CP violation in $K_L \to \pi^0 e^+ e^-$: Interference of one-photon and two-photon exchange

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The decay $K_L \to \pi^0 e^+ e^-$ can proceed through a CP-violating amplitude of order $G_F \alpha_{\rm em} \epsilon$ or a CP-conserving amplitude of order $G_F \alpha_{\rm em}^2$. The latter is shown to be significant, and probably the dominant contribution. Interference of the two pieces produces a large asymmetry between the e^+ and e^- energy spectra. Some comments are made on the $e:\mu$ ratio in $K_L \to \pi^0 l^+ l^-$.

I. INTRODUCTION

It was noted a long time ago[1,2] that in the limit of CP invariance the decay $K_2 \to \pi^0 e^+ e^-$ (where K_2 denotes the CP-odd state of the neutral K meson) cannot occur through a one-photon intermediate state. This raised the possibility that this reaction might be a sensitive probe of the CP-violating vertex $K_2 \to \pi^0 +$ virtual photon, insofar as the competing CP-conserving amplitude involving two-photon exchange was suppressed[2] (Fig. 1). This discussion has recently been revived[3,4] in the context of new experiments searching for rare decays of the K meson.

Quite generally, the one-photon amplitude for the transition $K_2(p) \to \pi^0(p') + e^-(k) + e^+(k')$ has the form

$$A(K_2 \to \pi^0 e^+ e^-)\,|_{1\gamma} = iF_{1\gamma}(w)\bar{u}(k)\not{p}v(k') , \quad (1)$$

where $F_{1\gamma}$ is a function of the invariant mass of the e^+e^- pair. We use as invariants

$$w = t + t', \quad \Delta = t - t' ,$$

where

$$t = (p-k)^2, \quad t' = (p-k')^2, \quad s = (k+k')^2 ,$$
$$s + t + t' = M^2 + \mu^2 + 2m_e^2 , \quad (2a)$$

with M, μ, and m_e denoting the masses of K, π, and an electron. In the rest frame of the K meson, w and Δ are related to the sum and difference of e^+ and e^- energies:

$$w = 2M^2 + 2m_e^2 - 2M(E_- + E_+), \quad \Delta = 2M(E_- - E_+) . \quad (2b)$$

Phase space is defined by

$$4m_e^2 < s < (M-\mu)^2, \quad -\Delta_0(s) < \Delta < +\Delta_0(s) ,$$

with

$$\Delta_0(s) = \left[1 - \frac{4m_e^2}{s}\right]^{1/2} [(M^2-\mu^2-s)^2 - 4\mu^2 s]^{1/2} . \quad (2c)$$

The two-photon-exchange amplitude has the general structure

$$A(K_2 \to \pi^0 e^+ e^-)\,|_{2\gamma} = F_{2\gamma}(w,\Delta)\bar{u}(k)\not{p}v(k') + \tilde{F}_{2\gamma}(w,\Delta)\bar{u}(k)v(k') , \quad (3)$$

where the form factors $F_{2\gamma}$ and $\tilde{F}_{2\gamma}$ are functions of both w and Δ. Because of chiral symmetry (i.e., invariance of the Lagrangian under $\psi_e \to e^{i\gamma_5}\psi_e$ in the limit $m_e \to 0$) the form factor $\tilde{F}_{2\gamma}$ must be an odd function of m_e, vanishing when $m_e = 0$. Accordingly we neglect $\tilde{F}_{2\gamma}$. The complete amplitude for $K_2 \to \pi^0 e^+ e^-$ is then

$$A(K_2 \to \pi^0 e^+ e^-) = [iF_{1\gamma}(w) + F_{2\gamma}(w,\Delta)]\bar{u}(k)\not{p}v(k') . \quad (4)$$

Because of CPT invariance, the CP-violating piece $iF_{1\gamma}$ and the CP-conserving piece $F_{2\gamma}$ must be out of phase by 90° if no absorptive parts are present. (This is the reason for inserting the factor i preceding $F_{1\gamma}$.) In actual fact,

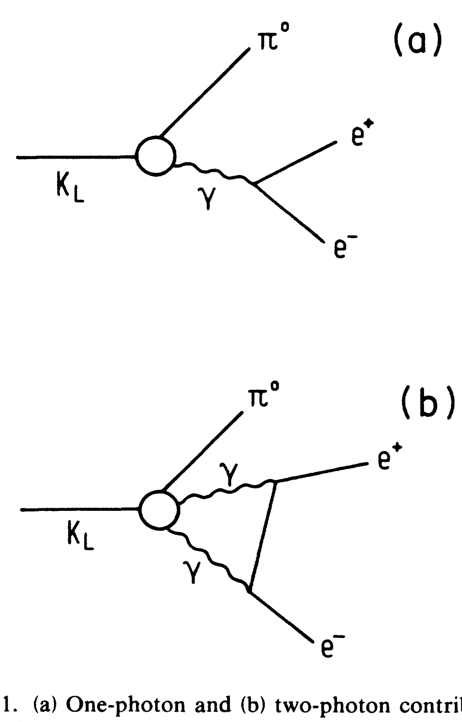

FIG. 1. (a) One-photon and (b) two-photon contributions to $K_L \to \pi^0 e^+ e^-$.

both the one- and two-photon amplitudes can have "unitarity" phases which are particularly evident for the form factor $F_{2\gamma}$ (Ref. 5). Thus the phase difference between the two terms in (4) is different from 90° which means they can interfere with each other, the interference being a function of the variables w and Δ. The fact that $F_{2\gamma}(w,\Delta)$ describes a transition to a CP-odd final state implies that $F_{2\gamma}$ must be an *odd* function of Δ, in first approximation proportional to Δ. This, together with the fact that $F_{1\gamma}$ is independent of Δ, means that the decay rate has the form

$$d\Gamma(w,\Delta)=a_0(w)+a_1(w)\Delta+a_2(w)\Delta^2$$

with a nonvanishing linear term in Δ, implying an asymmetry between the e^+ and e^- distributions. The purpose of this paper is to discuss quantitatively the one- and two-photon contributions to $K_L^0 \to \pi^0 e^+ e^-$ and the CP-violating asymmetry arising from their interference.

II. ONE-PHOTON CONTRIBUTION

The CP-violating one-photon-exchange amplitude has recently been calculated in two explicit models.[3,4] Taking into account the CP impurity in the K_L wave function as well as the violation in the decay amplitude, these models generate an amplitude of the form

$$A(K_L \to \pi^0 e^+ e^-)\,|_{1\gamma}=(e^{i\pi/4}+i\eta)f(w)\bar{u}(k)\not{p}v(k')\ , \tag{5}$$

where $f(w)$ is a slowly varying real function of w. The term $e^{i\pi/4}$ represents the phase associated with the parameter $\epsilon=(2.27\times10^{-3})e^{i\pi/4}$: this is the only term present in a superweak type of CP violation. The additional piece $i\eta$ is a pure imaginary contribution associated with intrinsic CP violation in the decay amplitude. The calculation of Donoghue, Holstein, and Valencia[3] uses the "electromagnetic penguin" diagram to generate an effective interaction of the form $[\bar{d}\gamma_\mu(1+\gamma_5)s](\bar{e}\gamma_\mu e)$. Their results can be summarized by

$$\eta\approx 2,\quad B(K_L \to \pi^0 e^+ e^-)\,|_{1\gamma}=3.7\times10^{-12}\ , \tag{6a}$$

which gives the value of the parameter η in Eq. (5), and the branching ratio due to one-photon exchange alone. The alternative calculation of Ecker, Pich, and de Rafael[4] is based on a meson Lagrangian incorporating the $\Delta\mathbf{I}=\frac{1}{2}$ rule for the nonleptonic weak interaction and chiral symmetry for the strong interaction. Two possible results are obtained, which may be stated as

$$\eta=1.7,\quad B(K_L \to \pi^0 e^+ e^-)\,|_{1\gamma}=9.4\times10^{-12}$$

or (6b)

$$\eta=-0.5,\quad B(K_L \to \pi^0 e^+ e^-)\,|_{1\gamma}=8.1\times10^{-12}\ .$$

We will use the results (6a) and (6b) as a general indication of the strength and the phase of the one-photon-exchange contribution to $K_L \to \pi^0 e^+ e^-$.

III. TWO-PHOTON CONTRIBUTION

Turning to the CP-conserving two-photon intermediate state, we first remark that this amplitude has no reason to vanish in the limit $m_e \to 0$ (Ref. 2). It is true that a special ansatz for the vertex $K_2 \to \pi^0\gamma\gamma$ (namely, $F_{\mu\nu}F^{\mu\nu'}$) produces an amplitude for $K_2 \to \pi^0 e^+ e^-$ that is proportional to m_e and therefore negligibly small. This was the form considered by Donoghue, Holstein, and Valencia.[3] However, there is a second invariant for this vertex (namely, $p_\alpha p_\beta F_{\mu\alpha}F^{\mu\beta'}$) which leads to a nonvanishing amplitude in the limit $m_e \to 0$. The existence of these two possible forms has been known for a long time,[6] and has been noted again by Ecker, Pich, and de Rafael.[4]

In fact, the question of the two-photon mediated rate for $K_2 \to \pi^0 e^+ e^-$ has a close parallel to the discussion of the decay mode $\eta \to \pi^0 e^+ e^-$ that took place in a different context nearly twenty years ago. At that time, there was speculation[7] that the electromagnetic interaction of hadrons might contain a C-violating part that would induce the reaction $\eta^0 \to \pi^0 e^+ e^-$ through a one-photon-exchange mechanism. The question then arose as to what the expected rate via the CP-conserving 2γ intermediate state was. Some early estimates[8] gave an amplitude proportional to m_e, and hence very small rates. The issue was finally settled by Cheng[9] who showed, with the help of a simple vector-meson-dominance model, that this decay had a nonzero amplitude in the limit $m_e \to 0$. The model yielded both the absorptive and real parts of the $\eta^0 \to \pi^0 e^+ e^-$ amplitude, and a ratio $\Gamma(\eta^0 \to \pi^0 e^+ e^-)/\Gamma(\eta^0 \to \pi^0\gamma\gamma)=1.0\times10^{-5}$. It is this model that we will adapt to the calculation of the 2γ contribution to $K_2 \to \pi^0 e^+ e^-$.

To connect $K_2 \to \pi^0 e^+ e^-$ to $\eta^0 \to \pi^0 e^+ e^-$, we assume a model in which the nonleptonic weak interaction is represented by the vertices $K^0 \to \pi^0$, $K^0 \to \eta^0$, and $K^0 \to \eta'$. These vertices can be related to each other by assuming a $\Delta\mathbf{I}=\frac{1}{2}$ quark operator $\bar{d}s+s\bar{d}$ such as generated by a "penguin" diagram. The relations are

$$\frac{f_{K^0\to\eta}}{f_{K^0\to\pi^0}}=\frac{1}{\sqrt{3}},\quad \frac{f_{K^0\to\eta'}}{f_{K^0\to\pi^0}}=-2(\tfrac{2}{3})^{1/2}\ , \tag{7}$$

where η and η' are taken to be SU(3)-octet and -singlet states, respectively. Similarly, we use vector-meson dominance and the quark model for $VP\gamma$ couplings to relate the vertices $\pi \to \pi\gamma\gamma$ and $\eta' \to \pi\gamma\gamma$ to $\eta \to \pi\gamma\gamma$, the relations being

$$\frac{f(\pi^0 \to \pi^0\gamma\gamma)}{f(\eta \to \pi^0\gamma\gamma)}=\frac{5}{\sqrt{3}},\quad \frac{f(\eta' \to \pi^0\gamma\gamma)}{f(\eta \to \pi^0\gamma\gamma)}=\sqrt{2}\ . \tag{8}$$

The 2γ contribution to $K_2 \to \pi^0 e^+ e^-$ may then be written as

$$A(K_2\to\pi^0e^+e^-)|_{2\gamma}=\sqrt{2}\left[\frac{f_{K^0\pi}}{m_K^2-m_\pi^2}\frac{f(\pi\to\pi\gamma\gamma)}{f(\eta\to\pi\gamma\gamma)}+\frac{f_{K^0\eta}}{m_K^2-m_\eta^2}+\frac{f_{K^0\eta'}}{m_K^2-m_\eta'^2}\frac{f(\eta'\to\pi\gamma\gamma)}{f(\eta\to\pi\gamma\gamma)}\right]A(\eta\to\pi^0e^+e^-)\Big|_{2\gamma}$$
$$=(\tfrac{2}{3})^{1/2}(2.07)\frac{f_{K^0\pi}}{m_K^2-m_\pi^2}A(\eta\to\pi^0e^+e^-)\Big|_{2\gamma}\ . \quad (9)$$

It is also possible to take account of the effects of η-η' mixing. Equation (7) is then replaced by[10]

$$\frac{f_{K^0\to\eta}}{f_{K^0\to\pi}}=\frac{1}{\sqrt{3}}(\cos\theta+2\sqrt{2}\sin\theta)\ ,$$
$$\frac{f_{K^0\to\eta'}}{f_{K^0\to\pi}}=-2(\tfrac{2}{3})^{1/2}\left[\cos\theta-\frac{1}{2\sqrt{2}}\sin\theta\right] \quad (10)$$

and Eq. (9) by

$$\frac{f(\pi\to\pi\gamma\gamma)}{f(\eta\to\pi\gamma\gamma)}=\frac{5}{\sqrt{3}}\frac{1}{\cos\theta-\sqrt{2}\sin\theta}\ ,$$
$$\frac{f(\eta'\to\pi\gamma\gamma)}{f(\eta\to\pi\gamma\gamma)}=\sqrt{2}\left[\cos+\frac{1}{\sqrt{2}}\sin\theta\right]\ . \quad (11)$$

With $\theta=-10°$ ($-20°$), the coefficient 2.07 in Eq. (9) gets replaced by 3.17 (3.06). Corrections can also arise from SU(3)-symmetry breaking. We will use the estimate of Eq. (9) as the nominal value, and allow for a factor-2 uncertainty in the amplitude. Using the value

$$f_{K^0\pi}=1.0\times10^{-7}m_K^2 \quad (12)$$

obtained by relating the $K\to\pi$ vertex to $K\to\pi\pi$ via PCAC (partial conservation of axial-vector current) (Ref. 10), we get

$$A(K_2\to\pi^0e^+e^-)|_{2\gamma}\approx2(\tfrac{2}{3})^{1/2}10^{-7}A(\eta\to\pi^0e^+e^-)|_{2\gamma}\ . \quad (13)$$

Note that because of the very similar masses of K and η, the above equality may be expected to hold approximately for the real and absorptive parts separately. Neglecting the small difference of phase space between $K\to\pi e^+e^-$ and $\eta\to\pi e^+e^-$ we have

$$\Gamma(K_2\to\pi^0e^+e^-)|_{2\gamma}=2.6\times10^{-14}\Gamma(\eta\to\pi^0e^+e^-)|_{2\gamma}\ . \quad (14)$$

Using the result of Cheng's calculation[9]

$$\frac{\Gamma(\eta\to\pi^0e^+e^-)}{\Gamma(\eta\to\pi^0\gamma\gamma)}=1.0\times10^{-5} \quad (15)$$

and the empirical value $\Gamma(\eta\to\pi^0\gamma\gamma)=0.8$ eV, we obtain finally

$$B(K_2\to\pi^0e^+e^-)|_{2\gamma}=1.5\times10^{-11}\ , \quad (16)$$

an estimate that we consider reliable to a factor 4. Comparison with the one-photon estimates given in Eqs. (6a) and (6b) then shows that the 2γ rate is at least comparable to the 1γ contribution, and may well be dominant.

The phase of the 2γ amplitude may also be determined from Cheng's analysis[9] of $\eta^0\to\pi^0e^+e^-$. The amplitude has the structure

$$A(K_2\to\pi^0e^+e^-)|_{2\gamma}=(A_{\rm disp}+iA_{\rm abs})\bar{u}(k)\not{p}v(k')\ , \quad (17)$$

where $A_{\rm disp}$ and $A_{\rm abs}$ are the dispersive and absorptive parts. Up to a common factor determined by the rate, these terms are given by

$$A_{\rm abs}\sim\Delta,\quad A_{\rm disp}\sim\Delta\left[a+b\ln\frac{s}{\Lambda^2}\right]\ , \quad (18)$$

where the latter is an approximate representation of the numerical results of Ref. 9, with $a\approx0.5$, $b\approx-0.05$, $\Lambda=0.45$ GeV, and $s=M^2+\mu^2-w$. The dispersive and absorptive parts contribute to the 2γ rate in the ratio

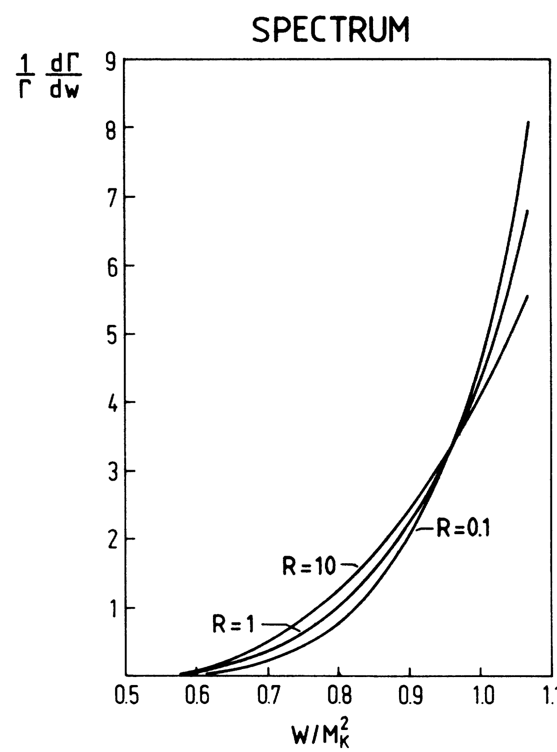

FIG. 2. Decay spectrum of $K_L\to\pi^0e^+e^-$ in the variable w, related to the invariant mass of the e^+e^- pair by $w=M^2+\mu^2+2m_e^2-s$, and to the π^0 energy in the K^0 rest frame by $w=2ME_\pi+2m_e^2$. (M and μ denote masses of K and π.)

$\Gamma_{disp}/\Gamma_{abs} \approx 0.4$.

We note in passing that the model described above also relates the $K_2 \to \pi^0\gamma\gamma$ process to $\eta^0 \to \pi^0\gamma\gamma$, yielding a branching ratio $B(K_2 \to \pi^0\gamma\gamma) = 1.6 \times 10^{-6}$. This is a factor 2.3 higher than the estimate 6.8×10^{-7} based on a $K_2 \to \pi^+\pi^-\pi^0 \to \pi^0\gamma\gamma$ mechanism[6] or a chiral Lagrangian model,[4] suggesting that intermediate vector-meson states play an important role in this decay.[11]

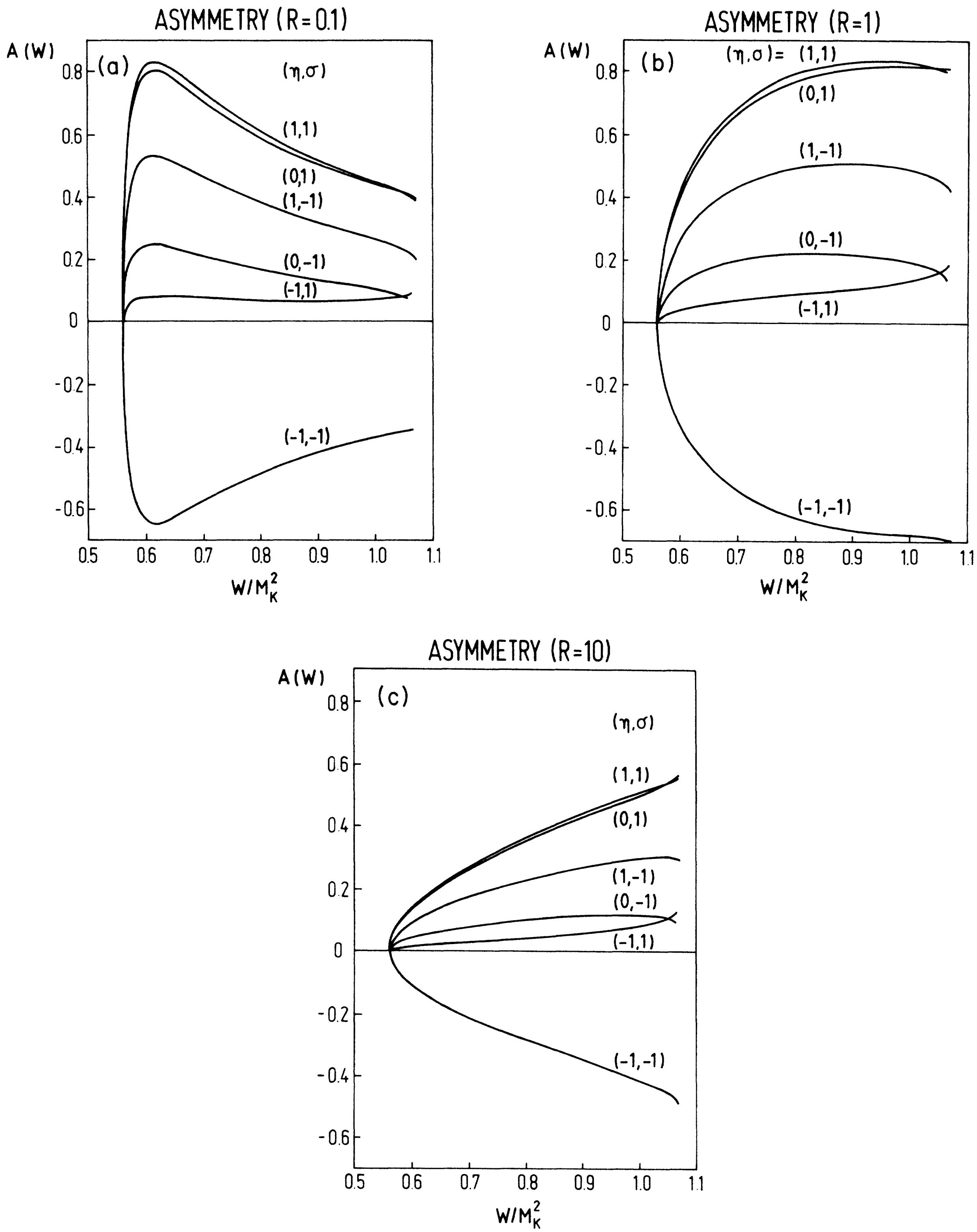

FIG. 3. Asymmetry between events with $E_- > E_+$ and $E_- < E_+$ [Eq. (21)] as a function of w for $R = \Gamma_{2\gamma}/\Gamma_{1\gamma} = 0.1$ (a), 1 (b), and 10 (c).

IV. ONE-PHOTON–TWO-PHOTON INTERFERENCE

Combining the amplitudes (5) and (17) we can calculate the full decay spectrum of $K_L \to \pi^0 e^+ e^-$ using

$$\frac{d\Gamma}{dw\, d\Delta} \propto |\, iF_{1\gamma} + F_{2\gamma}\,|^2 (w^2 - 4M^2\mu^2 - \Delta^2) \; . \tag{19}$$

Our main interest is the *CP*-violating asymmetry between e^+ and e^- arising from the presence in this spectrum of a term linear in Δ. To this end we define

$$\begin{aligned} \frac{d\Gamma(E_- > E_+)}{dw} &\equiv \int_0^{(w^2-4M^2\mu^2)^{1/2}} \frac{d\Gamma}{dw\, d\Delta} d\Delta \; , \\ \frac{d\Gamma(E_- < E_+)}{dw} &\equiv \int_{-(w^2-4M^2\mu^2)^{1/2}}^{0} \frac{d\Gamma}{dw\, d\Delta} d\Delta \end{aligned} \tag{20}$$

which denote densities in the two halves of the Dalitz plot, defined by $E_- > E_+$ and $E_- < E_+$, respectively. A measure of *CP* violation is then given by the asymmetry parameter

$$\mathcal{A}(w) = \frac{\dfrac{d\Gamma}{dw}(E_- > E_+) - \dfrac{d\Gamma}{dw}(E_- < E_+)}{\dfrac{d\Gamma}{dw}(E_- > E_+) + \dfrac{d\Gamma}{dw}(E_- < E_+)} \; . \tag{21}$$

We have calculated this asymmetry using the 1γ and 2γ amplitudes discussed in Secs. II and III. To take into account the uncertainty in the absolute values of the two contributions, we consider a ratio

$$R = \Gamma_{2\gamma}/\Gamma_{1\gamma} \tag{22}$$

and study the range $R = \frac{1}{10}, 1, 10$. The parameter η which defines the phase of the 1γ amplitude [Eq. (5)] is allowed to take values $-1, 0, +1$. The phase of the 2γ amplitude is defined by Eqs. (18) and (19); however, the relative sign of the absorptive and dispersive terms,

$$\sigma \equiv \mathrm{sgn}(A_{\mathrm{disp}}/A_{\mathrm{abs}}) \; , \tag{23}$$

is not deducible from the numerical results of Cheng,[9] and we allow $\sigma = \pm 1$. Finally, the overall relative sign of $F_{1\gamma}$ and $F_{2\gamma}$ is not known, *a priori,* and so the asymmetry that we calculate is defined only up to an overall sign.

The results are shown in Figs. 2 and 3. The spectrum $d\Gamma/dw$ integrated over all Δ (which eliminates the contribution of the *CP*-violating linear term) is plotted in Fig. 2. The spectrum is weighted towards large w (i.e., towards small invariant masses of the e^+e^- pair). The asymmetry $\mathcal{A}(w)$ is shown in Fig. 3 for the cases $R = \frac{1}{10}$, 1, and 10. We note that the asymmetry is sizable over the whole w domain. The effects are most striking when $R = \Gamma_{2\gamma}/\Gamma_{1\gamma}$ is unity and the parameter η is positive, in which case the asymmetry is of order 70–80 %.

Our conclusion is that the decay $K_L \to \pi^0 e^+ e^-$ has a probable branching ratio of 2×10^{-11} (with an uncertainty of a factor 3 or 4), and should exhibit a sizable *CP*-violating asymmetry between events with $E_- > E_+$ and $E_- < E_+$.

V. REMARKS ON THE $e{:}\mu$ RATIO IN $K_L \to \pi^0 l^+ l^-$

A discussion of the decay $K_L \to \pi^0 \mu^+ \mu^-$ is in most respects parallel to that of $K_L \to \pi^0 e^+ e^-$. Assuming a one-photon matrix element of the form (5) with a slowly varying $f(w)$ (such as that given by a K^* pole) the muon-to-electron ratio is

$$\left.\frac{B(\mu)}{B(e)}\right|_{1\gamma} = 0.2 \; . \tag{24}$$

The same ratio holds approximately for the 2γ-exchange matrix element [Eq. (4)] as long as the form factor $\tilde{F}_{2\gamma}$, which is proportional to the lepton mass, is neglected. In the case of $K_L \to \pi^0 \mu^+ \mu^-$, however, this form factor is not necessarily negligible, which can cause the $e{:}\mu$ ratio to be larger than 0.2. (As another consequence[4] the interference of $\tilde{F}_{2\gamma}$ with the one-photon amplitude $F_{1\gamma}$ can give rise to a transverse polarization of the muons in $K_L \to \pi^0 \mu^+ \mu^-$.)

An estimate of the $\tilde{F}_{2\gamma}$-dependent contribution to the 2γ rate for $K_2 \to \pi^0 \mu^+ \mu^-$ has been made by Ecker, Pich, and de Rafael,[4] using a model with an $\epsilon\cdot\epsilon'$ type matrix element for $K_2 \to \pi^0 \gamma\gamma$. The associated branching ratio (which adds incoherently to the contribution associated with the form factor $F_{2\gamma}$) is found to be 5.2×10^{-12}. It follows that the branching ratios of $K_L \to \pi^0 \mu^+ \mu^-$ and $K_L \to \pi^0 e^+ e^-$ are related by

$$B(\mu) \approx \tfrac{1}{5} B(e) + 5.2\times 10^{-12} \; . \tag{25}$$

A measurement of a $\mu{:}e$ ratio significantly higher than 1/5 would thus indicate the presence of the $\tilde{F}_{2\gamma}$ term in the matrix element of $K_L \to \pi^0 \mu^+ \mu^-$. As an illustration, if we take for the branching ratio of $K_L \to \pi^0 e^+ e^-$ the central value 2.0×10^{-11} and allow a factor-3 variation in either direction, the expected $\mu{:}e$ ratio is

$$\frac{B(\mu)}{B(e)} = \begin{cases} 0.94 \\ 0.46 \\ 0.28 \end{cases} \text{for } B(e) = \begin{cases} 0.7\times 10^{-11}, \\ 2.0\times 10^{-11}, \\ 6.0\times 10^{-11} \; . \end{cases} \tag{26}$$

It is clear that the $\mu{:}e$ ratio in $K_L \to \pi^0 l^+ l^-$ would be a valuable diagnostic in understanding the mechanisms underlying these decays.

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