5% error in the experimental moments. The parameter values for the various curves are $\cdots$, $m=4.15$ and $\sqrt{s_0}=11.7$; $*-*-*$, $m=4.16$ and $\sqrt{s_0}=11.4$; $---$ $m=4.16$ and $\sqrt{s_0}=11.7$; $-\cdot-\cdot-\cdot$ $m=4.17$ and $\sqrt{s_0}=11.6$; $+++$, $m=4.18$ and $\sqrt{s_0}=11.7$.

for $\Lambda=100-150$ MeV. We note that the discrepancy of (2.11) with the value found in Ref. 10 is entirely due to the changed situation for the experimental values of $\Gamma(e^+e^-)$, but the value (2.11) has smaller error bars since six resonances could be used in calculating M_n^{expt} compared to four resonances in Ref. 10. This implies a higher value of s_0 and smaller continuum contributions making the determination of m more reliable. The value (2.11) for m is the most accurate determination of any quark mass to date. A similar determination of the charmed-quark mass is also possible and has in fact been carried out in Ref. 7. However, the uncertainties in the charmed sector are larger than in the beauty sector, since gluon condensate contributions cannot be neglected.

In order to check whether the values (2.11) are particular to the moment method, we have performed a similar analysis for the Borel-transformed polarization function. In this case, instead of taking derivatives at $Q^2=0$ [as in (2.1)] we take the limit $Q^2,n\to\infty$ with $Q^2/n=M^2$, which gives

$$\Pi(M^2)=\frac{1}{\pi M^2}\int ds\, e^{-s/M^2}\text{Im}\Pi(s) . \qquad (2.12)$$

Again we compare $\Pi^{\text{expt}}(M^2)$ [calculated by substituting the resonances (2.6) into (2.12)] with the theoretical expression $\Pi^{\text{theor}}(M^2)$ [which is the integral (2.12) from $4m^2$ to s_0]. The theoretical expression of $\text{Im}\Pi(s)$ can be found, for instance, in Ref. 7.

Of course, we cannot expect that $\Pi^{\text{theor}}(M^2)$ and $\Pi^{\text{expt}}(M^2)$ coincide (within errors) for all M^2, but they should do so in a range of M^2, where the approximations make sense (α_s corrections small). In Table II we have tabulated Π^{expt} and Π^{theor} for $5\le M^2\le 100$ GeV^2. It can be seen from the table that for $M^2\ge 10$ GeV^2 the polarization functions agree within a few percent. Slight changes in m and/or s_0 would destroy this agreement, confirming the values (2.11). We note that for large M^2 (for $M^2\gtrsim 70$ GeV^2 the continuum contribution amounts to more than 50% of the total polarization function) the continuum contributions become dominant and we do not expect the two functions to coincide, while for $M^2<10$ GeV^2 the first-order α_s correction is too large for the approximation to make sense.

Using (2.11) in (2.10) we can calculate the value of the on-shell quark mass $m(p^2=+m^2)$. We find

$$m(p^2=+m^2)=4.55\pm0.05 \text{ GeV} . \qquad (2.13)$$

The error in (2.13) is due to the variation in α_s and in $m(p^2=-m^2)$.

In conclusion of this section we note that the errors in (2.11) are almost entirely due to the (relatively large) errors in the e^+e^- widths of the Υ and its radial excitations. The main theoretical uncertainty is the unknown second-order α_s correction.

III. CALCULATION OF f_B

To calculate the leptonic decay constant f_B we shall consider two sum rules. The first one, which has been

TABLE II. The experimental and theoretical Borel-transformed polarization functions for the parameter set $m(p^2=-m^2)=4.16$ GeV, $\sqrt{s_0}=11.60$ GeV, and $\Lambda=150$ MeV. This parameter set gives the best results in the case of Borel-transformed polarization functions.

M^2 in GeV^2	$\Pi^{\rm expt}(M^2)$	$\Pi^{\rm theor}(M^2)$	M^2 in GeV^2	$\Pi^{\rm expt}(M^2)$	$\Pi^{\rm theor}(M^2)$
5	9.13×10^{-9}	4.56×10^{-9}	55	0.200	0.203
10	8.31×10^{-5}	8.94×10^{-5}	60	0.232	0.236
15	1.87×10^{-3}	1.90×10^{-3}	65	0.264	0.269
20	9.11×10^{-3}	9.12×10^{-3}	70	0.294	0.300
25	2.38×10^{-2}	2.38×10^{-2}	75	0.323	0.330
30	4.54×10^{-2}	4.54×10^{-2}	80	0.351	0.359
35	7.21×10^{-2}	7.24×10^{-2}	85	0.377	0.387
40	0.102	0.103	90	0.403	0.413
45	0.134	0.136	95	0.427	0.439
50	0.167	0.169	100	0.450	0.463

used before in Refs. 6 and 7, is constructed from the two-point function of two pseudoscalar currents (1.3):

$$\Pi_1(q^2)=i\int d^4x\, e^{iqx}\langle 0\,|\,T[P(x)\bar{P}(0)]\,|\,0\rangle = \frac{1}{\pi}\int \frac{{\rm Im}\Pi_1(s)}{s-q^2}ds\ . \tag{3.1}$$

Expressions for the bare loop and the first-order α_s corrections to ${\rm Im}\Pi_1(q)$, as well as for the gluon condensate, quark condensate, and some higher-dimensional operator contributions to $\Pi_1(q)$ can be found in Ref. 6. [We note that we have corrected the sign of the (inessential) gluon condensate contribution.] Here we just give the final results for the Borel-transformed sum rule:

$$\Pi_1^{\rm res}(M^2)=\frac{3}{8\pi^2M^2}\int_{m^2}^{s_0}ds\, e^{-s/M^2}s\left[1-\frac{m^2}{s}\right]^2\left[1+\frac{4}{3}\frac{\alpha_s}{\pi}a\left[\frac{m^2}{s}\right]\right] -\frac{\langle m\bar{q}q\rangle}{M^2}e^{-m^2/M^2}+\frac{1}{12M^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle e^{-m^2/M^2}-\frac{m}{2M^4}\langle\bar{q}\sigma Gq\rangle\left[1-\frac{m^2}{2M^2}\right]e^{-m^2/M^2} -\frac{8\pi}{27M^4}\alpha_s(M^2)\langle\bar{q}q\rangle^2\left[2-\frac{m^2}{2M^2}-\frac{m^4}{6M^4}\right]e^{-m^2/M^2}\ . \tag{3.2}$$

$\Pi_1^{\rm res}(M^2)$ contains only the resonance contribution from M_B since we have taken into account the continuum contribution on the right-hand side of (3.2) by integrating from m^2 to s_0; s_0 is the continuum threshold and serves as one of the parameters to be determined. As noted in Ref. 6 the last two terms of (3.2) are very small and do not play a role of any significance in determining f_B. The function $a(x)$ reads[14,15]

$$a(x)=\tfrac{9}{4}+2l(x)+\ln x\,\ln(1-x)+\tfrac{3}{2}\ln\left[\frac{x}{1-x}\right]-\ln(1-x)-x\ln\left[\frac{x}{1-x}\right]-\frac{x}{1-x}\ln x\ , \tag{3.3}$$

where $l(x)=-\int_0^x dt\, t^{-1}\ln(1-t)$ is the Spence function. In this expression as well as in (3.2) the b-quark mass m is renormalized on shell, i.e., $m=m(p^2=+m^2)$.

To obtain the phenomenological expression for $\Pi_1^{\rm res}(M^2)$ we insert as usual a δ-function resonance for ${\rm Im}\Pi_1$ into the dispersion relation in (3.1). This gives

$$\Pi_1^{\rm res}(M^2)=\frac{f_B^2M_B^4}{m^2M^2}e^{-M_B^2/M^2}\ . \tag{3.4}$$

Equating (3.2) and (3.4) gives the first sum rule for f_B.

The second sum rule is derived from the two-point function of the currents (1.3) and (1.4):

$$-iq_\mu\Pi_2(q^2)=i\int d^4x\, e^{iqx}\langle 0\,|\,T[P(x)\bar{A}_\mu(0)]\,|\,0\rangle=\frac{iq_\mu}{\pi}\int ds\frac{{\rm Im}\Pi_2(s)}{s-q^2}ds\ . \tag{3.5}$$

Calculating the same contributions as for Π_1 we find, for the Borel-transformed sum rule,

$$\Pi_2^{\rm res}(M^2)=\frac{3m}{8\pi^2M^2}\int_{m^2}^{s_0}ds\, e^{-s/M^2}\left[1-\frac{m^2}{s}\right]^2\left[1+\frac{4}{3}\frac{\alpha_s}{\pi}a\left[\frac{m^2}{s}\right]\right]$$
$$-\frac{\langle\bar{q}q\rangle}{M^2}e^{-m^2/M^2}-\frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{1}{mM^2}(1-e^{-m^2/M^2})+\tfrac{1}{4}m^2\langle\bar{q}\sigma Gq\rangle\frac{1}{M^6}e^{-m^2/M^2}$$
$$+\frac{8\pi}{27M^6}m\,\alpha_s(M^2)\langle\bar{q}q\rangle^2\left[\frac{m^2}{6M^2}+1\right]e^{-m^2/M^2}\ . \tag{3.6}$$

Comparing (3.6) and (3.2) we see that

$$\frac{d}{d(-1/M^2)}M^2\Pi_2(M^2)=mM^2\Pi_1(M^2)\ , \tag{3.7}$$

which is a consequence of the relation $s\,{\rm Im}\Pi_2(s)=m\,{\rm Im}\Pi_1(s)$.

Again we saturate the dispersion relation in (3.5) by a δ-function resonance which gives, using the couplings of the B meson to the pseudoscalar and axial-vector currents,

$$\Pi_2^{\rm res}(M^2)=\frac{f_B^2M_B^2}{mM^2}e^{-M_B^2/M^2}\ . \tag{3.8}$$

Equating (3.8) and (3.6) gives the second sum rule for f_B.

Summarizing our two sum rules read, from $\Pi_1(q^2)$,

$$f_B^2M_B^4e^{-M_B^2/M^2}=\frac{3m^6}{8\pi^2}\int_1^{s_0/m^2}\frac{dx}{x}(1-x)^2e^{-xm^2/M^2}\left[1+\frac{4}{3\pi}\alpha_s a(x)\right]$$
$$-m^3\langle\bar{q}q\rangle e^{-m^2/M^2}+\tfrac{1}{12}m^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle e^{-m^2/M^2}-\frac{m^3}{2M^2}\langle\bar{q}\sigma Gq\rangle\left[1-\frac{m^2}{2M^2}\right]e^{-m^2/M^2}$$
$$-\frac{8\pi}{27}\frac{m^2}{M^2}\alpha_s(M^2)\langle\bar{q}q\rangle^2\left[2-\frac{m^2}{2M^2}-\frac{m^4}{6M^4}\right]e^{-m^2/M^2} \tag{3.9a}$$

and, from $\Pi_2(q^2)$,

$$f_B^2M_B^2e^{-M_B^2/M^2}=\frac{3m^4}{8\pi^2}\int_1^{s_0/m^2}\frac{dx}{x^2}(1-x)^2e^{-xm^2/M^2}\left[1+\frac{4\alpha_s}{3\pi}a(x)\right]$$
$$-m\langle\bar{q}q\rangle e^{-m^2/M^2}-\frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle(1-e^{-m^2/M^2})+\frac{m^3}{4M^4}\langle\bar{q}\sigma Gq\rangle e^{-m^2/M^2}$$
$$+\frac{8\pi}{27}\frac{m^2}{M^4}\alpha_s(M^2)\langle\bar{q}q\rangle^2\left[1+\frac{m^2}{6M^2}\right]e^{-m^2/M^2}\ . \tag{3.9b}$$

The ratio of (3.9a) and (3.9b) gives a sum rule for M_B^2, while both equations can be used to determine f_B.

As usual we search for a region in M^2 where f_B (or M_B) is approximately independent of M^2 and where the power and perturbative corrections are under control (i.e., small enough in order that corrections from higher-dimensional operators and from higher orders in α_s can be neglected). For a reliable determination of the resonance parameters we also have to demand that the continuum contributions be not too large.

For the vacuum expectation values of the various operators in (3.9) we employ the standard values (see, e.g., Ref. 7)

$$\langle\bar{q}q\rangle\simeq(-250\ {\rm MeV})^3\ ,$$
$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\simeq(360\ {\rm MeV})^4\ , \tag{3.10}$$
$$\langle\bar{q}\sigma Gq\rangle=m_0\langle\bar{q}q\rangle\ \ \text{with}\ m_0\simeq0.8\ {\rm GeV}^2\ .$$

For α_s we use formula (2.9) and in the perturbative corrections we take $\alpha_s(m^2)$. The continuum threshold s_0 will be determined by the sum-rule method itself; as we shall see the requirements of a stable region in M^2 and that s_0 be the same in (3.9a) and (3.9b) will fix s_0 almost uniquely.

We recall that the quark mass in (3.9) is renormalized on shell. In that case the α_s corrections turn out to be large (about 40% of the total perturbative correction). This is a serious defect and the QCD sum-rule calculations of f_B in Refs. 6 and 7 suffer from this. The usual remedy[11] in such a situation is to choose a different mass renormalization point, as in the previous section where the mass renormalization was shifted to $p^2=-m^2$. So, also here we search for a mass renormalization point at which the α_s corrections are small. At $p^2=-m^2$ the corrections are about 20% of the zeroth-order contribution (for $M^2 \gtrsim 7$ GeV^2) which is still tolerable. The choice of $m(p^2=-m^2)$ is especially convenient since its value (2.11) has a very small error. The α_s corrections are even smaller at $p^2=-\frac{1}{2}m^2$, 0, or $+\frac{1}{2}m^2$, but the corresponding masses at these points have larger errors than (2.11), since we have to use a formula similar to (2.10) to perform the shift.

As said before, for a reliable calculation of resonance parameters we need a sizable contribution of the resonance to the sum rule. Therefore, we restrict M^2 to $M^2 \lesssim 15$ GeV^2 for which the contribution of the B meson resonance is larger than $\sim 20\%$ (for $s_0 \simeq 32$ GeV^2). For the power corrections the situation is varying. The $\langle \bar{q}q \rangle^2$ terms are negligibly small for all M^2. The $\langle \bar{q}\sigma G q \rangle$ corrections are less than about 10% of the leading term for $M^2 \gtrsim 5$ GeV^2. For the first sum rule the gluon condensate gives negligible contributions for all M^2, while for the second sum rule it is $\lesssim 20\%$ for $M^2 \gtrsim 6$ GeV^2. For the quark condensate corrections the situation is the same for both sum rules ($\lesssim 30\%$ for $M^2 \gtrsim 6$ GeV^2).

So, from the above we conclude that it should be possible to determine the resonance parameters in the region

$$7 \lesssim M^2 \lesssim 15 \text{ GeV}^2 . \qquad (3.11)$$

In Fig. 2 we have plotted f_B as a function of M^2 for the two sum rules (3.9) (with the mass m renormalized at $p^2=-m^2$) for $m(p^2=-m^2)=4.17$ and for various values of s_0. It is clear from the figure that for this value of m the two sum rules coincide for $s_0=32$ GeV^2. For higher and lower s_0 the two sum rules start to diverge and we lose stability very quickly. Varying m within the limits of (2.11) makes very little difference, f_B changes only by a few MeV and stability is reached for the same s_0. Changing Λ to 150 MeV also has a very slight effect, it pushes down the value of f_B by 3 or 4 MeV. If we choose a different mass renormalization point we have to allow a larger error in the mass value and consequently a bigger spread in f_B. For instance, at $p^2=+\frac{1}{2}m^2$, where the α_s corrections are less than 10%, we have $m(p^2=+\frac{1}{2}m^2)=4.35\pm0.05$ GeV which results in a spread of about 10 MeV in the value of f_B. Stability is reached at the same s_0 as before. Further uncertainties are due to the neglect of second-order α_s corrections and higher power corrections. In total we believe that conservatively estimated the total error cannot be larger than about 10% which gives, as our final value,

$$f_B = 170 \pm 20 \text{ MeV} , \qquad (3.12)$$

which agrees within errors with (1.2), but has somewhat smaller error bars. However, we stress that, since we have two sum rules at our disposal, we have much better control of the parameters m and s_0. A drastic reduction of the error in (3.12) would be possible if the second-order α_s corrections to the two-point functions (3.1) and (3.5) were known.

IV. DETERMINATION OF M_B

As mentioned above we can also use the sum rules (3.9) to determine the mass of M_B. Of course, the values of the parameters m and s_0 have to be the same as for f_B in Sec. III. The first sum rule for M_B is given by the ratio of (3.9a) and (3.9b), a second sum rule can be constructed by taking the derivative of (3.9a) with respect to $-1/M^2$ and subsequently dividing the resulting expression by (3.9a). The more derivatives we take the more we enhance the continuum contribution in the sum rule. Therefore, we expect the second sum rule to be less reliable than the first. In Fig. 3 we have plotted M_B as a function of M^2 for $s_0=32$ GeV^2 and for several values of $m(p^2=-m^2)$. It can be seen that the dependence on the quark mass is very weak, and that in each case the curves reasonably converge to the same value which is in good agreement with the experimental value of M_B. The upper curves correspond to the first sum rule, which becomes stable at the end of the M^2 range shown in Fig. 3. The lower curves corresponding to the second sum rule

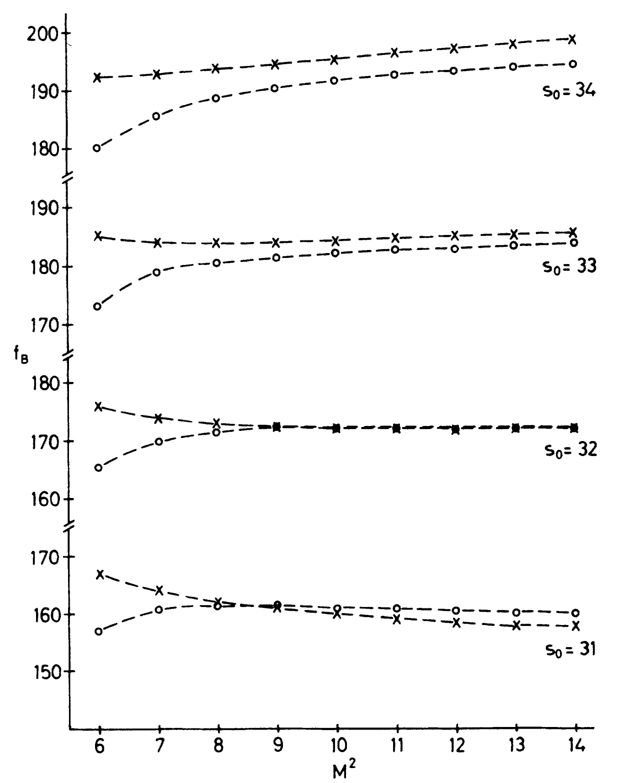

FIG. 2. f_B as a function of M^2 from the two sum rules (3.9a) (upper curves) and (3.9b) (lower curves) for $\Lambda=100$ MeV and various values of s_0.

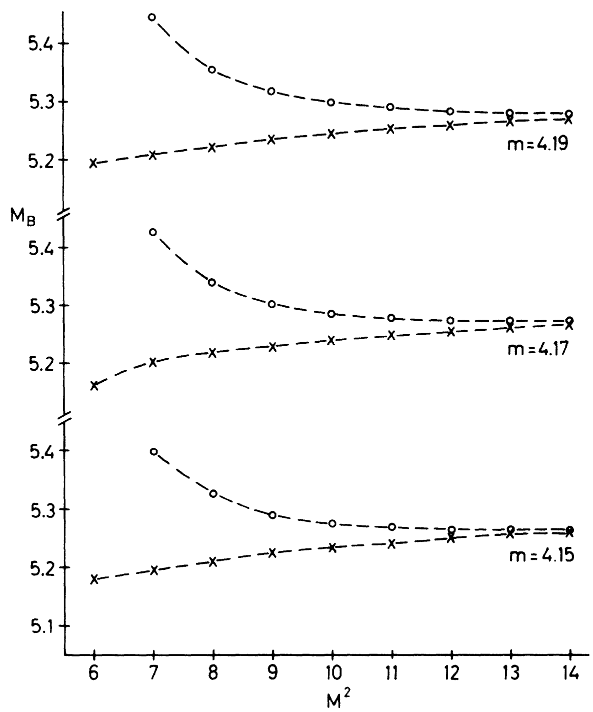

FIG. 3. The prediction for M_B from the two sum rules mentioned in the text for $s_0=32$ GeV^2, $\Lambda=150$ MeV, and various values of $m(p^2=-m^2)$.

do not show a stable plateau, but are ever increasing (be it very slowly), reflecting the poor reliability of the corresponding sum rule. As a function of s_0 the spread in the value for M_B is somewhat larger, but still rather small. As our final result we quote

$$M_B=5.25\pm0.15 \text{ GeV} , \tag{4.1}$$

which covers the range of parameter values $m(p^2=-m^2)=4.17\pm0.02$ GeV, $s_0=32\pm2$ GeV^2, and $\Lambda=100-150$ MeV.

We note that thanks to the new sum rule (3.9b) this is the first time that the mass of an open heavy flavor meson has been determined via Borel-transformed sum rules. The moment method has been used before in Ref. 5.

V. CONCLUSIONS

In this paper we have constructed two sum rules for the decay constant f_B of the B meson, one from the two-point function of two pseudoscalar currents, the other from the correlation function of a pseudoscalar and an axial-vector current. The latter sum rule is new and in combination with the first one made it possible to fix the sum rule parameters almost unambiguously.

We also reanalyzed the vector channel for the upsilon system with the aim of determining the b-quark mass, which led to the extremely accurate values (2.11) and (2.13). Apart from the errors in the experimental data the main uncertainty here is due to the unknown second-order α_s corrections. A calculation of these corrections would be of great interest.

In the sum rules for f_B we used the quark mass (2.11) renormalized at the point $p^2=-m^2$, which is different from Refs. 6 and 7 where the on-shell mass was used. We found that for $m(p^2=+m^2)$ the α_s corrections are too large for the sum rules to be reliable. Also here, a calculation of the second-order α_s corrections would be of great importance.

We have used the same sum rules for determining the mass of the B meson. The resulting value is in very good agreement with the data and provides a useful check on the calculation of f_B.

Our results can be summarized as

$$f_B=170\pm20 \text{ MeV}, \quad M_B=5.25\pm0.15 \text{ GeV} . \tag{5.1}$$

As mentioned in the Introduction the value of f_B is of special significance in connection with $B\bar{B}$ mixing in determining the matrix element (1.1). The value of f_B used is often considerably smaller than (5.1), which has led to the prediction of a large top-quark mass.[3] A definite conclusion on the value of the matrix element (1.1) can only be made if the parameter B is also known, the calculation of which is in progress.

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