$b \to s\gamma$ in the two-Higgs-doublet model

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The processes $b \to s\gamma$ and $b \to sg$ are examined within the context of the two-Higgs-doublet extension of the standard model. The usual W^- loop diagrams responsible for these decays are now supplemented by loops involving charged Higgs bosons (H^-). We find that the total amplitudes for both processes are substantially enhanced in this model for a wide range of top-quark and charged-Higgs-boson masses. For a reasonable range of the ratio of the vacuum expectation values of the two Higgs doublets, these decays can provide important tests of such models.

The standard model of the electroweak interactions is in complete agreement with all experimental data.[1] However, we remain ignorant of the detailed nature of the Higgs-boson sector of the model since it has yet to confront any direct experimental tests. In many extensions of the standard model (SM), such as models with spontaneous *CP* violation and horizontal symmetries[2] or involving supersymmetry,[3] the Higgs-boson sector is enlarged from a single doublet to two (or more) doublets which necessitates the existence of charged Higgs bosons. This possibility has already received substantial attention in the literature. The probing of loop-induced couplings provides a means of testing the detailed structure of the SM at the level of radiative corrections where Glashow-Iliopoulos-Maiani[4] (GIM) cancellations are important. Compared with *s*-quark loop decays, those involving *b* quarks are expected to be far more frequent, quite sensitive to the *t*-quark mass (since the *b* quark also lies in the third generation), and to have calculable QCD short-distance corrections.

In this paper, we will examine how the existence of charged Higgs bosons may influence the decay rates of loop-induced processes; in particular $b \to s\gamma$ (Ref. 5) and $b \to sg$ (Ref. 6). Both of these processes have been discussed at some length in recent literature. We hope to show that for a reasonable range of parameters, consistent with existing experiment, the charged-Higgs-boson diagram substantially modifies the conventional *W*-penguin contribution to both of the above processes. It should be noted that the existence of the charged-Higgs-boson-exchange diagram would not alter the usual SM expectation that $\Gamma(b \to d\gamma)/\Gamma(b \to s\gamma) = |V_{td}/V_{ts}|^2$ [and similarly for $\Gamma(b \to dg)/\Gamma(b \to sg)$], so that these modes are still expected to be suppressed in contrast with what may happen in models involving the fourth generation.

The diagrams responsible for the generic processes $b \to q\gamma$ and $b \to qg$ which result from charged-Higgs-boson exchange are shown in Fig. 1. The conventional *W*-loop amplitude for $b \to q\gamma$ is given by (assuming *t*-quark dominance)

$$A_W^\gamma = \frac{G_F}{\sqrt{2}} \left[\frac{e}{16\pi^2} \right] \bar{q} \sigma_{\lambda\nu} q^\nu (1+\gamma_5) b m_b V_{tb} V_{tq}^* \times [H(x) + Q_t I(x)] \epsilon^\lambda , \qquad (1)$$

where $x \equiv m_t^2/M_W^2$ and the functions H and I are given by[7]

$$H \equiv \frac{2-7x+11x^2}{2(1-x)^3} - 1 + \frac{3x^3}{(1-x)^4} \ln x ,$$
$$I \equiv \frac{1-5x-2x^2}{2(1-x)^3} - \frac{1}{2} - \frac{3x^2}{(1-x)^4} \ln x , \qquad (2)$$

and $Q_t = \frac{2}{3}$. The corresponding amplitude for $b \to qg (A_w^g)$ is obtainable from (1) via the substitutions $e \to g_s \lambda_a/2$, $\epsilon^\lambda \to \epsilon_a^\lambda$, $H=0$, and $Q_t = 1$ where $\lambda_a/2$ are the SU(3)-color generators with g_s being the strong coupling constant. Note that as $x \to \infty$, both H and $I \sim$ const apart from logarithmic corrections. In order to calculate the diagrams in Fig. 1 we note that the charged-Higgs-boson coupling to *t* quarks is given by

$$\frac{ig}{2\sqrt{2}} \frac{m_t}{M_W} V_{tq} (\tan\beta)^{-1} \bar{t}(a - b\gamma_5) q H^- + \text{H.c.} , \qquad (3)$$

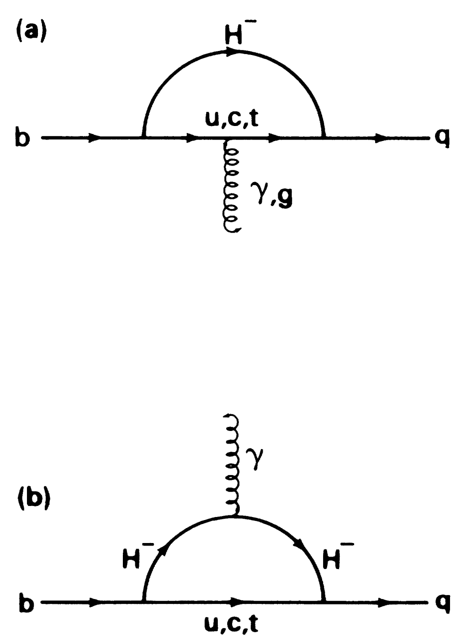

FIG. 1. Diagrams contributing to the processes $b \to q\gamma, qg$ where the $\gamma(g)$ is on shell.

where $a,b = 1 \pm (m_q/m_t)\tan^2\beta$ and V_{tq} is an element of the Kobayashi-Maskawa mixing matrix. $\tan\beta = v_2/v_1$ where v_1 (v_2) is the vacuum expectation value of the doublet which gives mass to the $Q = \frac{2}{3}$ ($Q = -\frac{1}{3}$) quarks. Since $m_b < m_t$ and $m_s < m_c$ one might expect $v_2 < v_1$ although we will not assume this in what follows. Naturalness suggests $v_2/v_1 \sim 1$ and limits from the K-$\bar{K}$ system[8] also suggest values near unity for this quantity. Note that in the limit $m_q/m_t \tan^2\beta \ll 1$ (which is clearly true for $q = d,s$) we find $a = b = 1$ so that H^- is chirally coupled.

Given (3) we find that the Higgs-boson-induced amplitude is given by

$$A_H^\gamma = \frac{G_F}{\sqrt{2}} \left[\frac{e}{16\pi^2} \right] \bar{q} \sigma_{\lambda\nu} q^\nu (1+\gamma_5) b m_b V_{tb} V_{tq}^* \times [G(\delta) + Q_t F(\delta)] \epsilon^\lambda , \quad (4)$$

where $\delta \equiv m_t^2/m_H^2$. The functions F and G take the forms $F(G) = F_1(G_1) + F_2(G_2)/\tan^2\beta$ where the integrals F_i and G_i ($i = 1,2$) are given in the Appendix. We note that for large values of δ, F_i and G_i both tend to constant values just like $H(x)$ and $I(x)$ do for large values of x. In what follows we will use the results of Ref. 1 which strongly indicates $25 \lesssim m_t \leq 200$ GeV and the e^+e^- experimental data[9] which imply $m_H \geq 20$ GeV. Taken together these two constraints imply $\sqrt{\delta} \lesssim 10$ where for a fixed value m_t, e.g., $m_t = 60$ (100) GeV we obtain $\sqrt{\delta} \lesssim 3$ (5). The corresponding Higgs-boson-induced amplitude for $b \to sg$ (A_H^g) can be obtained from (4) by the same substitutions as above with $G = 0$. Note that we have not included the QCD corrections in our discussion below.

Figure 2 shows the ratio $R_\gamma = | A_H^\gamma / A_W^\gamma |$ as a function of m_t/m_H with m_t fixed at 60 GeV for $\tan\beta = 0.5$, 1, and 2, respectively. Note that for light m_H ($m_t/m_H > 1$), R_γ is reasonably large for all values of $\tan\beta$. For large m_H ($m_t/m_H < 1$) we find that the relative enhancement is quite small, e.g., for $m_H = 150$ GeV we obtain $R_\gamma \simeq 1$. Figure 2 also shows the importance of keeping terms of order unity when v_2/v_1 is not greatly different from one. Note that even for very large values of $\tan\beta$ we still obtain a significant "irreducible" enhancement due to the terms of order unity. For light-Higgs-boson masses one can, in principle, use our results to put bounds on the value of $\tan\beta$ if one could reliably calculate the exclusive mode $B \to K^*\gamma$ (Ref. 5)

Figure 3 also shows R_γ but for $m_t = 100$ GeV. As one might expect, R_γ is reduced somewhat in this case since A_W^γ scales roughly as m_t^2. Again for light m_H there is a reasonably substantial enhancement in the $b \to s\gamma$ amplitude even for modest values of $\tan\beta$. Note that we obtain $R_\gamma \simeq 1$ for $m_H \simeq 200$ GeV and that there is still some enhancement even when $m_t/m_H \lesssim 1$.

It is clear from this discussion that if $m_t/m_H > 1$ and $\tan\beta \sim 1$ one may expect very sizable enhancements in the $b \to s\gamma$ branching ratio and that values in the range 3–10 are not unusual. Such enhancements may be somewhat larger than the effect of fourth-generation fermions.[10]

Figure 4 shows $R_g = | A_H^g / A_W^g |$ as a function of m_t/m_H with m_t fixed at 60 GeV for $\tan\beta = 0.5$, 1, and 2. Since $G = 0$ in this case (as is H) we expect the magnitude of the enhancement to be somewhat different for this process. As shown in Fig. 4, $R_g \simeq R_\gamma$ for $m_t = 60$ GeV. If $B(b \to sg)$ is 10^{-2} (3×10^{-3}) in the SM, and we demand that this branching ratio be less than 10^{-1} (so as not to

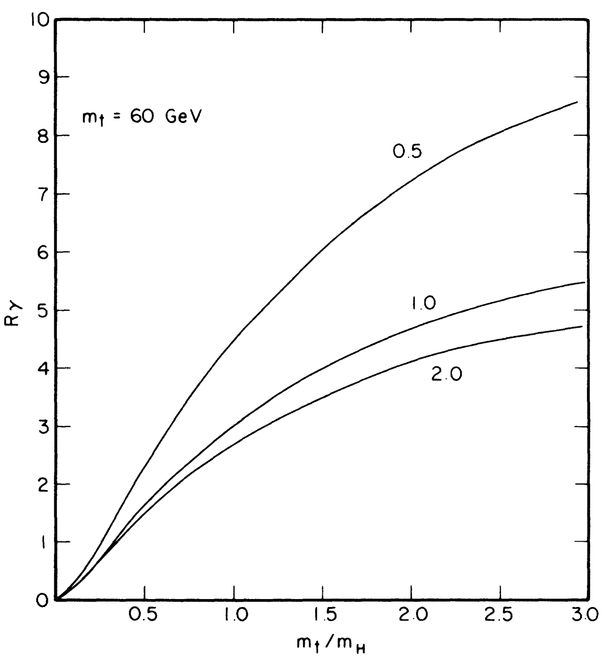

FIG. 2. The ratio of amplitudes, R_γ, for $m_t = 60$ GeV as a function of m_t/m_H. The values assumed for $\tan\beta$ are shown.

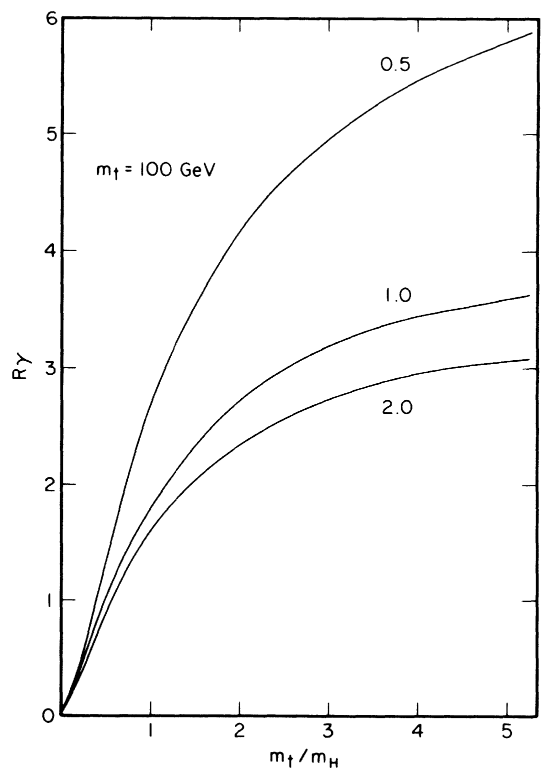

FIG. 3. Same as Fig. 2 but with $m_t = 100$ GeV.

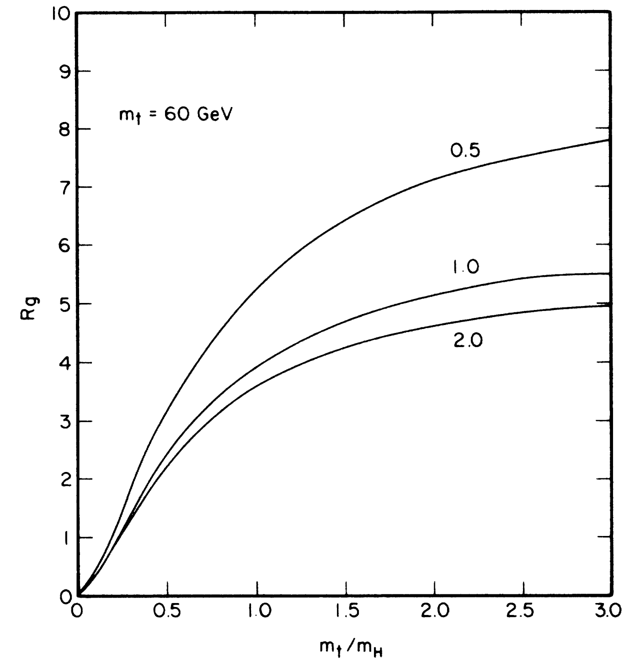

FIG. 4. The ratio of amplitudes, R_g, for $m_t=60$ GeV as a function of m_t/m_H. The values assumed for $\tan\beta$ are shown.

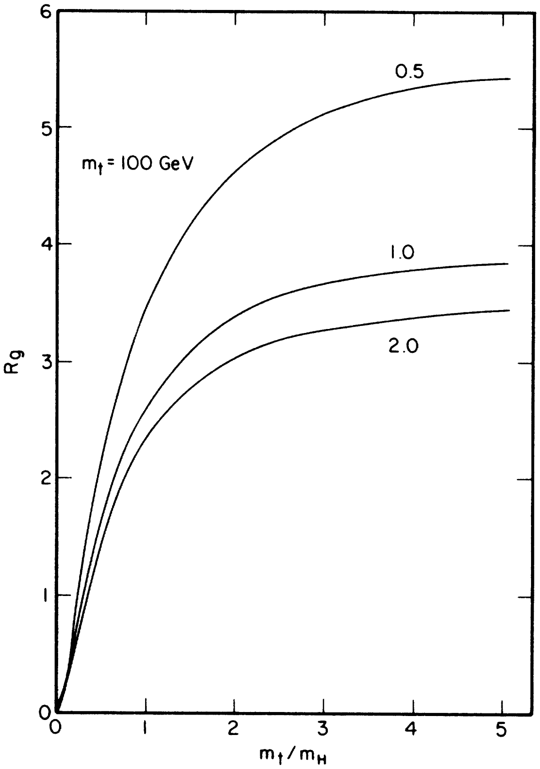

FIG. 5. Same as Fig. 4 but with $m_t=100$ GeV.

upset b-decay phenomenology too much) one can obtain a $\tan\beta$ bound dependent on $m_t/m_H \lesssim 0.7$ (1.0–3.0) for $m_t=60$ GeV; larger values of $\tan\beta$ only yield a limit on the branching ratio if the SM value is assumed to be 10^{-2}.

Figure 5 shows R_g for $m_t=100$ GeV as a function of m_t/m_H. As expected from the above discussion of R_γ, R_g is somewhat smaller in this case since the W contribution is enhanced. We again see that for reasonable values of the parameters the branching ratio for $b\to sg$ can be enhanced by an order of magnitude.

The main points of this paper are as follows: (i) The existence of charged-Higgs-boson scalars can substantially enhance the branching ratios for the rare processes $b\to s\gamma$ and $b\to sg$ for a wide range of parameters; (ii) the largest enhancement occurs when $m_H<m_t$; (iii) terms independent of v_1/v_2 and/or proportional to m_b cannot be neglected in calculating the Higgs-boson contribution to either process; (iv) for fixed m_t and m_H we find that $b\to sg$ is as enhanced as $b\to s\gamma$, i.e., $R_g\simeq R_\gamma$; (v) improvement in the present experimental upper limits on $B\to K^*\gamma$ may provide bounds on v_2/v_1 and m_H for fixed values of m_t if one could relate the inclusive $b\to s\gamma$ process to the exclusive $B\to K^*\gamma$ process.

Clearly, new experiments on rare decays may yield the first clue to the nature of the Higgs-boson sector of the standard model.

Since this paper was completed, several other authors[11,12] have also considered the effect of charged-Higgs-boson scalars on the $b\to s\gamma$ and $b\to sg$ processes. All of the results appear to be in general agreement.

The author would like to thank Ian Hinchliffe for discussions related to this work and for bringing the work of Grinstein and Wise[11] to his attention. The author would also like to thank J. L. Hewett for discussion on this and related works and bringing the results of Ref. 12 to my attention.

APPENDIX

In this appendix we give the explicit forms of the integrals F_i and G_i ($i=1,2$) used in the text. Explicitly we find that

$$\begin{aligned} G_1(\delta)&=2\delta\int_0^1 dt\,\frac{t(1-t)}{\delta+(1-\delta)t}\\ &=2\delta(1-\delta)^{-3}[\tfrac{1}{2}(1-\delta^2)+\delta\ln\delta]\\ &=1 \quad \text{as } \delta\to\infty\ , \end{aligned} \tag{A1}$$

$$\begin{aligned} G_2(\delta)&=\delta\int_0^1 dt\,\frac{t^2(1-t)}{\delta+(1-\delta)t}\\ &=\delta(1-\delta)^{-4}(\tfrac{1}{6}-\delta+\tfrac{1}{2}\delta^2+\tfrac{1}{3}\delta^3-\delta^2\ln\delta)\\ &=\tfrac{1}{3} \quad \text{as } \delta\to\infty\ , \end{aligned} \tag{A2}$$

$$\begin{aligned} F_1(\delta)&=2\delta\int_0^1 dt\,\frac{t^2}{1+(\delta-1)t}\\ &=-2\delta(1-\delta)^{-3}(\tfrac{3}{2}-2\delta+\tfrac{1}{2}\delta^2+\ln\delta)\\ &=1 \quad \text{as } \delta\to\infty\ , \end{aligned} \tag{A3}$$

$$F_2(\delta)=\delta\int_0^1 dt\frac{t^2(1-t)}{1+(\delta-1)t}$$
$$=\delta(1-\delta)^{-4}(\tfrac{1}{3}+\tfrac{1}{2}\delta-\delta^2+\tfrac{1}{6}\delta^3+\delta\ln\delta) \quad \text{(A4)}$$
$$=\tfrac{1}{6} \text{ as } \delta\rightarrow\infty \ .$$

It should be noted that in each case the large-δ limit yields a constant and not a decreasing power of δ as one might expect. This is due to the couplings of the charged Higgs scalar which are proportional to m_t for the situation being discussed here.

[1]See, for example, the recent review by U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987).

[2]There has been a great deal of work in this area over the last few years. For recent work on this subject see H. Neufeld, W. Grimus, and Ecker, University of Vienna Reports Nos. UWThPh-1987-19, -21, and -23 (unpublished); and R. N. Mohapatra, University of Maryland report, 1987 (unpublished), and references therein.

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