Analysis of the mixing matrix in a model with coincident quark electroweak and mass eigenstates

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(Received 3 February 1988)

A new approach to relating quark masses and mixing angles was proposed by del Águila, Kane, and Quirós, in which the mass matrix for the weak eigenstates was assumed to be diagonal in the absence of mixing with heavier quarks. The purpose of this paper is to examine in detail the constraints of *CP* violation and B^0-$\bar{B}^0$ mixing on the quark-mixing-angle matrix of the model and the range of m_t for which the description could hold. For the case where *CP* violation and B^0-$\bar{B}^0$ mixing arise from the quark mixing matrix the result is that, for at least some values of the parameters, m_t can be as small as 85 GeV but not less. In addition, $|V_{ub}|/|V_{cb}|$ is required to be larger than 0.11, an important constraint on the model. Mixing and *CP* violation arising from flavor-changing currents present in the model are also examined.

I. INTRODUCTION

Even though the standard model with three generations is in agreement with all the available data from low-energy phenomena it is still unable to explain some of the features of the world. The number of generations and the origin of the masses, among others, are still not understood.

In Ref. 1 an extension of the standard model with a heavy $SU(2)_L$-singlet down-type *D* quark was proposed, with the quark electroweak and mass eigenstates coinciding before mixing. If the hierarchy $m_u^0 > m_d^0$ is imposed to the electroweak states, the relation $m_d > m_u$, between the physical masses of the lightest family, arises from the mixing of *d* with the vectorlike *D* quark. In addition the Cabibbo-Kobayashi-Maskawa (CKM) matrix of the approach only depends on mass ratios and, as we will show later, is consistent with experimental constraints.

Models with extra *D* quarks have been considered in the literature[2-4] but the approach of Ref. 1 gives to the mixing matrix some special features (leading to predictions such as $|V_{ub}| \approx |V_{td}|$, for example), that make attractive the study of the mixing properties of the model with the present data on the K^0-$\bar{K}^0$ system, meson decays, and B^0-$\bar{B}^0$ mixing. Bounds to the matrix elements, as well as the range of m_t values that makes compatible the approach with the experimental data, will be extracted from the study.

The paper is organized as follows. We first briefly describe the approach and the mixing matrix. The extraction of constraints for the elements of the mixing matrix from the data on meson decays, $|\epsilon|$ parameter, and B^0-$\bar{B}^0$ mixing will be presented in some detail afterwards. Finally, we summarize the bounds to the matrix elements, coming from different data, and present the results and conclusions.

II. THE MODEL

We consider a model with three families of ordinary quarks and a vectorlike down-type quark *D*. We take the *u*-type-quark mass matrix to be diagonal whereas, for the *d* sector, we allow the *D* quark to mix with *d*, *s*, and *b*. Then the quark mass matrices have the form

$$M_u = \bar{u}_L \overset{u_R}{\begin{bmatrix} m_u^0 & 0 & 0 \\ 0 & m_c^0 & 0 \\ 0 & 0 & m_t^0 \end{bmatrix}} , \qquad (2.1)$$

$$M_d = \begin{matrix} \\ \bar{d}_L \\ \\ \bar{D}_L \end{matrix} \overset{\qquad d_R \qquad\qquad D_R}{\begin{bmatrix} m_d^0 & 0 & 0 & m_1' \\ 0 & m_s^0 & 0 & m_2' \\ 0 & 0 & m_b^0 & m_3' \\ m_1 & m_2 & m_3 & M \end{bmatrix}} , \qquad (2.2)$$

where D_L and D_R are $SU(2)_L$ singlets. We assume there is no mixing for the *u*-type quarks so m_u^0, m_c^0, and m_t^0 are the physical masses m_u, m_c, and m_t, respectively. For the *d*-type quarks we start with $m_d^0 < m_u^0$, $m_s^0 \approx m_s$, $m_b^0 \approx m_b$ and only m_d^0 will be qualitatively changed. As in Ref. 1 we will take the $M(\Delta I = 0)$ mass of the same order as the electroweak scale: $M \sim M_Z$ to within factors of 2. The $m_i'(\Delta I = \frac{1}{2})$ has to be suppressed with respect to M and we assume $m_i \sim 1$ GeV. For m_i, even though SU(2) breaking is not required, we will choose $m_i \sim 1$ GeV as in Ref. 1. Only mass ratios occur in the results, so different choices giving the same ratios could be made. The matrix M_d can be diagonalized by unitary matrices

$$D = U_L^d M_d U_R^{d\dagger} . \qquad (2.3)$$

In order to get a manageable expression for the matrices and eigenvalues it is convenient to give the result of the diagonalization as an expansion in powers of m_i/M and m_i'/M. Then the eigenvalues are

$$m_d \approx -m_1' m_1/M, \quad m_s \approx m_s^0, \quad m_b \approx m_b^0, \quad m_D \approx M , \qquad (2.4)$$

and the diagonalizing matrices are given by

$$U_L^d \approx \begin{pmatrix} 1 & \mu_{12}/m_s & \mu_{13}/m_b & -m_1'/M \\ -\mu_{12}^*/m_s & 1 & \frac{m_b}{m_b^2-m_s^2}\left[\mu_{23}+\frac{m_s}{m_b}\mu_{32}^*\right] & -m_2'/M \\ -\mu_{13}^*/m_b & \frac{-m_b}{m_b^2-m_s^2}\left[\mu_{23}^*+\frac{m_s}{m_b}\mu_{32}\right] & 1 & -m_3'/M \\ m_1'^*/M & m_2'^*/M & m_3'^*/M & 1 \end{pmatrix}, \tag{2.5}$$

where

$$\mu_{ij}=m_i'm_j/M \tag{2.6}$$

and

$$U_R^d=U_L^d(m_i\leftrightarrow m_i'^*) \; . \tag{2.7}$$

The weak-interaction Lagrangian reads as

$$\mathcal{L}=\frac{g}{\sqrt{2}}W_\mu^\dagger \bar{u}_{Li}\gamma^\mu d_{L\alpha}A_{i\alpha}'+\text{H.c.}+\frac{1}{2}\frac{g}{\cos\theta_W}Z_\mu(\bar{u}_{Li}\gamma^\mu u_{Li}-\bar{d}_{L\alpha}\gamma^\mu d_{L\beta}B_{\alpha\beta}-2\sin^2\theta_W J_{\text{em}}^\mu) \; , \tag{2.8}$$

where the mixing matrix $A_{i\alpha}'$ is the 3×4 sector of $U_L^{d\dagger}$:

$$A_{i\alpha}' \approx \begin{pmatrix} 1 & -\mu_{12}/m_s & -\mu_{13}/m_b & m_1'/M \\ \mu_{12}^*/m_s & 1 & -\frac{m_b}{m_b^2-m_s^2}\left[\mu_{23}+\frac{m_s}{m_b}\mu_{32}^*\right] & m_2'/M \\ \mu_{13}^*/m_b & \frac{m_b}{m_b^2-m_s^2}\left[\mu_{23}^*+\frac{m_s}{m_b}\mu_{32}\right] & 1 & m_3'/M \end{pmatrix} \tag{2.9}$$

and

$$B_{\alpha\beta}=\delta_{\alpha\beta}-(U_L^d)_{\alpha4}(U_L^{d*})_{\beta4} \; . \tag{2.10}$$

The last term in Eq. (2.10) is responsible for flavor-changing neutral currents (FCNC's) at the tree level in the model. The mixing matrix $A_{i\alpha}'$ (2.9) can be written in the form

$$A_{i\alpha}' \approx \begin{pmatrix} 1 & |V_{12}|e^{i\Phi_{12}} & |V_{13}|e^{i\Phi_{13}} & |V_{14}|e^{i\Phi_{14}} \\ -|V_{12}|e^{-i\Phi_{12}} & 1 & |V_{23}|e^{i\Phi_{23}} & |V_{24}|e^{i\Phi_{24}} \\ -|V_{13}|e^{-i\Phi_{13}} & -|V_{23}|e^{-i\Phi_{23}} & 1 & |V_{34}|e^{i\Phi_{34}} \end{pmatrix} \tag{2.11}$$

with

$$\begin{gathered} |V_{1i}|e^{i\Phi_{1i}}=-\mu_{1i}/m_s, \quad i=2,3, \quad |V_{i4}|e^{i\Phi_{i4}}=m_i'/M, \quad i=1,2,3 \; , \\ |V_{23}|e^{i\Phi_{23}}=-\frac{m_b}{m_b^2-m_s^2}\left[\mu_{23}+\frac{m_b}{m_s}\mu_{32}^*\right] \; . \end{gathered} \tag{2.12}$$

Some of the phases of the $A_{i\alpha}'$ matrix can be changed by redefining the phases of the physical fields

$$d_\alpha \to e^{i\rho_\alpha}d_\alpha, \quad \alpha=1,\ldots,4, \quad u_j\to e^{-i\eta_j}u_j, \quad j=1,\ldots,3 \; , \tag{2.13}$$

such that the mixing matrix becomes

$$A_{i\alpha}'\to A_{i\alpha}'e^{i(\eta_i+\rho_\alpha)}\equiv A_{i\alpha}, \quad i=1,2,3, \quad \alpha=1,2,3,4 \; . \tag{2.14}$$

To make the 3×3 sector of $A_{i\alpha}$ resemble as closely as possible the usual CKM matrix with three generations, we choose the elements A_{11}, A_{22}, A_{33}, A_{12}, and A_{23} to be real. Then, after rephasing as in (2.13), we get

$$A_{i\alpha}\approx\begin{bmatrix} 1 & |V_{12}| & |V_{13}|e^{-i\Phi} & |V_{14}|e^{i\Phi_1} \\ -|V_{12}| & 1 & |V_{23}| & |V_{24}|e^{i\Phi_2} \\ -|V_{13}|e^{i\Phi} & -|V_{23}| & 1 & |V_{34}|e^{i\Phi_3} \end{bmatrix} \tag{2.15}$$

with

$$\Phi=-\Phi_{13}+\Phi_{23}+\Phi_{12},\quad \Phi_1=\Phi_{14}+\gamma\ ,$$
$$\Phi_2=\Phi_{24}+\Phi_{12}+\gamma,\quad \Phi_3=\Phi_{34}+\Phi_{23}+\Phi_{12}+\gamma\ , \tag{2.16}$$

and γ is an arbitrary phase that we can choose to fix $\Phi_3=0$, for convenience.

It is worth noticing that for two families we can still have CP violation (in the FCNC sector for ordinary quarks, for instance) due to the remaining Φ phase

$$A^{(2)}_{i\alpha}=\begin{bmatrix} 1 & |V_{12}| & |V_{13}|e^{i\Phi} \\ -|V_{12}| & 1 & |V_{23}| \end{bmatrix}, \tag{2.17}$$

$$J^{\mu}_{\rm FC}=\frac{g}{2\cos\theta_W}\bar{d}_{Lj}\gamma^{\mu}d_{L_k}\,|V_{j3}|\,|V_{k3}|\,e^{i\Phi(k-j)},$$
$$k\neq j=1,2\ . \tag{2.18}$$

From (2.15) one can write the 3×3 mixing matrix as

$$V_{ij}=\begin{bmatrix} 1 & |V_{12}| & |V_{13}|e^{-i\Phi} \\ -|V_{12}| & 1 & |V_{23}| \\ -|V_{13}|e^{i\Phi} & -|V_{23}| & 1 \end{bmatrix}. \tag{2.19}$$

To compare with the standard CKM matrix, consider the Maiani[5] parametrization for small angles

$$U\simeq\begin{bmatrix} 1 & s_{12} & s_{13}e^{-i\delta'} \\ -s_{12}-s_{23}s_{13}e^{i\delta'} & 1 & s_{23} \\ s_{12}s_{23}-s_{13}e^{i\delta'} & -s_{23} & 1 \end{bmatrix}. \tag{2.20}$$

One could identify the parameters $|V_{12}|\sim s_{12}$ and $|V_{23}|\sim s_{23}$, but the condition $|U_{13}|=|U_{31}|$ fixes the CP phase,

$$\delta'=\arccos\frac{s_{12}s_{23}}{2s_{13}}\ , \tag{2.21}$$

whereas, in the V_{ij} matrix (2.19), the CP phase Φ remains absolutely free. In addition it turns out to be impossible to make compatible both conditions: $|V_{13}|=|V_{31}|$ and $V^*_{13}=-V_{31}$, with the elements of the U matrix. This shows the qualitatively different structure of our mixing matrix (2.19), even in the case of small mixing of the extra D quark with the rest of the d-type quarks,[3] compared with the three-generation standard CKM matrix. Another implication of the difference will appear from the analysis below, when we can conclude that the model of Ref. 1 can only be consistent with the data if $|V_{13}|/|V_{23}|\geq 0.11$, a result that does not hold for the general parametrization.

III. CONSTRAINTS

Let us now discuss the implications that the data on CP violation in the K^0-$\bar{K}^0$ system and B^0-$\bar{B}^0$ mixing have on the parameters of the mixing V_{ij} (2.19), assuming no effects from the FCNC part of the Lagrangian (2.8). As in the standard CKM matrix, we have three independent moduli and one phase; some of them can be directly determined from the experimental data on flavor-changing decay of mesons.

A. Constraints from meson decays

The analyses[6] of the hyperon β decay and semileptonic kaon decay[7] fixes the value of the $|V_{12}|$ element:

$$|V_{12}|=0.220\pm0.002\ . \tag{3.1}$$

The data on semileptonic B-meson decay allow the determination of $|V_{23}|$. Since the b quark is heavy compared with the scale of strong interaction, the B-meson decay can be approximated by the decay of a free b quark, and one can write[8]

$$|V_{23}|^2=\frac{B(b\to ce\nu)}{\tau_B}\left[\frac{192\pi^3}{G_F^2}\right]\frac{1}{m_b^5 f(m_c/m_b)}\frac{1}{\eta'_0}\ , \tag{3.2}$$

where the phase-space suppression factor f is given by

$$f(z)=1-8z^2+8z^6-z^8-24z^4\ln(z) \tag{3.3}$$

and η'_0 is a QCD correction[8] $\eta'_0=0.87$. From experimental data[9,10] we find

$$B(b\to ce\nu)=0.123\pm0.008\ ,$$
$$\tau_B=(1.11\pm0.16)\times10^{-12}\ {\rm s}\ . \tag{3.4}$$

Concerning the choice of values of m_c and m_b we follow Ref. 11 and take

$$m_c=1.5\pm0.2\ {\rm GeV}\ ,$$
$$m_b=5.0\pm0.3\ {\rm GeV}\ , \tag{3.5}$$
$$m_c/m_b=0.30\pm0.03\ ,$$

where the smaller range of variation in the ratio m_c/m_b is justified by the closeness of the physical quark and meson masses to the lower and upper limits of m_c and m_b; therefore, it seems very unlikely that m_c and m_b attain their opposite limits in the computation of the ratio m_c/m_b. Using (3.3) and (3.5) one finds

$$f(m_c/m_b)=0.52\pm0.07 \tag{3.6}$$

and with (3.2) and (3.4) the value of V_{23} that we get is

$$|V_{23}|^2=(2.3\pm0.8)\times10^{-3}\,,$$
$$|V_{23}|=0.048\pm0.008\,. \tag{3.7}$$

An upper bound to $|V_{13}|$ can be obtained from the experimental measurement of the ratio

$$\overline{R}\equiv\frac{\Gamma(b\to ue\nu)}{\Gamma(b\to ce\nu)}=\frac{|V_{13}|}{|V_{23}|}\frac{1}{f(m_c/m_b)}\frac{\eta_0''}{\eta_0'}\,, \tag{3.8}$$

where the QCD correction η_0' is given in the previous paragraph and $\eta_0''=0.82$ (Ref. 8). In the very conservative limit[12] $\overline{R}\leq0.08$ we get

$$|V_{13}|\leq0.012\,, \tag{3.9}$$

where both upper bounds to f (3.6) and $|V_{23}|$ (3.7) have been used. For our purpose, following Ref. 11, it is more interesting to use the ratio

$$q=\frac{|V_{13}|}{|V_{23}|}\leq0.22\,. \tag{3.10}$$

B. Constraints from K^0-$\overline{K}^0$ mixing

We will use the usual formula for the $|\epsilon|$ parameter

$$|\epsilon|=\frac{1}{\sqrt{2}}\left[\frac{\mathrm{Im}M_{12}}{\Delta m}+\xi\frac{2\,\mathrm{Re}M_{12}}{\Delta m}\right]\,, \tag{3.11}$$

where the ξ parameter, which takes into account CP violation in the $K^0\to\pi\pi$ amplitude, on being proportional to $\mathrm{Im}(V_{td}^*V_{ts})=q\,|V_{23}|^2\sin\Phi$, is much smaller than the rest of the contributions coming from the first term in Eq. (3.11) and can safely be dropped.

The Lagrangian (2.8), unlike the standard model, allows a tree contribution to the $|\epsilon|$ parameter through the exchange of a single Z. The off-diagonal matrix element due to this diagram is given by

$$M_{12}^Z=\frac{2G_F}{3\sqrt{2}}(f_K^2B_K)m_K\eta(B_{12})^2 \tag{3.12}$$

and the usual box contribution is read as

$$M_{12}^{\mathrm{box}}=\frac{G_F^2}{12\pi^2}m_KM_W^2(f_K^2B_K)[iD(x_c,x_t)+F(x_c,x_t)]\,. \tag{3.13}$$

The functions D and F are given in Refs. 8 and 13, in terms of the elements of the mixing matrix, and $x_i=m_i/M_W$. m_K is measured experimentally, and we will take $\Delta m/m_K=0.71\times10^{-14}$, $f_K=160$ MeV, and $M_W=82$ GeV. The "bag factor" B_K has values, depending on estimations, ranging from 0.2 to 1 (Ref. 14). We will use $B_K=0.9$ and results with other values will be studied. η is a QCD correction equal to 0.76 (Ref. 8), and B_{12}, given in Eq. (2.10), is $B_{12}=-|V_{14}|\,|V_{24}|e^{i(\Phi_1-\Phi_2)}\equiv-|V_{14}|\,|V_{24}|e^{i\beta}$.

If the main contribution to ϵ comes from the Z exchange, the present value $\epsilon=2.3\times10^{-3}$ imposes

$$\mathrm{Im}[(B_{12})^2]=|B_{12}|^2\sin(2\beta)\simeq2.4\times10^{-10} \tag{3.14}$$

in agreement with Ref. 4. On the other hand, $K\to\mu^+\mu^-$ restricts $|B_{12}|<2\times10^{-5}$ (Ref. 15) so that $\beta\gtrsim18°$ in order to satisfy (3.14), and there is no bound to the Φ phase of V_{ij} (2.19). However, we are interested in the opposite limit, where the box diagrams are responsible for the value of $|\epsilon|$. Therefore we impose

$$|B_{12}|^2\sin(2\beta)<2.4\times10^{-10}\,. \tag{3.15}$$

This constraint can be easily compatible with our assumption of $m_i'/M\sim10^{-2}$ and, probably, small angle β.

In this limit, and with the parametrization (2.19), the $|\epsilon|$ parameter can be written as

$$|\epsilon|=\frac{G_F^2}{6\pi^2}\frac{m_K}{\sqrt{2}}\frac{M_W^2}{\Delta m}f_K^2B_K\,|V_{23}|\,q\sin\Phi$$
$$\times[\,|V_{12}|\,\eta_3S(x_c,x_t)-|V_{23}|^2q\cos\Phi\,\eta_2S(x_t)] \tag{3.16}$$

with the QCD corrections $\eta_2=0.6$, $\eta_3=0.4$, and $S(x_c,x_t),S(x_t)$ taken from Ref. 8.

In Figs. 1 and 2 we show the allowed region, in the Φ-q plane, for a top-quark mass of 50 and 150 GeV, respectively. The upper (lower) curve corresponds to minimum (maximum) values of $|V_{23}|$ and m_c. (Φ,q) pairs which fit the data are in the region between curves and $q\leq0.22$ (shaded area).

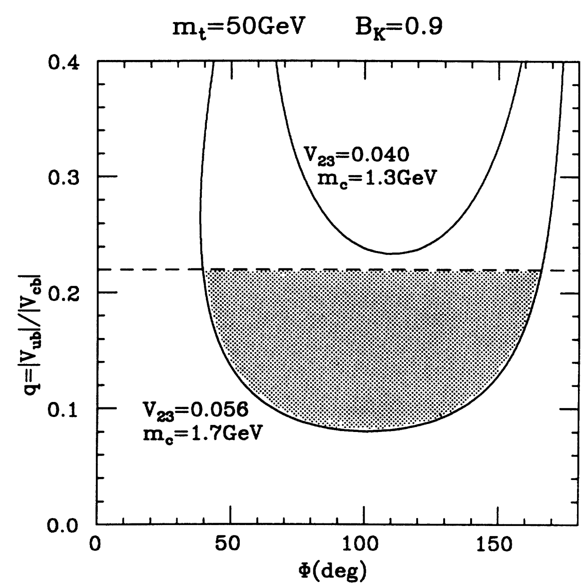

FIG. 1. Bounds from $|\epsilon|$: Allowed values of the CP phase Φ and q ($=|V_{ub}|/|V_{cb}|$), for $m_t=50$ GeV and $B_K=0.9$. Solid lines are the upper ($V_{23}=0.040$, $m_c=1.3$ GeV) and lower ($V_{23}=0.056$, $m_c=1.7$ GeV) bounds to the values allowed by $|\epsilon|$. The dashed line is the upper limit on q (≤0.22). Shaded area is the range of (q,Φ) pairs permitted by $|\epsilon|$ data.

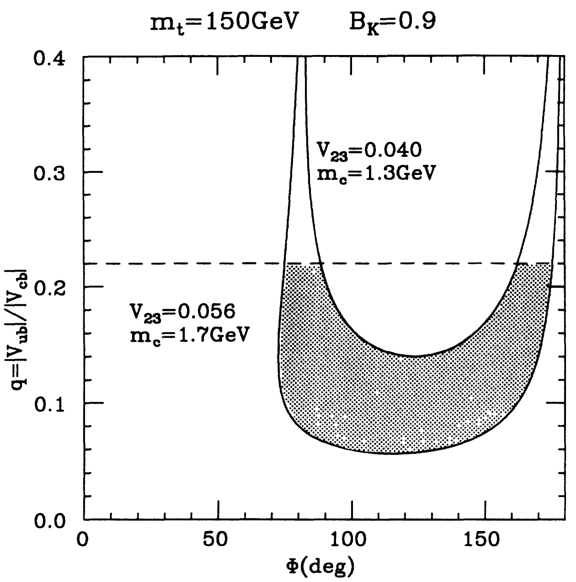

FIG. 2. The same as Fig. 1 but for $m_t = 150$ GeV and $B_K = 0.9$.

C. Constraints from $(B^0\text{-}\bar{B}^0)_d$ mixing

The degree of mixing r can be seen as the probability that B^0 oscillates into a $\bar{B}^0$ relative to the probability that it remains a B^0 (Ref. 13):

$$r = \frac{P(B^0 \to \bar{B}^0)}{P(B^0 \to B^0)} \simeq \frac{x^2}{2+x^2} \qquad (3.17)$$

with $x = 2\Delta M/(\Gamma_L + \Gamma_S)$. To an accuracy of 1% (Ref. 8), we will identify $\Delta M = 2\,|M_{12}|$.

The Lagrangian (2.8), through the FCNC sector, can correctly explain the amount of the $B^0\text{-}\bar{B}^0$ mixing only if

$$|B_{db}| = |B_{13}| \approx 6\times 10^{-4}\,. \qquad (3.18)$$

This is about 30 times larger than $|B_{12}|$ is allowed to be, so presumably the observed $B^0\text{-}\bar{B}^0$ mixing should not be interpreted as due to this tree-level effect. On the other hand, values of $|B_{db}|$ at least one order of magnitude smaller than (3.18) are still compatible with our approach and lead us to consider the usual box diagrams as mainly responsible for the mixing. In this assumption, and in the parametrization (2.19) for the mixing matrix, the evaluation of the transition matrix element gives

$$x \equiv \frac{\Delta M}{\Gamma} \simeq \tau_B\, |V_{23}|^2 \frac{G_F^2}{6\pi^2} m_B (B_B f_B^2) q^2 \eta S(x_t)\ . \qquad (3.19)$$

We will use $m_B = 5.275$ GeV, $\eta = 0.85$ (Ref. 8), and

$$|V_{23}|^2 \tau_B = (3.81 \pm 1.3)\times 10^9\ \text{GeV}^{-1} \qquad (3.20)$$

from (3.2), (3.4), and (3.6).

The "bag parameter" B_B is expected to be close to 1 since the vacuum insertion becomes more accurate with

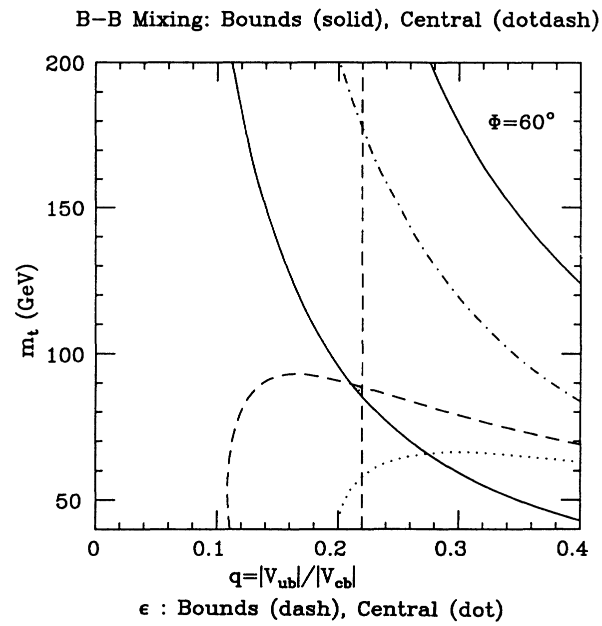

FIG. 3. Combination of $|\epsilon|$ and $B^0\text{-}\bar{B}^0$ bounds, for a *CP* phase $\Phi = 60°$ and $B_K = 0.9$. Solid lines give the upper ($x = 0.73 + 0.18$, $\Delta = 5.9 - 2.0$) and lower ($x = 0.73 - 0.18$, $\Delta = 5.9 + 8.3$) bounds to the ($m_t, q = |V_{ub}|/|V_{cb}|$) region allowed by $B^0\text{-}\bar{B}^0$ mixing; the dot-dashed line corresponds to the central value ($x = 0.73$, $\Delta = 5.9$). The dashed line is the upper bound ($V_{23} = 0.040$, $m_c = 1.3$ GeV) from $|\epsilon|$, and the dotted line gives the central value ($V_{23} = 0.048$, $m_c = 1.5$ GeV). The dashed vertical line gives the upper limit on q (≤ 0.22). Shaded area is the range of (m_t, q) values allowed by both $|\epsilon|$ and $B^0\text{-}\bar{B}^0$ data.

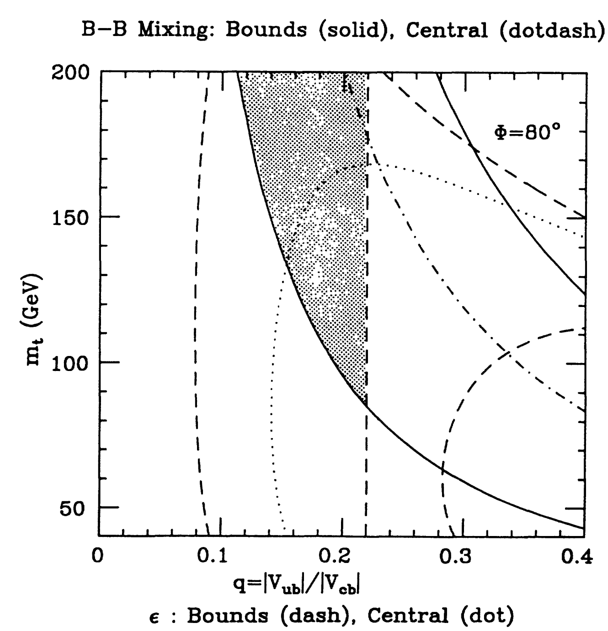

FIG. 4. The same as Fig. 2 but for $\Phi = 80°$.

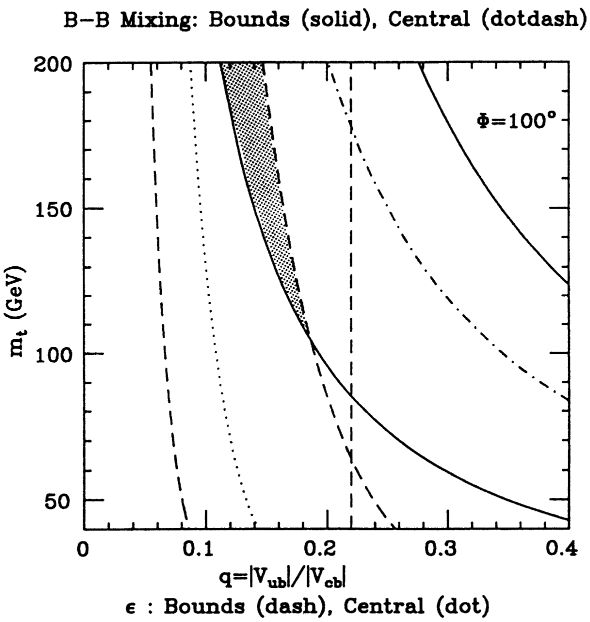

FIG. 5. The same as Fig. 2 but for $\Phi=100°$.

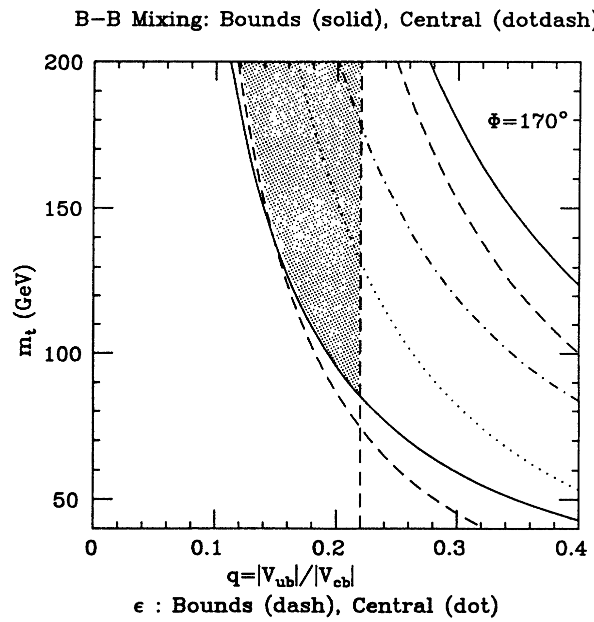

FIG. 6. The same as Fig. 2 but for $\Phi=170°$.

the increase of the meson mass. For the evaluation of f_B QCD sum rules[16] give values between 130 and 190 MeV, depending on the mass of the b quark. Other calculations[17] give values in the region $50 \le f_B \le 150$ MeV. We take[11]

$$B_B f_B^2 = (150 \pm 50 \text{ MeV})^2 . \tag{3.21}$$

Writing Eq. (3.19) in the form

$$x = q^2 S(x_t)\Delta \tag{3.22}$$

the limits to Δ,

$$\Delta = 5.9^{+8.3}_{-2.0} , \tag{3.23}$$

can be extracted from (3.20) and (3.21).

The quantity that the ARGUS group[18] found for the mixing is

$$r_d = 0.21 \pm 0.008 \tag{3.24}$$

which gives

$$x = 0.73 \pm 0.18 . \tag{3.25}$$

The formula (3.22) with the value (3.25) will define the allowed region, in the m_t-q plane, for different extremes (3.23). In Figs. 3–6 (solid and dot-dashed lines) we present the results, taking ($\Delta=5.9$, $x=0.73$) as central value (central curve) and ($\Delta=14.2$, $x=0.53$), ($\Delta=3.9$, $x=0.91$) as upper and lower bounds, respectively, in the m_t-q plane. In the same figures (3– 6), dashed and dotted lines show the region of (m_t, q) pairs compatible with the $|\epsilon|$ data.

IV. RESULTS

We will present the results of our analysis in a collection of figures which will restrict the allowed values of $q = |V_{13}| / |V_{23}|$ and Φ parameters. We work in the range $50 \le m_t \le 200$ GeV and will require $q \le 0.22$.

As we show in Figs. 1 and 2, the combination of data for the ϵ parameter (3.16) restricts q to be in the range $0.06 \le q \le 0.22$, depending on the value of the CP phase Φ. Small phases are not allowed for large m_t and $\Phi \ge 40°$ has to be imposed if $m_t > 50$ GeV. This lower bound to Φ increases up to 70° for $m_t > 150$ GeV.

Variations of the "bag parameter" B_K do not change the shape of the curves and the only effect on them is to increase (decrease) the values of q, for smaller (larger) B_K.

Because the x parameter of the B^0-$\bar{B}^0$ mixing (3.22) does not depend on the CP phase Φ, one can get direct constraints on m_t and q ($= |V_{ub}| / |V_{cb}|$) from the data of ARGUS. The analysis determines $q \ge 0.11$ (Figs. 3–6) if $m_t < 200$ GeV. This is an important constraint on the approach of Ref. 1 and illustrates again how different is the mixing matrix we are using, from the general one.

In Figs. 3–6 we combine the B^0-$\bar{B}^0$ and $|\epsilon|$ results to see the intervals of m_t, q, and Φ that can fit simultaneously both data. Values $\Phi < 60°$ are forbidden (Fig. 3), even though this limit can be slightly decreased by increasing B_K.

Figures 3 and 4 show that $m_t \approx 85$ GeV is allowed for $60° \le \Phi \le 90°$ and $q \approx 0.22$. Larger m_t (≥ 100 GeV) and smaller q (≈ 0.19) are required for $100° \le \Phi \le 140°$, as we see in Fig. 5. From Fig. 6 we conclude that values of m_t as small as 85 GeV (if $q \approx 0.22$) are allowed for $150° \le \Phi \le 175°$.

Results on Fig. 6 (5) are only shown for $\Phi=170°$ ($\Phi=100°$) but the preceding qualitative conclusions can be extended to the range $100° \le \Phi \le 140°$ ($150° \le \Phi \le 175°$) as we verified.

V. CONCLUSIONS

We have analyzed the mixing matrix of Ref. 1 with the data of the K^0-$\bar{K}^0$ system, meson decays, and B^0-$\bar{B}^0$ mixing. We have found the matrix compatible with the experimental measurements for a mass of the top quark not less than 85 GeV. The ratio $q = |V_{ub}|/|V_{cb}|$ is restricted to be in the range [0.11, 0.22] and it is not possible to have a CP phase Φ smaller than 40°–60°.

ACKNOWLEDGMENTS

I thank G. L. Kane for fruitful discussions and help. I also appreciate helpful conversations with J. Wudka. The author is indebted to the Consejo Superior de Investigaciones Científicas, Spain for financial support.

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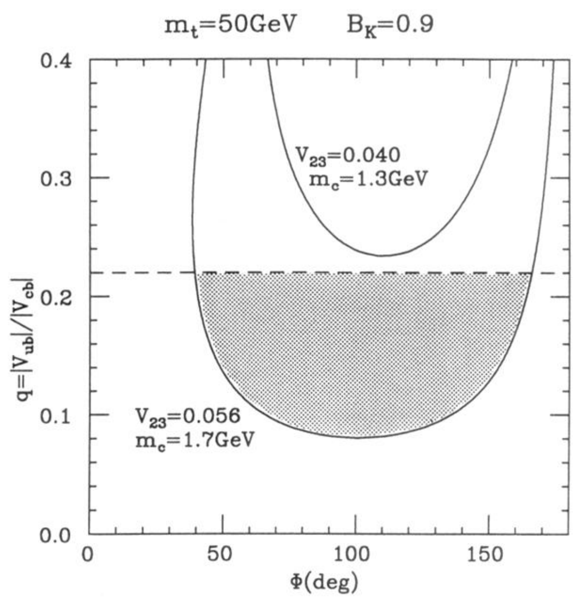

FIG. 1. Bounds from $|\epsilon|$: Allowed values of the CP phase Φ and q $(=|V_{ub}|/|V_{cb}|)$, for $m_t=50$ GeV and $B_K=0.9$. Solid lines are the upper ($V_{23}=0.040$, $m_c=1.3$ GeV) and lower ($V_{23}=0.056$, $m_c=1.7$ GeV) bounds to the values allowed by $|\epsilon|$. The dashed line is the upper limit on q (≤ 0.22). Shaded area is the range of (q,Φ) pairs permitted by $|\epsilon|$ data.

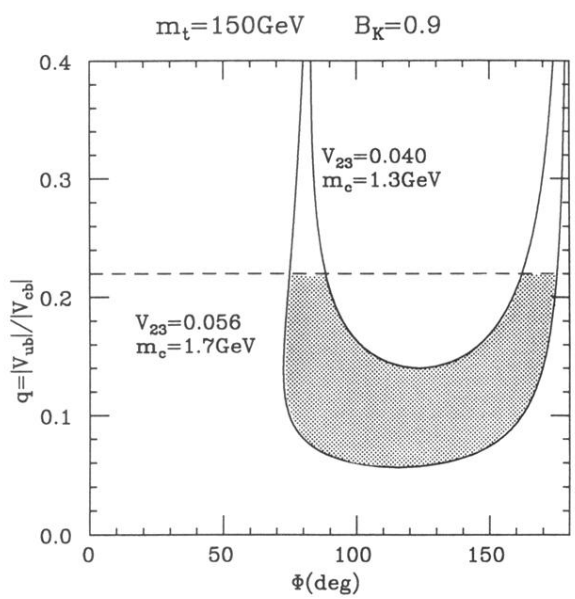

FIG. 2. The same as Fig. 1 but for $m_t = 150$ GeV and $B_K = 0.9$.

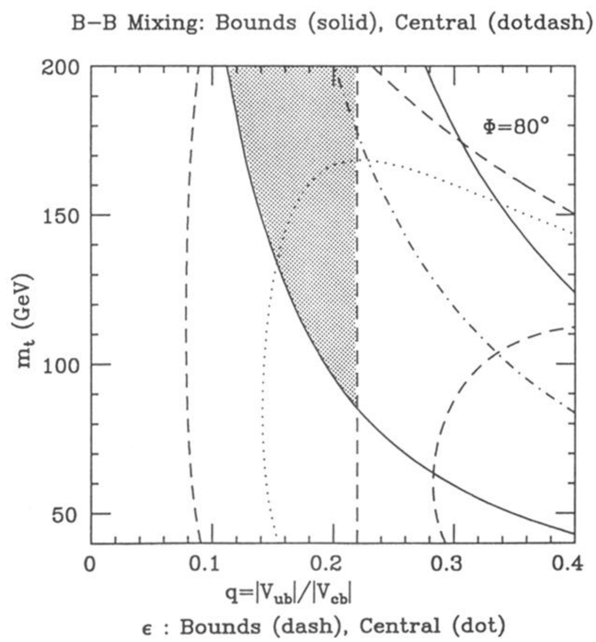

FIG. 4. The same as Fig. 2 but for $\Phi = 80°$.

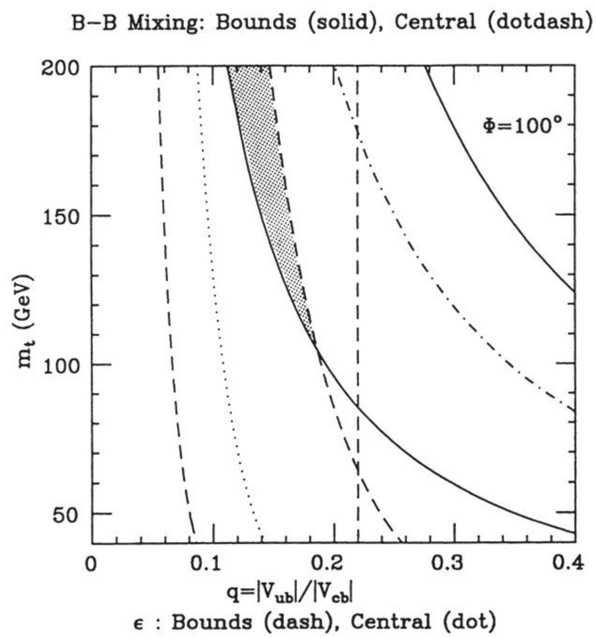

FIG. 5. The same as Fig. 2 but for $\Phi = 100°$.

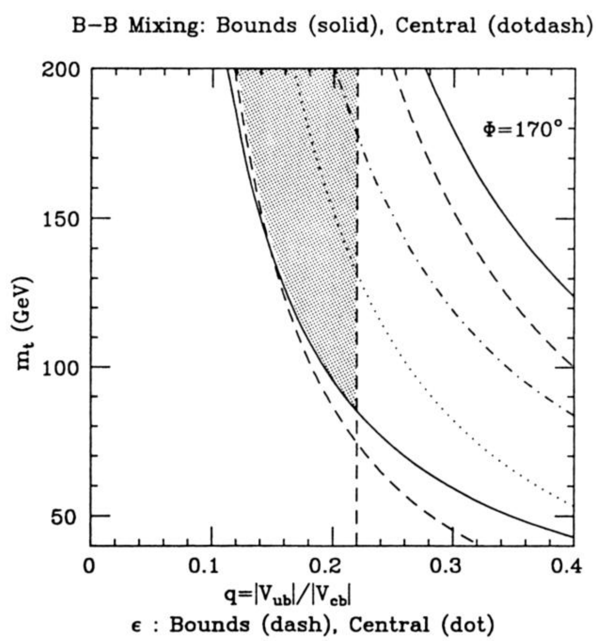

FIG. 6. The same as Fig. 2 but for $\Phi = 170°$.