PHYSICAL REVIEW D VOLUME 38, NUMBER 3 1 AUGUST 1988

Quark and lepton masses in superstring-type models with mirror families

Alexander L. Kagan*
Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637

Carl H. Albright
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510
and Department of Physics, Northern Illinois University, De Kalb, Illinois 60115[†]
(Received 28 December 1987)

We investigate ordinary-mirror fermion mass splitting, quark isospin breaking, and both Majorana and Dirac neutrino mass seesaw mechanisms in the framework of E_6-based superstring-type models with mirror families. Our results suggest that actual realizations containing vectorlike generations should not be overlooked in the search for a realistic low-energy phenomenology.

I. INTRODUCTION

Superstring theory,[1] whether based on the heterotic version[2] formulated in ten dimensions which must be compactified down to four dimensions or on its more recent versions[3] formulated directly in four dimensions, has provided a rich new framework within which one can attempt to embed the low-energy standard model of Glashow, Salam, and Weinberg. Various issues are amenable to investigation which heretofore lay outside the purview of the standard model. The quark-lepton generation pattern with its puzzling mass spectrum is one aspect which is of interest to us here.

In the case of the ten-dimensional heterotic superstring, for Calabi-Yau compactification,[4–6] the number of chiral quark and lepton families is related to the difference of two Betti-Hodge numbers $b_{2,1}-b_{1,1}$. The single number $b_{1,1}$, on the other hand, yields an upper bound on the number of vectorlike matter families which may exist in nature. It has been popular in the literature[4,5,7] to consider manifolds which minimize the latter, i.e., $b_{1,1}=1$, since no evidence has appeared in nature which suggests the existence of mirror families. However, the current lower bound[8] on such quarks and charged-lepton masses is only 22 GeV. Since such mirror fermions have $SU(2)_L$ quantum numbers, their masses are bounded from above to be $\lesssim M_W$, so they may be discovered or eliminated completely in the near future. In this context, we investigate here what quark and lepton mass spectrum is viable in the superstring framework when at least one mirror family of quarks and leptons is present. The interesting case of large mixing of ordinary and mirror families, leading to quark and lepton seesaw mass contributions, is considered.[9] Such mixing will be generated via intermediate mass scales. Though, in general, ordinary families will mix with each other and with all mirror families present, for purposes of illustration we shall consider mass matrices involving only one ordinary family and one mirror family.

In what follows, we shall focus our attention on the heterotic superstring and on the maximal subgroup of E_6, $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$ and its subgroups, but the various mechanisms considered will be presented in as gauge group independent a manner as possible. We consider both standard and exotic matter field content, i.e., the $\mathbf{16}+\overline{\mathbf{16}}$ of SO(10) and $\mathbf{27}+\overline{\mathbf{27}}$ of E_6, respectively. Based on group-theoretic considerations we anticipate what mechanisms may be present to obtain a realistic low-energy phenomenology.

By way of an overview, we descriptively list our major assumptions and results. The Higgs superfields are assumed to arise only from the vectorlike sector of the theory surviving compactification, with no colored Higgs superfields present. We impose ordinary supersymmetric R parity or M parity (under which matter superfields are odd). We further assume that R parity or M parity remains unbroken in the low-energy theory, i.e., no matter scalar vacuum expectation values (VEV's) exist. These two assumptions taken together guarantee the absence of both gaugino, and Higgsino, matter fermion mass mixing. Neither assumption alone is sufficient to guarantee either. R parity or M parity together with the assumption of no colored Higgs superfields in the theory further prevents rapid proton decay.[10] With these assumptions spelled out in the ensuing sections, we construct the most general form of the quark and lepton mass matrices, with and without mirror fermions present.

Within the framework outlined above, the following results are obtained. There exist mechanisms for splitting ordinary and mirror charged-fermion masses and for obtaining the observed pattern of quark isospin breaking. The quarks and leptons will receive both seesaw and radiative mass contributions. Generalized neutrino mass seesaw mechanisms[11] of both the Majorana and Dirac type emerge in the presence of mirror families, where intermediate-mass-scale entries will be required for all but one of them. The Dirac case with no intermediate mass scale was treated earlier in Ref. 12 as a special case. The Dirac schemes will be viable only if $SU(2)_R$ is not part of the low-energy gauge group. Our results suggest that actual realizations containing mirror families may well play a role.

Missing from our analysis is a discussion of the renormalization-group constraints on the particle field content.[13] It is interesting to note, however, that the presence of additional quarks from mirror families will tend to make supersymmetric $SU(3)_c$ (SQCD) asymptotically divergent, since the β function for SQCD becomes positive for five or more quark families, not including the exotic isosinglets of E_6. It follows that supersymmetric models with mirror families provide a natural setting for the attractive nonperturbative scenario of Maiani, Parisi, and Petronzio[14] (MPP). In this scenario the value of the gauge coupling $g(\mu)$ at the low-energy scale μ is almost completely determined by the scale Λ at which it becomes $\gtrsim 1$, being very insensitive to $g(\Lambda)$. Depending on the choice of field content, Λ can be close to the compactification scale.[14] Furthermore, it has been shown that the ten-dimensional (10D) heterotic superstring is likely to be strongly coupled,[15] a result which is consistent with the large initial couplings $g(\Lambda)$ required for the MPP scenario.

The organization of our paper is as follows. In Sec. II we present the general field content, couplings, and vacuum expectation values that contribute to the quark and lepton mass matrices. Approximate mass eigenvalues and eigenvectors are given in Sec. III for the charged fermions of one generation. Discussion of $SU(2)_L$ breaking in the vectorlike Higgs sector, splitting of standard and mirror fermion masses, along with comments on interfamily and intrafamily quark mass splittings are also presented. Neutrino mass seesaw mechanisms of the Dirac and Majorana types are considered in Sec. IV for chiral and vectorlike families, with and without exotic leptons. We conclude with a summary of our results together with a discussion of the realization in superstring theories of the field content required for the various mechanisms considered.

II. GENERAL MODEL-INDEPENDENT FRAMEWORK

A. Field content of the $E_8 \otimes E_8'$ theory

We begin by elaborating the matter and gauge field content in the compactified $E_8 \otimes E_8'$ theory.[2] In the zero-slope limit of the superstring in ten dimensions prior to compactification,[4] the representation content of the massless sector is just

$$(\mathbf{248},1)\oplus(1,\mathbf{248}) , \tag{2.1a}$$

where the chiral fermions and bosons observed in nature belong to the fundamental (and adjoint) representation **248** of E_8, while the "shadow" world resides in the **248** of E_8'. Upon compactification of six of the spatial dimensions in a Kähler manifold with $SU(3)_H$ holonomy, the E_8 group is broken down to $E_6 \otimes SU(3)_H$ with $N=1$ supersymmetry, whereby

$$\mathbf{248} \to (27,3)+(\overline{27},\overline{3})+(78,1)+(1,8) . \tag{2.1b}$$

The **27** supermultiplet contains matter fields and Higgs bosons associated with the "standard" fermions; the $\overline{\mathbf{27}}$ contains those associated with "mirror" fermions; the $\overline{\mathbf{78}}$ contains vector gauge bosons and gauginos, and the **1** contains a Higgs scalar and Higgsino fermion. For Calabi-Yau manifolds[4,6] the number of chiral families is given by

$$n_{27} - n_{\overline{27}} = \tfrac{1}{2} \, |\chi(K_6)| , \tag{2.2}$$

where χ is the Euler characteristic of K_6, while the number of vectorlike matter families is governed by the Betti-Hodge number $b_{1,1}$.

In general, the number of families obtained in this compactification is too large, and further breaking of E_6 is difficult to accomplish. Both difficulties are surmounted by the Wilson-loop mechanism[16] of Hosotani and Witten, whereby the E_8 is broken directly to a subgroup of E_6. It should be noted, however, that while the chiral families are protected by an index theorem, the vectorlike families are not. Hence, whereas for chiral families all components of the **27** survive, in general this is not true for vectorlike families. We shall therefore consider several combinations of survivors for vectorlike matter families in what follows.

Since our primary focus is on rank-6 subgroups of E_6, it is convenient to classify[17] the supermultiplets according to the $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$ maximal subgroup of E_6. For subgroups of $[SU(3)]^3$ the appropriate submultiplets should be understood. Thus

$$\mathbf{27} = (3,3,1)+(\overline{3},1,\overline{3})+(1,\overline{3},3) \tag{2.3a}$$

with $\Psi_{27} \equiv \hat{A} + \hat{B} + \hat{C}$,

$$\overline{\mathbf{27}} = (\overline{3},\overline{3},1)+(3,1,3)+(1,3,\overline{3}) \tag{2.3b}$$

with $\Psi_{\overline{27}} \equiv \hat{\hat{A}} + \hat{\hat{B}} + \hat{\hat{C}}$,

$$\mathbf{78} = (8,1,1)+(1,8,1)+(1,1,8)+(3,\overline{3},\overline{3})+(\overline{3},3,3) \tag{2.3c}$$

with $\Phi_{78} \equiv \hat{g} + \hat{G}_L + \hat{G}_R + \hat{\chi}_L + \hat{\chi}_R$, where the carets indicate superfields. The matter superfields $\hat{A}_M$, $\hat{B}_M$, and $\hat{C}_M$ with content

$$\hat{A}_M \supset A_M, \tilde{A}_M, \quad \hat{B}_M \supset B_M, \tilde{B}_M, \quad \hat{C}_M \subset C_M, \tilde{C}_M ,$$

involve quarks and squarks, antiquarks and antisquarks, and leptons and sleptons, respectively. The standard quarks and leptons can be represented by 3×3 matrices of $SU(3)\otimes SU(3)$ according to

$$A_M = \begin{bmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \\ h_1 & h_2 & h_3 \end{bmatrix} , \tag{2.4a}$$

$$B_M = \begin{bmatrix} h_1^c & d_1^c & u_1^c \\ h_2^c & d_2^c & u_2^c \\ h_3^c & d_3^c & u_3^c \end{bmatrix} , \tag{2.4b}$$

$$C_M = \begin{bmatrix} e & \nu & n^c \\ E & \nu_E & N^c \\ N_E^c & E^c & e^c \end{bmatrix} , \tag{2.4c}$$

while their superpartners $\tilde{A}_M$, $\tilde{B}_M$, and $\tilde{C}_M$ are given by similar matrices with tildes indicating the corresponding scalar fields. The entries in the matrices corresponding to (2.3b) are primed and conjugated. The $SU(2)_L \otimes SU(2)_R$ submatrices of C_M given by

$$\begin{bmatrix} E & \nu_E \\ N_E^c & E^c \end{bmatrix}, \quad (e \quad \nu), \quad \begin{bmatrix} N^c \\ e^c \end{bmatrix}, \quad n^c \tag{2.4c$'$}$$

lie in part of the **10**, **16**, **16**, and **1** representations, respectively, in the SO(10) decomposition of the **27** of E_6: $\mathbf{27}=\mathbf{16}+\mathbf{10}+\mathbf{1}$.

In addition, Higgs superfields $\hat{C}_H \supset C_H, \tilde{C}_H$ (or submultiplets for subgroups of $[SU(3)]^3$) containing Higgs scalars and Higgsinos will also be present. These will be assumed to arise from the vectorlike sector in general,[18] which, for Calabi-Yau manifolds corresponds to those representations among the $b_{1,1}$ pairs of **27**'s and $\overline{\mathbf{27}}$'s which survive compactification. For example, if the low-energy group is $[SU(3)]^3$, the Higgs superfields would arise from an equal excess of $\hat{C}$'s and $\hat{\bar{C}}$'s compared to the number of $\hat{A}_M$'s, $\hat{B}_M$'s and $\hat{\bar{A}}_M$'s, $\hat{\bar{B}}_M$'s, respectively. Note that we have assumed that there do not exist any colored Higgs superfields. How this assumption may be realized is discussed in Sec. V B. As a result, imposition of either R parity or matter parity[10] will prevent rapid proton decay (see Sec. II B). Although it is important to distinguish between matter and Higgs supermultiplets, as will be discussed in the next section, we shall use the slepton labels for both to specify the transformation properties of the scalars.

We need not spell out in detail the representation content of the **78** superfields containing gauge fields and their associated gauginos. Although it is possible that one or more E_6 singlets,[19] $\hat{\phi}$, will survive compactification, for Calabi-Yau backgrounds[20] they will obtain a mass comparable to the Planck mass $m_{\rm Pl}$, due to world-sheet instanton effects.

B. Yukawa and nonrenormalizable couplings

The low-energy groups we consider are $[SU(3)]^3$ and and its subgroups. The Yukawa part of the superpotential can be written generally as

$$W=\sum_{i,j,k}(\hat{C}_i\hat{C}_j\hat{C}_k+\hat{A}_i\hat{A}_j\hat{A}_k+\hat{B}_i\hat{B}_j\hat{B}_k+\hat{A}_i\hat{B}_j\hat{C}_k) + \overline{\mathbf{27}}^3 \text{ terms} . \tag{2.5}$$

Expanding in components, the Yukawa terms of interest can include

$$\hat{A}\hat{B}\hat{C} \supset uh^c\tilde{e}+ud^c\tilde{E}+du^c\tilde{E}^c+hu^c\tilde{e}^c+uu^c\tilde{N}_E^c+dh^c\tilde{\nu}+dd^c\tilde{\nu}_E+hh^c\tilde{n}^c+hd^c\tilde{N}^c , \tag{2.6a}$$

$$\begin{aligned}\hat{C}\hat{C}\hat{C} \supset & -eE^c\tilde{N}^c+\nu\tilde{N}^cN_E^c+\tilde{n}^cEE^c-\tilde{n}^c\nu_EN_E^c+e\tilde{\nu}_Ee^c-e\tilde{E}^cN^c-\tilde{\nu}e^cE+n^cE\tilde{E}^c+\tilde{e}\nu_Ee^c+\nu N^c\tilde{N}_E^c \\ & -\nu e^c\tilde{E}-n^c\nu_E\tilde{N}_E^c+(e\nu_E\tilde{e}^c-\nu\tilde{e}^cE+\tilde{\nu}N^cN_E^c-n^c\tilde{\nu}_EN_E^c-\tilde{e}E^cN^c+n^c\tilde{E}E^c)\end{aligned} \tag{2.6b}$$

plus similar expressions for the $\overline{\mathbf{27}}^3$ terms, where the slepton labels here refer to Higgs scalars with the same transformation properties. In (2.6b) we have separated out the Dirac terms from the Majorana terms.[21] The couplings implicit above are related only by the low-energy group.[5] If Higgs singlets $\hat{\phi}$ were present, Yukawa terms of the following type would also rise in the superpotential:

$$\sum_{i,j,k}(\hat{A}_i\hat{\bar{A}}_j\hat{\phi}_k+\hat{B}_i\hat{\bar{B}}_j\hat{\phi}_k+\hat{C}_i\hat{\bar{C}}_j\hat{\phi}_k+\hat{\phi}_i\hat{\phi}_j\hat{\phi}_k) . \tag{2.7}$$

In this paper we study matter fermion mass matrices for which there is no mixing of matter with Higgsinos and gauginos. This greatly simplifies the search for effective seesaw solutions to the light neutrino problem. (In fact, solutions such as those that we have found may not exist if Higgsinos and gauginos are included[22] in the neutral-lepton mass matrix.) To ensure that such mixing does not occur, we impose either ordinary supersymmetric R parity[23] (under which matter fermions and Higgs scalars are even, while matter scalars and Higgs fermions are odd) or matter parity[10] under which matter superfields are odd while Higgs superfields are even. Both of these discrete symmetries forbid Yukawa couplings involving only one matter superfield and so prevent direct mixing of matter fermions and Higgsinos through Higgs VEV's. We must further assume that, following renormalization-group evolution, those scalar mass inequalities which guarantee absence of matter scalar VEV's are satisfied, i.e., R parity or M parity remain unbroken in the low-energy theory. This prevents direct mixing of matter fermions with gauginos via supersymmetric (SUSY) kinetic energy terms, as well as direct mixing of the former with Higgsinos through matter scalar VEV's. Either R parity or matter parity implies only one type of Yukawa coupling for matter superfields, $\Psi_M\Psi_M\Psi_H$, and an extra one for Higgs superfields, $\Psi_H\Psi_H\Psi_H$. If the latter type couplings are typically larger than the former, we expect Higgs-scalar masses to evolve more rapidly than matter scalar masses, perhaps justifying our assumption that only Higgs scalars acquire VEV's. It is interesting that R parity or matter parity will also prevent the troublesome A_M^3, B_M^3, $\bar{A}_M^3$, and $\bar{B}_M^3$ terms. With no colored Higgs superfields present, as will be assumed throughout, this ensures absence of rapid proton decay.[10] The resulting Yukawa superpotential will then consist of the couplings

$$W=\hat{C}_M\hat{C}_M\hat{C}_H+\hat{A}_M\hat{B}_M\hat{C}_H+\hat{C}_H\hat{C}_H\hat{C}_H+\overline{27}^3 \text{ terms}$$
$$+\hat{\phi}^3+(\hat{A}_M\hat{\bar{A}}_M+\hat{B}_M\hat{\bar{B}}_M+\hat{C}_M\hat{\bar{C}}_M+\hat{C}_H\hat{\bar{C}}_H)\hat{\phi}\ , \quad (2.8)$$

where, as usual, the appropriate representations and couplings should be understood for subgroups of $[\mathrm{SU}(3)]^3$. Though indices have been suppressed, in general several multiplets of each kind may appear. The first two terms plus their mirror counterparts lead to ordinary $\Delta I=\frac{1}{2}$ matter fermion masses, and if intermediate scales exist, to large entries in the neutral-lepton mass matrix crucial for all but one of our light neutrino seesaw solutions. If ϕ's obtain VEV's (see the next section), the last set of terms will lead to important standard-mirror matter fermion mixing.

In addition to Yukawa couplings, nonrenormalizable terms of dimension 5 or higher may be present in the superpotential. For example, it has been shown that for Calabi-Yau backgrounds, world-sheet instantons will generate such terms.[20] Imposing either R parity or matter parity, we find that the nonrenormalizable superpotential up to and including dimension-7 contributions consists of the terms

$$W_{\rm NR}=\frac{1}{m_{\rm Pl}}\hat{C}_H\hat{\bar{C}}_H(\hat{C}_M\hat{\bar{C}}_M+\hat{A}_M\hat{\bar{A}}_M+\hat{B}_M\hat{\bar{B}}_M) \quad (2.9a)$$
$$+\frac{1}{m_{\rm Pl}}(\hat{C}_H\hat{C}_H\hat{\bar{C}}_M\hat{\bar{C}}_M+\hat{\bar{C}}_H\hat{\bar{C}}_H\hat{C}_M\hat{C}_M) \quad (2.9b)$$
$$+\frac{1}{m_{\rm Pl}}\hat{C}_H\hat{\bar{C}}_H\hat{C}_H\hat{\bar{C}}_H$$
$$+\frac{1}{m^3_{\rm Pl}}(\hat{C}_H\hat{C}_H\hat{C}_H)(\hat{\bar{C}}_H\hat{\bar{C}}_H\hat{\bar{C}}_H) \quad (2.9c)$$
$$+\frac{1}{m^3_{\rm Pl}}(\hat{C}_H\hat{\bar{C}}_H\hat{C}_H\hat{\bar{C}}_H)(\hat{C}_M\hat{\bar{C}}_M+\hat{A}_M\hat{\bar{A}}_M+\hat{B}_M\hat{\bar{B}}_M) \quad (2.9d)$$

plus other dimension-5, -6, and -7 terms with an even number of matter superfields. The couplings in (2.9c) can be important in lifting flat directions, thus leading to intermediate mass scales[24] as discussed in Sec. II C below. They will also play an important role in a radiative scheme for splitting ordinary and mirror matter fermion masses as discussed in Sec. III. Couplings of (2.9a) and (2.9d) will lead to important mixing between ordinary and mirror quarks and leptons while (2.9b) may lead to large right-handed Majorana neutrino masses essential in one of our light-neutrino solutions.

C. Vacuum expectation values

We now list the vacuum expectation values which can arise from the vectorlike Higgs sector. For convenience we label fields in question by their sneutrino counterparts:

$$\tilde{\nu}_E,\tilde{N}^c_E,\tilde{\nu},\tilde{N}^c,\tilde{n}^c\subset C_H,\quad \tilde{\nu}'^c_E,\tilde{N}'_E,\tilde{\nu}'^c,\tilde{N}',\tilde{n}'\subset\bar{C}_H\ , \quad (2.10)$$

where, in general, several copies of each are possible following compactification. Nonzero VEV's for $\tilde{\nu}_E,\tilde{N}^c_E,\tilde{\nu}$ and $\tilde{\nu}'^c_E,\tilde{N}'_E,\tilde{\nu}'^c$ with break $\mathrm{SU}(2)_L$ and contribute mass to ordinary and mirror fermions via the couplings of (2.8). How such VEV's may be obtained will be discussed in Sec. III B. Given a pair $(\tilde{n}^c,\tilde{n}')$ or $(\tilde{N}^c,\tilde{N}')$, depending on the relative sizes of SUSY-breaking soft terms at low energy, its contribution to the scalar potential can be positive definite along D-flat directions. For such a pair it may be possible for one or both elements to obtain VEV's $\lesssim 1$ TeV (the observable SUSY-breaking scale) radiatively, in much the same way that $\mathrm{SU}(2)_L$-breaking VEV's are generated.

Intermediate mass scale VEV's for at least one pair $(\tilde{n}^c,\tilde{n}')$ or $(\tilde{N}^c,\tilde{N}')$ will be essential in all our ultralight-neutrino seesaw mechanisms. In addition, such VEV's can affect the complete breaking of the low-energy group down to the standard model at a high scale Λ_I, although the phase transition[25] will actually occur at temperatures $T\sim 1$ TeV. There are several ways in which flat directions may become lifted at intermediate-mass scales. For example, this can happen purely radiatively at one-loop level without introduction of nonrenormalizable terms.[26] Typically, this will result in $\langle\tilde{n}^c\rangle,\langle\tilde{n}'\rangle$ or $\langle\tilde{N}^c\rangle,\langle\tilde{N}'\rangle$ or both of $\sim 10^{14}$ GeV.

Addition of the nonrenormalizable couplings

$$\frac{(n^cn')^3}{m^3_{\rm Pl}} \quad \text{or} \quad \frac{(N^cN')^3}{m^3_{\rm Pl}}$$

contained in the second term of (2.9c) will also lift the flat directions at $\Lambda_I\sim 10^{14}$ GeV, while nonrenormalizable couplings

$$\frac{(n^cn')^2}{m_{\rm Pl}} \quad \text{or} \quad \frac{(N^cN')^2}{m_{\rm Pl}}$$

contained in the first term of (2.9c) will lead to $\Lambda_I\sim 10^{10}-10^{11}$ GeV, so that two intermediate mass scales are possible.[27]

If E_6 singlets ϕ exist, it again becomes possible to lift flat directions without the presence of nonrenormalizable terms, this time leading to $\Lambda_I\sim 10^{10}-10^{11}$ GeV. To see this, consider for simplicity only one E_6-singlet superfield $\hat{\phi}$ and one pair of $(\hat{n}^c,\hat{n}')$ superfields together with the following superpotential plus soft terms:

$$\hat{\phi}\hat{n}^c\hat{n}'_F+\hat{\phi}^3_F+m_{\rm Pl}\hat{\phi}^2_F-m^2|\tilde{n}^{c2}|+|\tilde{n}^{c2}-\tilde{n}'^2|^2_{D^2}\ . \quad (2.11)$$

The second and third terms will be generated by world-sheet instantons as previously pointed out,[20] while the fourth term will be driven negative as a result of renormalization-group effects with $m\sim m_0\sim 1$ TeV. The F^2 terms resulting from the first and third couplings will lift the flat directions and lead to $\langle\tilde{n}^c\rangle=\langle\tilde{n}'\rangle\sim(mm_{\rm Pl})^{1/2}$, $\langle\phi\rangle\sim m$.

D. General mass matrices

We now present the general form of the mass matrices for a vectorlike family of quarks and leptons with all components of the 27 and $\overline{27}$ present. If fewer components survive, just set the relevant entries in the matrices equal to zero. These matrices are sufficient for studying the basic effects of mirror families on the mass spectrum. The effects of quark mass mixing can be incorporated by perturbating about the generation-diagonal limit. No mixing with gauginos and Higgsinos appears under our assumptions of R parity or matter parity together with the absence of matter scalar VEV's.

We first identify the matrix entries with the possible Higgs VEV's where coupling strengths are implicit throughout, and slepton labels serve to identify the transformation properties of the Higgs scalars. Transformation properties under $SU(3)_c \times SU(2)_L \otimes SU(2)_R \otimes U(1) \otimes U(1)'$ are indicated below in parentheses. Entries in the mass matrices which can be $\gtrsim 10^{10}$ GeV or ~ 1 TeV transform as

$$\begin{aligned} s_2 &\sim V_2 \equiv \langle \tilde{n}^c \rangle \sim (1,1,1)_{2/3,-2/3} , \\ s_2' &\sim V_2' \equiv \langle \tilde{n}' \rangle \sim (1,1,1)_{-2/3,2/3} , \\ s_1 &\sim V_1 \equiv \langle \tilde{N}^c \rangle \sim (1,1,2)_{2/3,1/3} , \\ s_1' &\sim V_1' \equiv \langle \tilde{N}' \rangle \sim (1,1,2)_{-2/3,-1/3} . \end{aligned} \tag{2.12a}$$

Weak scale entries transform as

$$\begin{aligned} m_1 &\sim \langle \tilde{N}_E^c \rangle \sim (1,2,2)_{-1/3,1/3} , \\ m_1' &\sim \langle \tilde{N}_E' \rangle \sim (1,2,2)_{1/3,-1/3} , \\ m_2 &\sim \langle \tilde{\nu}_E \rangle \sim (1,2,2)_{-1/3,1/3} , \\ m_2' &\sim \langle \tilde{\nu}_E'^c \rangle \sim (1,2,2)_{1/3,-1/3} , \\ m_3 &\sim \langle \tilde{\nu} \rangle \sim (1,2,1)_{-1/3,-2/3} , \\ m_3' &\sim \langle \tilde{\nu}'^c \rangle \sim (1,2,1)_{1/3,2/3} . \end{aligned} \tag{2.12b}$$

Entries which can result from couplings of dimension 5 or 7 (given in that order) and from $\phi \times 27_M \times \overline{27}_M$ if ϕ exists are of the Dirac type,

$$\begin{aligned} r &\sim \frac{1}{m_{Pl}} V_i' V_i , \frac{1}{m_{Pl}^3} (V_i V_i')(V_j V_j'), \langle \phi \rangle \sim (1,1,1)_{0,0} , \\ u &\sim \frac{1}{m_{Pl}} V_2 V_1' , \frac{1}{m_{Pl}^3} (V_2 V_1')(V_i V_i') \sim (1,1,2)_{0,-1} , \\ u' &\sim \frac{1}{m_{Pl}} V_1 V_2' , \frac{1}{m_{Pl}^3} (V_1 V_2')(V_i V_i') \sim (1,1,2)_{0,1} , \end{aligned} \tag{2.13a}$$

and of the Majorana type,

$$\begin{aligned} t_i &\sim \frac{1}{m_{Pl}} V_i^2 , \frac{1}{m_{Pl}^3} V_i^2 V_j V_j' \sim (1,1,3)_{4/3,(2/3,-4/3)} , \\ t_{12} &\sim \frac{1}{m_{Pl}} V_1 V_2 , \frac{1}{m_{Pl}^3} V_1 V_2 V_j V_j' \sim (1,1,2)_{4/3,-1/3} , \\ t_i' &\sim \frac{1}{m_{Pl}} V_i'^2 , \frac{1}{m_{Pl}^3} V_i'^2 V_j V_j' \sim (1,1,3)_{-4/3,(-2/3,4/3)} , \\ t_{12}' &\sim \frac{1}{m_{Pl}} V_1' V_2' , \frac{1}{m_{Pl}^3} V_1' V_2' V_j V_j' \sim (1,1,2)_{-4/3,1/3} , \end{aligned} \tag{2.13b}$$

where $i,j=1$ or 2. It should be understood that in general there may be more than one Higgs VEV with the same transformation properties entering the above expressions. Order-of-magnitude estimates for the entries of (2.13a) and (2.13b) will depend on whether they originate from dimension-5 or -7 terms and on which scales enter the above expressions:

$$Q=\pm\tfrac{2}{3}: \quad B^u = \{u, u^c, u'^c, u'\}_L .$$

In terms of the two-component Weyl spinor basis, the up-quark mass matrix is

$$M^u = \begin{bmatrix} M_{27}^u & r \\ r & M_{\overline{27}}^u \end{bmatrix} = \begin{bmatrix} 0 & m_1 & r_1 & 0 \\ m_1 & 0 & 0 & r_2 \\ r_1 & 0 & 0 & m_1' \\ 0 & r_2 & m_1' & 0 \end{bmatrix} , \tag{2.14}$$

where r_i mixes standard and mirror families. If a standard family is unpaired with a mirror family, clearly just the upper block-diagonal submatrix enters:

$$Q=\pm\tfrac{1}{3}: \quad B^d = \{d^c, d, h^c, h, d', d'^c, h', h'^c\}_L ,$$

$$Q=\pm 1: \quad B^e = \{e, e^c, E, E^c, e'^c, e', E'^c, E'\}_L .$$

The down-quark and charged-lepton mass matrices have the identical forms if the bases are ordered as above:

$$M^{d,e} = \begin{bmatrix} 0 & m_2 & 0 & s_1 & r_1' & 0 & u' & 0 \\ m_2 & 0 & m_3 & 0 & 0 & r_2' & 0 & 0 \\ 0 & m_3 & 0 & s_2 & u & 0 & r_3' & 0 \\ s_1 & 0 & s_2 & 0 & 0 & 0 & 0 & r_4' \\ r_1' & 0 & u & 0 & 0 & m_2' & 0 & s_1' \\ 0 & r_2' & 0 & 0 & m_2' & 0 & m_3' & 0 \\ u' & 0 & r_3' & 0 & 0 & m_3' & 0 & s_2' \\ 0 & 0 & 0 & r_4' & s_1' & 0 & s_2' & 0 \end{bmatrix} , \tag{2.15}$$

$$Q=0: \quad B^N = \{\nu, \nu_E, N_E^c, \nu'^c, \nu_E'^c, N_E', N^c, n^c, N', n'\}_L .$$

Here the ordering is chosen for convenience in determining the form of the eigenvalue and eigenvector solutions given in Sec. IV below:

$$M^N=\begin{pmatrix} 0 & 0 & s_1 & r_1'' & u_1' & 0 & m_{11} & 0 & 0 & 0 \\ 0 & 0 & s_2 & u_1 & r_2'' & 0 & 0 & m_{12} & 0 & 0 \\ s_1 & s_2 & 0 & 0 & 0 & r_3'' & m_3 & m_2 & 0 & 0 \\ r_1'' & u_1 & 0 & 0 & 0 & s_1' & 0 & 0 & m_{11}' & 0 \\ u_1' & r_2'' & 0 & 0 & 0 & s_2' & 0 & 0 & 0 & m_{12}' \\ 0 & 0 & r_3'' & s_1' & s_2' & 0 & 0 & 0 & m_3' & m_2' \\ m_{11} & 0 & m_3 & 0 & 0 & 0 & t_1' & t_{12}' & r_4'' & u_2 \\ 0 & m_{12} & m_2 & 0 & 0 & 0 & t_{12}' & t_2' & u_2' & r_5'' \\ 0 & 0 & 0 & m_{11}' & 0 & m_3' & r_4'' & u_2' & t_1 & t_{12} \\ 0 & 0 & 0 & 0 & m_{12}' & m_2' & u_2 & r_5'' & t_{12} & t_2 \end{pmatrix} \quad (2.16)$$

With this form most of the higher-dimensional terms are grouped in the lower diagonal block. Equality of different r, m, and u entries in the above mass matrices depends on the choice of gauge group.

Again we emphasize that the above matrices are of the most general form in the absence of Higgsino, gaugino, and family mixing.

III. QUARK AND CHARGED-LEPTON MASSES

We organize this section into four parts. In Sec. III A we give the quark and charged-lepton mass eigenvalues and eigenstates corresponding to the matrices of Sec. II C under certain simplifying assumptions. In Sec. III B we discuss in some detail the origin of $SU(2)_L$-breaking VEV's entering the expressions of Sec. III A. In Sec. III C we suggest mechanisms for splitting the masses of ordinary and mirror charged matter fermions and discuss important radiative mass contributions to quarks and leptons. The multigeneration case and quark isospin breaking is discussed in Sec. III D.

A. Quark and charged-lepton masses and eigenvectors

We now present the approximate mass eigenvalues and eigenvectors for the quarks and charged leptons. Although we are more interested in the vectorlike case, we include for the sake of comparison the chiral case as well. For ease of presentation we denote all r entries in the preceding mass matrices as r.

$Q=\pm\frac{2}{3}$: ***27 only***

The eigenvalues and eigenvectors

$$\pm m_1:\ \frac{1}{\sqrt{2}}(u_L \pm u_L^c) \quad (3.1)$$

represent two degenerate Majorana masses which are equivalent to a Dirac mass and four-component spinor

$$m_1:\ \psi_u=\begin{pmatrix} u_L \\ i\sigma_2 u_L^{c*}\end{pmatrix}. \quad (3.2)$$

$Q=\pm\frac{2}{3}$: ***27+$\overline{27}$***

With both standard and mirror families present, the results are now

$$\pm m_1':\ \frac{1}{\sqrt{2}}\left[u_L'+\frac{r}{m_1'}u_L \pm \left[u_L'^c+\frac{r}{m_1'}u_L^c\right]\right], \quad (3.3a)$$

$$\pm\frac{|r^2-m_1m_1'|}{m_1'}:\ \frac{1}{\sqrt{2}}\left[u_L-\frac{r}{m_1'}u_L'\pm\left[u_L^c-\frac{r}{m_1'}u_L'^c\right]\right], \quad (3.3b)$$

where we have taken $m_i, r < m_i'$, cf. Sec. III C. Numerically for the first family, $m_1 \sim 10^{-3}$ GeV, $m_1' > 22$ GeV.

$Q=\pm\frac{1}{3}$: ***27 only***

Here the results are

$$\pm(s_1^2+s_2^2)^{1/2}:\ \frac{1}{\sqrt{2}}\left[h_L\pm\frac{1}{(s_1^2+s_2^2)^{1/2}}(s_1d_L^c+s_2h_L^c)\right], \quad (3.4a)$$

$$\pm\frac{|m_2s_2-m_3s_1|}{(s_1^2+s_2^2)^{1/2}}:\ \frac{1}{\sqrt{2}}\left[d_L\pm\frac{1}{(s_1^2+s_2^2)^{1/2}}(s_2d_L^c-s_1h_L^c)\right]. \quad (3.4b)$$

$Q=\pm\frac{1}{3}$: ***27+$\overline{27}$***

Now we find approximately, with $m_i,u,u',r < m_i' \ll s_1,s_1' < s_2,s_2'$,

$$\pm(s_1^2+s_2^2)^{1/2}:\ \frac{1}{\sqrt{2}}\left[h_L\pm\frac{1}{(s_1^2+s_2^2)^{1/2}}(s_1 d_L^c+s_2 h_L^c)\right], \tag{3.5a}$$

$$\pm(s_1'^2+s_2'^2)^{1/2}:\ \frac{1}{\sqrt{2}}\left[\frac{1}{(s_1'^2+s_2'^2)^{1/2}}(s_1' d_L'+s_2' h_L')\pm h_L'^c\right], \tag{3.5b}$$

$$\pm\frac{|m_2's_2'-m_3's_1'|}{(s_1'^2+s_2'^2)^{1/2}}:\ \frac{1}{\sqrt{2}}\left\{\frac{1}{(s_1'^2+s_2'^2)^{1/2}}\left[s_2'\left[d_L'+\frac{r}{m_2'}d_L\right]-s_1'h_L'\right]\pm\left[d_L'^c+\frac{r}{m_2'}d_L^c\right]\right\}, \tag{3.5c}$$

$$\pm\frac{(r^2-m_2m_2')s_2s_2'+(r^2-m_3m_3')s_1s_1'-(ru'-m_2m_3')s_1's_2-(ru-m_2'm_3)s_1s_2'}{(s_1^2+s_2^2)^{1/2}|m_2's_2'-m_3's_1'|}:$$
$$\frac{1}{\sqrt{2}}\left\{\left[d_L-\frac{r}{m_2'}d_L'\right]\pm\frac{1}{(s_1^2+s_2^2)^{1/2}}\left[s_2\left[d_L^c-\frac{r}{m_2'}d_L'^c\right]-s_1h_L^c\right]\right\}. \tag{3.5d}$$

In the above we have, for simplicity, set $u_i=u_i'=0$ and only kept terms up to $O(r/m')$ in the eigenstates, neglecting terms of $O(rs_1/m's_2)$. The charged-lepton masses and eigenstates are simply obtained from (3.4) and (3.5) above by the appropriate basis substitutions in Sec. II. It is apparent from (3.4a) and (3.5a) that we must take $s_2/s_1 \gtrsim 10$ in order to ensure that the heavy-mass eigenstate is mostly the exotic $Q=+\frac{1}{3}$ singlet member of the **27**, while the lightest-mass eigenstate is mostly the ordinary doublet member d_L^c. It is also interesting to note that in the limit $s_2\sim s_2'\gg s_1,s_1'$, (3.5c) and (3.5d) will yield mass eigenstates and eigenvalues for d quarks, charged leptons, and their mirrors exactly of the same form as in the simpler u-quark case (3.3a) and (3.3b).

Finally we consider the possibility that the full **27** and $\overline{\mathbf{27}}$ representations of E_6 do not survive compactification, i.e., only the would-be SO(10) representations **16** and $\overline{\mathbf{16}}$ remain massless. In this situation, to be elaborated upon in Sec. V, the $Q=\frac{2}{3}$ mass matrix remains unchanged, while the $Q=-\frac{1}{3}$ and $Q=-1$ mass matrices in (2.15) reduce to the form of (2.14) with m_1 and m_1' replaced by m_2 and m_2', respectively. The mass eigenvalues and eigenvectors corresponding to (3.4) and (3.5) are then replaced by the following.

$Q=\pm\frac{1}{3}$: ***16***

We have

$$\pm m_2:\ \frac{1}{\sqrt{2}}(d_L\pm d_L^c)\,. \tag{3.4'}$$

$Q=\pm\frac{1}{3}$: ***16***+$\overline{\mathit{16}}$

We have

$$\pm m_2':\ \frac{1}{\sqrt{2}}\left[d_L'+\frac{r}{m_2'}d_L\pm\left[d_L'^c+\frac{r}{m_2'}d_L^c\right]\right], \tag{3.5a'}$$

$$\pm\frac{|r^2-m_2m_2'|}{m_2'}:\ \frac{1}{\sqrt{2}}\left[d_L-\frac{r}{m_2'}d_L'\pm\left[d_L^c-\frac{r}{m_2'}d_L'^c\right]\right]. \tag{3.5b'}$$

B. SU(2)$_L$ breaking in the vectorlike Higgs sector

The heavier quarks and leptons can obtain the bulk of their mass via mixing with their mirrors,[28] see Sec. III D. Therefore, an acceptable VEV pattern may be $\langle\tilde{\nu}_E'^c\rangle,\langle\tilde{N}_E'\rangle\sim M_W$; $\langle\tilde{\nu}_E\rangle,\langle\tilde{N}_E^c\rangle\sim 0$. The former may simply be a consequence of $m^2_{\tilde{\nu}_B'^c,N_B'}<0$. These inequalities are easily attainable as mirror fermions are experimentally constrained to have large Yukawa couplings with $\tilde{\nu}_E'^c$ and $\tilde{N}_E'$, leading to large renormalization of the latter's masses. We shall see that even with $m^2_{\tilde{\nu}_E'^c},m^2_{N_B'}$ both less than zero, the scalar potential is not generally unbounded from below. It follows that SUSY-breaking scalar bilinears such as $A\mu\tilde{\nu}_E\tilde{N}_E^c$, $A\mu'\tilde{\nu}_E'^c\tilde{N}_E'$ (where μ and μ' are the coupling strengths of the corresponding superfield bilinears resulting from trilinear terms in the Higgs superpotential in E_6-based models, and A, the SUSY-breaking coupling, also has dimensions of mass and is determined by details of the SUSY-breaking mechanism) are not strictly required in order to obtain acceptable SU(2)$_L$ breaking and fermion masses. This is to be contrasted with the case of minimal supergravity[29] (SUGRA), where the first term is required in order to obtain $\langle\tilde{\nu}_E\rangle,\langle\tilde{N}_E\rangle\lesssim M_W$ and ensure the absence of an electroweak axion. For vectorlike models, the assumption of low-energy ordinary-mirror quark mixing guarantees the latter.

In light of the above discussion, we are free to consider two cases in what follows: $A\ll m_0$, and $A\sim m_0$, Refs. 30 and 31, respectively, where m_0 (taken to be ~ 1 TeV) is the mass typically acquired by all scalars in the theory as a result of SUSY breaking. We shall see below and in more detail in Sec. III C that there will be additional mechanisms available for splitting ordinary and mirror fermion masses when $A\ll m_0$.

We begin by giving the relevant low-energy Higgs-scalar potential. For simplicity, consider just one copy each of the Higgs superfields $\hat{\nu},\hat{N}_E^c,\hat{\nu}_E$ and their mirrors $\hat{\nu}'^c,\hat{\nu}_E'^c,\hat{N}_E'$, while in general allowing for several pairs of

Higgs scalars $(\hat{n}^c,\hat{n}')$ and $(\hat{N}^c,\hat{N}')$. The couplings of Eqs. (2.8) and (2.9) involving only the above Higgs superfields together with D^2 terms and SUSY-breaking soft terms will lead, in general, to the following relevant low-energy Higgs-scalar potential, which we give for future reference:

$$\langle\tilde{n}^c_i\rangle^2(|\nu_E|^2+|N^c_E|^2)+\langle\tilde{n}'_i\rangle^2(|\nu'^{c}_E{}^2+|N'_E|^2)+\langle\tilde{N}^c_i\rangle^2(|\nu|^2+|N^c_E|^2)+\langle\tilde{N}'_i\rangle^2(|\nu'^c|^2+|N'_E|^2)$$
$$+\langle\phi\rangle^2(|\nu_E|^2+|\nu'^c_E|^2)+\langle\phi\rangle^2(|N^c_E|^2+|N'_E|^2)+\langle\phi\rangle^2(|\nu|^2+|\nu'^c|^2) \qquad (3.6a)$$

$$+|N^c_E\nu_E|^2+|\nu'^c_E N'_E|^2+|\nu N^c_E|^2+|\nu'^c N'_E|^2+|\nu_E\nu'^c_E|^2+|N^c_E N'_E|^2+|\nu\nu'^c|^2$$
$$+\frac{g^2}{8}\left|\,|\nu|^2-|\nu'^c|^2+|\nu_E|^2-|\nu'^c_E|^2-|N^c_E|^2+|N'_E|^2\right|^2$$
$$+\text{similar } D^2 \text{ terms for other generators of the low-energy gauge group} \qquad (3.6b)$$

$$+m_1^2|N^c_E|^2+m_2^2|\nu_E|^2+m_3^2|\nu|^2+m_1'^2|N'_E|^2+m_2'^2|\nu'_E|^2+m_3'^2|\nu'^c|^2$$
$$+A\langle\tilde{n}^c_i\rangle\nu_E N^c_E+A\langle\tilde{N}^c_i\rangle\nu N^c_E+A\langle\tilde{n}'_i\rangle\nu'^c_E N'_E+A\langle\tilde{N}'_i\rangle\nu'^c N'_E+A\langle\phi\rangle\nu\nu'^c+A\langle\phi\rangle N^c_E N'_E+A\langle\phi\rangle\nu_E\nu'^c_E$$
$$+A\frac{1}{m_{\rm Pl}}\langle\tilde{n}^c_i\tilde{n}'_j,\tilde{N}^c_i\tilde{N}'_j\rangle(\nu\nu'^c,N^c_E N'_E,\nu_E\nu'^c_E)+A\frac{1}{m^3_{\rm Pl}}\langle(\tilde{n}^c_i\tilde{n}'_j,\tilde{N}^c_i\tilde{N}'_j)(\tilde{n}^c_k\tilde{n}'_l,\tilde{N}^c_k\tilde{N}'_l)\rangle(\nu\nu'^c,N^c_E N'_E,\nu_E\nu'^c_E)\,. \qquad (3.6c)$$

Clearly, the coupling strengths implicit above will be related by choice of the low-energy gauge group. Note that we have assumed that in addition to Yukawa couplings, nonrenormalizable terms in the superpotential will also lead to SUSY-breaking soft terms in the scalar potential, see (3.6c). The coefficients of all such soft terms are referred to generically as A.

Recall that for Higgs pairs $(\tilde{n}^c,\tilde{n}')$ and $(\tilde{N}^c,\tilde{N}')$ there are three possible ranges of VEV's: ~ 1 TeV, $\sim 10^{10}$–10^{11} GeV or $\gtrsim 10^{14}$ GeV, cf. Sec. II C. For $(\tilde{n}^c,\tilde{n}')$ or $(\tilde{N}^c,\tilde{N}')$ with intermediate-mass scale VEV's, some or all of the Higgs couplings

$$\hat{\nu}^c\hat{\nu}_E\hat{N}^c_E+\hat{n}'\hat{\nu}'^c_E\hat{N}'_E+\hat{N}^c\hat{N}^c_E\hat{\nu}+\hat{N}'\hat{N}'_E\hat{\nu}'^c \qquad (3.7a)$$

must be eliminated from the Higgs superpotential so that some or all of the Higgs scalars $\tilde{\nu}_E,\tilde{N}^c_E,\tilde{\nu}$ and mirrors remain light enough to acquire $SU(2)_L$-breaking VEV's, see (3.6a). An example of a discrete symmetry which accomplishes this exists for the extensively studied Yau three-family models.[32] For pairs $(\tilde{n}^c,\tilde{n}')$ or $(\tilde{N}^c,\tilde{N}')$ acquiring VEV's $\gtrsim 10^{14}$ GeV, in addition to the couplings of (3.7a) the couplings

$$\frac{n^c n'}{m_{\rm Pl}}(\nu_E\nu'^c_E,N^c_E N'_E,\nu\nu'^c) \quad\text{or}\quad \frac{N^c N'}{m_{\rm Pl}}(\nu_E\nu'^c_E,N^c_E N'_E,\nu\nu'^c) \qquad (3.7b)$$

will also have to be forbidden if the corresponding isodoublet Higgs scalars are to remain light.

We now return to a comparison of the two cases mentioned at the beginning of this section: $A\sim m_0$, $A\ll m_0$. In the former case, which leads to a low-energy potential very similar to those of the Inouè type,[33] for suitable ranges of parameters negative scalar masses will not be required for generating some or all $SU(2)_L$-breaking VEV's. In the latter case ($A\ll m_0$) with those terms appearing in the last three lines of (3.6c) essentially absent from the Higgs-scalar potential, the opposite is true—generation of non-negligible $SU(2)_L$-breaking VEV's necessitates negative scalar masses for all of the corresponding Higgs fields. This property may facilitate standard-mirror fermion mass splitting, cf. Sec. III C. As a result, F^2 quartic terms (3.6b) are generally necessary to ensure stability of the potential. For example, if $m^2_{\nu'^c_B,N'_B}<0$ is to lead to $\langle\tilde{\nu}'^c_E\rangle,\langle\tilde{N}'_E\rangle\sim M_W$, the quartic term $|N'_E\nu'^c_E|^2$, originating from $(\hat{n}'\hat{N}'_E\hat{\nu}'^c_E)_F$, must be present in order to stabilize the scalar potential along the D-flat direction $|\langle\tilde{\nu}'^c_E\rangle|=|\langle\tilde{N}'_E\rangle|$.

C. Splitting the masses of standard and mirror fermions and radiative contributions to quark and lepton masses

Charged mirror fermions are experimentally constrained[8] to have masses larger than 22 GeV. Therefore, if a known generation is vectorlike, a mechanism must exist for splitting its ordinary and mirror quark and charged-lepton masses. The necessity for such splitting, together with flavor-changing neutral-current (FCNC) and weak universality constraints, requires[34] $m_{1,2,3}\ll m'_{1,2,3}$ and $r\ll m'_{1,2}$, respectively; see Eqs. (3.3), (3.5), and (3.5$'$). We give in this section several alternative sources for the former inequality, not mutually exclusive, one of which requires $A\ll m_0$. Each mechanism involves different explicit breakings of ordinary-mirror interchange symmetry in the Yukawa sector. The section concludes with a discussion of important radiative contributions to $m_{1,2,3}$, alluded to previously.

First we elaborate on a feature of E_6-based models which naturally lends itself to ordinary-mirror mass splitting. This is the fact that ordinary fermions and their mirrors are provided with their own sets of mass giving Higgs fermions, ordinary and mirror Higgs fermions appearing in the **27** and $\overline{\mathbf{27}}$ representations, respectively. Because of gauge invariance, there are no tree-level Yukawa couplings between ordinary matter fermions and mirror Higgs fermions or vice versa. One-loop Yukawa couplings of this type are discussed later in this section. As a result, the standard-mirror fermion mass hierarchy may originate from a hierarchy of standard-mirror Higgs VEV's rather than standard-mirror Yukawa couplings or from a combination of the two. Before addressing these possibilities in some detail, we note that the same can be achieved for SO(10)-based models (or for its subgroups) possessing two copies of the complex Higgs representation **10** (or submultiplets) if one introduces an extra global symmetry. A $U(1)_{PQ}$ is the natural choice[35] which distinguishes between them. The $U(1)_{PQ}$ ensures that at the tree level one **10** couples exclusively to **16**'s (ordinary matter), while the other couples exclusively to $\overline{\mathbf{16}}$'s (mirror matter), hence resulting in the same feature attributed above to E_6-based models.

We now return to a discussion of mechanisms for obtaining $m_{1,2,3} < m'_{1,2,3}$. Let us assume that the Yukawa couplings of the Higgs superfields $\hat{\nu}_E, \hat{N}^c_E, \hat{\nu}$ of type $C_M C_M C_H$, $A_M B_M C_H$, or C_H^3 are typically smaller than those of their mirror counterparts (for Calabi-Yau manifolds this is a possibility, as the **27**'s and $\overline{\mathbf{27}}$'s come from two distinct cohomology classes); then, in general, due to renormalization-group effects we expect

$$(\langle\tilde{\nu}_E\rangle, \langle\tilde{N}^c_E\rangle, \langle\tilde{\nu}\rangle) < (\langle\tilde{\nu}'^c_E\rangle, \langle\tilde{N}'_E\rangle, \langle\tilde{\nu}'^c\rangle) \ . \quad (3.8a)$$

From Eq. (3.8a) and the hierarchy of standard-mirror Yukawa couplings assumed above, one also trivially obtains

$$m_{1,2,3} \ll m'_{1,2,3} \ . \quad (3.8b)$$

However, for $A \sim m_0$, the burden of mass splitting falls almost entirely on the hierarchy of standard versus mirror matter Yukawa couplings. This is because the VEV hierarchy of Eq. (3.8a) cannot be very large, $\lesssim 10$, due to the presence of scalar bilinears, proportional to A, which couple ordinary and mirror $\Delta I = \frac{1}{2}$ Higgs fermions; see the last six terms of (3.6c).

For $A/m_0 \ll 1$, the standard-mirror or overall Yukawa hierarchy need not be as large since the standard-mirror VEV hierarchy can be greatly increased. In fact, we shall see in Sec. III D that, including radiative effects, when the latter is the case the overall Yukawa coupling hierarchy required need not exceed 10^2 for certain models containing vectorlike generations compared to 10^5 for the standard model. The increased standard-mirror VEV hierarchy follows from an observation made in the previous section—that for $A \ll m_0$, a large $\Delta I = \frac{1}{2}$ VEV can only arise if the correspondings Higgs-boson mass squared is negative. As an example, again consider[30] $A/m_0 \lesssim 10^{-4}$. If the largest Yukawa couplings of ν_E, N^c_E, or ν are as little as a factor of 3 or 4 smaller than those of their mirrors, then $m^2_{\nu_E, N^c_E, \nu}$ and $m^2_{\nu'^c_E, N'_E, \nu'^c}$ may be positive ($\lesssim m_0^2$) and negative ($\sim -M_W^2$), respectively, at M_W following renormalization-group evolution. We then expect the following VEV hierarchy from Eq. (3.6):

$$\begin{aligned} &\langle\tilde{\nu}'^c_E\rangle, \langle\tilde{N}'_E\rangle, \langle\tilde{\nu}'^c\rangle \le M_W \ , \\ &\langle\tilde{\nu}_E\rangle, \langle\tilde{N}^c_E\rangle, \langle\tilde{\nu}\rangle \le 10^{-4} \frac{m_0}{m^2_{\nu_E, N^e_E, \nu}} \left[\langle\phi\rangle, \frac{m_I^2}{m_{\rm Pl}} \right] (\langle\tilde{\nu}'^c_E\rangle, \langle\tilde{N}'_E\rangle, \langle\tilde{\nu}'^c\rangle) \\ &\qquad \le 10^{-4} (\langle\tilde{\nu}'^c_E\rangle, \langle\tilde{N}'_E\rangle, \langle\tilde{\nu}'^c\rangle) \sim 10 \text{ MeV} \ , \end{aligned} \quad (3.9)$$

where the appropriate couplings are implicit, and $\langle\phi\rangle \lesssim 1$ TeV, $m_I \sim 10^{10}-10^{11}$ GeV. Interestingly, the VEV's $\langle\tilde{\nu}_E\rangle$, $\langle\tilde{N}^c_E\rangle$, and $\langle\tilde{\nu}\rangle$ given above are large enough to be of significance for the first-generation quarks and charged leptons.

Finally, a large standard-mirror VEV hierarchy may be due to a global or discrete symmetry; a $U(1)_{PQ}$ is used in Ref. 35. For example, for a pair $(\tilde{n}^c, \tilde{n}')$ with $\langle\tilde{n}^c\rangle, \langle\tilde{n}'\rangle \sim M_I$, such a symmetry may ensure that the couplings $\hat{n}^c \hat{\nu}_E \hat{N}^c_E, \hat{n}^c \hat{E}\hat{E}^c$, and $\hat{n}' \hat{\nu}'_E \hat{N}'_E, \hat{n}' \hat{E}' \hat{E}'^c$ are, respectively, present and absent in the superpotential. As a result, only $\tilde{N}^c_E, \tilde{\nu}_E$ would acquire masses $\sim M_I$, making their VEV's negligible, while the VEV's of $\tilde{N}'^c_E, \tilde{\nu}'^c_E$ would be $\lesssim M_W$ as before. The analogous suppression is possible for $\langle\tilde{\nu}\rangle$ and m_3, if there exists a pair $(\tilde{N}^c, \tilde{N}')$ with VEV's $\sim M_I$. With the VEV's $\langle\tilde{\nu}_E\rangle, \langle\tilde{N}^c_E\rangle, \langle\tilde{\nu}\rangle$ negligibly small, m_1, m_2, and m_3 will be predominantly radiative in origin and will not be large enough to generate heavy-quark masses—at least for t and b. The latter will instead obtain the bulk of their masses via large mixing with their mirrors—see Sec. III D.

For superstring-type models based on Calabi-Yau compactification, implementation of the above scenario requires discrete symmetries whose charge assignments differ for the **27** and $\overline{\mathbf{27}}$ sectors. There is *a priori* no reason why this could not happen for two distinct cohomology classes.[36] In fact, such a discrete symmetry, if it existed, would likely have different charge assignments for standard and mirror quarks so that it could lead automatically to a $U(1)_{PQ}$ (together with an invisible axion and an acceptable solution to the strong *CP* problem if $M_{PQ} \sim M_I \sim 10^{10}-10^{12}$ GeV).

The remainder of this section details the important radiative contributions[37] to m_1, m_2, m_3, cf. Fig. 1. Other diagrams which we do not discuss here involving $\hat{E}, \hat{E}^c$ Higgs-boson exchange will also contribute. They will, however, be negligible if $\hat{E}, \hat{E}^c$ obtain intermediate-mass scale masses; cf. the previous paragraph. How the trilinear scale vertex for m_1^Q in Fig. 1(a) is generated can be seen by considering the following terms in the superpotential [cf. (2.6a), (2.6b), (2.8), and (2.9c)], where the neutral superfields are Higgs superfields while the charged superfields are matter:

$$\lambda_1 \hat{u}\hat{u}^c \hat{N}^c_E + \lambda_2 \left[\hat{\phi}, \frac{\hat{n}^c \hat{n}'}{m_{Pl}}, \frac{\hat{N}^c \hat{N}'}{m_{Pl}} \right] \hat{N}^c_E \hat{N}'_E \; . \quad (3.10)$$

Note that we must require

$$\lambda_2 \left[\langle \phi \rangle, \frac{\langle \tilde{n}^c \tilde{n}' \rangle}{m_{Pl}}, \frac{\langle \tilde{N}^c \tilde{N}' \rangle}{m_{Pl}} \right] \lesssim 10^2 - 10^3 \text{ GeV}$$

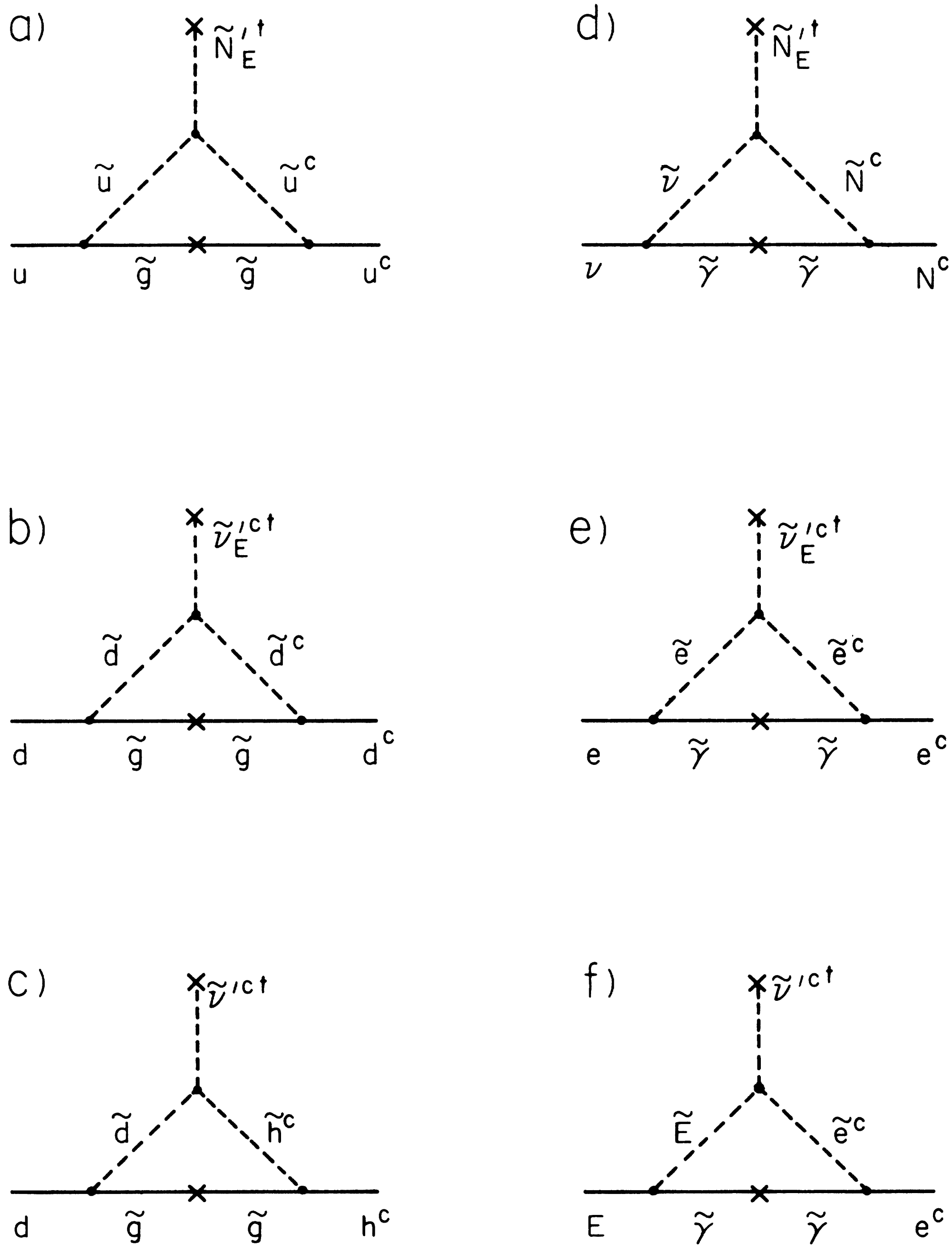

FIG. 1. Diagrams (a), (b), and (c) depict the radiative contributions to $m^Q_{1,2,3}$ and (d), (e), and (f) those to $m^L_{1,2,3}$.

so that $\tilde{N}'_E$ remains light enough to obtain a VEV $\leq M_W$. This relation is naturally satisfied for $\langle\phi\rangle \lesssim 1$ TeV or $\langle\tilde{n}^c\rangle\langle\tilde{n}'\rangle, \langle\tilde{N}^c\rangle, \langle\tilde{N}'\rangle \sim M_I \sim 10^{10}-10^{11}$ GeV. If $M_I \sim 10^{14}$ GeV, cf. Sec. II C, then we must replace the dimension-5 terms in Eq. (3.10) with dimension-7 terms. The scalar trilinear vertex is then obtained via

$$|F_{\tilde{N}_B^c}|^2 \supset \lambda_1\lambda_2 \left[\langle\phi\rangle, \frac{\langle\tilde{n}^c\tilde{n}'\rangle}{m_{\rm Pl}}, \frac{\langle\tilde{N}^c\tilde{N}'\rangle}{m_{\rm Pl}}\right] \tilde{u}\tilde{u}^c\tilde{N}'^\dagger_E \, . \tag{3.11}$$

Vertices for the other diagrams of Fig. 1 will be generated in analogous fashion.

One then expects radiative contributions to m_1^Q to be

$$m_1^Q \sim \frac{\alpha_s}{4\pi}\lambda_1\lambda_2 \left[\langle\phi\rangle, \frac{\langle\tilde{n}^c\tilde{n}'\rangle}{m_{\rm Pl}}, \frac{\langle\tilde{N}^c\tilde{N}'\rangle}{m_{\rm Pl}}\right] \langle\tilde{N}'_E\rangle$$
$$\times m_{\tilde{g}}[\max(m^2_{\tilde{u},\tilde{u}^c}, m^2_{\tilde{g}})]^{-1} \, , \tag{3.12}$$

where $m_{\tilde{g}}$ is the SUSY-breaking gluino mass $\sim m_{1/2}$ and $m_{\tilde{u},\tilde{u}^c}$ are SUSY-breaking squark masses $\sim m_0$. For $\alpha_s/4\pi \sim 0.01$ and $m_{1/2}, m_0 \sim 10^3$ GeV, we obtain

$$m_1^Q \lesssim 1 \text{ GeV} \times \lambda_1 \, . \tag{3.13}$$

Estimates for the other diagrams of Fig. 1 are obtained in a similar manner. Those of $m^L_{1,2,3}$ will be $\sim 0.1 m^Q_{1,2,3}$, not taking into account differences in quark and lepton Yukawa coupling strengths, since α_s is replaced by $\alpha_{\rm em}$ or α_W in Eq. (3.12), corresponding to photinos or Z-inos in the loop. Contributions to $m_{1,2}$ and similarly to m_3 (if $m'_3 \neq 0$) will certainly be large enough to be of significance in generating light-quark masses. In fact, if they provide the dominant contributions to the d quark and electron, one obtains $m_e/m_d = O(\alpha_W/\alpha_s) \sim 0.1$ as observed, even under the assumption of global or local leptoquark symmetry. These radiative contributions will also be important in obtaining quark mass mixing since, in general, the Yukawa couplings leading to the scalar vertices associated with Figs. 1(a)–(1c) will not be generation diagonal. We return to these radiative contributions in the following section, where we discuss a mechanism for obtaining the observed pattern for quark isospin breaking when the known generations are vectorlike.

D. The multigeneration case and quark isospin breaking

In this section we make some remarks concerning the multigeneration case with one or more mirror families present. For simplicity we work in the limit where quark mixing effects are obtainable as a perturbation about the generation-diagonal case, so the generations are essentially either chiral or vectorlike. The expressions for the fermion masses and mass eigenstates given by Eqs. (3.3), (3.5), and (3.5′) can then be used to study an important feature of the charged-fermion mass spectrum, i.e., the generation dependence of the intrafamily quark mass splittings. In particular, we shall point out a possible origin of the observed inequalities, $m_t > m_b$, $m_c > m_s$, $m_d > m_u$, which is linked to the low-energy vacuum structure and standard-mirror quark mixing. For a more detailed discussion, including comments on quark-lepton mass splittings, see Ref. 35.

First we show that with at least one mirror family present, the overall Yukawa coupling hierarchy need not exceed 10^2. The b and t quarks and τ lepton can acquire the bulk of their masses via large mixing with the mirrors, see Eqs. (3.3) and (3.5) and the next paragraph. A large standard-mirror VEV hierarchy is then favored, reducing the required Yukawa coupling hierarchy. In fact, the readily attainable VEV's $\langle\tilde{\nu}_E\rangle, \langle\tilde{N}^c_E\rangle, \langle\tilde{\nu}\rangle \sim 0$, are acceptable since radiative contributions can account for the remaining charged-fermion masses. The c-quark Yukawa coupling should be ~ 1, while the first-generation quark and lepton couplings should not exceed $\sim 10^{-2}$, see Eqs. (3.12) and (3.13) and recall that for leptons one replaces α_s with α_W or $\alpha_{\rm EM}$ in (3.12).

When three mirror families are present, a nice mechanism emerges which explains the observed pattern of quark isospin breaking. The point of view is taken that Yukawa couplings for up and down quarks will not be sufficiently different to explain $m_d \gtrsim m_u$, $m_{c,t} \gg m_{s,b}$, so that an additional mechanism relying on the vacuum structure of the theory is required. This is certainly true if $SU(2)_R$ is part of the four-dimensional gauge group, since then isospin is a symmetry of the Lagrangian. We assume that one of the mechanisms of Sec. III C for splitting ordinary and mirror fermion masses is operative, with $m_{1,2,3}$ primarily radiative in origin. The masses of up and down quarks for a vectorlike family are given in Eqs. (3.3b) and (3.5d), respectively, for $m_i, r < m'_i$. for the down quarks consider, for simplicity, the case where $s_2, s'_2 \gg s_1, s'_1$ and the first term in the numerator of (3.5d) is the dominant one. In this limit, one then obtains

$$m_d \sim \frac{|r^2 - m_2 m'_2|}{m'_2} \tag{3.14}$$

which is of the same form as the expression for m_u. The situation is the same if there are no exotic quarks in the theory. Hence m_u and m_d then consist of a seesaw part $\sim r^2/m'_{1,2}$ and a direct part $\sim m_{1,2}$. The seesaw part will clearly be the dominant source of mass for the third family of quarks[38] and probably for the charmed quark as well. The direct part may be relevant for the strange quark but will certainly be competitive for the first family of quarks, cf. (3.13). It will also be an important source of quark mixing. The important one-loop contributions to m_1 and m_2 are proportional to $\langle\tilde{N}'_E\rangle$ and $\langle\tilde{\nu}'^c_E\rangle$, respectively. The same is also true of the small (≤ 10 MeV) tree-level contributions of Eq. (3.9). If $\langle\tilde{\nu}'^c_E\rangle > \langle\tilde{N}'_E\rangle$, then we expect $m'_2 > m'_1$, i.e., mirror down quarks more massive than mirror up quarks, and $m_2 > m_1$ for all three families,[39] so that the seesaw contribution is larger for up quarks while the direct contribution is larger for down quarks. As a result, we expect $m_t > m_b$, $m_c > m_s$ (since the seesaw mechanism dominates for these families), while $m_d > m_u$ becomes possible, since we expect the direct contributions to be competitive for the first family.

A central requirement of the above model is $\langle\tilde{\nu}'^c_E\rangle > \langle\tilde{N}'_E\rangle$. This inequality or $\langle\tilde{\nu}_E\rangle > \langle\tilde{N}^c_E\rangle$ could be due, simply, to gauge invariance under the extended

E_6-based electroweak group, whereas the reverse inequalities could not. What is required is a pair of Higgs superfields $(\tilde{N}^c, \tilde{N}')$ for which $\langle \tilde{N}^c \rangle, \langle \tilde{N}' \rangle \lesssim (m_0) \sim 1$ TeV. The F^2 terms resulting from the couplings $\hat{N}^c \hat{\nu} \hat{N}_E^c$, $\hat{N}' \hat{\nu}'^c \hat{N}'_E$, i.e., $|N^c|^2 |N_E^c|^2$ and $|N'|^2 |N'_E|^2$, would lead to mass contributions of order $\langle \tilde{N}^c \rangle^2, \langle \tilde{N}' \rangle^2$ to $\tilde{N}_E^c$ and $\tilde{N}'_E$, respectively. By contrast, because of E_6 gauge invariance, no Yukawa couplings exist between $\hat{N}^c$ and $\hat{\nu}_E$ or $\hat{N}'$ and $\hat{\nu}'^c_E$, see Eq. (2.6), so that $\tilde{\nu}_E$ and $\tilde{\nu}'^c_E$ cannot receive mass contributions from the VEV's of $\tilde{N}^c$ and $\tilde{N}'$ in the F^2 sector. As a result, $m^2_{N_B^c}$ and $m^2_{N'_B}$ would be larger than $m^2_{\tilde{\nu}_B}$ and $m^2_{\tilde{\nu}'^c_E}$, respectively, and the desired VEV inequality follows. Supersymmetry is crucial for obtaining this result. Global symmetries cannot ensure the absence of the term $|\nu_E^{c\prime}|^2 |N'|^2$ in the scalar potential of a nonsupersymmetric model.

Generally, the number of quark families receiving seesaw mass contributions cannot exceed the number of mirror families; therefore, two mirror families are required above to explain $m_t > m_b$ and $m_c > m_s$. The inversion resulting from the "direct" radiative masses is probably too large in the absence of a partially compensating seesaw, which is why we included a third mirror family above.

In the case of models discussed above, because $m^{Q,L}_{1,2,3}$ are primarily radiative in origin, the Yukawa couplings of first-generation quarks or the electron need not be smaller than 10^{-2}, cf. Eqs. (3.12) and (3.13). Since m_b/m_e or m_t/m_u is of $\sim 10^4$, the hierarchy of couplings leading to standard-mirror mixing need not exceed 10^2, since seesaw mass contributions to ordinary fermion masses are approximately proportional to the coupling squared. Therefore, the overall coupling hierarchy responsible for the observed family structure need not exceed 10^2 which is a dramatic improvement over 10^5 required for the standard model, or 10^4 for the two-Higgs-doublet models; cf. del Aguila, Ref. 9.

We conclude this section with some phenomenological remarks. The models considered require large mixing of heavy quarks with mirror quarks. The large τ mass may also be generated via mixing with a mirror lepton. It has been checked in Ref. 35 that the mixings required are consistent with flavor-changing neutral-current (FCNC) constraints. Such large mixing leads to smaller forward-backward asymmetries in Z decay than expected in the standard model, as there will be a significant admixture of $V+A$ in the neutral currents. Such deviation may be observable at the Stanford Linear Collider (SLC) or CERN LEP, especially for $e^+e^- \to b\bar{b}$, for which LEP should be able to measure A_{FB} to within $\pm$ 3% (Ref. 40). Also, because $\langle \tilde{N}'_E \rangle < \langle \tilde{\nu}'^c_E \rangle$ (or $m'_u < m'_d$), we expect larger standard-mirror mixing and, therefore, smaller forward-background asymmetry for the u quark than for the d quark within a given family; see Eqs. (3.3), (3.5), and (3.5′).

IV. NEUTRAL LEPTONS

We shall begin our discussion of the neutral leptons with the Dirac case, for there more entries in the neutrino mass matrix are required to vanish. This, in turn, has some implications for the charged-lepton mass matrix. The conditions for an ultralight neutrino are considerably relaxed in the general Majorana case; moreover, such solutions persist in the Majorana case when the full 27 and $\overline{27}$ representations of E_6 do not survive compactification. For ease of presentation, we refer to all r entries in M^N generically as r, and take $m_{11} \sim m_{12} \equiv m_1$, $m'_{11} \sim m'_{12} \equiv m'_1$.

A. Dirac case: $m_2 = m'_2 = m_3 = m'_3 = 0$

The Dirac case for the neutrino mass matrix is distinguished by the vanishing of m_2, m'_2, m_3, and m'_3, as well as all t's and t's, since these elements lead to Majorana contributions as indicated in Secs. IV B and IV C. As we shall see, this case cannot be implemented for models with $SU(3)_L$ or $SU(2)_R$ invariance because of resulting implications for the charged-lepton masses.

$Q=0$: 27 only

The solutions here are given approximately by

$$\pm(s_1^2+s_2^2)^{1/2}: \quad \frac{1}{\sqrt{2}}\left[\frac{1}{(s_1^2+s_2^2)^{1/2}}(s_1\nu_L+s_2\nu_{EL}) \pm N_{EL}^c\right], \tag{4.1a}$$

$$\pm m_1: \quad \frac{1}{\sqrt{2}}\frac{1}{(s_1^2+s_2^2)^{1/2}}[(s_2\nu_L-s_1\nu_{EL}) \pm(s_2N_L^c-s_1n_L^c)] , \tag{4.1b}$$

$$0: \quad \frac{1}{(s_1^2+s_2^2)^{1/2}}(s_1N_L^c+s_2n_L^c) . \tag{4.1c}$$

These results are unsatisfactory, for the ordinary doublet neutrino is not ultralight but has a mass comparable to the charged-lepton mass.[41]

$Q=0$: $27+\overline{27}$

Here we can find a suitable solution with an ultralight mass for the doublet neutrino if we note the determinant of the mass matrix is given by

$$\mathrm{Det}M^N = -[r(r^2-m_1m'_1)^2 - r(r^2-m_1m'_1-uu')(s_1s'_1+s_2s'_2) + m_1m'_1(s'_1s_2u'_2+s_1s'_2u_2) + (s'_2s_1u_1+s'_1s_2u'_1) - u_1u'_1r(r^2-u_2u'_2) - u_1u'_1r(r^2-u_2u'_2) - rm_1m'_1(u_1u'_2+u'_1u_2)]^2 \tag{4.2}$$

and impose on the entries in M^N the conditions

$$(a)\quad s_1s_1'+s_2s_2'\simeq 0 \;, \tag{4.3a}$$

$$(b)\quad s_1's_2u'+s_1s_2'u\simeq 0 \;, \tag{4.3b}$$

$$(c)\quad s_2's_1u_1+s_1's_2u_1'\simeq 0 \;. \tag{4.3c}$$

Presumably these three conditions can result from extra discrete symmetries in the model. In particular, discrete symmetries may lead to $u_1'=u_2'=s_1=s_2'=0$ in the matrix of (2.16) as well as $m_2=m_2'=m_3=m_3'=0$. Then the approximate solutions become

$$\pm s_2:\quad \frac{1}{\sqrt{2}}(\nu_{EL}\pm N_{EL}^c) \;, \tag{4.4a}$$

$$\pm s_1':\quad \frac{1}{\sqrt{2}}(N_{EL}'\pm \nu_L'^c) \;, \tag{4.4b}$$

$$\pm(r^2+m_1'^2)^{1/2}:\quad \frac{1}{\sqrt{2}}\left[n_L'\pm\left[\nu_{EL}'^c+\frac{r}{m_1'}n_L^c\right]\right] \;, \tag{4.4c}$$

$$\pm r:\quad \frac{1}{\sqrt{2}}\left[\left[N_L'+\frac{m_1}{r}\nu_L\right]\pm N_L^c\right] \;, \tag{4.4d}$$

$$\pm\frac{(r^2-m_1m_1')^2}{(r^2+m_1'^2)^{1/2}s_2s_1'}:\quad \frac{1}{\sqrt{2}}\left[\left[\nu_L-\frac{m_1}{r}N_L'\right]\pm\left[n_L^c-\frac{r}{m_1'}\nu_{EL}'^c\right]\right] \;. \tag{4.4e}$$

In (4.4a)–(4.4e) we have, for simplicity, set $u_1=u_2=0$ and again have assumed that $m_i, r \ll m_i' \ll s_i, s_i'$. To satisfy weak universality constraints, we also require $m_1 \ll r$, see (4.4e).

It is important to check that the Dirac neutrino mass conditions, $m_2^N=m_2'^N=m_3^N=m_3'^N=0$ and $u_1'^N=u_2'^N=s_1^N=s_2'^N=0$, where the superscripts refer to the corresponding mass matrix, do not impose undesirable results for the charged leptons. The Dirac neutrino conditions together with $SU(2)_L$ invariance imply $s_1^e=s_2'^e=u'^e=0$. $SU(2)_R$ invariance would further imply $m_3^e=m_3'^e=0$. The determinant of the charged-lepton mass matrix would then become

$$\det M^e=[r_3'(m_2m_2'r-r_1'r_2'r)]^2 \;.$$

Since there will be two Dirac eigenstates with masses $\simeq s_1'$ and s_2, respectively, the above equation would imply the existence of an ultralight charged lepton. $SU(3)_L$ invariance would lead to $m_3^e=m_3'^e=m_2^e=m_2'^e=0$ and again, as for $SU(2)_R$ invariance, there would be at least one ultralight charged lepton. It is therefore apparent that a seesaw mechanism for an ultralight Dirac neutrino is not a viable option for models possessing $SU(3)_L$ or $SU(2)_R$ invariance.

In contrast, if $SU(2)_R$ or $SU(3)_L$ are not contained in the gauge group, the light electron mass eigenstate follows from (3.5d) and is given approximately by

$$\pm m_2:\quad \frac{1}{\sqrt{2}}\left[\left[e_L-\frac{r}{m_2'}e_L'\right]\pm\left[e_L^c-\frac{r}{m_2'}e_L'^c\right]\right] \tag{4.5}$$

while the ultralight Dirac neutrino corresponds to (4.4e). To satisfy weak universality constraints, we set $m_1 \ll r$ in (4.4e). For purposes of numerical illustration, we set $m_1\sim m_2\sim 1$ MeV, $m_1'\sim m_3'\sim 100$ GeV, and $r\sim 1$ GeV, where r can be obtained from

$$r\sim\frac{V_iV_i'}{m_{\rm Pl}}\sim\lambda_r\frac{(10^{10}-10^{11})^2}{10^{18}}\ {\rm GeV} \;. \tag{4.6}$$

We then find $m_e\sim 1$ MeV and

$$m_{\rm light}^{\nu}\simeq\frac{(r^2-m_1m_1')^2}{m_1's_2s_1'}\sim 10^{-15}-10^{-13}\ {\rm eV} \tag{4.7a}$$

$$\sim 10^{-8}-10^{-6}{\rm eV} \tag{4.7b}$$

$$\sim 0.1-10\ {\rm eV} \tag{4.7c}$$

according to whether one ($10^{10}-10^{11}$ GeV), two ($10^{10}-10^{11}$ and 10^3-10^4 GeV), or one (10^3-10^4 GeV) intermediate scales are present, respectively, in the denominator. The latter case was the one discussed previously by one of us in Ref. 12.

B. Majorana case with $t_i=t_i'=t_{12}=t_{12}'=0$

$Q=0$: 27 only

Now we find

$$\pm(s_1^2+s_2^2)^{1/2}:\quad \frac{1}{\sqrt{2}}\left[\frac{1}{(s_1^2+s_2^2)^{1/2}}(s_1\nu_L+s_2\nu_{EL})\pm N_{EL}^c\right], \tag{4.8a}$$

$$\pm m_1:\quad \frac{1}{\sqrt{2}}\frac{1}{(s_1^2+s_2^2)^{1/2}}[(s_2\nu_L-s_1\nu_{EL})\pm(s_2N_L^c-s_1N_L^c)] \;, \tag{4.8b}$$

$$2m_1\frac{m_2s_2+m_3s_1}{s_1^2+s_2^2}:\quad \frac{1}{(s_1^2+s_2^2)^{1/2}}(s_1N_L^c+s_2n_L^c) \;. \tag{4.8c}$$

Here again as in the Dirac case with 27 only, the wrong neutrino is ultralight.

$Q=0$: $27+\overline{27}$

Here the same conditions (4.3) as for the Dirac case must obtain to achieve an ultralight-neutrino mass. With these conditions satisfied via $u_1'=u_2'=s_1=s_2'=0$, we find the determinant of M^N is given approximately by

$$\operatorname{Det}M^N\simeq 2m_1m_1'2(r^2-m_1m_1')^2(m_3s_1+m_2s_2)\times(m_3's_1'+m_2's_2') \;. \tag{4.9}$$

The eight heaviest masses and eigenstates are nearly Dirac type and are given approximately by (4.4a)–(4.4d), where again, for simplicity, we have taken $u_1=u_2=0$. The lightest Dirac pair (4.4e), however, splits into two Majorana masses:

$$2\frac{m_1m_3'}{s_1'}\frac{r^2-m_1m_1'}{r^2}:\ \nu_L-\frac{m_1}{r}N_L'\ , \tag{4.10a}$$

$$2\frac{m_1'm_2}{s_2}\frac{r^2-m_1m_1'}{r^2+m_1'^2}:\ n_L^c-\frac{r}{m_1'}\nu_{EL}'^c\ . \tag{4.10b}$$

The light electron is still given by Eq. (4.5), and the correct mass is obtained with $m_1\sim m_2\sim 1$ MeV, $m_1'\sim m_3'\sim 100$ GeV, $r\sim 1$ GeV, and $u'^e\sim 0$ as before.

Estimates for the two ultralight-neutrino masses in (4.10) follow with $s_2\sim s_1'\sim 10^{10}-10^{11}$ GeV: for example,

$$m_{\rm light}\simeq 2\frac{m_1m_3'}{s_1'}\sim 2\times(10^{-3}-10^{-2})\ \text{eV}\ , \tag{4.11a}$$

$$m_{\rm lightest}\simeq 2\frac{m_1'm_2}{s_2}\frac{r^2-m_1m_1'}{m_1'^2}\sim 2\times(10^{-7}-10^{-6})\ \text{eV}\ . \tag{4.11b}$$

The doublet neutrino is ultralight but can be noticeably heavier than the lightest one which transforms mainly as the O(10)-singlet member of the 27.

C. General Majorana case

Q=0: 27 only

This case was treated previously by Nandi and Sarkar.[42] Here the determinant of the mass matrix is given by

$$\text{Det}M^N=m_1^2(s_2^2t_2'+s_1^2t_1'+2s_1s_2t_{12}')-2m_1^3(m_2s_2+m_3s_1)\ ; \tag{4.12}$$

with

$$m_i\ll t_j',t_{12}'\ll s_k \tag{4.13a}$$

the doublet neutrino can be made ultralight. If we take $s_1\lesssim 0.1s_2$ so as to satisfy weak universality, from (2.13b) it is reasonable to assume

$$t_1'<t_{12}'<t_2'\ . \tag{4.13b}$$

The masses and eigenvectors are then given approximately by

$$\pm(s_1^2+s_2^2)^{1/2}:\ \frac{1}{\sqrt{2}}\left[\frac{1}{(s_1^2+s_2^2)^{1/2}}(s_1\nu_L+s_2\nu_{EL})\pm N_{EL}^c\right], \tag{4.14a}$$

$$t_2'-\frac{t_{12}'^2}{t_1'-t_2'}:\ \frac{1}{\left[1+\left[\frac{t_{12}'}{t_1'-t_2'}\right]^2\right]^{1/2}}\times\left[-\frac{t_{12}'}{t_1'-t_2'}N_L^c+n_L^c\right], \tag{4.14b}$$

$$t_1'+\frac{t'^2}{t_1'-t_2'}:\ \frac{1}{\left[1+\left[\frac{t_{12}'}{t_1'-t_2'}\right]^2\right]^{1/2}}\times\left[N_L^c+\frac{t_{12}'}{t_1'-t_2'}n_L^c\right], \tag{4.14c}$$

$$\frac{m_1^2(s_2^2t_2'+s_1^2t_1'+2s_1s_2t_{12}')}{(s_1^2+s_2^2)(t_{12}'^2-t_1't_2')}:\ \frac{1}{(s_1^2+s_2^2)^{1/2}}\times(s_2\nu_L-s_1\nu_{EL})\ . \tag{4.14d}$$

Estimates here reveal that the ultralight neutrino has a mass of order

$$m_{\rm light}\simeq\frac{m_1^2t_2'}{t_{12}'^2-t_1't_2'}\sim 1\ \text{eV} \tag{4.15}$$

with $t_2'\sim 10^4$ GeV, although a very small result is obtained if we would take $s_2\sim 10^{14}$ GeV so that $t_2'\sim 10^{10}$ GeV.

Q=0: 27+$\overline{27}$

In the full general Majorana case with pairing of the standard and mirror families, as discussed below, an ultralight doublet neutrino is possible if

$$\text{(a)}\ \ t_1\simeq t_{12}\simeq 0\ , \tag{4.16a}$$

$$\text{(b)}\ \ r,m_i,m_i',u_i,u'\ll t_1',t_{12}',t_2',t_2\ . \tag{4.16b}$$

Condition (a) can presumably be satisfied with extra discrete symmetries. Since $t,t'\ll s,s'$, see (2.13b), leading contributions to the determinant of the mass matrix, taking (4.16) into account, are of order

$$\text{Det}M^N\simeq s^4t^2\times(\text{terms quartic in } u_i,u_i',m_i,m_i',r)\ . \tag{4.17}$$

Clearly there will be four eigenvalues $\sim s$, and three eigenvalues $\sim t$, see upper left 6×6 corner and lower right 4×4 corner of (2.16), respectively. Equations (4.16b) and (4.17) therefore imply the existence of an ultralight neutrino. We shall require $s_1\lesssim 0.1s_2$ to satisfy weak universability constraints, see (4.18g), so that again (4.13b) will be assumed in what follows. With conditions (4.13b), and (4.16) the full neutral spectrum becomes

$$\pm(s_1^2+s_2^2)^{1/2}:\ \frac{1}{\sqrt{2}}\left[\frac{1}{(s_1^2+s_2^2)^{1/2}}(s_1\nu_L+s_2\nu_{EL})\pm N_{EL}^c\right], \tag{4.18a}$$

$$\pm(s_1'^2+s_2'^2)^{1/2}:\quad \frac{1}{\sqrt{2}}\left[N_{EL}'\pm\frac{1}{(s_1'^2+s_2'^2)^{1/2}}(s_1'\nu_L'^c+s_2'\nu_{EL}'^c)\right], \tag{4.18b}$$

$$t_2:\ n_L', \tag{4.18c}$$

$$t_1'+\frac{t_{12}'^2}{t_1'-t_2'}:\quad \frac{1}{\left[1+\left[\frac{t_{12}'}{t_1'-t_2'}\right]^2\right]^{1/2}}\left[N_L^c-\frac{r}{m_1'}\nu_L'^c+\frac{t_{12}'}{t_1'-t_2'}n_L^c\right], \tag{4.18d}$$

$$t_2'-\frac{t_{12}'^2}{t_1'-t_2'}:\quad \frac{1}{\left[1+\left[\frac{t_{12}'}{t_1'-t_2'}\right]^2\right]^{1/2}}\left[-\frac{t_{12}'}{t_1'-t_2'}\left[N_L^c-\frac{r}{m_1'}\nu_L'^c\right]+n_L^c\right], \tag{4.18e}$$

$$\pm(r^2+m_1'^2)^{1/2}:\quad \frac{1}{\sqrt{2}}\left\{\left[N_L'+\frac{r}{m_1'}\nu_L\right]\pm\frac{1}{(s_1^2+s_2^2)^{1/2}}\left[s_2'\left[\nu_L'^c+\frac{r}{m_1'}N_L^c\right]-s_1'\nu_{EL}'^c\right]\right\}, \tag{4.18f}$$

$$-\frac{(r^2-m_1m_1')^2t_2'}{(r^2+m_1'^2)(t_1't_2'-t_{12}'^2)}:\quad \frac{1}{(s_1^2+s_2^2)^{1/2}}\left[s_2\left[\nu_L-\frac{r}{m_1'}N_L'\right]-s_1\nu_{EL}\right]. \tag{4.18g}$$

In the above, we have set $u_i=u_i'=0$ for simplicity and only kept terms up to $O(r/m')$. An estimate of the ultralight-neutrino mass is

$$m_{\text{light}}\sim 0.1-10\ \text{eV} \tag{4.19a}$$

$$\sim 10^{-7}-10^{-5}\ \text{eV} \tag{4.19b}$$

when $t_1'\sim 10^4$ GeV and $\sim 10^{10}$ GeV is taken, respectively.

D. $16+\overline{16}$

Here again we entertain the possibility that the full 27 and $\overline{27}$ representations of E_6 do not survive compactification. In particular, we examine numerically the case where the would-be 16 and $\overline{16}$ representations of SO(10) remain massless. The neutral mass matrix of (2.16) is then replaced by the following in the basis

$$Q=0:\ B^N=\{\nu,\nu'^c,N^c,N'\}_L$$

We have

$$M^N=\begin{bmatrix}0 & r & m_1 & 0\\ r & 0 & 0 & m_1'\\ m_1 & 0 & t_1' & r\\ 0 & m_1' & r & t_1\end{bmatrix}. \tag{4.20}$$

Only the general Majorana case yields ultralight neutrinos as follows.

Q=0: 16 only

We have

$$t_1':\ N_L^c, \tag{4.21a}$$

$$\frac{m_1^2}{t_1'}:\ \nu_L. \tag{4.21b}$$

Q=0: $16+\overline{16}$

We set $t_1=0$ in order to obtain a seesaw mechanism for ν. Then we obtain approximately:

$$t_1':\ N_L^c, \tag{4.22a}$$

$$\pm(r^2+m_1'^2)^{1/2}:\quad \frac{1}{\sqrt{2}}\left[\frac{1}{(r^2+m_1'^2)^{1/2}}(r\nu_L+m_1'N_L')\pm\nu_L'^c\right], \tag{4.22b}$$

$$-\frac{(r^2-m_1m_1')^2}{(r^2+m_1'^2)t_1'}:\quad \frac{1}{(r^2+m_1'^2)^{1/2}}(m_1'\nu_L-rN_L'). \tag{4.22c}$$

We thus see that the ultralight-neutrino mass predictions persist in the Majorana case when only the standard-matter representations remains light. This is not true in the Dirac case, for the ultralight mass there relies crucially on the structure of the large 10×10 matrix.

We conclude this section by noting that all of our light-neutrino solutions predict a mirror Dirac neutrino with mass $(r^2+m_1'^2)^{1/2}\lesssim M_W$. The phenomenology of such neutrinos has already been discussed in the literature.[43] In particular, Z decay can lead to a monojet $+\not{p}_T$, whereas W decay can lead to an isolated charged lepton plus jets.

V. SUMMARY

A. Discussion of results

We begin by summarizing the quark and charged-lepton results. Our basic assumptions have already been listed in Sec. I. The $Q=\pm\frac{2}{3}$ quark mass matrix involves just the u_L and u_L^c quarks of the 16 and possibly the mirror quarks $u_L'^c$ and u_L' of the $\overline{16}$. The $Q=\pm\frac{1}{3}$ mass matrix will involve the exotic quarks h_L, h_L^c and possibly $h_L', h_L'^c$ if **27**'s and $\overline{\mathbf{27}}$'s are present in the theory. However, in the limit $s_2 \gg s_1$, see (2.15), the mass eigenvalues and eigenstates of the light down quark and its mirror reduce to the same form as those of the $\mathbf{16}+\overline{\mathbf{16}}$ case. Similar statements hold for the charged leptons.

Mechanisms are possible for splitting of ordinary and mirror quark and charged-lepton masses with the mirrors easily acquiring masses >22 GeV. Mixing of ordinary and mirror quarks can easily generate heavy quarks and τ lepton masses via seesawlike contributions, while the radiative masses of Fig. 1 will be of significance for the light quarks and leptons. These same diagrams will also be important in generating the Kobayashi-Maskawa (KM) mixing angles.

If at least one mirror family is present, the overall coupling hierarchy responsible for the fermion mass hierarchy need not exceed 10^2 compared to 10^5 for the standard model. If quark isospin is an exact or nearly exact symmetry of the Lagrangian, an aesthetically attractive possibility, the observed breaking must be primarily due to vacuum alignment. The inequalities $m_t > m_b$, $m_c > m_s$, $m_d > m_u$ then favor a vectorlike model with seesaw and radiative mass contributions playing crucial roles. The Yukawa couplings of isodoublet u and d quarks need not differ, while the overall coupling hierarchy responsible for the fermion mass hierarchy need not exceed 10^2 compared to 10^5 for the standard model. The presence of additional quarks from mirror families may provide a natural setting for the attractive MPP scenario[14] which is insensitive to the initial $\lesssim 1$ gauge couplings at $M_{\rm GUT}$. Clearly, all of the above features would facilitate the task of the correct string vacuum being correct.

Turning to the neutrino results, we have observed that no seesaw mechanism is possible for Dirac neutrinos with the **27** alone; however, if both standard and mirror neutrinos are present in the **27** and $\overline{\mathbf{27}}$, and if $SU(2)_R$ is not part of the low-energy gauge group after compactification, a seesaw mechanism can exist to yield ultralight Dirac neutrinos. Clearly the Dirac neutrino case requires the most restrictive global symmetries, as 16 entries are required to vanish in (2.16). Unless high mass scale entries are absent in the neutral lepton mass matrix, their mass is typically in the $10^{-15}-10^{-6}$ eV range, which is exceedingly small. Since the Dirac seesaw mechanism relies heavily on the features of the 10×10 matrix, if only the SO(10) **16** and $\overline{\mathbf{16}}$ representations are present rather than the **27** and $\overline{\mathbf{27}}$, an ultralight neutrino mass cannot be obtained.

It is interesting to note that the right-handed component of the allowed ultralight Dirac neutrino transforms principally as an SO(10) singlet, rendering it nearly sterile. Hence it is not in conflict with astrophysical limits on the number of light-neutrino degrees of freedom consistent with the observed abundance of the elements from primordial nucleosynthesis. Also, we observe that a new Dirac neutrino is predicted with mass r easily greater than the cosmological lower bound[44] of 2 GeV. This neutrino couples mainly to $SU(2)_R$ weak bosons, so that it would have escaped detection given present-day limits.[45]

For the Majorana seesaw mechanism, we have observed that an ultralight neutrino is possible under more general conditions. In particular, for the 10×10 neutral-lepton matrix (2.16) only two entries are required to vanish, cf. (4.16a). In this general case with t entries of order 10^4 GeV (10^{10} GeV), a light neutrino in the range $0.1-10$ eV ($10^{-7}-10^{-5}$ eV) can be obtained in either the standard fermion case as shown by Nandi and Sarkar,[42] or in the standard-mirror paired case. Unlike the Dirac case, it is of considerable interest to note that this light-neutrino solution persists even if only $\mathbf{16},\overline{\mathbf{16}}$ matter fermions are present. In the restricted Majorana case with vanishing t entries, a light doublet neutrino of mass $10^{-3}-10^{-2}$ eV is possible with mirrors present; however, the primary singlet n_L^c is several orders of magnitude lighter. Again it is nearly sterile and hence not in conflict with astrophysical limits on the number of light-neutrino degrees of freedom.

B. Realizations in superstring theories

In this section we discuss, very generally, how the different possibilities for matter superfield content we have suggested may be realized in actual superstring scenarios. The Calabi-Yau compactifications are primarily discussed in this context, but a brief remark is included concerning four-dimensional heterotic superstrings, which are a promising alternative.

We first briefly review the flux breaking mechanism[16] together with the associated determination of survivors from the vectorlike sector. For a discrete group G acting freely on a simply connected manifold $K_{\rm sc}$, one can obtain a nonsimply connected manifold $K = K_{\rm sc}/G$. In this space there may exist nontrivial E_6 gauge field configurations contributing to Wilson loops given by $U_g = P\exp(i\int_\Gamma A_m dx^m)$, where $g\in G$ and Γ is a noncontractible loop in K. The U_g form a discrete subgroup $\overline{G}$ of E_6, which is homomorphic to G. The group E_6 is broken to the subgroup H under the action of U_g, where $[H, U_g]=0$, for every $g\in G$. For G Abelian, the U_g can be parametrized[46] in the $SU(3)_c\times SU(3)_L\otimes SU(3)_R$ basis as

$$U_g = 1\times\begin{bmatrix}\alpha & 0 & 0\\ 0 & \alpha & 0\\ 0 & 0 & \alpha^{-2}\end{bmatrix}\times\begin{bmatrix}\beta\delta & 0 & 0\\ 0 & \beta\delta^{-1} & 0\\ 0 & 0 & \beta^{-2}\end{bmatrix}, \tag{5.1a}$$

whereas for $\overline{G}$ non-Abelian, the U_g can be parametrized as

$$U_g = 1\times\begin{bmatrix}\alpha & 0 & 0\\ 0 & \alpha & 0\\ 0 & 0 & \alpha^{-2}\end{bmatrix}\times\begin{bmatrix}\beta & 0 & 0\\ 0 & v_{11} & v_{12}\\ 0 & v_{21} & v_{22}\end{bmatrix} \tag{5.1b}$$

with $\beta \det v = 1$. The above parametrizations reflect the fact that H must contain $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$. The field content of the vectorlike sector following compactification is given by those representations of H contained in the paired **27**'s and $\overline{\mathbf{27}}$'s which are singlets under $G\oplus\overline{G}$.

In what follows next we point out how, given a Calabi-Yau manifold with $SU(3)_H$ holonomy and vectorlike matter sector (chiral matter sector optional), compactification may lead to the different combinations of matter and Higgs representations which we have considered. In doing so, we elucidate on some properties which such manifolds must possess. As previously mentioned, the number of paired **27**'s and $\overline{\mathbf{27}}$'s is given by the Betti-Hodge number $b_{1,1}$ of K_{sc}. For Calabi-Yau spaces there will always exist at least one pair which is invariant under G, so that the survivors must be invariant under $\overline{G}$. It is not difficult to see from the parametrization of U_g given above that, for such pairs, the standard and mirror quarks contained in the **16** and $\overline{\mathbf{16}}$ of SO(10) cannot survive in the presence of Wilson symmetry breaking, i.e., for $U_g \neq 1$. For this reason we do not consider $b_{1,1}=1$ a viable option for realizing models with one vectorlike generation.

On the other hand, Calabi-Yau manifolds with $b_{1,1}>1$ are promising for realizations of the kinds of models we have been discussing. Here there exists the possibility that the additional $\mathbf{27}+\overline{\mathbf{27}}$ representations are not G singlets. The condition that they contain survivors after flux breaking, i.e., that they be $G+\overline{G}$ singlets can now, in principle, be satisfied for some or all of their components.[47] In particular, it may be that for the vectorlike sector either full matter **27**'s and $\overline{\mathbf{27}}$'s or only nonexotic matter fields contained in **16**'s and $\overline{\mathbf{16}}$'s of SO(10) survive compactification, both possibilities having been discussed earlier. It is also possible that no Higgs color triplets survive compactification. This was already pointed out for $b_{1,1}=1$ in Ref. 18. In light of the above, if $\chi=0$ (cf. next paragraph) an attractive possibility arises. As now there are no **27**'s protected by the index theorem, it follows that the exotic color triplets which could mediate fast proton decay may not exist at all in the compactified low-energy theory.

Compactifications on Calabi-Yau manifolds can certainly lead to the presence of mirror families. An example with $\chi=0$ and large $b_{1,1}$ is known to exist[48] and would lead to a vectorlike low-energy theory. Calabi-Yau manifolds with $\chi=-2$ and -4 have also been constructed.[7,48] Perhaps these can lead to low-energy models possessing at least one or two mirror families, respectively, along with three ordinary families.

Four-dimensional heterotic superstring constructions with $N=1$ supersymmetry[49] can also contain mirror families for suitable choice of boundary conditions.[50] The gauge groups for such models can contain SO(10) or one of its subgroups among its factor. One expects standard-matter representations with respect to such factors, corresponding to **16**'s and $\overline{\mathbf{16}}$'s of SO(10).

Given the attractive features of $N=1$ SUSY model possessing mirror families, it would be interesting to check if any of the above candidate string vacua lead to their realization.

ACKNOWLEDGMENTS

We wish to thank Jon Rosner for encouraging us to pursue this problem, for his interest in the early stages of this work, and for his critical reading of the manuscript. One of us (A.L.K.) thanks Rabi Mohapatra for useful conversations. It is also a pleasure to acknowledge the kind hospitality of the Fermilab Theory Group, where much of this research was carried out. Fermilab is operated by Universities Research Association, Inc. under contract with the United States Department of Energy. The research of C.H.A. was supported in part by Grants Nos. PHY-8304880 and PHY-8704240 from the National Science Foundation. The research of A.L.K. was supported in part by the United States Department of Energy under Contract No. DE-AC02-82ER-40073. This work was submitted by A.L.K. in partial fulfillment of requirements for the Ph.D. in Physics at the University of Chicago.

---

*Present address: Department of Physics and Astronomy, University of Maryland, College Park, MD 20742.

†Permanent address.

[1]M. Green and J. Schwarz, Phys. Lett. **149B**, 117 (1984); **151B**, 21 (1985).

[2]D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985); Nucl. Phys. **B256**, 253 (1985).

[3]R. Bluhm and L. Dolan, Phys. Lett. **169B**, 347 (1986); H. Kawai, D. C. Lewellen, and S. H. H. Tye, Phys. Rev. Lett. **57**, 1832 (1986); I. Antoniadis, C. P. Bachas, and C. Kounnas, Nucl. Phys. **B289**, 87 (1987).

[4]P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985); in *Anomalies, Geometry and Topology*, proceedings of the Symposium, Argonne, Illinois, 1985, edited by W. A. Bardeen and A. R. White (World Scientific, Singapore, 1985).

[5]E. Witten, Nucl. Phys. **B258**, 75 (1985).

[6]E. Witten, Nucl. Phys. **B268**, 79 (1986).

[7]A. Strominger and E. Witten, Commun. Math. Phys. **101**, 341 (1985).

[8]D. Errede *et al.*, Phys. Lett. **149B**, 519 (1984); M. L. Perl *et al.*, Phys. Rev. D **32**, 2859 (1985); Mark J Collaboration, B. Adeva *et al.*, Phys. Lett. **152B**, 439 (1985).

[9]Upper bounds for such mixing can be obtained from FCNC constraints. See, for example, F. del Aguila, Ann. Phys. (N.Y.) **165**, 237 (1985); F. del Aguila and M. J. Bowick, Nucl. Phys. **B224**, 107 (1983); I. Umemura and K. Yamamoto, Phys. Lett. **108B**, 37 (1972); A. Kagan (in preparation).

[10]This was noticed for matter parity in J. D. Breit, B. A. Ovrut, and G. C. Segrè, Phys. Lett. **158B**, 33 (1985).

[11]M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe*, Tsukuba, Ibaraki, Japan, 1979, edited by A. Sawada and S. Sugamoto (KEK, Tokyo, 1979).

[12]C. H. Albright, Phys. Lett. B **178**, 219 (1986).

[13]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974); M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, Nucl. Phys. **B259**, 549 (1985).

[14]L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. **B136**, 115 (1978). For a supersymmetric treatment, cf. N. Cabibbo and G. R. Farrar, Phys. Lett. **110B**, 353 (1982); L. Maiani and R. Petronzio, Phys. Lett. B **176**, 120 (1986).

[15]M. Dine and N. Seiberg, Phys. Rev. Lett. **55**, 366 (1985); V. Kaplunovsky, *ibid.* **55**, 1033 (1985).

[16]Y. Hosotani, Phys. Lett. **126B**, 309 (1983); **129B**, 193 (1983); E. Witten (Ref. 5).

[17]J. L. Rosner, Comments Nucl. Part. Phys. **15**, 195 (1986); in *Proceedings of the International Symposium on Particle and Nuclear Physics,* Beijing, China, 1985, edited by N. Hu and C.-S. Wu (World Scientific, Singapore, 1987).

[18]Breit, Ovrut, and Segrè (Ref. 10); A. Sen, Phys. Rev. Lett. **55**, 33 (1985); Witten (Ref. 5).

[19]Witten (Ref. 6); T. Hubsch, Commun. Math. Phys. **108**, 291 (1987).

[20]M. Dine, N. Seiberg, X. G. Wen, and E. Witten, Nucl. Phys. **B278**, 769 (1986); **B289**, 319 (1987).

[21]The Dirac mass terms are operationally defined as those which lead to pairs of degenerate Majorana masses.

[22]R. N. Mohapatra, Phys. Rev. Lett. **56**, 561 (1986); J. Pulido, Z. Phys. C **36**, 131 (1987); J. W. F. Valle, Phys. Lett. B **186**, 73 (1987).

[23]J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, NJ, 1983).

[24]Dine *et al.* (Ref. 13).

[25]K. Yamamoto, Phys. Lett. **168B**, 341 (1986); G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, Phys. Rev. Lett. **56**, 557 (1986); O. Bertolam and G. G. Ross, Phys. Lett. B **183**, 163 (1987).

[26]A. Font, F. Quevedo, and M. Quiros, Phys. Lett. B **188**, 75 (1987); G. Costa, F. Feruglio, F. Gabbiani, and F. Zwirner, Nucl. Phys. **B286**, 325 (1987).

[27]See Dine *et al.* (Ref. 13); F. del Aguila, G. A. Blair, M. Daniel, and G. G. Ross, Nucl. Phys. **B272**, 413 (1986).

[28]The importance of these seesawlike contributions for generating large quark masses has previously been discussed in del Aguila (Ref. 9); F. del Aguila, M. Dugan, B. Grinstein, L. Hall, G. G. Ross, and P. West, Nucl. Phys. **B250**, 225 (1985).

[29]For a review, see H. P. Nilles, Phys. Rep. **110**, 1 (1984).

[30]See, Font, Quevedo, and Quiros (Ref. 26).

[31]J. Ellis, A. B. Lahanas, D. V. Nanopoulos, M. Quiros, and F. Zwirner, Phys. Lett. B **188**, 408 (1987).

[32]S. T. Yau, in *Anomalies, Geometry and Topology* (Ref. 4); B. R. Greene, K. H. Kirklin, P. J. Miron, and G. G. Ross, Nucl. Phys. **B278**, 667 (1986); Phys. Lett. B **180**, 69 (1986); Nucl. Phys. **B292**, 606 (1987); R. B. Mann and U. Sarkar, University of Toronto Report No. UTPT-87/03 (unpublished); S. Kalara and R. N. Mohapatra, Phys. Rev. D **36**, 3474 (1987).

[33]K. Inouè, A. Kakuto, and S. Takeshita, Prog. Theor. Phys. **67**, 1889 (1982); **68**, 927 (1982).

[34]These conditions can be relaxed for third-family quarks for which large ordinary-mirror splitting is not essential, and for which weak universality and FCNC constraints are not very restrictive.

[35]A. Kagan (in preparation).

[36]For example, the Yau three-generation models of Ref. 32 possess discrete symmetries which have different charge assignments for the **27** and $\overline{\mathbf{27}}$ sectors.

[37]Similar radiative contributions were found for $N=2$ SUSY models by F. del Aguila *et al.* (Ref. 28).

[38]If the seesaw part is to be the dominant contribution to third-family quark masses, the approximation $r \ll m'_{1,2}$ will no longer be valid, so that m_t, m_b will have more complicated forms than given; however, this does not change our results qualitatively.

[39]We have assumed that the Yukawa couplings of u and d quarks will not differ sufficiently to change the direction of the inequality for any generation.

[40]A. Blondel (private communication).

[41]Though we have shown that we cannot obtain ultralight Dirac neutrino masses by a seesaw mechanism for **27** only, A. Masiero, D. V. Nanopoulos, and A. I. Sanda, Phys. Rev. Lett. **57**, 663 (1986) obtain light Dirac neutrinos for **27** only in a radiative scheme.

[42]S. Nandi and U. Sarkar, Phys. Rev. Lett. **56**, 564 (1986).

[43]H. Haber and M. H. Reno, Phys. Rev. D **34**, 2732 (1986); L. Hall, J. E. Kim, and A. Nelson, Phys. Rev. Lett. **54**, 2285 (1985).

[44]B. W. Lee and S. Weinberg, Phys. Rev. Lett. **39**, 165 (1977).

[45]C. Wendt *et al.*, Phys. Rev. Lett. **58**, 1810 (1987).

[46]R. Holman and P. B. Reis, Phys. Lett. **166B**, 305 (1986).

[47]See del Aguila, Blair, Daniel, and Ross (Ref. 27).

[48]P. Candelas, A. M. Parke, C. A. Lütken, and R. Schimmrigk, Nucl. Phys. **B298**, 493 (1988).

[49]Kawai, Lewellen, and Tye (Ref. 3); Antoniadis, Bachas, and Kounnas (Ref. 3).

[50]H. Kawai and S.-H. H. Tye (private communication).