Strange-baryon production via charge-changing weak currents in high-energy electron-capture reactions

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We investigate strange-baryon Λ and Σ^0 production via charge-changing weak currents on a proton target induced by high-energy electrons. To evaluate the various weak form factors, including vector, weak-magnetism, and scalar form factors for the polar-vector current and the axial-vector, pseudoscalar, and weak electric form factors for the axial-vector current, we adopt the flavor-SU(6) wave functions with quarks described as confining Dirac particles. The quark wave function adopted is of the form given by the MIT bag model, with or without the sharp surface smoothed out. In the few-GeV range, it is found that the cross section for large-angle Λ production is in the range of 10^{-40} cm^2 while that for Σ^0 is smaller by about an order of magnitude. The cross section is sensitive to contributions due to vector, weak-magnetism, and axial-vector form factors. Measurements of these form factors in the few-GeV range provide tests of quark models for nucleons and strange baryons. Some important aspects, such as recoil effects, are briefly discussed.

I. INTRODUCTION

The forthcoming construction of high-energy ($\gtrsim 1$ GeV) and high-intensity electron accelerators will make it feasible to study weak processes in a region where quark degrees of freedom are expected to be dominant, but where confinement effects cannot be neglected. The cross section for an exclusive semileptonic weak process increases with the beam energy but necessarily falls off with q^2 due to the hadronic form factors. Thus, an exclusive semileptonic weak process can be studied best with an electron beam of a few GeV. In this paper we study strange-baryon Λ and Σ^0 production via charge-changing weak currents, $e^- + p \rightarrow \Lambda + \nu_e$ and $e^- + p \rightarrow \Sigma^0 + \nu_e$, which complement the strangeness-conserving reaction $e^- + p \rightarrow n + \nu_e$ and offer an opportunity of probing the behavior of the strangeness-changing weak transition form factors in the few-GeV range.

The study of these weak reactions will be difficult, despite the fact that the detection of the two charged particles (p and π^-) from the weak decay of the Λ helps in the reconstruction of both the four-momentum of the decaying Λ and the missing (neutrino) four-momentum and thus provides useful vetoing mechanisms against background processes. These studies will allow us to extract the various $\Lambda \rightarrow N$ or $\Sigma^0 \rightarrow N$ transition weak form factors at different q^2, which cannot be obtained by other means, and are of importance for unraveling the quark structure of low-lying baryons.

The rest of this paper is organized as follows. In Sec. II, we introduce the $p \rightarrow \Lambda$ and $p \rightarrow \Sigma^0$ weak transition form factors and describe a quark-model calculation of these form factors. Sample numerical predictions are presented in Sec. III while some general discussion is given in Sec. IV.

II. FORMULATION

In what follows we use the reaction $e^- + p \rightarrow \Lambda + \nu_e$ as our primary example of the weak processes of interest. The results for the reaction $e^- + p \rightarrow \Sigma^0 + \nu_e$ will be mentioned whenever necessary.

The transition amplitude for the strangeness-changing electron-capture reaction $e^- + p \rightarrow \Lambda + \nu_e$ is given by[1]

$$T(e^- + p \rightarrow \Lambda + \nu_e) = i\bar{u}(p_\nu)\gamma_\lambda(1+\gamma_5)u(p_e)(2^{-1/2}G_F \sin\theta_C)\langle \Lambda(p') \,|\, [V_\lambda(0) + A_\lambda(0)] \,|\, p(p) \rangle \;, \tag{1}$$

where G_F and θ_C are, respectively, the Fermi coupling constant and the Cabibbo angle, and $V_\lambda(x)$ and $A_\lambda(x)$ are the charge-lowering strangeness-changing polar-vector and axial-vector currents, respectively. We use Dirac spinors and Dirac matrices as in the work of Primakoff.[1] Treating the proton and Λ as "elementary particles," we may write

$$\langle \Lambda(p') \,|\, V_\lambda(0) \,|\, p(p) \rangle = i\bar{u}_\Lambda(p') \left[\gamma_\lambda f_V(q^2) + \frac{\sigma_{\lambda\eta} q_\eta}{2m_p} f_M(q^2) - \frac{i q_\lambda}{2m_p} f_S(q^2) \right] u_p(p) \tag{2a}$$

and

$$\langle \Lambda(p') \,|\, A_\lambda(0) \,|\, p(p) \rangle = i\bar{u}_\Lambda(p') \left[\gamma_\lambda \gamma_5 f_A(q^2) + \frac{\sigma_{\lambda\eta} q_\eta \gamma_5}{2m_p} f_E(q^2) - \frac{2iM q_\lambda \gamma_5}{m_\pi^2} f_P(q^2) \right] u_p(p) \tag{2b}$$

with $q_\lambda \equiv (p-p')_\lambda$ and $M=(m_p+m_\Lambda)/2$. The masses m_p, m_Λ, and m_π are, respectively, those of the proton, Λ, and pion; and $f_V(q^2)$, $f_M(q^2)$, $f_S(q^2)$, $f_A(q^2)$, $f_E(q^2)$, and $f_P(q^2)$ are, respectively, the vector, weak magnetism, scalar, axial-vector, weak electric, and pseudoscalar form factors.[1] For the sake of clarity, we divide our presentation into several subsections, including (a) the differential cross section, (b) the quark-only impulse approximation, and (c) evaluation of the $p \to \Lambda$ and $p \to \Sigma^0$ weak transition form factors. Detailed formulas related to the quark-only impulse approximation, and needed to determine the various form factors, are relegated to the Appendix.

A. The differential cross section

In the laboratory frame where the target proton is at rest, we may rewrite Eqs. (2a) and (2b) as follows (with χ_Λ and χ_p two-component Pauli spinors in the hadron's own rest frame):[2]

$$\langle \Lambda(p') | \mathbf{V}(0) | p(p) \rangle = \chi_\Lambda^\dagger [-i(\boldsymbol{\sigma}\times\mathbf{q}/|\mathbf{q}|)G_M - (\mathbf{q}/|\mathbf{q}|)G_S]\chi_p , \tag{3a}$$

$$\langle \Lambda(p') | V_0(0) | p(p) \rangle = \chi_\Lambda^\dagger \chi_p G_V , \tag{3b}$$

$$\langle \Lambda(p') | \mathbf{A}(0) | p(p) \rangle = \chi_\Lambda^\dagger [-\boldsymbol{\sigma} G_A - (\mathbf{q}\,\boldsymbol{\sigma}\cdot\mathbf{q}/|\mathbf{q}|^2)G_P]\chi_p , \tag{3c}$$

$$\langle \Lambda(p') | A_0(0) | p(p) \rangle = \chi_\Lambda^\dagger (-\boldsymbol{\sigma}\cdot\mathbf{q}/|\mathbf{q}|)\chi_p G_E , \tag{3d}$$

with

$$G_M = \xi\{a f_M(q^2) + b[f_V(q^2) - c f_M(q^2)]\} , \tag{4a}$$

$$G_S = \xi b[f_V(q^2) - c f_M(q^2)] - \xi a f_S(q^2) , \tag{4b}$$

$$G_V = \xi[f_V(q^2) - ab f_M(q^2)] - \xi c f_S(q^2) , \tag{4c}$$

$$G_A = \xi[f_A(q^2) + c f_E(q^2) - ab f_E(q^2)] , \tag{4d}$$

$$G_P = \xi[(2M|\mathbf{q}|/m_\pi^2) b f_P(q^2) + ab f_E(q^2)] , \tag{4e}$$

$$G_E = \xi\{-a f_E(q^2) - b[f_A(q^2) - (2Mq_0/m_\pi^2) f_P(q^2)]\} . \tag{4f}$$

Here we have used

$$\xi \equiv [(E_\Lambda + m_\Lambda)/2E_\Lambda]^{1/2} , \tag{5a}$$

$$a \equiv |\mathbf{q}|/2m_p , \tag{5b}$$

$$b \equiv |\mathbf{q}|/(E_\Lambda + m_\Lambda) , \tag{5c}$$

$$c \equiv -q_0/2m_p . \tag{5d}$$

The differential cross section (in the laboratory frame) for the electron-capture reaction $e^- + p \to \Lambda + \nu_e$ is determined by

$$d\sigma = (2\pi)^{-2} d^3p_\nu d^3p_\Lambda \delta^4(p_\nu + p' - p_e - p) \sum_{\text{av}} |T|^2 . \tag{6}$$

Here $\sum_{\text{av}}$ denotes both summation over final discrete states (such as spins) and averaging over the initial discrete states. If we choose to integrate over the unobserved neutrino three-momentum and define the differential cross section $d\sigma/d\Omega_\Lambda$ we obtain,[2] in the case of an unpolarized beam on an unpolarized target with the final Λ polarization undetected,

$$\begin{aligned} d\sigma/d\Omega_\Lambda &= (2\pi)^{-2} E_\nu p_\Lambda \{E_\Lambda/[E_p + (E_e/p_\Lambda)(p_\Lambda - E_\Lambda\cos\theta_\Lambda)]\} \sum_{\text{av}} |T|^2 \\ &= [G_F E_\nu \cos\theta_C/(2\pi)]^2 (p_\Lambda/E_\nu)\{E_\Lambda/[E_p + (E_e/p_\Lambda)(p_\Lambda - E_\Lambda\cos\theta_\Lambda)]\}(D_0/2) . \end{aligned} \tag{7}$$

Here D_0 can be expressed in terms of the form factors G as

$$\begin{aligned} D_0 = {} & 2\sin^2(\theta_\nu/2)\{(G_M^2 + 2G_A^2) + 4\cos^2(\theta_\nu/2)(E_e E_\nu/|\mathbf{q}|^2)(G_A^2 + G_M^2) + 4[(E_e + E_\nu)/|\mathbf{q}|]G_A G_M\} \\ & + 2\cos^2(\theta_\nu/2)\{[(G_A + G_P)(q_0/|\mathbf{q}|) - G_E]^2 + [G_S(q_0/|\mathbf{q}|) + G_V]^2\} . \end{aligned} \tag{8}$$

Equations (4)–(8) allow us to predict the differential cross section $d\sigma/d\Omega_\Lambda$ for the electron-capture reaction $e^- + p \to \Lambda + \nu_e$, once the various weak form factors are given.

B. The quark-only impulse approximation

We note that the polar-vector current is conserved. That is,

$$\partial_\lambda V_\lambda(x) = 0$$

or

$$q_\lambda \langle \Lambda(p') | V_\lambda(0) | p(p) \rangle = 0 . \tag{9a}$$

This implies

$$(m_\Lambda - m_p) f_V(q^2) + [q^2/(2m_p)] f_S(q^2) = 0 , \tag{9b}$$

so that there are only two independent polar-vector form factors. To evaluate the $p \to \Lambda$ (and $p \to \Sigma^0$) weak transition form factors, we assume that the proton and Λ are described by flavor-SU(6) wave functions with quarks treated as confined Dirac particles. We consider in the next section two different possibilities for quark wave functions. The first possibility is the one given by the MIT bag model,[3] which is also used in chiral bag models with a sharp boundary. The second possibility, which we concentrate on a little more later in this paper, is a quark wave function based on Dirac particles moving in a harmonic-oscillator confining potential (consisting of a Lorentz scalar and the time component of a Lorentz four-vector). As we shall see in the next section, the hadronic form factors calculated in the harmonic-oscillator quark model get damped very rapidly as q^2 becomes large compared to $(1/R)^2$, where R is the confinement scale, so that cross sections arising from one-body currents become negligibly small for q^2 greater than about 50 fm^{-2}. As is well known in electronuclear physics (where Gaussian-type wave functions are used for nucleons), contributions from two-body currents (often referred to as meson-exchange currents in nuclear physics) and others become dominant at large q^2. On the other hand, the sharp boundary in the original MIT quark wave function gives rise to a slowly damped oscillating behavior of the form factors at large q^2 and the predicted cross section due to one-body currents remains fairly sizable at large q^2.

We note that the matrix elements which we need to determine for the reaction $e^- + p \to \Lambda + \nu_e$ are

$$v_\lambda = \langle \Lambda(p') \mid V_\lambda(0) \mid p(p) \rangle \tag{10a}$$

and

$$a_\lambda = \langle \Lambda(p') \mid A_\lambda(0) \mid p(p) \rangle \ . \tag{10b}$$

To carry out a quark-model calculation of these matrix elements, we need to know (1) the initial and final baryon wave functions expressed in terms of quarks and other constituents (including the confining field) and (2) the operators which characterize the reaction mechanism at the quark level. For the purpose of this paper, we shall assume that the quark wave functions of the initial or final baryons at rest are determined by the flavor-SU(6) symmetry. For instance, the quark part of the proton wave function at rest is given by[4]

$$\begin{aligned} |p(\uparrow)\rangle_Q = 18^{-1/2}[&2u^{(1)}(+)u^{(2)}(+)d^{(3)}(-) \\ &-u^{(1)}(+)u^{(2)}(-)d^{(3)}(+) \\ &-u^{(1)}(-)u^{(2)}(+)d^{(3)}(+) \\ &+(1\leftrightarrow 3)+(2\leftrightarrow 3)] \ . \end{aligned} \tag{11a}$$

Analogously, the quark part of the Λ wave function at rest is specified by[4]

$$\begin{aligned} |\Lambda(\uparrow)\rangle_Q = 12^{-1/2}[&u^{(1)}(+)d^{(2)}(-)s^{(3)}(+) - u^{(1)}(-)d^{(2)}(+)s^{(3)}(+) - d^{(1)}(+)u^{(2)}(-)s^{(3)}(+) \\ &+d^{(1)}(-)u^{(2)}(+)s^{(3)}(+) + s^{(1)}(+)u^{(2)}(+)d^{(3)}(-) - s^{(1)}(+)u^{(2)}(-)d^{(3)}(+) \\ &-s^{(1)}(+)d^{(2)}(+)u^{(3)}(-) + s^{(1)}(+)d^{(2)}(-)u^{(3)}(+) + d^{(1)}(-)s^{(2)}(+)u^{(3)}(+) \\ &-d^{(1)}(+)s^{(2)}(+)u^{(3)}(-) - u^{(1)}(-)s^{(2)}(+)d^{(3)}(+) + u^{(1)}(+)s^{(2)}(+)d^{(3)}(-)] \ . \end{aligned} \tag{11b}$$

Here and in what follows, we suppress color indices wherever possible and use a shorthand notation such as

$$u^{(a)}(+) \equiv \psi(r^{(a)}; s_z = \tfrac{1}{2}, I_3 = \tfrac{1}{2})$$

and

$$u^{(a)}(-) \equiv \psi(r^{(a)}; s_z = -\tfrac{1}{2}, I_3 = \tfrac{1}{2}) \ .$$

More specifically, the quark wave function which we adopt in this paper is of the relativistic form[3]

$$\psi(\mathbf{r}; s) = \begin{bmatrix} u(r) \\ i\sigma\cdot\hat{\mathbf{r}} v(r) \end{bmatrix} \chi_s \tag{12}$$

with $r \equiv |\mathbf{r}|$ and $\mathbf{r}$ the quark coordinate expressed relative to the center of the bag. χ_s is the Pauli spinor with s the z component of spin.

The baryon wave functions in Eqs. (11) do not give any information beyond the quark part, such as the gluonic components or the confining field. It is also of importance to note that only baryon wave functions at rest are given. Since the hadron energy-momentum is not carried entirely by the quark constituents (as suggested for instance by high-energy deep-inelastic lepton-proton scattering experiments), we will discuss briefly the relativistic center-of-mass problem and obtain the quark part of the hadron wave function in motion as though quarks were free (as suggested by the asymptotic-free nature of QCD as well as by general successes of bag models such as the simple MIT bag model[3] at low energies). Since the specific way which we choose to handle recoil effects requires some justification, we follow Ref. 5 in discussing briefly recoil effects in bag models.

The reaction $e^- + p \to \Lambda + \nu_e$ involves the one-body charge-lowering strangeness-changing quark currents[6]

$$V_\lambda(x) = \sum_{a=1}^{3} (i\tau_-\gamma_4\gamma_\lambda)^{(a)} \delta^{(3)}(x - r^{(a)}) \tag{13a}$$

and

$$A_\lambda(x) = \sum_{a=1}^{3} (i\tau_-\gamma_4\gamma_\lambda\gamma_5)^{(a)} \delta^{(3)}(x - r^{(a)}) \ , \tag{13b}$$

which take this simple form since quarks are assumed to be pointlike Dirac particles. It is expected[6] that the impulse approximation in terms of Eqs. (13a) and (13b) is valid, above all, in the Breit frame, in which the initial and final hadron three-momenta are equal in magnitude but opposite in sign. This assumption is justified[6] in, e.g., the determination of the proton charge and anomalous magnetic form factors $e_p(q^2)$ and $\mu_p(q^2)$ where adoption of the Breit frame ensures both that $e_p(q^2)$ and $\mu_p(q^2)$ are functions of q^2 alone and that there is no additional spurious form factor. Thus, we take the same assumption for the sake of consistency whenever Eqs. (13a) and (13b) are used. Using Eqs. (13a) and (13b), we obtain[6]

$$\langle \Lambda(\mathbf{p}') \mid V_\lambda(0) \mid p(\mathbf{p}) \rangle = \left\langle \Lambda(\mathbf{p}') \left| \sum_{a=1}^{3} \exp(-i\mathbf{q}\cdot\mathbf{r}^{(a)})(i\tau_- \gamma_4 \gamma_\lambda)^{(a)} \right| p(\mathbf{p}) \right\rangle , \tag{14a}$$

$$\langle \Lambda(\mathbf{p}') \mid A_\lambda(0) \mid p(\mathbf{p}) \rangle = \left\langle \Lambda(\mathbf{p}') \left| \sum_{a=1}^{3} \exp(-i\mathbf{q}\cdot\mathbf{r}^{(a)})(i\tau_- \gamma_4 \gamma_\lambda \gamma_5)^{(a)} \right| p(\mathbf{p}) \right\rangle . \tag{14b}$$

Equations (14a) and (14b) assume that, in the impulse approximation, it is adequate to know only the quark part of the baryon wave function, such as Eq. (11a) or (11b). Specifically, we find, with η the overlap integral for a spectator quark,

$$\langle \Lambda(\mathbf{p}';J_z=\tfrac{1}{2}) \mid V_\lambda(0) \mid p(\mathbf{p};J_z=\tfrac{1}{2}) \rangle = (\tfrac{3}{2})^{1/2}\eta^2 \int d^3r\, \psi_s^\dagger(r;+) \exp(-i\mathbf{q}\cdot\mathbf{r}) S_f^\dagger (i\gamma_4\gamma_\lambda) S_i \psi_u(r;+) , \tag{15a}$$

$$\langle \Lambda^0(\mathbf{p}';J_z=-\tfrac{1}{2}) \mid V_\lambda(0) \mid p(\mathbf{p};J_z=\tfrac{1}{2}) \rangle = (\tfrac{3}{2})^{1/2}\eta^2 \int d^3r\, \psi_s^\dagger(r;-) \exp(-i\mathbf{q}\cdot\mathbf{r}) S_f^\dagger (i\gamma_4\gamma_\lambda) S_i \psi_u(r;+) . \tag{15b}$$

The corresponding formulas for the charge-lowering axial-vector current $A_\lambda(x)$ are identical with Eqs. (15a) and (15b), except that the operator $i\gamma_4\gamma_\lambda$ is replaced by $i\gamma_4\gamma_\lambda\gamma_5$. Here the boost operators S_f and S_i are introduced such that $S_f\psi(r)$ and $S_i\psi(r)$, with $\psi(r)$ the quark wave function in the rest frame of the hadron, are, respectively, the final- and initial-quark wave functions as seen in the Breit frame.

To describe the weak reaction $e^- + p \to \Sigma^0 + \nu_e$, we need to replace Eqs. (15a) and (15b) by the formulas

$$\begin{aligned} \langle \Sigma^0(\mathbf{p}';J_z=\tfrac{1}{2}) \mid V_\lambda^+(0) \mid p(\mathbf{p};J_z=\tfrac{1}{2}) \rangle = -(2^{-1/2}/3)\eta^2 \int d^3r [\psi_s^\dagger(r;+) \exp(-i\mathbf{q}\cdot\mathbf{r}) S_f^\dagger (i\gamma_4\gamma_\lambda) S_i \psi_u(r;+) \\ + 2\psi_s^\dagger(r;-) \exp(-i\mathbf{q}\cdot\mathbf{r}) S_f^\dagger (i\gamma_4\gamma_\lambda) S_i \psi_u(r;-)] , \end{aligned} \tag{16a}$$

$$\langle \Sigma^0(\mathbf{p}';J_z=-\tfrac{1}{2}) \mid V_\lambda^\dagger(0) \mid p(\mathbf{p};J_z=\tfrac{1}{2} \rangle = (2^{-1/2}/3)\eta^2 \int d^3r\, \psi_s^\dagger(r;-) \exp(-i\mathbf{q}\cdot\mathbf{r}) S_f^\dagger (i\gamma_4\gamma_\lambda) S_i \psi_u(r;+) . \tag{16b}$$

Equations (15a) and (15b) and (16a) and (16b), together with appropriate boost operators S_i and S_f, allow us to evaluate the $p \to \Lambda$ and $p \to \Sigma^0$ weak transition form factors. We wish to refer to these equations as the "quark-only impulse approximation" (QOIA).

C. Evaluation of the weak transition form factors

To determine the quark contribution to the various form factors, we follow the procedure used by Hwang and Ernst[6] in which one selects out a sufficient number of independent Breit-frame matrix elements so that each form factor can be determined unambiguously. The general problem of Lorentz covariance[7] and recoil effects[8] has been studied by Krajcik and Foldy and by others. Once a choice is made for how to boost the quark wave functions, the matrix elements appearing in Eqs. (15) and (16) can be evaluated in the QOIA. The detailed formulas for evaluating the various $p \to \Lambda$ and $p \to \Sigma^0$ weak transition form factors are described in some detail in the Appendix. Using the results as input, we then use formulas (4)–(8) to evaluate cross sections.

When q^2 becomes large, the cross section is dominated by two-body or higher-body, rather than by one-body, currents (as specified by QOIA) just as in electron-nucleus scattering. It may therefore be tempting to take the form factors evaluated near $q^2=0$ and scale them with q^2 through a dipole form of some sort. Although this could result in cross sections which may be closer to data, we believe that it is important to contrast the data with the QOIA predictions in order to assess the importance of two-body currents and other effects.

III. NUMERICAL PREDICTIONS

In this section, we present selected numerical predictions for both the reactions $e^- + p \to \Lambda + \nu_e$ and $e^+ + p \to \Sigma^0 + \nu_e$, by means of the formalism developed in Sec. II. We consider two choices for the quark wave functions, both of which are in the form of Eq. (12). The first choice is from the MIT bag model.[3] The upper and lower components $u(r)$ and $v(r)$ are given by

$$u(r) = \begin{cases} N[(\omega+m)/(4\pi\omega)]^{1/2} j_0(xr/R) & \text{for } r \le R , \\ 0 \quad \text{for } r > R , \end{cases} \tag{17a}$$

$$v(r) = \begin{cases} N[(\omega-m)/(4\pi\omega)]^{1/2} j_1(xr/R) & \text{for } r \le R , \\ 0 \quad \text{for } r > R , \end{cases} \tag{17b}$$

$$\begin{aligned} (N^2R^3)^{-1} = j_0^2(x)\{2\omega[\omega-(1/R)] \\ +(m/R)\}[\omega(\omega-m)]^{-1} , \end{aligned} \tag{17c}$$

with x and ω determined by

$$\tan x = x\{1 - mR - [x^2+(mR)^2]^{1/2}\}^{-1} , \tag{17d}$$

$$\omega = (1/R)[x^2+(mR)^2]^{1/2} . \tag{17e}$$

This quark wave function has also been used in most chiral bag models where a sharp boundary is used. It gives rise to a slowly damped oscillating behavior in the form factors and the resultant cross section does not fall very rapidly as q^2 increases. As an alternative, we have also considered the case in which the quark (as a Dirac particle) is confined by a potential $(1+\gamma_4)kr^2/4 + (a+b\gamma_4)$. The solution is then given by[9]

$$u(r) = c \exp(-r^2/R^2) , \tag{18a}$$

$$v(r) = \xi rc \exp(-r^2/R^2) , \tag{18b}$$

with

$$\xi = kR^2/4 , \tag{18c}$$

$$c^2R^3(\pi/2)^{3/2} = (1+\tfrac{3}{4}\xi^2R^2)^{-1} . \tag{18d}$$

Here we may adjust the constants a and b so that both the mass and the eigenenergy can be fixed to desired values. In addition, the strength of the lower component relative to the upper one can be adjusted by choosing a value for kR^2 [cf. Eq. (18c)].

In Table I, we attempt to relate the two models by adjusting the parameters $\{\xi, R\}$ in the Dirac harmonic-oscillator model (DHOM) to reproduce the same $\langle r^2\rangle_N$ and f_M as in the MIT bag model. We use Eqs. (17) and (18) to evaluate the various $p \to \Lambda$ weak form factors by choosing $E_e=300$ MeV and $\theta_\Lambda=16°$, which yields $q^2=1.088$ fm^{-2}. For the sake of simplicity, we have adopted in the rest of this paper the boost operators as specified by Eqs. (A1) and (A2) given in the Appendix. We also use, for the bag-model calculation, $R_N=0.987$ fm, $R_\Lambda=0.977$ fm, and $m=10$ MeV. Correspondingly, we use $R_N=0.763$ fm and $R_\Lambda=0.756$ fm in the DHOM. It is clear from Table I (as well as from Ref. 9) that the DHOM with $\xi=1.6$ fm^{-1} yields results very close to the MIT bag model. We note that our value for f_A (or g_A) agrees well with what Horvat *et al.*[10] have obtained.

Once the various form factors have been determined, we use the formulas given in Sec. II to obtain cross sections. In what follows, we present sample numerical results. All the input parameters are the same as those used in Table I. For the sake of clarity, we divide our results into two subsections according to the specific reaction of interest.

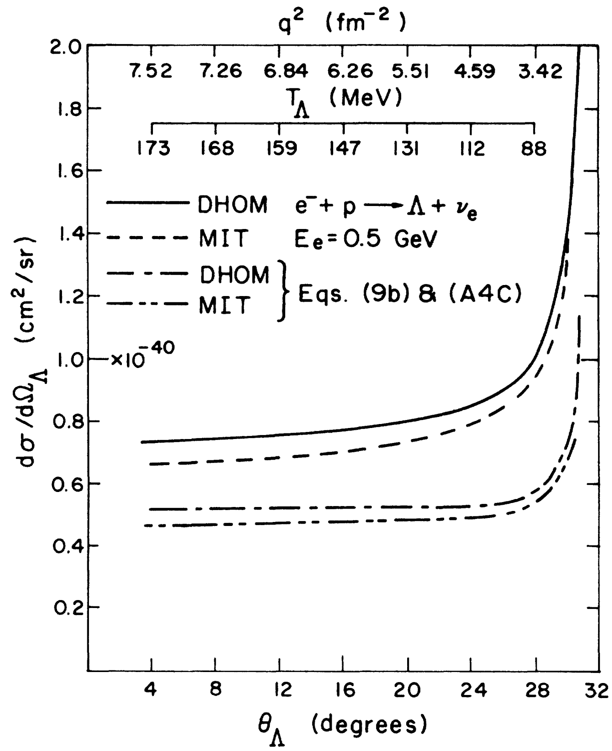

FIG. 1. The predicted weak production cross section of Λ^0 at $E_e=0.5$ GeV.

A. $e^- + p \to \Lambda + \nu_e$

In Fig. 1 we present numerical predictions as functions of the recoiling Λ angle (in the laboratory frame) for an electron beam energy of 0.5 GeV. We have also listed in this figure the corresponding q^2 and the recoiling hadron kinetic energy T_Λ for each given recoiling Λ angle. We note that the predicted differential cross section reaches a value of about 40×10^{-40} cm^2/sr for the recoiling Λ angle near 30.97°. We also note that the predictions for the two models are very similar.

The solid curve (DHOM) and the dashed curve (MIT) shown in Fig. 1 are obtained with the formulas listed in the Appendix. We have also considered possible ramifications of the constraint Eq. (9b) due to current conservation by using Eq. (9b) and the calculated $f_S(q^2)$ to determine $f_V(q^2)$ [instead of Eq. (A4a)]. This procedure has been shown[11] to yield results similar to what may be obtained via implementation of Siegert's theorem. The predicted cross sections are also shown in Fig. 1. Note that this procedure results in a considerably smaller $f_V(q^2)$ and thus a somewhat smaller cross section. Nevertheless, general characteristics of the results remain similar so that, to obtain a reasonable estimate for the cross section, we choose to ignore the gauge-invariance constraint [Eq. (9b)] and adopt the formulas given in the Appendix for making other predictions presented in this paper.

TABLE I. The predicted $p\to\Lambda$ weak form factors at $q^2=1.088$ fm^{-2}, which corresponds to $E_e=300$ MeV and $\theta_\Lambda=16°$. The boost operators specified by Eqs. (A1) and (A2) are assumed for the sake of simplicity. For the bag-model calculation, we use $R_N=0.987$ fm, $R_\Lambda=0.977$ fm, and $m=10$ MeV. Accordingly, we use $R_N=0.763$ fm, $R_\Lambda=0.756$ fm and $\xi=1.6$ fm^{-1} in the Dirac harmonic-oscillator model.

	DHOM	MIT bag
$\langle r^2\rangle_N$	0.517	0.517
f_V	1.42	1.36
f_M	2.21	2.20
f_S	0.183	0.227
f_A	0.938	0.925
f_P	−0.039	−0.035
f_E	0.670	0.547

In Fig. 2 we present numerical predictions as functions of the recoiling Λ angle (in the laboratory frame) for an electron beam energy of 4.0 GeV. It is clear that, for a 0.5-GeV electron beam, the available phase space is rather limited. On the other hand, the range for the allowed q^2 is considerably enlarged at $E_e=4.0$ GeV so that both the rapid Gaussian falloff of the predicted cross section for the Dirac harmonic-oscillator model and its slowly damped oscillatory behavior for the bag model are clearly displayed by this figure. In fact, predictions from the two models differ considerably from each other, although they agree qualitatively at $E_e=0.5$ GeV.

It should be pointed out that, just like electron-nucleus scattering, the cross section is dominated by two-body or higher-body currents rather than by one-body currents as q^2 becomes sufficiently large. Therefore, it is expected that the present DHOM predictions should be modified

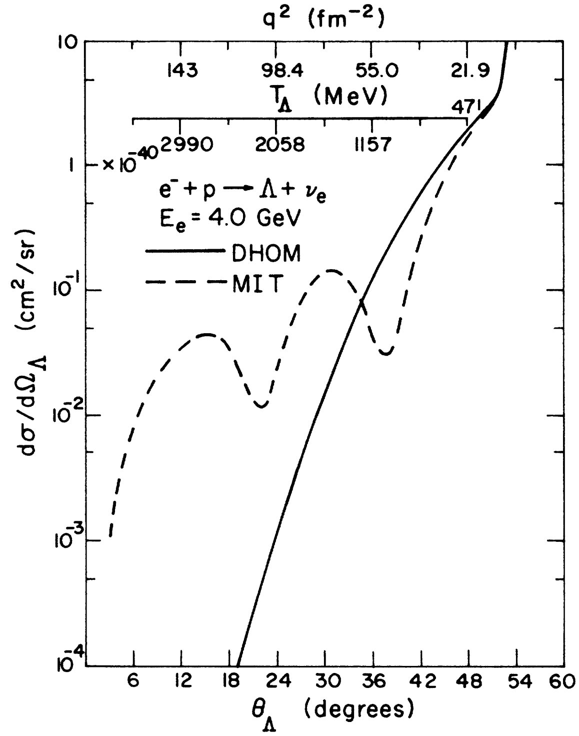

FIG. 2. The predicted weak production cross section of Λ^0 at $E_e=4.0$ GeV.

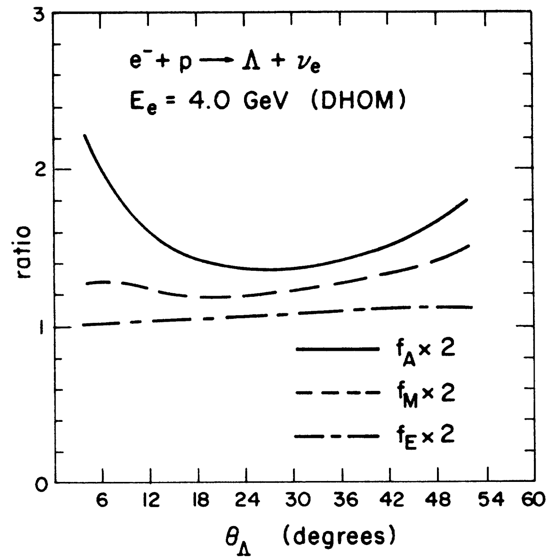

FIG. 3. Sensitivity of the Λ^0 weak production cross section to the various $p\to\Lambda^0$ weak form factors. Here the ratio of the newly predicted cross section (e.g., by increasing f_A by a factor of 2) to that given in Fig. 2 is shown.

beyond a certain q^2, but this is not an indication of a failure of the model. Rather, the smallness of the large-q^2 one-body predictions in the DHOM makes room for two-body contributions and eventually for the perturbative QCD behavior at a sufficiently large q^2. (It is not clear whether we can do the same thing with a bag model.)

The sensitivity of our predictions to the various weak form factors are displayed in Fig. 3, where we arbitrarily multiply the indicated form factors by 2 while keeping the rest the same as in Fig. 2. We consider only the DHOM prediction. To exhibit the sensitivity we plot the ratio of the newly predicted cross section to that given by Fig. 2. It is clear from this figure that contributions from the weak-magnetism and axial-vector form factors are of numerical importance while those from the weak electric form factors are negligibly small. Note that, in view of the factor q_λ, the scalar and pseudoscalar form factors do not contribute to cross sections at all. We also note that, at a fixed q^2, one may perform experiments at several different electron-beam energies so that the measurements may be inverted to yield those form factors which contribute significantly.

B. $e^-+p\to\Sigma^0+\nu_e$

In Fig. 4 we show the predicted cross section for the electron-capture reaction $e^-+p\to\Sigma^0+\nu_e$ at the electron beam energy of 0.5 GeV. As before, the DHOM prediction is made with $\xi=1.6\ \text{fm}^{-1}$. As in the case of Fig. 1 for the $p\to\Lambda$ transition, the predictions from the two models are rather similar at these energies. Note that the predicted cross section is at best in the vicinity of $1\times10^{-41}\ \text{cm}^2$, which is smaller than the predicted Λ production cross section by about an order of magnitude. This is so partly because the calculated $f_A(q^2)$ in the $p\to\Sigma^0$ transition is considerably smaller than that in the $p\to\Lambda$ transition; for instance, at $E_e=0.5$ GeV and $\theta_{\Lambda,\Sigma}=2°$, we find $f_A(q^2)=0.146$ for $p\to\Sigma^0$ and 0.765 for $p\to\Lambda$ while $f_M(q^2)=0.908$ for $p\to\Sigma^0$ and 1.485 for $p\to\Lambda$.

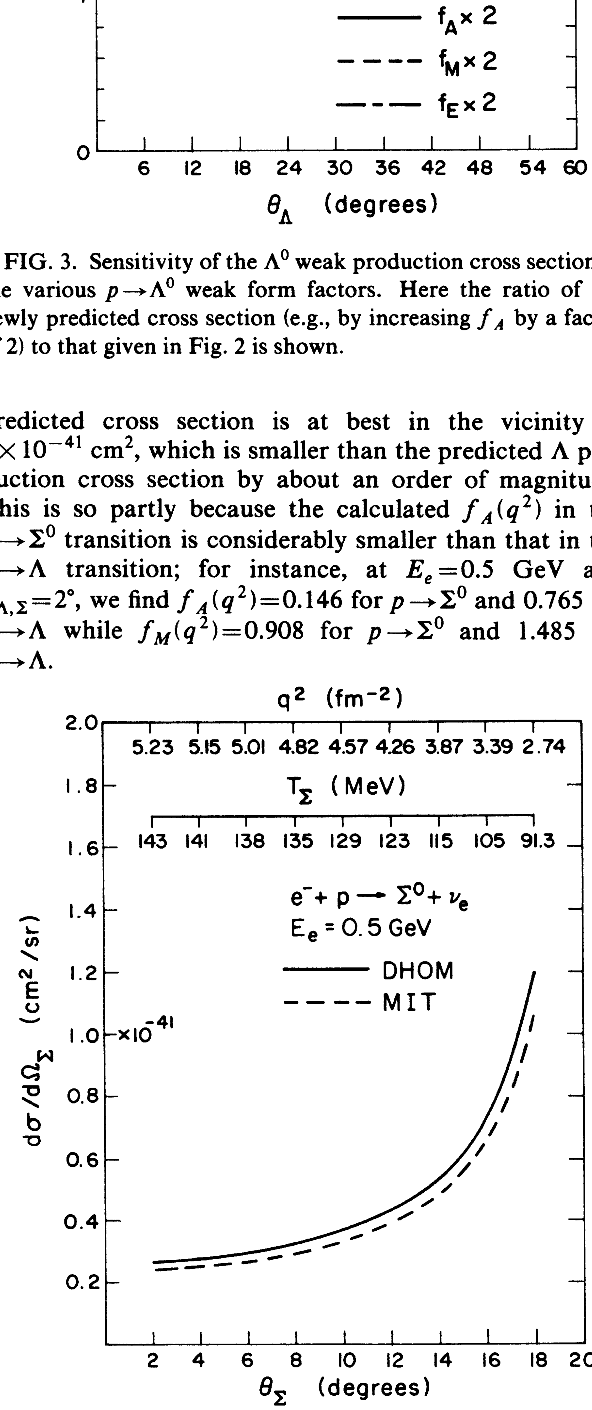

FIG. 4. The predicted weak production cross section of Σ^0 at $E_e=0.5$ GeV.

In Fig. 5 we show the predicted cross section at the electron beam energy of 4.0 GeV. The messages revealed by this figure are very similar to those by Fig. 2. Again, it is interesting to note that the predicted cross section in the $p \to \Sigma^0$ transition is smaller than that in the $p \to \Lambda$ transition.

Comparing Figs. 4 and 5, respectively, with Figs. 1 and 2, we note that the predicted cross section for $e^- + p \to \Sigma^0 + \nu_e$ is consistently smaller than that for $e^- + p \to \Lambda + \nu_e$ by about an order of magnitude. Since Σ^0 decays electromagnetically into $\Lambda + \gamma$, the smallness of the Σ^0 production cross section makes it possible to neglect such process as a possible source for background events in an experimental study of the reaction $e^- + p \to \Lambda + \nu_e$.

IV. DISCUSSION

So far we have described in Sec. II how the differential cross sections for the weak reactions $e^- + p \to \Lambda + \nu_e$ and $e^- + p \to \Sigma^0 + \nu_e$ are determined in quark models. We have also described in Sec. III sample numerical results for an electron beam of 0.5 or 4.0 GeV. The cross section for weak production of Λ is in the range of 10^{-40} cm^2/sr, while that for weak production of Σ^0 is smaller by about an order of magnitude.

It may be possible to measure a cross section of order of 10^{-40} cm^2/sr since there is little background for Λ's below the kaon associated production threshold (i.e., $e^- + p \to e^- + K^+ + \Lambda$). Even above the threshold, the missing-mass squared for an associated produced Λ is

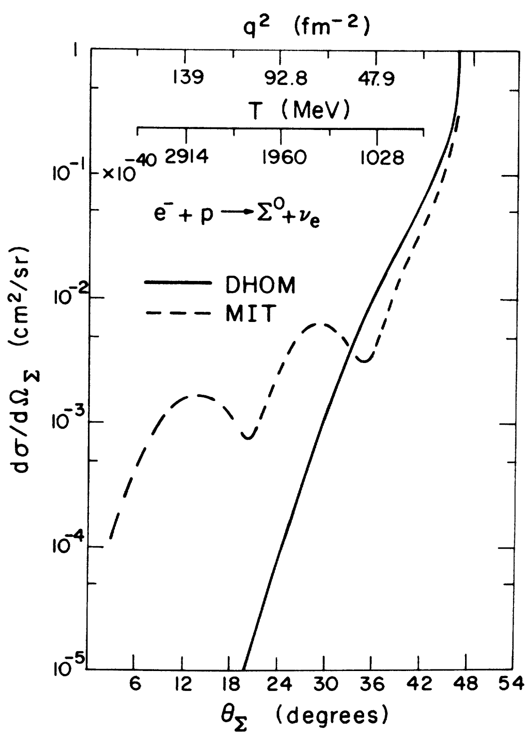

FIG. 5. The predicted weak production cross section of Σ^0 at $E_e = 4.0$ GeV.

given by

$$m_X^2 \equiv -(p_e + p_p - p_\Lambda)^2 = -(p_K + p_e')^2 = m_K^2 + m_e^2 + 2(E_K E_e' - \mathbf{p}_K \cdot \mathbf{p}_e') > m_K^2 \ . \tag{19}$$

It is quite likely that the Λ's four-momentum can be reconstructed well enough by a high-resolution measurement of the decaying products $p + \pi^-$. Specifically, we may rewrite the missing-mass squared as

$$m_X^2 = m_e^2 + m_p^2 + m_\Lambda^2 + 2E_e m_p - 2E_\Lambda m_p + 2(E_e p_\Lambda \cos\theta_\Lambda - E_e E_\Lambda) \ ,$$

so that

$$\delta m_X^2 = (2m_p - 2E_\Lambda + 2p_\Lambda \cos\theta_\Lambda)\delta E_e - [2m_p + 2E_e - 2E_e(E_\Lambda / p_\Lambda)\cos\theta_\Lambda]\delta E_\Lambda - 2E_e p_\Lambda \sin\theta_\Lambda \delta\theta_\Lambda \ . \tag{20}$$

For an electron beam of 4 GeV with the Λ recoiling angle of 52° we have

$$\delta m_X^2 = (0.1170 \text{ GeV})\delta E_e - (1.5267 \text{ GeV})\delta E_\Lambda - (5.1372 \text{ GeV}^2)\delta\theta_\Lambda \ . \tag{21a}$$

Suppose that we achieve only 1° in angular resolution and 10 MeV in energy resolution. We find, neglecting the first term in Eq. (21a) (which is clearly negligible),

$$|\delta m_X^2| = 0.1049 \text{ GeV}^2 \ , \tag{21b}$$

which can easily be separated from the associated produced background events which have the missing-mass squared greater than

$$m_K^2 = (0.4937 \text{ GeV})^2 = 0.2437 \text{ GeV}^2 \ . \tag{21c}$$

In other words, the resolution requirement which allows for a clean identification of genuine weak-interaction events may be a modest one.

We wish to conclude the present paper by appending a couple of remarks.

(1) We have described in Sec. II the formulas for (i) the differential cross section (expressed in terms of the various weak transition form factors) and (ii) the weak form factors (expressed in terms of the Breit-frame matrix elements as well as in terms of the integrals over the quark wave functions). The quark-only impulse approximation, which describes one-body currents, has also been introduced. In Sec. III we have described sample numerical results obtained via the formulas given in the previous section. Specifically, we have obtained a cross section of the order of 10^{-40} cm^2/sr for the reaction $e^- + p \to \Lambda + \nu_e$ and 10^{-41} cm^2/sr for the reaction $e^- + p \to \Sigma^0 + \nu_e$. It seems that Λ production via charge-changing weak currents is feasible with the present experimental technique.

(2) At present, the only information[10] concerning the various $p \to \Lambda$ and $p \to \Sigma^0$ weak transition form factors comes from β decay of Λ or Σ^0. This involves values of

q^2 which are very close to zero. Measurements of the weak reactions studied in this paper will yield information about these form factors for a much wider range of q^2. This will make it possible to test ideas such as CVC (conserved vector current) and PCAC (partially conserved axial-vector current), which are essential ingredients of the standard $SU(3)_c \times SU(2)_w \times U(1)$ model of the strong, electromagnetic, and weak interactions.

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APPENDIX: EVALUATION OF THE $p \to \Lambda$ AND $p \to \Sigma^0$ WEAK TRANSITION FORM FACTORS

We assume that the boost operators can be parametrized as

$$S_i = a_i + b_i \boldsymbol{\alpha}\cdot(\mathbf{q}/|\mathbf{q}|) , \tag{A1a}$$

$$S_f = a_f + b_f \boldsymbol{\alpha}\cdot(\mathbf{q}/|\mathbf{q}|) . \tag{A1b}$$

For example, if the free boost operators are used, we find

$$a_i = [(E_i^B + M_i)/(2M_i)]^{1/2} , \tag{A2a}$$

$$b_i = [(E_i^B - M_i)/(2M_i)]^{1/2} , \tag{A2b}$$

$$a_f = [(E_f^B + M_f)/(2M_f)]^{1/2} , \tag{A2c}$$

$$b_f = -[(E_f^B - M_f)/(2M_f)]^{1/2} . \tag{A2d}$$

Here and in what follows, we use the superscript B to indicate explicitly those quantities which are defined in the Breit frame. It is not clear whether free boost operators can be adopted at all. This and the general problem of Lorentz covariance[7] and recoil[8] has been studied by Krajcik and Foldy. We will present a detailed analysis in a future paper. We find that Eqs. (17) and (18) are a reasonable approximation and are the leading approximation to generate the boosted quark wave functions. We adopt them for use in the remainder of this appendix.

With the procedure of Hwang and Ernst,[6] one selects out independent Breit-frame matrix elements to determine each form factor. The results are summarized immediately below. We define

$$I_0 \equiv [2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}\langle \Lambda(-\mathbf{q}^B/2, J_z'=\tfrac{1}{2})\,|\,V_0(0)\,|\,p(\mathbf{q}^B/2, J_z=\tfrac{1}{2})\rangle , \tag{A3a}$$

$$I_x \equiv -i[2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(2m_p/|\mathbf{q}|)\langle \Lambda(-q^B\hat{\mathbf{y}}/2, J_z'=\tfrac{1}{2})\,|\,V_x(0)\,|\,p(q^B\hat{\mathbf{y}}/2, J_z=\tfrac{1}{2})\rangle , \tag{A3b}$$

$$I_z \equiv -[2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(2m_p/|\mathbf{q}|)\langle \Lambda(-q^B\hat{\mathbf{z}}/2, J_z'=\tfrac{1}{2})\,|\,V_z(0)\,|\,p(q^B\hat{\mathbf{z}}/2, J_z=\tfrac{1}{2})\rangle . \tag{A3c}$$

Then we have

$$f_V = [(1+\delta)^2+\alpha]^{-1} \times ((1+\delta)\{I_0+[q_0/(2m_p)]I_z\}+\beta I_x) , \tag{A4a}$$

$$f_M = [(1+\delta^2)+\alpha]^{-1} \times ((1+\delta)I_x-\beta'\{I_0+[q_0/(2m_p)]I_z\}) , \tag{A4b}$$

$$f_S = (1+\delta')^{-1}\{+I_z+\beta'' f_V-[q_0/(2m_p)]\beta' f_M\} , \tag{A4c}$$

where we have introduced

$$1+\delta = 1-|\mathbf{q}|^2/[4(E_\Lambda+m_\Lambda)(E_p+m_p)] - q_0/[2(E_p+m_p)]+q_0/[2(E_\Lambda+m_\Lambda)] , \tag{A5a}$$

$$\alpha = (q^2/4)[(E_p+m_p)^{-1}+(E_\Lambda+m_\Lambda)^{-1}]^2 , \tag{A5b}$$

$$\beta = (q^2/4m_p)[(E_p+m_p)^{-1}+(E_\Lambda+m_\Lambda)^{-1}] , \tag{A5c}$$

$$\beta' = m_p[(E_p+m_p)^{-1}+(E_\Lambda+m_\Lambda)^{-1}] , \tag{A5d}$$

$$\beta'' = m_p[(E_p+m_p)^{-1}-(E_\Lambda+m_\Lambda)^{-1}] , \tag{A5e}$$

$$\delta' = |\mathbf{q}|^2/[4(E_\Lambda+m_\Lambda)(E_p+m_p)] . \tag{A5f}$$

Analogously, we define

$$K_0 \equiv [2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2} \times [2E_p/(E_p+m_p)]^{1/2}(2m_p/|\mathbf{q}|) \times \langle \Lambda(-q^B\hat{\mathbf{z}}/2, J_z'=\tfrac{1}{2})\,|\,A_0(0)\,|\,p(q^B\hat{\mathbf{z}}/2, J_z=\tfrac{1}{2})\rangle , \tag{A6a}$$

$$K_z \equiv [2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2} \times \langle \Lambda(-q^B\hat{\mathbf{z}}/2, J_z'=\tfrac{1}{2})\,|\,A_z(0)\,|\,p(q^B\hat{\mathbf{z}}/2, J_z=\tfrac{1}{2})\rangle , \tag{A6b}$$

$$K_z' \equiv [2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2} \times \langle \Lambda(-q^B\hat{\mathbf{x}}/2, J_z'=\tfrac{1}{2})\,|\,A_z(0)\,|\,p(q^B\hat{\mathbf{x}}/2, J_z=\tfrac{1}{2})\rangle . \tag{A6c}$$

Accordingly, we have

$$f_A=-((q^2/m_\pi^2)(1+\delta')^2+(2m_p/m_\pi)^2\{[q_0/(2m_p)](1-\delta')-[\,|\mathbf{q}|/(2m_p)]^2\beta''\}^2)^{-1}$$
$$\times(K_z'(q^2/m_\pi^2)(1+\delta')+(K_z 2m_p q_0/m_\pi^2-K_0\,|\mathbf{q}|^2/m_\pi^2)$$
$$\times\{[q_0/(2m_p)](1+\delta')-[\,|\mathbf{q}|/(2m_p)]^2\beta''\})\ , \tag{A7a}$$

$$f_P=[(q^2/m_\pi^2)(M/m_p)\beta']^{-1}(-K_z+[q_0/(2m_p)]K_0-f_A\{(1-\delta')-[q_0/(2m_p)]\beta''\})\ , \tag{A7b}$$

$$f_E=((q^2/m_\pi^2)(1+\delta')^2+(2m_p/m_\pi)^2\{[q_0/(2m_p)](1-\delta')-[\,|\mathbf{q}|/(2m_p)]^2\beta''\}^2)^{-1}$$
$$\times(K_z'\{[q_0/(2m_p)](1-\delta')-[\,|\mathbf{q}|/(2m_p)]^2\beta''\}(2m_p/m_\pi)^2$$
$$-(K_z 2m_p q_0/m_\pi^2-K_0\,|\mathbf{q}|^2/m_\pi^2)(1+\delta'))\ . \tag{A7c}$$

Using Eqs. (12), (15a), and (15b) in the text, we obtain, for the $p\to\Lambda$ case with $p\equiv|q^B|r$,

$$I_0=[2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(\tfrac{3}{2})^{1/2}\eta^2 4\pi\int dr\, r^2 j_0(\rho)[u_f(r)u_i(r)+v_f(r)v_i(r)]\ , \tag{A8a}$$

$$I_x=[2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(2m_p/|\mathbf{q}|)(\tfrac{3}{2})^{1/2}\eta^2$$
$$\times 4\pi\int dr\, r^2((a_f a_i-b_f b_i)j_1(\rho)[u_f(r)v_i(r)+v_f(r)u_i(r)]$$
$$-(b_f a_i-a_f b_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]+\tfrac{2}{3}j_2(\rho)v_f(r)v_i(r)\})\ , \tag{A8b}$$

$$I_z=-[2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(2m_p/|\mathbf{q}|)(\tfrac{3}{2})^{1/2}\eta^2$$
$$\times 4\pi\int dr\, r^2\{(a_f a_i+b_f b_i)j_1(\rho)[u_f(r)v_i(r)-v_f(r)u_i(r)]+(b_f a_i+a_f b_i)j_0(\rho)[u_f(r)u_i(r)+\tfrac{1}{3}v_f(r)v_i(r)]\}\ , \tag{A8c}$$

$$K_0=[2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(2m_p/|\mathbf{q}|)(\tfrac{3}{2})^{1/2}\eta^2$$
$$\times 4\pi\int dr\, r^2((a_f a_i+b_f b_i)j_1(\rho)[u_f(r)v_i(r)-v_f(r)u_i(r)]$$
$$-(a_f b_i+b_f a_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]-2j_2(\rho)v_f(r)v_i(r)\})\ , \tag{A8d}$$

$$K_z=[2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(\tfrac{3}{2})^{1/2}\eta^2$$
$$\times 4\pi\int dr\, r^2(-(a_f a_i+b_f b_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]-2j_2(\rho)v_f(r)v_i(r)\}$$
$$+(a_f b_i+b_f a_i)j_1(\rho)[u_f(r)v_i(r)-v_f(r)u_i(r)])\ , \tag{A8e}$$

$$K_z'=[2E_\Lambda/(E_\Lambda+m_\Lambda)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(\tfrac{3}{2})^{1/2}\eta^2$$
$$\times 4\pi\int dr\, r^2(-(a_f a_i-b_f b_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]+\tfrac{2}{3}j_2(\rho)v_f(r)v_i(r)\}$$
$$-(a_f b_i-b_f a_i)j_1(\rho)[u_f(r)v_i(r)+v_f(r)u_i(r)])\ . \tag{A8f}$$

Similarly, we obtain, from Eqs. (12) and (16a) and (16b) in the text, in the case of $e^-+p\to\Sigma^0+\nu_e$,

$$I_0=-[2E_\Sigma/(E_\Sigma+m_\Sigma)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(\tfrac{1}{2})^{1/2}\eta^2 4\pi\int dr\, r^2 j_0(\rho)[u_f(r)u_i(r)+v_f(r)v_i(r)]\ , \tag{A9a}$$

$$I_x=[2E_\Sigma/(E_\Sigma+m_\Sigma)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(2m_p/|\mathbf{q}|)(\tfrac{1}{2})^{1/2}(\tfrac{1}{3})\eta^2$$
$$\times 4\pi\int dr\, r^2((a_f a_i-b_f b_i)j_1(\rho)[u_f(r)v_i(r)+v_f(u)u_i(r)]$$
$$-(b_f a_i-a_f b_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]+\tfrac{2}{3}j_2(\rho)v_f(r)v_i(r)\})\ , \tag{A9b}$$

$$I_z=[2E_\Sigma/(E_\Sigma+m_\Sigma)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(2m_p/|\mathbf{q}|)(\tfrac{1}{2})^{1/2}\eta^2$$
$$\times 4\pi\int dr\, r^2\{(a_f a_i+b_f b_i)j_1(\rho)[u_f(r)v_i(r)-v_f(r)u_i(r)]+(b_f a_i+a_f b_i)j_0(\rho)[u_f(r)u_i(r)+\tfrac{1}{3}v_f(r)v_i(r)]\}\ , \tag{A9c}$$

$$K_0=[2E_\Sigma/(E_\Sigma+m_\Sigma)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(2m_p/|\mathbf{q}|)(\tfrac{1}{2})^{1/2}(\tfrac{1}{3})\eta^2$$
$$\times 4\pi\int dr\, r^2((a_f a_i+b_f b_i)j_1(\rho)[u_f(r)v_i(r)-v_f(r)u_i(r)]$$
$$-(a_f b_i+b_f a_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]-2j_2(\rho)v_f(r)v_i(r)\})\ , \tag{A9d}$$

$$K_z=[2E_\Sigma/(E_\Sigma+m_\Sigma)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(\tfrac{1}{2})^{1/2}\tfrac{1}{3}\eta^2$$
$$\times 4\pi\int dr\, r^2(-(a_fa_i+b_fb_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]-2j_2(\rho)v_f(r)v_i(r)\}$$
$$+(a_fb_i+b_fa_i)j_1(\rho)[u_f(r)v_i(r)-v_f(r)u_i(r)])\ , \qquad \text{(A9e)}$$

$$K_z'=[2E_\Sigma/(E_\Sigma+m_\Sigma)]^{1/2}[2E_p/(E_p+m_p)]^{1/2}(\tfrac{1}{2})^{1/2}\tfrac{1}{3}\eta^2$$
$$\times 4\pi\int dr\, r^2(-(a_fa_i-b_fb_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]+\tfrac{2}{3}j_2(\rho)v_f(r)v_i(r)\}$$
$$-(a_fb_i-b_fa_i)j_1(\rho)[u_f(r)v_i(r)+v_f(r)u_i(r)])\ . \qquad \text{(A9f)}$$

In concluding this appendix, we note that Eqs. (A3)–(A9), together with a choice of the boost operators [Eqs. (A1) with Eqs. (A2) as an example], allow us to determine the various weak transition form factors for both $e^-+p\rightarrow\Lambda+\nu_e$ and $e^-+p\rightarrow\Sigma^0+\nu_e$. Using the results as input, we then use formulas (4)–(8) to evaluate cross sections.

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