Strange-quark content of the proton

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The usual evidence for the strange-quark content of the proton comes from chiral-symmetry breaking as measured in the pion-nucleon σ term, which involves the proton matrix elements of the scalar-quark densities. Another piece of evidence for it, which involves the proton matrix elements of the pseudoscalar-quark densities, is presented.

It has been known for quite some time that the pattern of chiral-symmetry breaking in QCD is not compatible[1] with the fact that the proton is entirely made of up and down quarks. This conclusion is inferred from the consideration of the Σ term in pion-nucleon scattering which is given by

$$\Sigma_{\pi N}(0)=\tfrac{1}{2}\langle p \mid [F^5_{1-i2},[F^5_{1+i2},H_M]] \mid p \rangle \ , \tag{1}$$

where F^5_i are the axial-vector charges and H_M is the chiral-symmetry-breaking Hamiltonian in QCD:

$$\begin{aligned} H_M &= m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \\ &= \sqrt{6} m_0 S_0 + \frac{2}{\sqrt{3}}(\overline{m}-m_s)S_8 + (m_u - m_d)S_3 \ . \end{aligned} \tag{2}$$

Here $m_0=(m_u+m_d+m_s)/3$, $\overline{m}=(m_u+m_d)/2$, and $S_i=\bar{q}(\lambda_i/2)q$, where λ_i $(i=0,1,\ldots,8,\ \lambda_0=\sqrt{2/3}I)$ are the Gell-Mann matrices and $\bar{q}=(\bar{u}\bar{d}\bar{s})$. Using the equal-time commutators in the quark model,

$$\begin{aligned} [F^5_i,S_j] &= i d_{ijk} P_k \ , \\ [F^5_i,P_j] &= -i d_{ijk} S_k \ , \end{aligned} \tag{3}$$

where $P_i=i\bar{q}\gamma_5(\lambda_i/2)q$, one obtains

$$\begin{aligned} \Sigma_{\pi N}(0) &= \overline{m}\left\langle p \left| \frac{1}{\sqrt{3}}(S_8+\sqrt{2}S_0) \right| p \right\rangle \\ &= \overline{m}\langle p \mid \bar{u}u+\bar{d}d \mid p \rangle \ . \end{aligned} \tag{4}$$

Let us write

$$\langle p \mid \sqrt{2}S_0 \mid p \rangle = 2(1+c_s)\langle b \mid S_8 \mid p \rangle \tag{5a}$$

or

$$\langle p \mid \bar{s}s \mid p \rangle = \tfrac{2}{3}c_s \frac{1}{1+\frac{2}{3}c_s}\langle p \mid \tfrac{1}{2}(\bar{u}u+\bar{d}d) \mid p \rangle \ , \tag{5b}$$

so that

$$\Sigma_{\pi N}(0)=\sqrt{3}\overline{m}(1+\tfrac{2}{3}c_s)\langle p \mid S_8 \mid p \rangle \ . \tag{5c}$$

Now $\langle p \mid S_8 \mid p \rangle$ is entirely determined from the Gell-Mann–Okubo mass splitting caused by the second term of (2) as[1]

$$\langle p \mid S_8 \mid p \rangle = \sqrt{3}\frac{m_\Xi - m_\Lambda}{m_s-\overline{m}} \ . \tag{6}$$

Thus, finally one obtains

$$\Sigma_{\pi N}(0)=\frac{\overline{m}}{m_s-\overline{m}}3(m_\Xi-m_\Lambda)(1+\tfrac{2}{3}c_s) \ . \tag{7}$$

Now $\overline{m}/(m_s-\overline{m})\simeq 0.04$ as determined from the standard SU(3) PCAC (partial conservation of axial-vector current) analysis of pseudoscalar masses.[2] Thus

$$\Sigma_{\pi N}(0)\simeq 25(1+\tfrac{2}{3}c_s)\ \text{MeV} \ . \tag{8}$$

Now in a valence-quark model if the proton primarily consists of up and down quarks, one would expect[3]

$$\begin{gathered} \langle p \mid \bar{s}s \mid p \rangle \ll \langle p \mid \tfrac{1}{2}(\bar{u}u+\bar{d}d) \mid p \rangle \\ \text{or} \\ c_s \simeq 0 \ , \end{gathered} \tag{9}$$

giving

$$\Sigma_{\pi N}(0)\simeq 25\ \text{MeV} \ , \tag{10}$$

which is about half the value of $\Sigma_{\pi N}(0)$ extracted from low-energy pion-nucleon scattering: namely,[1,4]

$$\Sigma_{\pi N}(0)=51\pm 5\ \text{MeV} \ .$$

One possible explanation of this discrepancy is that the assumption (9) is wrong[5] and in fact one would require that $c_s\approx\frac{3}{2}$, which is very large, or, from (5b),

$$\langle p \mid \bar{s}s \mid p \rangle \simeq \tfrac{1}{2}\langle p \mid \tfrac{1}{2}(\bar{u}u+\bar{d}d) \mid p \rangle \ ; \tag{11}$$

i.e., the proton has about 50% probability of containing strange $\bar{s}s$ quark pairs as compared to the average of $\bar{u}u$ and $\bar{d}d$ pairs.[5]

The purpose of this paper is to show that a similar conclusion about the strange $\bar{s}s$-quark content of the proton is reached if one considers the effect of the isospin-violating part (i.e., the third term) of the Hamiltonian (2) on the pion-nucleon vertex function. Let us consider

$$\Gamma=\langle p\pi^0 \mid H_{\rm QCD}^{\Delta I=1} \mid p\rangle = -\delta g\bar{u}(p')i\gamma_5 u(p) \ . \qquad (12)$$

In the soft-pion limit one obtains [on using (3)]

$$\begin{aligned}\Gamma &= -\frac{i}{F_\pi}\langle p \mid [F_3^5, H_{\rm QCD}^{\Delta I=1}] \mid p\rangle \\ &= -\frac{i}{F_\pi}(m_u-m_d)\langle p \mid [F_3^5, S_3] \mid p\rangle \\ &= \frac{i}{F_\pi}(m_d-m_u) i d_{33k}\langle p \mid P_k \mid p\rangle \\ &= -\frac{m_d-m_u}{F_\pi}\frac{1}{\sqrt{3}}\langle p \mid (P_8+\sqrt{2}P_0) \mid p\rangle \ . \qquad (13)\end{aligned}$$

Note that the same combination of the pseudoscalar densities enters as that for the scalar densities in the $\Sigma_{\pi N}$. Now we also have

$$\begin{aligned}\partial_\mu A_{8\mu} &= -i[F_8^5, H_M] \\ &= \tfrac{2}{3}(2\bar{m}+m_s)P_8+\frac{2\sqrt{2}}{3}(\bar{m}-m_s)P_0 \ . \qquad (14)\end{aligned}$$

As in (5b), let us write

$$\langle p \mid \sqrt{2}P_0 \mid p\rangle = 2(1+c_p)\langle p \mid P_8 \mid p\rangle \ , \qquad (15a)$$

or

$$\langle p \mid \bar{s}i\gamma_5 s \mid p\rangle = \tfrac{2}{3}c_p\frac{1}{1+\tfrac{2}{3}c_p}\langle p \mid \tfrac{1}{2}(\bar{u}i\gamma_5 u+\bar{d}i\gamma_5 d) \mid p\rangle \ . \qquad (15b)$$

Thus

$$\Gamma = -\frac{m_d-m_u}{F_\pi}\sqrt{3}(1+\tfrac{2}{3}c_p)\langle p \mid P_8 \mid p\rangle \qquad (16a)$$

and

$$\langle p \mid \partial_\mu A_{8\mu} \mid p\rangle = \tfrac{2}{3}[(\bar{m}+2m_s)+2(1+c_p)(\bar{m}-m_s)] \times \langle p \mid P_8 \mid p\rangle \ , \qquad (16b)$$

so that

$$\Gamma = -\frac{m_d-m_u}{F_\pi}\sqrt{3}(1+\tfrac{2}{3}c_p) \times \frac{\langle p \mid \partial_\mu A_{8\mu} \mid p\rangle}{\tfrac{2}{3}[(\bar{m}+2m_s)+2(1+c_p)(\bar{m}-m_s)]} \ . \qquad (16c)$$

Now

$$\langle p \mid \partial_\mu A_{8\mu} \mid p\rangle = 2m\bar{u}(p')i\gamma_5 u(p) g_A \left[\frac{3F-D}{2\sqrt{3}(F+D)}\right] \ , \qquad (17)$$

where $F+D=1$ and we have used the flavor-SU(3) parametrization for the matrix elements $\langle p \mid \partial_\mu A_{i\mu} \mid p\rangle$, $i=3$ and 8. Using the Goldberger-Treiman relation $mg_A/F_\pi = g_\pi$ (g_π being the pion-nucleon coupling constant) and (12), we obtain, from Eqs. (16c) and (17), the isospin-violating correction due to $H_{\rm QCD}^{\Delta I=1}$:

$$\frac{\delta g}{g_\pi} = (m_d-m_u)\frac{3F/D-1}{F/D+1} \times \frac{1+\tfrac{2}{3}c_p}{\tfrac{2}{3}[(\bar{m}+2m_s)+2(1+c_p)(\bar{m}-m_s)]} \ . \qquad (18)$$

Note that if $c_p \neq 0$, $\delta g/g_\pi$ does vanish in the chiral SU(2)×SU(2) limit as it should. If, however, we assume that the proton consists of primarily up and down quarks, so that, from Eq. (15a),

$$c_p \approx 0 \ ,$$

and we obtain, from Eq. (18),

$$\frac{\delta g}{g_\pi} = \sqrt{3}\frac{m_d-m_u}{m_d+m_u}\left[\frac{1}{\sqrt{3}}\frac{3F/D-1}{F/D+1}\right] \ , \qquad (19)$$

which shows that $\delta g/g_\pi$ does not vanish in the chiral-SU(2)×SU(2) limit which cannot be so since δg as defined in Eq. (12) does not have any pion pole. Thus one may conclude that $c_p \neq 0$, i.e., the proton has the strange-quark content when one evaluates the proton matrix elements of pseudoscalar-quark densities.

Now in the chiral-SU(3)×SU(3) limit, $\langle p \mid P_8 \mid p\rangle$ is dominated by the η_8 pole while $\langle p \mid P_0 \mid p\rangle$ does not have any such pole (note that the ninth pseudoscalar meson η_1 is not a Nambu-Goldstone boson). Therefore, it follows from Eq. (15a), that in the chiral SU(3)×SU(3) limit

$$c_p \approx -1 \ , \qquad (20)$$

which is rather a very large value. This gives

$$\frac{\delta g}{g_\pi} \simeq \sqrt{3}\frac{m_d-m_u}{2(\bar{m}+2m_s)}\left[\frac{1}{\sqrt{3}}\left[\frac{3F}{D}-1\right]\Big/\left[\frac{F}{D}+1\right]\right] \simeq 3.75\times 10^{-3} \ . \qquad (21)$$

The numerical value in Eq. (21) has been obtained by using[2]

$$(m_d-m_u)/(\bar{m}+2m_s) \approx 1.37\times 10^{-2}$$

and[6]

$$(3F/D-1)/\sqrt{3}(F/D+1) \approx 0.316 \ .$$

It may be that the chiral-SU(3)×SU(3) limit is badly broken so that the actual value of c_p is not -1 but numerically smaller. For example, $c_p \approx -\tfrac{3}{4}$ and $c_p \approx -\tfrac{1}{2}$ give $\delta g/g_\pi$ about two and four times the value given in Eq. (21) (as $\bar{m} \ll m_s$). But with such large values of $\delta g/g_\pi$ one may be in trouble[7] with small charge asymmetry allowed experimentally in $^1S_0$ pp and nn scattering lengths. However, it is also possible that the breaking of chiral symmetry gives c_p numerically larger than 1. For example, if $c_p = -\frac{5}{4}$, $\delta g / g_\pi$ would be about 0.4 times the value in Eq. (21), consistent with the small charge asymmetry allowed in $^1S_0$ pp and nn scattering lengths.

Finally, since, in the nonrelativistic quark model,

$$\bar{q} i \gamma_5 q \simeq \bar{q} \frac{\sigma \cdot (\mathbf{p}' - \mathbf{p})}{2 m_q} q \ ,$$

where m_q is the constituent-quark mass, the negative sign for c_p, which implies [cf. Eq. (15b) with $c_p \approx -1$]

$$\langle p \mid -\bar{s} i \gamma_5 s \mid p \rangle \simeq 2 \langle p \mid \tfrac{1}{2} (\bar{u} i \gamma_5 u + \bar{d} i \gamma_5 d) \mid p \rangle \ , \quad (22)$$

indicates that the strange quarks are polarized opposite to the proton's spin while the up and down quarks are polarized along the proton's spin. Probably this can be tested in the sum rule of the type recently considered in Ref. 6.

To summarize, the rather large values of c_s and c_p imply that as the proton matrix elements of scalar and pseudoscalar density operators are concerned, the proton behaves in the valence-quark model as if it had large strange-quark content. Otherwise, the quark-model commutation relations in Eq. (3) may need modification.

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