Anomalous moments of W bosons in broken-supersymmetric models

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(Received 12 January 1988)

We present limits on the anomalous magnetic dipole and electric quadrupole moments of the W boson in the broken-supersymmetric version of the standard model. We give the separate contributions to these moments for each pair of fields that contribute to the loop. The main result is that supersymmetry is not likely to induce anomalous moments larger than 1%.

Supersymmetry (SUSY) is a very appealing extension of the usual $SU(2)_L \times U(1)_Y$ standard model[1] (SM). It solves one of the major problems associated with grand unified theories (i.e., the fine-tuning problem) by introducing a fermion-boson symmetry. As a consequence of this symmetry, many new degrees of freedom corresponding to the supersymmetric partners of the ordinary particles are expected to exist. In addition, SUSY is currently the only known path towards the unification of all the forces in nature. Clearly, it is of crucial interest to explore all of the phenomenological implications of SUSY theories in order to eventually confront experiment.

In this paper we focus our attention on the contributions to the anomalous magnetic dipole moment parameter κ and electric quadrupole moment parameter ΔQ of the W boson in the broken-SUSY version of the SM. These parameters are related to the static magnetic dipole and electric quadrupole moments μ and Q, respectively, via the expressions[2]

$$\mu=(1+\kappa-\Delta Q)e/2M_W, \quad Q=-(\Delta Q+\kappa)e/M_W^2 \; . \tag{1}$$

κ and ΔQ arise from the trilinear γWW vertex. The fixed form of this triple-gauge-boson vertex is the hallmark of gauge theories since any deviation from this form, apart from radiative corrections, prevents renormalizability. In the SM, $\kappa=1$ and $\Delta Q=0$ at the tree level and radiative corrections to these quantities are of order α/π. Deviations of κ from unity, $\Delta\kappa$, and of ΔQ from zero can serve as useful probes for physics beyond the SM. For example, large values of κ, such as $|\kappa| \sim 3$ (Ref. 3), can occur in composite models of the W boson. In contrast, heavy-fermion contributions to the one-loop corrections yield an upper bound[4] of $\Delta\kappa=1.5\times10^{-2}$ and $\Delta Q=2.5\times10^{-3}$, and in models with two Higgs doublets[5] the maximum deviation has been found[6] to be $\Delta\kappa=1.6\times10^{-2}$ and $\Delta Q=2.8\times10^{-3}$. Bilchak, Gastmans, and van Proeyen[7] have calculated the corrections to these same quantities in the unbroken-SUSY version of the SM (i.e., degenerate particle and sparticle masses) and found that ΔQ canceled among the members of each supermultiplet and that $\Delta\kappa$ can be nonvanishing, however, numerical estimates of $\Delta\kappa$ were not given. Here we consider these contributions in the more realistic case of broken-SUSY models, where particle and sparticle masses are no longer degenerate, and evaluate them numerically.

The prospects for experimental measurements of κ at future colliders have been discussed in the literature[4,8] so we will only briefly mention them here. The processes $e^+e^-\to W^+W^-$ and $e^+e^-\to We\nu$ are sensitive to deviations in κ, but also receive contributions from the ZWW coupling and separating the two vertices will be difficult. Determination of κ from $q\bar{q}\to W^+W^-$ and $\bar{u}d\to W^-\gamma$ will be plagued by large backgrounds at hadronic colliders. The processes $\gamma e\to W\nu$, $\bar{\nu}e\to W\gamma$, and $W\to e\bar{\nu}\gamma$ are clean, depend only on the γWW vertex, and could provide the most sensitive measurements. However, it is doubtful that any of the above reactions will determine $\Delta\kappa$ better than 5–10 % in the near future.

The one-loop corrections to the γWW vertex which yield nonzero contributions to $\Delta\kappa$ and ΔQ are shown in Fig. 1, where a and b generically represent the different particles which can appear in the loop. The diagrams involving sparticles are the same as the one given in Ref. 7. The most general CP- and electromagnetic-gauge-invariant γWW vertex is given by[9]

$$\Gamma^{\lambda\mu\nu}=ie\{A[2p^\lambda g^{\mu\nu}+4(Q^\nu g^{\lambda\mu}-Q^\mu g^{\lambda\nu})]+2(\kappa-1)(Q^\nu g^{\lambda\mu}-Q^\mu g^{\lambda\nu})+4(\Delta Q/M_W^2)p^\lambda Q^\mu Q^\nu\} \; , \tag{2}$$

where A is a real constant equal to unity in lowest order in the SM, and the kinematics are as defined in Fig. 1. The SUSY couplings which are relevant to this calculation are given by the following interaction Lagrangians.[1,7] In the gaugino and Higgsino sector we have

$$\mathcal{L}_{GH}=gW_\mu^+[\bar{\widetilde{W}}_2\gamma^\mu(\tfrac{1}{2}P_L+\cos\theta_W P_R)\widetilde{Z}-\bar{\widetilde{Z}}\gamma^\mu(\tfrac{1}{2}P_L+\cos\theta_W P_R)\widetilde{W}_1]$$
$$+eW_\mu^+(\bar{\widetilde{\gamma}}\gamma^\mu P_R\widetilde{W}_1-\bar{\widetilde{W}}_2\gamma^\mu P_R\widetilde{\gamma})+eA_\mu(\bar{\widetilde{W}}_1\gamma^\mu\widetilde{W}_1-\bar{\widetilde{W}}_2\gamma^\mu\widetilde{W}_2)-\tfrac{1}{2}gW_\mu^+(\bar{\widetilde{h}}\gamma^\mu P_L\widetilde{W}_1+\bar{\widetilde{W}}_2\gamma^\mu P_L\widetilde{h})+\text{H.c.} \; , \tag{3}$$

where $P_L=\frac{1}{2}(1-\gamma_5)$, $P_R=\frac{1}{2}(1+\gamma_5)$, θ_W is the weak mixing angle, $\tilde{h}(\tilde{W},\tilde{Z})$ is the supersymmetric partner of the Higgs (W,Z) boson, and $\tilde{\gamma}$ is the superpartner of the photon. For the squark sector we have

$$\mathcal{L}_{\tilde{q}\tilde{q}V}=\frac{-ig}{\sqrt{2}}[W_\mu^+(\tilde{u}_L^*\overleftrightarrow{\partial}^\mu\tilde{d}_L)+W_\mu^-(\tilde{d}_L^*\overleftrightarrow{\partial}^\mu\tilde{u}_L)]-ieA_\mu\sum_i e_i\tilde{q}_i^*\overleftrightarrow{\partial}^\mu\tilde{q}_i \ , \tag{4}$$

where $\tilde{q}_{L(R)}$ denotes left- (right-)handed squarks, the sum extends over $\tilde{u}_L$, $\tilde{u}_R$, $\tilde{d}_L$, and $\tilde{d}_R$, and e_i is the electric charge of sparticle i. The Lagrangian for the slepton sector may be obtained from Eq. (4) by substituting $\tilde{u}\to\tilde{\nu}$ and $\tilde{d}\to\tilde{e}$.

In Table I we present the various SUSY contributions to $\Delta\kappa$, in units of $a\equiv g^2/96\pi^2$. For completeness we also include the contributions from heavy fermions, two Higgs doublets, and the SM. We define

$$\delta\equiv(M_a/M_W)^2,\quad \epsilon\equiv(M_b/M_W)^2,\quad F\equiv1+\epsilon-\delta \ , \tag{5}$$

and $x_W=\sin^2\theta_W=0.23$. U (D) denotes a heavy charged $+\frac{2}{3}$ $(-\frac{1}{3})$ quark and L (ν_L) represents a heavy charged (neutral) lepton. In the case of the W-ino contributions we have performed a sum over both W-ino states. Also in the case of the pure Higgs contribution, we have summed over all the neutral Higgs bosons, which for simplicity we have taken to be degenerate. Numerically, the dependence of $\Delta\kappa$ and ΔQ on these masses are logarithmic; hence, this is not a drastic assumption. Notice that the sum of the quark and lepton contributions to $\Delta\kappa$ cancels in the case of degenerate quark and lepton masses, and likewise for squarks an sleptons. However, $\Delta\kappa$ does not vanish in the gaugino and Higgsino sector, even in this degenerate mass limit. Our results agree with those presented in Ref. 7. As a consistency check on our results, we have independently calculated the one-loop contributions to the anomalous magnetic moment of the W-ino. We found that all of the different contributions

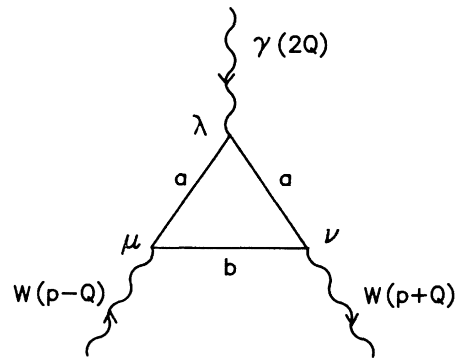

FIG. 1. The one-loop Feynman diagram for the γWW vertex.

agreed with those derived in Ref. 7, and correctly sum such that the anomalous magnetic moment of both W-ino states equal $\Delta\kappa$ in the exact SUSY limit.

ΔQ is given by the following integral:

$$\Delta Q=-Ca\int_0^1 dt\frac{t^3(1-t)}{t^2+\epsilon-t(1+\epsilon-\delta)} \ , \tag{6}$$

where ϵ, δ, and a are as defined above. We display in Table II the value of the coefficient C for the various loop particles which give rise to ΔQ. Again we have summed over both W-ino states in the W-ino contributions, and over all the neutral Higgs bosons in the pure Higgs-boson contribution. It is clear from Table II that in the unbroken-SUSY limit ΔQ vanishes exactly. We would like to point out that in the degenerate mass case the fermion and sfermion contributions to $\Delta\kappa$ and ΔQ vanish independently. This is because in this case the integrals may be factorized, leaving a summation over the electric and color charges, which must sum to zero due to the Adler-Bell-Jackiw anomaly cancellation.

The numerical results for $\Delta\kappa$ and ΔQ for the various SUSY contributions are shown in Figs. 2–6. In our numerical calculations we used $M_W=81$ GeV, $M_Z=94$ GeV, and $\alpha=\frac{1}{128}$. In Figs. 2(a) and 2(b) we present the squark contribution to $\Delta\kappa$ and ΔQ, respectively, summed over the two sets of squarks which may appear in the loop, $\tilde{u}\tilde{u}\tilde{d}$ and $\tilde{d}\tilde{d}\tilde{u}$, in units of a, as a function of the u-squark mass. The dotted curve corresponds to a d-squark mass of 1 TeV, the short-dashed curve to 500 GeV, the dotted-dashed curve to 250 GeV, the long-dashed curve to 100 GeV, and the solid curve to 50 GeV. As can be seen from the figure, the maximum squark contribution to $\Delta\kappa$ and ΔQ is $-4.9a$ and $-1.2a$, respectively. ΔQ is always negative and tends towards zero as the squark masses increase. The slepton contribution to $\Delta\kappa$ and ΔQ in units of a is shown in Figs. 3(a) and 3(b). The upper bound for $\Delta\kappa$ is $2a$ and for ΔQ is $1.2a$. Note that for degenerate squark and slepton masses, both $\Delta\kappa$ and ΔQ clearly cancel. We display in Figs. 4(a) and 4(b) the $\tilde{W}\tilde{W}\tilde{Z}$ contribution to $\Delta\kappa$ and ΔQ. The maximum contributions are given by $10.0a$ for $\Delta\kappa$ and $-4.6a$ for ΔQ. The $\tilde{W}\tilde{W}\tilde{\gamma}$ contributions are given in Figs. 5(a) and 5(b); the maximum value for $\Delta\kappa$ is $-1.8a$ and is $-1.1a$ for ΔQ. In Figs. 6(a) and 6(b) we show the $\tilde{W}\tilde{W}\tilde{h}$ contributions to $\Delta\kappa$ and ΔQ; the upper bounds on these quantities are $-2.0a$ and $-1.2a$, respectively.

The maximum value for $\Delta\kappa$ from the SUSY contributions only in the broken-SUSY model occurs when the W-ino mass is 1 TeV, $M_{\tilde{Z}}=M_{\tilde{\gamma}}=M_{\tilde{h}}=50$ GeV, $M_{\tilde{u}}=M_{\tilde{\nu}}=50$ GeV, $M_{\tilde{d}}=M_{\tilde{e}}=1$ TeV, and is given by

TABLE I. The contributions to $\Delta\kappa$, in units of a, from the various contributing particles, aab, in the Feynman diagram in Fig. 1.

Particles in loop			Contribution to $\Delta\kappa$
a	b	c	
U	U	D	$\begin{bmatrix} 6 \\ -3 \\ -3 \end{bmatrix} \int_0^1 dt \frac{t^4+(F-2)t^3+(2\delta-\epsilon)t^2}{t^2-tF+\epsilon}$
D	D	U	
L	L	ν_L	
W	W	Z	$6\left[\frac{20}{3\epsilon}-\frac{5}{6}+\int_0^1 dt \frac{\frac{1}{2}t^4+5t^3-18t^2+16t-8}{t^2-t\epsilon+\epsilon}\right]$
W	W	$\gamma+WW$	$40x_W$
W	W	H^0	$3\int_0^1 dt \frac{2t^4-(2+\epsilon)t^3+(4+\epsilon)t^2}{t^2-t\epsilon+\epsilon}$
H^+	H^+	(H^0+h^0)	$-6\int_0^1 dt \frac{-2t^4+(2+F)t^3-Ft^2}{t^2-tF+\epsilon}$
$\tilde{u}$	$\tilde{u}$	$\tilde{d}$	$\begin{bmatrix} 12 \\ -6 \\ -6 \end{bmatrix} \int_0^1 dt \frac{-2t^4+(2+F)t^3-Ft^2}{t^2-tF+\epsilon}$
$\tilde{d}$	$\tilde{d}$	$\tilde{u}$	
$\tilde{e}$	$\tilde{e}$	$\tilde{\nu}$	
$\tilde{W}$	$\tilde{W}$	$\tilde{Z}$	$-12\int_0^1 dt \frac{\sqrt{\epsilon\delta(1-x_W)}(2t-4t^2)+(\frac{5}{4}-x_W)[t^4+(F-2)t^3+(1+\delta-F)t^2]}{t^2-tF+\epsilon}$
$\tilde{W}$	$\tilde{W}$	$\tilde{\gamma}$	$-12x_W\int_0^1 dt \frac{t^4+(F-2)t^3+(1+\delta-F)t^2}{t^2-tF+\epsilon}$
$\tilde{W}$	$\tilde{W}$	$\tilde{h}$	$-3\int_0^1 dt \frac{t^4+(F-2)t^3+(1+\delta-F)t^2}{t^2-tF+\epsilon}$

$$(\Delta\kappa)_{\max}=-16.8a=-8.4\times10^{-3} . \tag{7}$$

The maximum positive values for $\Delta\kappa$ is given when the W-ino mass is 50 GeV, $M_{\tilde{Z}}=M_{\tilde{\gamma}}=M_{\tilde{h}}=1$ TeV, $M_{\tilde{u}}=M_{\tilde{\nu}}=1$ TeV, $M_{\tilde{d}}=M_{\tilde{e}}=50$ GeV, and equals

$$(\Delta\kappa)_{\max}=14.8a=7.4\times10^{-3} . \tag{8}$$

In the case of ΔQ, the maximum value for SUSY occurs when the relevant sparticle masses are light, yet distinct,

TABLE II. The various values of the numerical coefficient C in the expression for ΔQ.

Particles in loop			C
a	b	c	
U	U	U	-8
D	D	U	4
L	L	ν_L	4
W	W	Z	$-2[1+8(1-x_W)]$
W	W	γ	$-16x_W$
W	W	H^0	-2
H^+	H^+	(H^0+h^0)	-4
$\tilde{u}$	$\tilde{u}$	$\tilde{d}$	8
$\tilde{d}$	$\tilde{d}$	$\tilde{u}$	-4
$\tilde{e}$	$\tilde{e}$	$\tilde{\nu}$	-4
$\tilde{W}$	$\tilde{W}$	$\tilde{Z}$	$4[1+4(1-x_W)]$
$\tilde{W}$	$\tilde{W}$	$\tilde{\gamma}$	$16x_W$
$\tilde{W}$	$\tilde{W}$	$\tilde{h}$	4

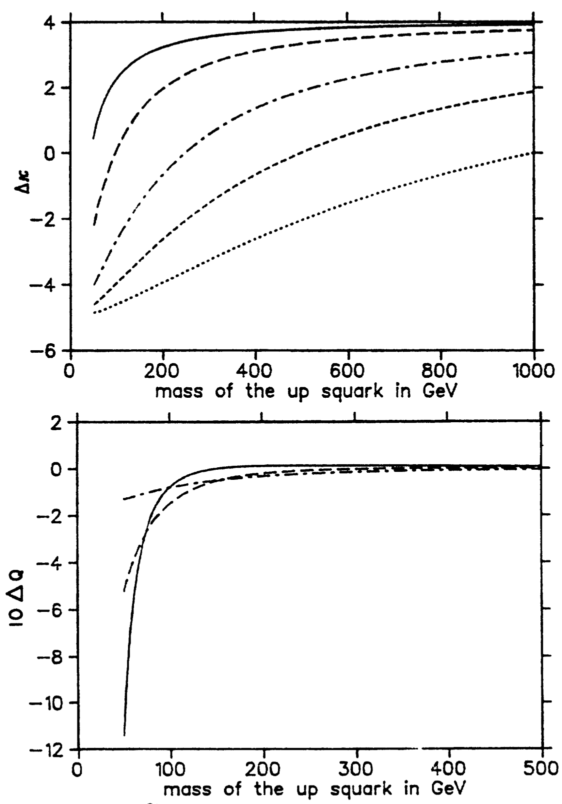

FIG. 2. The $\tilde{u}\tilde{u}\tilde{d}$ contributions to (a) $\Delta\kappa$ and (b) ΔQ as a function of the u-squark mass in units of a. The d-squark mass is 1000 GeV for the dotted curve, 500 GeV for the short-dashed curve, 250 GeV for the dotted-dashed curve, 100 GeV for the long-dashed curve, and 50 GeV for the solid curve.

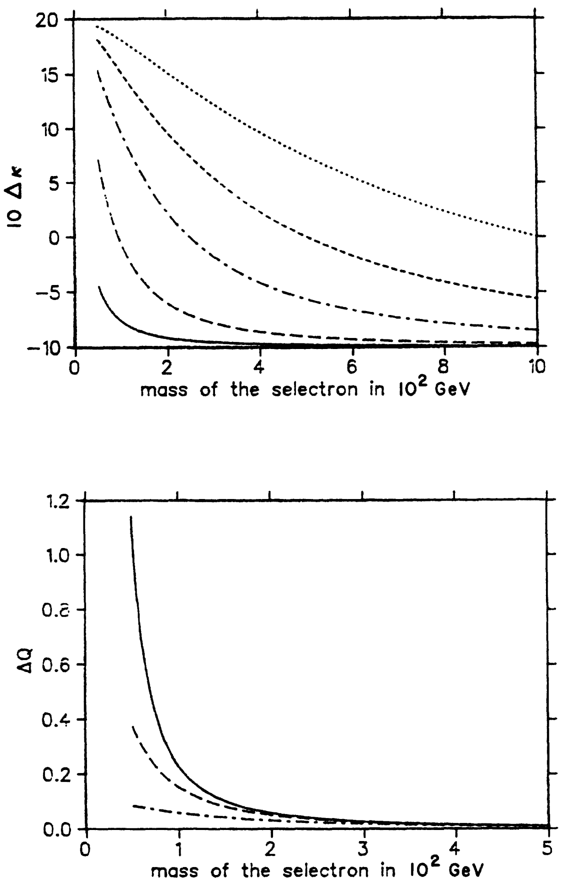

FIG. 3. The $\tilde{e}\tilde{e}\tilde{\nu}$ loop contributions, in units of a, to (a) $\Delta\kappa$ and (b) ΔQ as a function of the selectron mass. The legend for the sneutrino mass is the same as for the d-squark mass in Fig. 2.

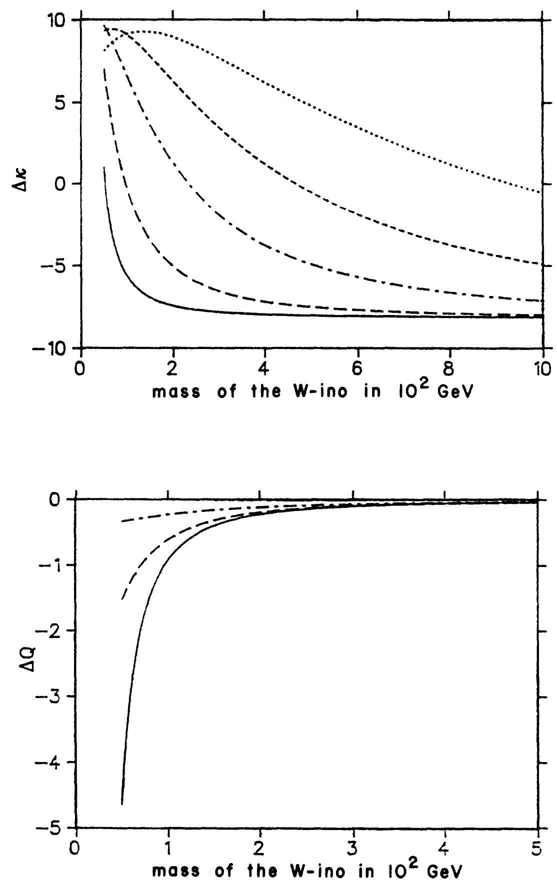

FIG. 4. (a) $\Delta\kappa$ and (b) ΔQ, in units of a, as a function of the W-ino mass for the $\tilde{W}\tilde{W}\tilde{Z}$ loop contribution. The dotted curve corresponds to a Z-ino mass of 1000 GeV, the short-dashed curve to 500 GeV, the dotted-dashed curve to 250 GeV, the long-dashed curve to 100 GeV, and the solid curve to 50 GeV.

and is given by

$$(\Delta Q)_{\max} = -8.0a = -4.0\times 10^{-3} . \tag{9}$$

As we shall see, this maximum contribution is larger than that obtained from other models.

These values for $\Delta\kappa$ and ΔQ in the broken-SUSY model should be explicitly compared to previous results for these quantities in other models. In the SM with a heavy top quark the maximum contributions are[4]

$$(\Delta\kappa)_{\max} = 30a = 1.5\times 10^{-2} ,$$
$$(\Delta Q)_{\max} = 5a = 2.5\times 10^{-3} . \tag{10}$$

The upper bounds in the two-Higgs-doublet model are given by[6]

$$(\Delta\kappa)_{\max} = 32a = 1.6\times 10^{-2} ,$$
$$(\Delta Q)_{\max} = 5.6a = 2.8\times 10^{-3} . \tag{11}$$

These values are the sum of the maximum contributions from the SM with heavy fermions and from models with two Higgs doublets. When the broken-SUSY contributions are added to that of the SM and the two-Higgs-doublet model, we see that the SUSY results can either add or subtract from the previous values, depending on the sparticle masses, but the total numbers are still reasonably tiny. The differences in these quantities between these three different models is small and will probably be indistinguishable in future experiments.

In conclusion, we have calculated the contributions to the anomalous magnetic moment and electric quadrupole moment parameters in the broken-SUSY model. We have numerically evaluated the separate SUSY contributions and have found the total maximum bound on these parameters. The total SUSY contribution strongly depends on the masses of the sparticles. The W-ino–Z-ino term is the largest and decides the sign of the SUSY contribution. However, for any sparticle mass combination, SUSY will not induce anomalous moments larger than 1%. If a large measurement of $\Delta\kappa$ or ΔQ were obtained,

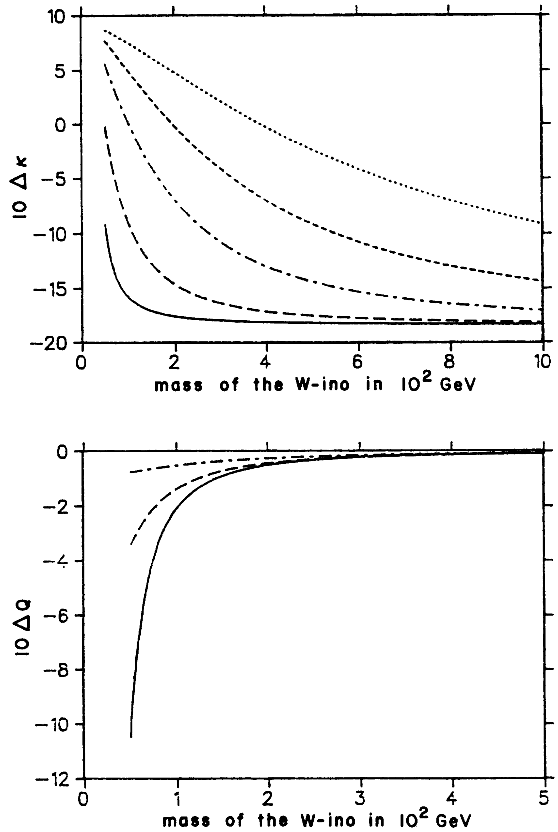

FIG. 5. Same as in Fig. 4, except for the $\tilde{W}\tilde{W}\tilde{\gamma}$ loop contributions. The labels for the photino masses are the same as for the Z-ino in Fig. 4.

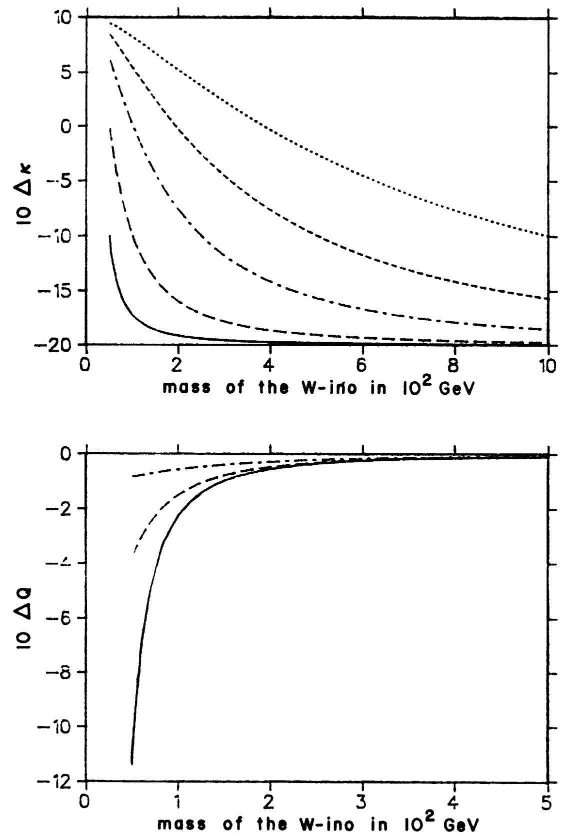

FIG. 6. Same as in Fig. 4, except for the $\tilde{W}\tilde{W}\tilde{h}$ loop contributions. The legend for the Higgsino mass is the same as for the Z-ino in Fig. 4.

it would not signal the existence of SUSY, but would be an indication of some other new physics, such as compositeness or strongly interacting Higgs bosons.

J.H. and T.R. would like to thank the theory group at TRIUMF for their hospitality while this work was begun. The work of J.H. was supported by the American Association of University Women and by ZONTA International. The work of J.N. and G.C. was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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