Radiative decays of the K^-p atom

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We use an extension of the cloudy bag model to $SU(3)\times SU(3)$ to calculate the radiative decay of the K^-p atom. Our result differs from earlier work in a fundamental way because of the inclusion of coupling to the open $\Sigma\pi$ channel. The branching ratio into $\Sigma^0\gamma$ is predicted to be of order 10–20 % bigger than that into $\Lambda\gamma$, which should make it accessible to a current experiment.

I. INTRODUCTION

The improved quality of modern low-energy K^- beam lines, has led to renewed interest in the radiative decay of the K^-p atom. An experiment currently underway at Brookhaven National Laboratory[1] should be sensitive to the $\Lambda\gamma$ and $\Sigma^0\gamma$ decay channels at the level of a few parts in 10^4. If this proves feasible it will be a dramatic improvement on the existing measurements: namely,

$$R_{\Lambda\gamma}=\Gamma(K^-p\to\Lambda\gamma)/\Gamma(K^-p\to\text{anything})$$
$$=(3.0\pm1.5)\times10^{-3}\ \ (\text{Ref. 2})\ ,$$
$$R_{\Lambda\gamma}<4\times10^{-4}$$

and

$$R_{\Sigma\gamma}=\Gamma(K^-p\to\Sigma\gamma)/\Gamma(K^-p+\text{anything})$$
$$<4\times10^{-3}\ \ (\text{Ref. 3})$$

and $R_{\Lambda\gamma}=(2.8\pm0.8)\times10^{-3}$ (Ref. 4). The experimental difficulties arise from both the low branching ratio and the continuum photons associated with background reactions.

Because the reactions

$$K^-p\to\Lambda\gamma\ , \tag{1.1}$$

$$K^-p\to\Sigma^0\gamma\ , \tag{1.2}$$

are dominated by the intermediate state $\Lambda^*(1405)$, just below the K^-p threshold, the theoretical study of this subject has been closely related to the quark structure of $\Lambda^*(1405)$. In the nonrelativistic quark model of Isgur and Karl,[5] the $\Lambda^*(1405)$ is predominantly an SU(3) singlet. Darewych, Horbatch, and Koniuk[6] (DHK), using this model to calculate the decay widths of $\Lambda^*(1405)\to\Lambda\gamma$ and $\Lambda^*(1405)\to\Sigma^0\gamma$, gave 143 and 91 keV, respectively. Then, Kaxiras, Moniz, and Soyeur[7] (KMS) estimated these radiative widths in both the MIT bag model and the Isgur-Karl nonrelativistic quark model (NRQM). In the MIT model they found the widths to be 60 and 18 keV, respectively, for a Λ_1^* at energy 1364 MeV and 17 keV and 2.7 keV for Λ_2^* at 1446 MeV. [Since the latter is predominantly a flavor singlet it is the result most relevant to the $\Lambda(1405)$.] They also found widths of 154 and 72 keV (respectively) in the nonrelativistic quark model.

Whereas the early theoretical paper dealing with the K^-p atom by Korenman and Popov[8] omitted the $\Lambda^*(1405)$ altogether, Burkhardt *et al.*[9] recently added a phenomenological transition moment for the process $\Lambda^*\to\Lambda\gamma$ to the external emission diagrams. They found two possible values for the radiative width $\Lambda^*\to\Lambda\gamma$, 6 ± 6 or 74 ± 22 keV, depending on the phenomenological transition moment. (We note, however, that this work has recently been criticized by Workman and Fearing.[10]) Finally, we observe that Darewych, Koniuk, and Isgur[11] used the NRQM to estimate the $K^-p\to\Lambda\gamma$ and $K^-p\to\Sigma^0\gamma$ branching ratios explicitly (rather than just Λ^*). These tend to be in the ratio 0.64 to 0.77, depending on which other resonances are included.

We have recently extended the cloudy bag model[12] (CBM) to chiral SU(3) (Ref. 13) and used it to study low-energy KN (Ref. 14) and $\bar{K}N$ (Ref. 15) scattering. We reached rather surprising conclusions regarding the $\Lambda^*(1405)$ resonance: namely, that it is predominantly a K^-p bound state. For $\bar{K}N$ scattering near the threshold, the three-quark state corresponding to $Y=0$, $I=0$, $J^\pi=\frac{1}{2}^-$ is above 1600 MeV (it will henceforth be referred to as Λ'), and contributes only 14% of the strength of the $\Lambda^*(1405)$. Since the $\Lambda^*(1405)$ is a serious problem for most models of hadron spectroscopy, this result is not

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unwelcome. It seems timely to investigate the radiative capture processes using this same model. An initial report on the results of this calculation has already been published as a Letter.[16] Here we shall present the details of the calculations, and correct a numerical error in the previous work.

In Sec. II after making the usual minimal-coupling substitution, we derive the appropriate Hamiltonian for low-energy radiative capture, the corresponding t matrices, and the decay widths of the processes $K^-p\to\Lambda\gamma$ and $K^-p\to\Sigma^0\gamma$. The results and a comparison with earlier work are presented in Sec. III. Finally, we make some concluding remarks in Sec. IV.

II. FORMALISM

After the minimal-coupling substitution and to order ϕ^2, the Lagrangian density for the SU(3)×SU(3) cloudy bag model with volume coupling is

$$\begin{aligned}\mathcal{L}_{\rm CBM}=&(i\bar{q}\partial\!\!\!/ q-B)\theta_v-\tfrac{1}{2}\bar{q}q\delta_s-\tfrac{1}{2}m_\phi^2\phi_j^2+\tfrac{1}{2}(\partial\phi_\mu)_j^\dagger(\partial^\mu\phi)_j-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\\&+\frac{1}{2f}\bar{q}\gamma^\mu\gamma_5\vec{\lambda}q\cdot\partial_\mu\vec{\phi}\theta_v-\frac{1}{4f^2}\bar{q}\vec{\lambda}\gamma^\mu q\cdot(\vec{\phi}\times\partial_\mu\vec{\phi})\theta_v-e_Q\bar{q}Aq\theta_v-ie_\phi(\phi_j^\dagger\partial^\mu\phi_j-\phi_j\partial_\mu\phi_j^\dagger)A_\mu\\&+\frac{i}{2f}e_\phi\bar{q}\gamma^\mu\gamma_5\vec{\lambda}q\cdot\vec{\phi}A_\mu\theta_v-ie_\phi\frac{\theta v}{4f^2}\bar{q}\gamma^\mu\vec{\lambda}q\cdot(\vec{\phi}\times\vec{\phi})A_\mu+e_\phi^2A_\mu A^\mu\phi_j^\dagger\phi_j\ .\end{aligned}\tag{2.1}$$

Here $q(x)$, $\phi(x)$, and $A(x)$ are the quark, meson-octet, and photon fields, B is the bag constant, f is the meson-octet decay constant, $\vec{\lambda}$ are the SU(3) matrices of Gell-Mann, and e_Q and e_ϕ are the charges of the corresponding quark and meson, respectively. The SU(3) dot and cross product are

$$\begin{aligned}\vec{\lambda}\cdot\vec{\phi}&=\sum_{j=1}^{8}\lambda_j\phi_j\ ,\\ \vec{\lambda}\cdot(\vec{\phi}\times\partial_\mu\vec{\phi})&=\sum_{abc}f_{abc}\lambda_a\phi_b\partial_\mu\phi_c\ ,\end{aligned}\tag{2.2}$$

where f_{abc} are the SU(3) structure constants.[17] The Hamiltonian corresponding to (2.1) can be written in the form

$$\hat{H}=H_0+\hat{H}_{\rm int},\tag{2.3}$$

where $\hat{H}_0$ describes the free bags, meson, and photons, and $H_{\rm int}$ the interactions between or among them.

A. The interaction Hamiltonian

In this paper we chose to work in the Coulomb gauge [so that $A^\mu=(0,\mathbf{A})$]. The Fourier transforms of the meson-octet and electromagnetic fields are

$$\phi_j(\mathbf{r})=\int\frac{d^3k}{[(2\pi)^3 2\omega_k]^{1/2}}[a_j(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}+a_j^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}]\ ,\quad j=1,2,\ldots,8\ ,\tag{2.4}$$

$$\mathbf{A}(\mathbf{r})=\sum_\lambda\int\frac{d^3q}{[(2\pi)^3 2q]^{1/2}}\boldsymbol{\epsilon}(\mathbf{q}\lambda)[a(\mathbf{q}\lambda)e^{i\mathbf{q}\cdot\mathbf{r}}+a^\dagger(\mathbf{q}\lambda)e^{-i\mathbf{q}\cdot\mathbf{r}}]\ ,$$

where $a_j(\mathbf{k})$ $[a_j^\dagger(\mathbf{k})]$ and $a(\mathbf{q}\lambda)$ $[a^\dagger(\mathbf{q}\lambda)]$ represent the annihilation [creation] operators of the meson octet and photon, respectively. As usual we must construct the complex fields for the charged meson from $\phi_j(r)$. For example, the annihilation operators for the charged pions are

$$\begin{aligned}C_{\pi^+}(\mathbf{k})&=-\frac{1}{\sqrt{2}}[a_1(\mathbf{k})-ia_2(\mathbf{k})]\ ,\\ C_{\pi^-}(\mathbf{k})&=\frac{1}{\sqrt{2}}[a_1(\mathbf{k})+ia_2(\mathbf{k})]\ .\end{aligned}\tag{2.5}$$

In our calculations, the relevant baryons are N, Λ^0, Σ $(\Sigma^\pm,\Sigma^0)$, and $\Lambda'(\frac{1}{2}^-)$. Of course, N, Λ^0, and Σ are members of the baryon octet composed of u, d, and s quarks in the $1s_{1/2}$ state. For a static spherical bag of radius R, this $1s_{1/2}$ wave function of the quark is[15]

$$q_{1s}(\mathbf{r},t)=\frac{N_s}{\sqrt{4\pi}}\begin{bmatrix}j_0(\omega_s r)\\ ij_1(\omega_s r)\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\end{bmatrix}e^{-i\omega_s t}b\theta(R-r)\ ,\tag{2.6}$$

where b denotes the spin-isospin wave function of the quark, and $\omega_s=2.0428/R$ represents the energy of the quark ground state. $\Lambda'(\frac{1}{2}^-)$ is composed of u, d, and s quarks in an (assumed) SU(3) singlet with one quark excited to a $1p_{1/2}$ state. As mentioned in the Introduction, $\Lambda(\frac{1}{2}^-)$ is to be distinguished from $\Lambda^*(1405)$, which is predominantly a K^-p bound state. Here the $1p_{1/2}$ wave function of the quark can be written as[15]

$$q_{1p_{1/2}}(\mathbf{r},t)=\frac{N_p}{\sqrt{4\pi}}\begin{bmatrix}-j_1(\omega_p r)\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\\ ij_0(\omega_p r)\end{bmatrix}e^{-i\omega_p t}b\theta(R-r)\ ,\tag{2.7}$$

where $\omega_p=3.8115/R$ is the energy of the first excited quark state. The normalization factors are

$$N_{s,p}^2=\frac{\omega_{s,p}R}{2R^3j_0^2(\omega_{s,p}R)(\omega_{s,p}R\mp1)}\ .\tag{2.8}$$

As usual, the interaction Hamiltonian $\hat{H}_{\rm int}$ can be projected onto the space of colorless, nonexotic baryon states[12] [here we retain only N, Λ, and $\Lambda'(\frac{1}{2}^-)$], leading to the following interactions.

1. The quark-meson interaction or Yukawa term

This interaction is

$$\hat{H}_s=\int d^3r\left[-\frac{\theta_v}{2f}\bar{q}\gamma^\mu\gamma_5\vec{\lambda}q\cdot\partial_\mu\vec{\phi}\right] . \tag{2.9}$$

Using the Dirac equation and the linear boundary condition on the surface of the quark bag, we do the integration by parts and get

$$H_s=\int\left[\frac{i}{2f}\bar{q}\gamma_5\vec{\lambda}q\cdot\vec{\phi}\delta_s-\frac{\theta v}{2f}\partial_0(\bar{q}\gamma^0\gamma_5\vec{\lambda}q\cdot\vec{\phi})\right.$$
$$\left.+\frac{i}{f}m_q\theta_v\bar{q}\gamma_5\vec{\lambda}q\cdot\vec{\phi}\right]d^3r , \tag{2.10}$$

where m_q is the mass of the quark. In our calculation we consider only massless quarks, so the third term in Eq. (2.10) gives no contribution. (This has been shown to be a good approximation in our earlier work.[14,15])

The interaction Hamiltonian for the transitions $BM\to\Lambda'$ and $B\to M\Lambda'$, shown in Fig. 1(a), are given by

$$H_s=\sum_j\int d^3k[\langle\Lambda'\mid V_{sj}(\mathbf{k})\mid B\rangle\Lambda'^{\dagger}BC_j(\mathbf{k})$$
$$+\langle\Lambda'\mid V_{sj}^{\dagger}(\mathbf{k})\mid B\rangle\Lambda'^{\dagger}BC_j^{\dagger}(\mathbf{k})] , \tag{2.11}$$

where j labels the type of meson (including its charge state), $\Lambda'^{\dagger}$ is the creation operator for the Λ', and B is the annihilation operator for a three-quark bag of type B. Following Veit *et al.*[14] the vertex function is written in the form

$$\langle\Lambda'\mid V_{sj}(\mathbf{k})\mid B\rangle=\frac{\lambda_{\alpha\Lambda'}}{2f}\frac{1}{[(2\pi)^3 2\omega_k]^{1/2}}C_{I_B I_M 0}^{i_B i_M 0}U_{\alpha\Lambda'}^{s}(kR) . \tag{2.12}$$

Here α labels the meson-baryon pair (e.g., B,M), I_B and I_M are the isospin of the baryon and the meson and i_B, i_M their projections. For s-wave scattering, the form factor is

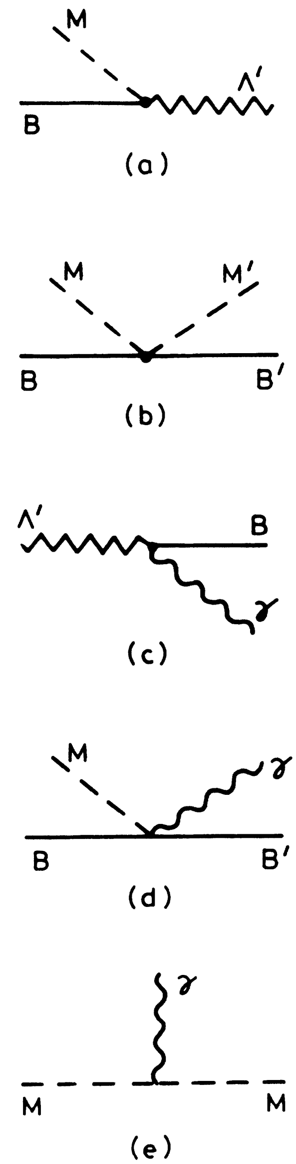

FIG. 1. Diagrams representing the various interaction terms derived in Sec. II: (a) the Yukawa term (2.11); (b) the contact term (2.16); (c) the $\Lambda'\to B\gamma$ vertex function (2.19); (d) the four-point coupling (2.22) involving baryon, meson, and photon; (e) the charged-meson–photon coupling.

$$U_{\alpha\Lambda'}^{s}(kR)=N_sN_p\left[2R^3j_0(\omega_sR)j_0(\omega_pR)j_0(kR)-(\omega_s-\omega_p+\omega_k)\int_0^R r^2[j_0(\omega_sr)j_0(\omega_pr)+j_1(\omega_sr)j_1(\omega_pr)]j_0(kr)dr\right] . \tag{2.13}$$

In the present work the coupling constants are taken to be $\lambda_{\bar{K}N,\Lambda'}=\sqrt{2}$ and $\lambda_{\Sigma\pi,\Lambda'}=\sqrt{3}$, which correspond to the assumption of exact SU(3)-flavor symmetry.

For the transition $BM\to B'$, where B and B' both belong to the baryon octet, conservation of parity means we find only p-wave vertex functions. These can be neglected because we are considering only s-wave scattering in the present work.

2. The four-point interaction (or the contact term)

At the quark-meson level this interaction takes the form

$$\hat{H}_c=\int d^3r\frac{\theta_v}{4f^2}\bar{q}\gamma^\mu\vec{\lambda}q\cdot(\vec{\phi}\times\partial_\mu\vec{\phi}) . \tag{2.14}$$

For s-wave scattering the spatial derivative part of this

TABLE I. The coupling constants $\lambda^I_{\alpha\beta}$ used in this work.

α \ β	$I=0$ $\bar{K}N$	$\Sigma\pi$	$I=1$ $\bar{K}N$	$\Sigma\pi$	$\Lambda^0\pi$
$\bar{K}N$	$-\frac{3}{2}$	$-\sqrt{6}/4$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\sqrt{6}/4$
$\Sigma\pi$	$-\sqrt{6}/4$	-2	$-\frac{1}{2}$	-1	0
$\Lambda^0\pi$	0	0	$\sqrt{6}/4$	0	0

interaction does not contribute, and the time derivative part for transitions between baryon-meson-octet members can be represented as

$$\hat{H}_c=\sum_{ij}\int d^3k\int d^3k'\langle B'M'\mid V^c_{ij}(\mathbf{k},\mathbf{k}')\mid B,M\rangle \times B'^{\dagger}BC_i^{\dagger}(\mathbf{k})C_j(\mathbf{k}') \ . \tag{2.15}$$

The vertex function is

$$\langle\alpha'\mid V^c_{ij}(\mathbf{k},\mathbf{k}')\mid\beta\rangle =\sum_{I,i}\frac{\lambda^I_{\alpha\beta}}{2f^2}\frac{1}{[(2\pi)^3 2\omega_M(k)]^{1/2}}\frac{1}{[(2\pi)^3 2\omega_{M'}(k')]^{1/2}} \times C^{i_B i_M i}_{I_B I_M I}C^{i_{B'} i_{M'} i}_{I_{B'} I_{M'} I}U^c_{\alpha\beta}(k,k',R) \ , \tag{2.16}$$

with the form factor

$$U^c_{\alpha\beta}(k,k',R)=N_s^2[\omega_M(k)+\omega_{M'}(k')] \times\int_0^R r^2[j_0^2(\omega_s r)+j_1^2(\omega_s r)] \times j_0(kr)j_0(k'r)dr \tag{2.17}$$

and the coupling constants are given in Table I.

3. The quark-photon interaction

This interaction is

$$\hat{H}_{Qe}=e_Q\int d^3r\,\bar{q}Aq\theta_v=\int d^3r\,j^{\mu}_Q A_\mu \ , \tag{2.18}$$

where $j^\mu_Q=e_Q\bar{q}\gamma^\mu q$ is the vector current of the quark. Considering s-wave scattering only, for the transition $\Lambda'\to B\gamma$ the vertex function [Fig. 1(c)] is

$$\langle B\mid \mathbf{V}_{Qe}(\mathbf{q})\epsilon(\mathbf{q}\lambda)\mid\Lambda'\rangle=\frac{i}{[(2\pi)^3 2q]^{1/2}}\lambda_{\Lambda' B}C^{m_B m m_{\Lambda'}}_{S_B 1 S_{\Lambda'}} \times\epsilon^*_m(\mathbf{q}\lambda)u_{Qe}(q,R) \ , \tag{2.19}$$

with the form factor

$$u_{Qe}(q,R)=N_sN_p\int_0^R r^2\{j_0(qr)[j_0(\omega_s r)j_0(\omega_p r)-\tfrac{1}{3}j_1(\omega_s r)j_1(\omega_p r)]+\tfrac{2}{3}j_2(qr)j_1(\omega_s r)j_1(\omega_p r)\}dr \ , \tag{2.20}$$

and the coupling constants $\lambda_{\Lambda'\Lambda^0}=-\frac{1}{2}$, $\lambda_{\Lambda',\Sigma^0}=-\sqrt{3}/2$.

4. The quark-meson-photon interaction

Once again at the quark-meson level this interaction is

$$\hat{H}_{Q\phi e}=-\frac{ie_\phi}{2f}\int d^3r\,\theta_v\bar{q}\gamma^\mu\gamma_5\vec{\lambda}q\cdot\vec{\phi}A_\mu \tag{2.21}$$

and if we restrict ourselves to s-wave scattering we find the following vertex function [Fig. 1(d)]:

$$\langle B'\mid \mathbf{V}_{Q\phi e}(\mathbf{k},\mathbf{q}\lambda)\cdot\epsilon(\mathbf{q}\lambda)\mid B\rangle=-\frac{ie_\phi}{2f}\frac{1}{[(2\pi)^3 2\omega(\mathbf{k})]^{1/2}[(2\pi)^3 2q]^{1/2}}\lambda_{\alpha B'}C^{m_B m m_{B'}}_{S_B 1 S_{B'}}C^{i_B i_M i_{B'}}_{I_B I_M I_{B'}}e^*_m u_{Q\phi e}(k,q,R) \tag{2.22}$$

with the form factor

$$u_{Q\phi e}(k,q,R)=N_s^2\int_0^R r^2\{j_0(qr)j_0(kr)[j_0^2(\omega_s r)-\tfrac{1}{3}j_1^2(\omega_s r)]+\tfrac{2}{3}j_2(qr)j_0(kr)j_1^2(\omega_s r)\}dr \tag{2.23}$$

and the coupling constants $\lambda_{\bar{K}N,\Lambda^0}=3\sqrt{2}$, $\lambda_{\bar{K}N,\Sigma^0}=-\sqrt{6}/3$, $\lambda_{\Sigma\pi,\Lambda^0}=-2\sqrt{3}$, and $\lambda_{\Sigma^0\pi,\Sigma}=\frac{4}{3}\sqrt{6}$.

5. The charged-meson–photon interaction

As usual, this interaction [shown in Fig. 1(e)] is

$$\hat{H}_{\phi e}=ie_\phi\int d^3r(\phi^\dagger\partial^\mu\phi-\phi\partial_\mu\phi^\dagger)A_\mu=\int d^3r\,j^\mu_\phi A_\mu \ . \tag{2.24}$$

The corresponding vertex function is

$$\mathbf{V}_{\phi e}(\mathbf{k},\mathbf{k}',\mathbf{q}\lambda)\cdot\epsilon=-\frac{e_\phi(\mathbf{k}+\mathbf{k}')\cdot\epsilon(\mathbf{q}\lambda)}{[(2\pi)^3 2\omega_M(k)]^{1/2}[(2\pi)^3 2\omega_{M'}(k')]^{1/2}[(2\pi)^3 2q]^{1/2}}(2\pi)^3\delta(\mathbf{k}-\mathbf{k}'-\mathbf{q}) \ . \tag{2.25}$$

The other interactions contained in Eq. (2.1) are not discussed here, because they are not required in our calculation.

B. The two-body t matrices

Before investigating the decay $K^-p \to \Lambda^0\gamma$ and $K^-P \to \Sigma^0\gamma$, we should estimate the momentum distribution of the kaon in the $\bar{K}^-p$ atom, because our whole calculation is done in momentum space. In fact, a K^-p system bound by the Coulomb force has only very-low-momentum components. For example, in the ground state the probability of finding $K=2.37$ MeV/c is only 25% of the probability for $k=0$. Although the principle quantum number n of the atomic state ns from which the decay takes place is unknown, it is certain to take place from an s state with more than 99% probability.[18] For $n \geq 2$, the momentum distribution around $k=0$ is narrower than for the ground state. So we can make the approximation that the kaon momentum in the K^-p atom is zero. Thus the matrix element can be factorized as follows:

$$\int d^3k\, \psi_s(\mathbf{k}) t_{fi}(\mathbf{k}) \simeq t_{fi}(0) \int d^3k\, \psi_s(\mathbf{k}) , \tag{2.26}$$

where $\psi_s(\mathbf{k})$ is the s-wave bound-state wave function in momentum space, and $t_{fi}(\mathbf{0})$ is the t matrix for the zero-energy $\bar{K}N$-$\Sigma\pi$ system. Since the same integral also appears in the calculation of the decay $K^-p \to$anything, it actually cancels in the branching ratio

$$R_{B\gamma} = \frac{\Gamma(K^-p \to B\gamma)}{\Gamma(K^-p \to \text{anything})} .$$

The S matrices for the processes $K^-p \to \Lambda^0\gamma$ and $\bar{K}p \to \Sigma^0\gamma$ shown in Fig. 1 can be expressed as[19]

$$S_{fi} = \delta_{fi} - 2\pi i \delta(E_f - E_i) T_{fi} ,$$

where f,i label the final and initial state, and the corresponding T matrix is

$$T_{fi} = \langle B \mid [a(\mathbf{q}\lambda), H_{\text{int}}] C_{\bar{K}}^\dagger(k=0) \mid p \rangle + \langle B \mid [a(\mathbf{q}\lambda), H_{\text{int}}] \times (E_i^+ - H)^{-1} [H_{\text{int}}, C_{\bar{K}}^\dagger(k=0)] \mid p \rangle . \tag{2.27}$$

In these expressions, initial and final energies are

$$E_i = (M_p^2 + k^2)^{1/2} + (M_{\bar{K}}^2 + k^2)^{1/2} , \quad E_f = (M_B^2 + q^2)^{1/2} + q , \tag{2.28}$$

where M_B and M_M are the masses of the baryon and meson, respectively. Returning to Eq. (2.3) where $H = H_0 + H_{\text{int}}$ we now have

$$H_0 = \sum_B (M_B^2 + k^2)^{1/2} B^+ B + \int d^3k' (M_M^2 + k'^2)^{1/2} C_M^\dagger(\mathbf{k}') C_M(\mathbf{k}') + \sum_\lambda \int d^3q\, q a^\dagger(\mathbf{q}\lambda) a(\mathbf{q}\lambda) , \tag{2.29}$$

and

$$H_{\text{int}} = H_s + H_c + H_{Qe} + H_{Q\phi e} + H_{\phi e} . \tag{2.30}$$

After projecting the interactions onto the space of colorless nonexotic baryons,[12] radiative K^- capture at rest ($k=0$) is described by the diagrams shown in Fig. 2.

The T matrices corresponding to the diagrams shown in Fig. 2 are as follows:

$$T_a = \langle B \mid \mathbf{V}_{Q\phi e}(\mathbf{k}=0) \cdot \boldsymbol{\epsilon}(\mathbf{q}\lambda) \mid P \rangle , \tag{2.31a}$$

$$T_b = \sum_{B'} \int d^3k' \langle B \mid \mathbf{V}_{Q\phi e}(\mathbf{k}') \cdot \boldsymbol{\epsilon}(\mathbf{q}\lambda) \mid B' \rangle [E_i - M_{B'}(\mathbf{k}') - \omega_{M'}(\mathbf{k}')]^{-1} t_{\beta\alpha}(\mathbf{k}', \mathbf{k}=0, E_i) , \tag{2.31b}$$

$$T_c = \langle B \mid \mathbf{V}_{Qe}(\mathbf{q}) \cdot \boldsymbol{\epsilon}(\mathbf{q}\lambda) \mid \Lambda' \rangle (E_i - M_{\Lambda'})^{-1} \langle \Lambda' \mid V_{sK^-}^\dagger(k=0) \mid p \rangle , \tag{2.31c}$$

$$T_d = \sum_{B'} \int d^3k' \langle B \mid \mathbf{V}_{Qe}(\mathbf{q}) \cdot \boldsymbol{\epsilon}(\mathbf{q}\lambda) \mid \Lambda \rangle (E_i - M_{\Lambda'})^{-1} \langle \Lambda' \mid V_{sM'}^\dagger(\mathbf{k}') \mid B' \rangle [E_i - M_{B'}(k') - \omega_{M'}(\mathbf{k}')]^{-1} t_{\beta\alpha}(\mathbf{k}', \mathbf{k}=0, E_i) , \tag{2.31d}$$

$$T_e = \sum_{B'} \int d^3k' \int d^3k'' \langle B \mid V_{sM''}^\dagger(\mathbf{k}'') \mathbf{V}_{\phi e}(\mathbf{k}', \mathbf{k}'', \mathbf{q}) \cdot \boldsymbol{\epsilon}(\mathbf{q}\lambda) \mid B' \rangle \times [M_B(q) - M_{B'}(k'') - \omega_M(k'')]^{-1} [E_i - M_{B'}(k') - \omega_{M'}(k')]^{-1} t_{\beta\alpha}(\mathbf{k}', \mathbf{k}=0, E_i) . \tag{2.31e}$$

Here B may be Λ^0 or Σ^0, $\beta = (B'M')$ may be $(N\bar{K})$ or (Σ, π) and $\alpha = (p, \bar{K})$. The two-body t matrix, $t_{\beta\alpha}(\mathbf{k}', \mathbf{k}=0, E_i)$, is represented by a circled t in Fig. 2. (It is the half-off-shell t matrix for $\bar{K}N \to \bar{K}N$ and $\bar{K}N \to \Sigma\pi$.) We define the effective potential $V_{\beta\alpha}$ as

$$V_{\beta\alpha}(\mathbf{k}', \mathbf{k}, E) = \sum_c \langle \beta \mid H_s \mid B' \rangle (E - M_c)^{-1} \langle B' \mid H_s \mid \alpha \rangle + \langle \beta \mid H_c \mid \alpha \rangle , \tag{2.32}$$

where for s-wave scattering we retain only the contact term (H_c) and the Λ'. The effect of crossed-meson lines is relatively small for s-wave meson-nucleon scattering and for this reason we neglect it here. The half-off-shell t matrix of this effective potential satisfies the Lippmann-Schwinger equation:

$$t_{\beta\alpha}(\mathbf{k}',\mathbf{k},E)=V_{\beta\alpha}(\mathbf{k}',\mathbf{k},E)+\sum_{\gamma}\int d^3k''V_{\beta\gamma}(\mathbf{k}',\mathbf{k}'',E)\times[E-E_\gamma(k'')+i\epsilon]^{-1}\times t_{\gamma\alpha}(\mathbf{k}'',\mathbf{k},E)\ , \tag{2.33}$$

where

$$E_\gamma(k'')=(M_{B\gamma}^2+k''^2)^{1/2}+(M_{M_\gamma}^2+k''^2)^{1/2} \tag{2.34}$$

is the energy of the intermediate baryon-meson system. The expansion in partial waves is made as usual:

$$V_{\beta\alpha}(\mathbf{k}',\mathbf{k},E)=\sum_{lm}Y_{lm}^*(\hat{\mathbf{k}}')Y_{lm}(\hat{\mathbf{k}})V_{\beta\alpha}^l(k',k,E)\ ,$$
$$t_{\beta\alpha}(\mathbf{k}',\mathbf{k},E)=\sum_{lm}Y_{lm}^*(\hat{\mathbf{k}}')Y_{lm}(\hat{\mathbf{k}})t_{\beta\alpha}^l(k',k,E)\ . \tag{2.35}$$

For the s wave ($l=0$) and including an explicit isospin index, we find

$$t_{\beta\alpha}^s(\mathbf{k}',\mathbf{k},E)=\frac{1}{4\pi}t_{\beta\alpha}^{l=0}(k',k,E)=\frac{1}{4\pi}\sum_{I,i}C_{I_BI_MI}^{i_Bi_Mi}C_{I_{B'}I_{M'}I}^{i_{B'}i_{M'}i}t_{\beta\alpha}^I(\mathbf{k}',\mathbf{k},E)\ . \tag{2.36}$$

In order to avoid the singularity in the denominator of, e.g., (2.31) and (2.33), we make a principal-value subtraction.[20]

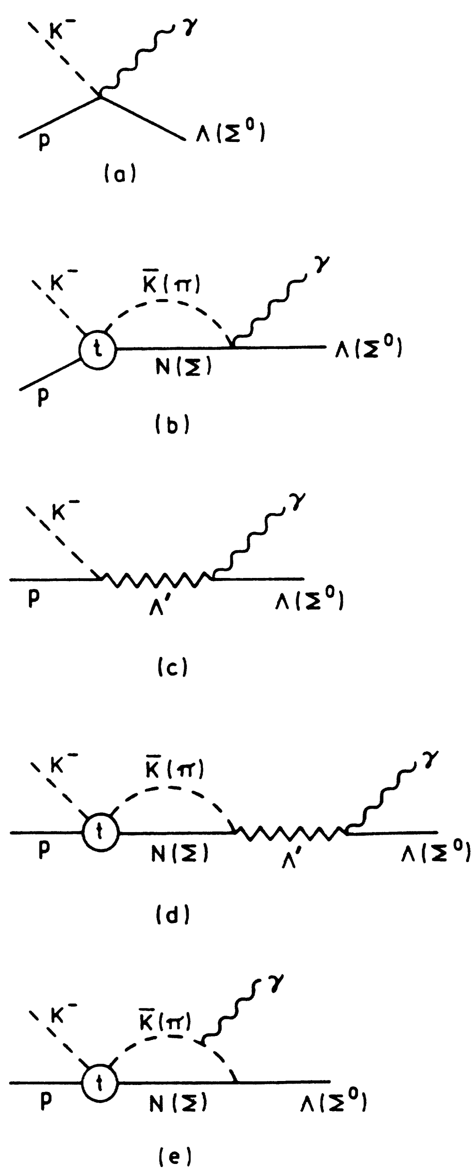

FIG. 2. Diagrammatic representation of the various contributions to the radiative decay of the $\bar{K}p$ atom. The contributions (a)–(e) correspond to Eqs. (2.31a)–(2.31e).

C. The decay widths

In order to calculate the ratio $R_{\Lambda\gamma}$ and $R_{\Sigma^0\gamma}$, we should calculate three decay widths $\Gamma(K^-p\to\Lambda^0\gamma)$, $\Gamma(K^-p\to\Sigma^0\gamma)$, and $\Gamma(K^-p\to\text{anything})$. (Here the initial K^-p denotes the K^-p atom.) As mentioned above, the t matrix for $K^-p\to B\gamma$ (where $B=\Lambda^0$ or Σ^0) can be factorized into

$$T_\gamma\simeq\left[\int d^3k\,\psi_s(\mathbf{k})\right]\sum_j T_j(K^-p\to B\gamma),\quad J=a,b,c,d,e\ , \tag{2.37}$$

where $T_j(K^-p\to B\gamma)$ ($j=a,b,\ldots,e$) are given in Eq. (2.31). The width in the c.m. system for this process can be written as

$$\Gamma(K^-p\to B\gamma)=\frac{8\pi^2}{2S_p+1}\sum_{M_pM_B}\sum_{\lambda}\left[\int d^3k\psi_s(\mathbf{k})\right]^2\frac{q^2M_B(q)}{M_B(q)+q}\times\left|\sum_j T_j(K^-p\to B\gamma)\right|^2\ . \tag{2.38}$$

(Here $S_p=\frac{1}{2}$ is the spin of proton, M_p and M_B are the spin projections of the proton and type-B baryon, respectively.)

Next we note that the decay width for $K^-p\to\Sigma\pi$ ($\Sigma^+\pi^-,\Sigma^-\pi^+,\Sigma^0\pi^0$) makes up more than 90% of the total width. For this reason we use the width of $K^-p\to\Sigma\pi$ instead of the total width in order to calculate branching ratios. That is, we use

$$\Gamma(K^-p\to\text{anything})=\frac{1}{4}\left[\int d^3k\,\psi_s(\mathbf{k})\right]^2\frac{k_0\omega_\pi(k_0)M_\Sigma(k_0)}{M_\Sigma(k_0)+\omega_\pi(k_0)}\times(|t_{\bar{K}p\to\Sigma\pi}^{I=0}|^2+|t_{\bar{K}p\to\Sigma\pi}^{I=1}|^2)\ , \tag{2.39}$$

where $t^{I=0,1}_{K^-p\to\Sigma\pi}$ are on the on-shell t matrices, which can be found by solving the Lippmann-Schwinger equation (2.33) [$I=(0,1)$ is the isospin of the K^-p system]. Once again, the final pion momentum can be obtained from the δ function:

$$k_0^2=(E_i+M_\Sigma+M_\pi)(E_i+M_\Sigma-M_\pi)\times(E_i-M_\Sigma-M_\pi)(E_i-M_\Sigma-M_\pi)/(4E_i^2) . \quad (2.40)$$

III. RESULTS AND DISCUSSION

In our calculation, there are only three parameters namely, the bag radius R, the meson decay constant f and M_Λ the mass of the SU(3)-singlet baryon Λ'. Using the same model, Veit *et al.*[15] found two sets of these parameters from an analysis of s-wave $\bar{K}N$ scattering:

Set A, $R=1.0$ fm, $M_0=1630$ MeV ,

$f^{I=0}=120$ MeV, $f^{I=1}_{\pi\Sigma\pi\Lambda}=110$ MeV ,

$f^{I=1}_{\bar{K}N}=100$ MeV ;

Set B, $R=1.1$ fm, $M_0=1650$ MeV ,

$f^{I=0}=110$ MeV, $f^{I=1}_{\pi\Sigma\pi\Lambda}=105$ MeV ,

$f^{I=1}_{\bar{K}N}=95$ MeV .

Using parameter set A, but f taken as the average value (e.g., $R=1.0$ fm, $M_0=1630$ MeV, $f=110$ MeV), we find the results shown in Table II [for the five amplitudes to T_j (j=a,b, . . . , e) corresponding to Figs. 2(a)–2(e)]. It is remarkable that whereas the $\Lambda\gamma$ amplitude is almost pure imaginary that for $\Sigma^0\gamma$ is predominantly real. This fact means that the $\Lambda^*(1405)$ resonance, just below the $\bar{K}N$ threshold, plays a very important role as we will discuss below.

The respective theoretical branching ratios (BR) are 1.9×10^{-3} for $\Lambda\gamma$ and 2.3×10^{-3} for $\Sigma^0\gamma$. The former is within one standard deviation of the existing measurement[4] and the latter is tantalizingly close to the experimental upper limit[3] for this process. [Note that the results of Table II differ from our previously reported results by an unimportant, overall phase for $\Lambda\gamma$, and a factor of $\sqrt{3}$ in Fig. 2(d) for $K^-p\to\Sigma^0\gamma$ which was omitted in the earlier computer calculations.]

We have made a number of calculations which test the reliability of these results. First we used the alternate parameter set B of Veit *et al.* (e.g., $R=1.1$ fm, $M_0=1650$ MeV, $f=105$ MeV), this gave $R_{\Lambda\gamma}=1.78\times10^{-3}$ and $R_{\Sigma\gamma}=1.93\times10^{-3}$, both within 10% of the earlier results. We also studied small variations in the parameters around sets A and B, and these results are collected together in Table III. We note that even though the $\Lambda\gamma$ and $\Sigma^0\gamma$ branching ratios may vary by as much as 30%, the relative branching ratios (last column of Table III) are remarkably stable.

We also wish to test the sensitivity of the radiative decay calculations to the fact that our K^-p scattering length gave a rather large imaginary part in isospin zero $a_0=-1.03+1.89i$ (fm) compared with that obtained from dispersion relations.[20] As a crude test we rescaled the half-off-shell t matrices for $\bar{K}N$ to $\bar{K}N$ and to $\Sigma\pi$ in order to give the dispersion relation value on shell for $\bar{K}N$, while remaining consistent with unitarity. This rather dramatic change lowered the branching ratios for both $\Lambda\gamma$ and $\Sigma^0\gamma$ by about 30%, but did not change the ratio of the two significantly. It should be pointed out that because of the sharp cusplike behavior in the $\bar{K}N$ t matrix at threshold,[15] we have doubts about the accuracy of the conventional dispersion relation analysis (see also Ref. 21), so this second experiment is in no sense meant as an improvement on the calculations quoted above.

Finally we note that in the rather different approach to this problem by Isgur and Maltman[22] the inclusion of finite meson size tended to decrease the $Kp\to\Sigma\pi$ rate and thereby raise the photon branching ratios. The CBM

TABLE II. Amplitudes for the various contributions to radiative $\bar{K}$ capture on the proton shown in Fig. 2.

	$\bar{K}p\to\Lambda\gamma$	$\bar{K}p\to\Sigma^0\gamma$
T_a	-0.086	0.017
T_b	$+0.062-0.107i$	$0.081-0.025i$
T_c	-0.011	-0.019
T_d	$+0.030-0.037i$	$0.050-0.063i$
T_e	$+0.007-0.005i$	$0.020-0.010i$
T_{tot}	$0.002-0.149i$	$0.149-0.098i$
BR	1.9×10^{-3}	2.3×10^{-3}
BR (expt)	$(2.8\pm0.8)\times10^{-3}$ [a]	$<4\times10^{-3}$ [b]

[a]Reference 4.
[b]Reference 3.

TABLE III. Sensitivity of the results to small changes in parameters (with $t_{\alpha\beta}$ fixed).

R (fm)	m_Λ (MeV)	f (MeV)	10^3BR($\Lambda\gamma$)	10^{+3}BR($\Sigma^0\gamma$)	BR($\Sigma^0\gamma$)/BR($\Lambda\gamma$)
1.0	1630	100	2.36	2.76	1.17
1.0	1630	110	1.94	2.28	1.17
1.0	1630	120	1.63	1.91	1.17
1.0	1640	100	2.30	2.67	1.16
1.0	1640	105	2.08	2.42	1.16
1.0	1640	110	1.90	2.21	1.16
1.0	1650	110	1.85	2.14	1.15
1.1	1650	105	1.78	1.93	1.09

used here does not explicitly include finite meson size, although that is implicit in the truncation to low-lying baryons. In our rormulation the strength of the various couplings are dictated by chiral symmetry and including finite meson size would have a very small effect through a slight softening of the various form factors involved.

IV. CONCLUSION

We have seen that the CBM makes a definite prediction for the relative decay rates of the $\bar{K}p$ atom into $\Lambda\gamma$ and $\Sigma^0\gamma$ $[R(\Sigma^0\gamma/\Lambda\gamma)\sim 1.1-1.2]$. This prediction is quite different from most other models (for example, Darewych, Koniuk, and Isgur[11] find 0.64–0.77 in the NRQM), and deserves to be tested experimentally.

It should also be stressed that the present work differs in a fundamental way from all earlier calculations which have included the $\Lambda^*(1405)$. In particular, the coupling of the $\Lambda^*(1405)$ to the open $\Sigma\pi$ channel has been carefully incorporated. As we discussed in connection with Table II this leads to quite dramatic differences between the amplitudes for $K^-p\rightarrow\Lambda\gamma$ and $K^-p\rightarrow\Sigma\gamma$. Indeed, in our approach the former is almost pure imaginary and the latter mostly real. On the other hand, in conventional quark-model calculations both amplitudes would usually have the same phase. Clearly a meaningful comparison between this calculation and others cannot be made until those others include explicit channel coupling effects in a reliable way. Indeed in view of our results any future calculation which ignores channel coupling will require justification.

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